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Anti-plane analysis of functionally graded materials weakened by several moving cracks [†]

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Abstract

This paper considers several finite moving cracks in a non-homogeneous material. The shear modulus and mass density of the plane are considered for a class of functional forms for which equilibrium equation has analytical solutions. The distributed dislocation technique is used to carry out stress analysis in a non-homogeneous plane containing moving cracks under anti-plane loading. The solution of a moving screw dislocation is obtained in a non-homogeneous plane by means of Fourier transform method. The stress components reveal the familiar Cauchy singularity at the location of dislocation. The solution is employed to derive integral equations for a plane weakened by moving cracks. Finally several examples are solved to show the effects of the material non-homogeneity and speed of cracks on the stress intensity factors.

Keywords: Anti-plane; Functionally graded material; Screw dislocation; Multiple moving crack

1. Introduction

The advent of materials with continuously varying volume fractions, the so-called functionally graded materials and their technological potential have stimulated a fair amount research in this area. Spatial variation of elastic properties and inclination of property gradation direction to the propagating crack make analytical solutions to the elastodynamic equations extremely difficult. With the introduction of functionally graded materials, research on various aspects of fracture of these non-homogeneous materials has generated extensive interest. The knowledge of crack growth and propagation in functionally graded materials is important in designing components of FGMs and improving its fracture toughness.

Especially, the influence of the crack moving speed on the stress intensity factors was a popular subject in classical elastodynamics. Problems of crack propagation at constant speed can be classified into three classes depending on the boundary conditions [1]. The first class is the steady-state crack growth. Here, the crack tip moves at constant speed for all the time and the mechanical fields are invariant with respect to an observer moving with the crack tip. The prototype problem in this category is the two-dimensional Yoffe problem of a crack of fixed length propagating in a body subjected to uniform far field tensile loading [2]. The second class of problems is the self-similar crack growth subject to time-independent loading. In this case, the crack tip has been moving at constant speed since some initial instant, and certain mechanical fields are invariant with respect to an observer moving steadily away from the process being observed. The third category of problems corresponds to crack in a body initially at rest and subjected to time-independent loading. Compared to quasi-static problem of FGMs, the dynamic fracture of FGMs has received much less attention from researcher. Atkinson and List [3] studied the crack propagation in materials with spatially varying elastic constants using integral transforms. Sih et al. [4] studied the dynamic behavior of a moving crack in layered composites. The dynamic crack propagation in the functionally graded particle dispersed material under dynamic loading are investigated by Nakagaki et al. [5].Wang and Meguid [6] proposed a theoretical and numerical treatment of a finite crack propagating in an interfacial layer with spatially varying elastic properties under the anti-plane loading. The problem of a functionally graded piezoelectric material with a constant velocity Yoffe-type moving crack studied by Li and Weng [7]. Meguid et al. [8] studied the dynamic crack propagation in FGMs under the plane elastic deformation using Fourier transforms technique. Jiang and Wang [9] analyzed the dynamic plane behavior of a Yoffe type crack propagating in a functionally graded interlayer bonded to dissimilar half planes. The dynamic stress intensity factor and strain energy density for moving crack in an infinite strip of functionally graded material subjected to anti-plane shear was determined by Bi et

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al. [10]. Li [11] solved the dynamic problem of an impermeable crack of length $2a$ propagating in a piezoelectric strip. The problem for the crack propagating at constant speed in a functionally graded piezoelectric ceramic strip under combined anti-plane shear and in-plane electrical loadings was studied by Kwon [12]. A finite crack with constant length propagating in the functionally graded orthotropic strip under in plane loading was investigated by Ma et al. [13]. The effects of material properties, the thickness of the functionally graded orthotropic strip and the speed of the crack propagating upon the dynamic fracture behavior were studied. Das [14], Considered the interaction between three moving collinear Griffith cracks under anti-plane shear stress situated at the interface of an elastic layer overlying a different half plane. The problem of a Griffith crack of constant length propagating at a uniform speed in a non-homogeneous plane under uniform load is investigated by Singh et al. [15]. Baolin et al. [16], considered the problem of a moving crack in a non-homogeneous material strip. They found that the maximum anti-plane shear stress around the crack tip is a suitable failure criterion for moving cracks. The finite crack with constant length (Yoffe-type crack) propagating in a functionally graded strip with spatially varying elastic properties between two dissimilar homogeneous layers under in-plane loading was studied by Cheng et al. [17]. Yan [18], investigated the problem of a propagating finite crack in functionally graded piezoelectric materials. The solution procedures devised in all above studies are neither capable of handling multiple cracks nor arbitrary arrangement. Ayatollahi et al. [19] investigated the problem of several finite cracks with constant length propagating in an orthotropic strip. The effects of the geometric parameters, the thickness of the orthotropic strip, the crack size and speed of the cracks on the stress intensity factors were studied.

The objective of present study is to provide a theoretical analysis of the dynamic behavior of multiple moving cracks with arbitrary patterns in functionally graded material under anti-plane traction. The present work is based on the use of dislocation method to formulate integral equations for an infinite plane weakened by several moving cracks. The integral equations are of Cauchy singular types which are solved numerically for the dislocation density on the crack. To confirm the validity of formulations, numerical values of dynamic stress intensity factors for a crack is compared with the results in literature. Several examples of cracks are solved to study the effect of the speed of the crack on the stress intensity factor to illustrate the applicability of the procedure.

2. Formulation of the problem

In crack problems a so called ‘distributed dislocation technique’ is often used in treating multiple cracks with smooth geometries. The method relies on the knowledge of stress field due to a single dislocation in the region of interest.

Let us consider a plane made up of FGM, where the elastic shear modulus μ varies continuously in the thickness direction. At a continuum level, the properties at any given point in an FGM can be assumed to be same in all directions, hence FGMs, can be treated as an isotropic non-homogeneous solid. The only nonzero displacement component under anti-plane deformation is the out of plane component $W(X,Y,t)$. Consequently, the constitutive relationships may be written as

$$\begin{aligned} \sigma_{zx}(X,Y,t) &= \mu(Y)\frac{\partial W}{\partial X} \\ \sigma_{zy}(X,Y,t) &= \mu(Y)\frac{\partial W}{\partial Y} \end{aligned} \tag{1}$$

where $\mu(Y)$ are the shear modulus of the material. The equilibrium equations in view of Eq. (1) reduce to

$$\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\mu'(Y)}{\mu(Y)}\frac{\partial W}{\partial Y} = \frac{\rho(Y)}{\mu(Y)}\frac{\partial^2 W}{\partial t^2}\,. \tag{2}$$

The elastic shear modulus and material mass density of FGM is considered as:

$$\begin{aligned} \mu(Y) &= \mu_0 e^{2\xi Y} \\ \rho(Y) &= \rho_0 e^{2\xi Y} \end{aligned} \tag{3}$$

where μ_0, ρ_0 are the shear modulus and the mass density of the medium and ξ represents the gradient of material properties. Substituting Eq. (3) into Eq. (2), we arrive at

$$\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + 2\zeta\frac{\partial W}{\partial Y} = \frac{\rho_0}{\mu_0}\frac{\partial^2 W}{\partial t^2}\,. \tag{4}$$

For the present problem of a crack propagating at constant velocity V along the X – direction, it is convenient to introduce a Galilean transformation such as

$$X = x + Vt, \quad Y = y, \quad \frac{\partial}{\partial t} = -V\frac{\partial}{\partial x} \tag{5}$$

with X and Y being a translating coordinate system, which is attached to the propagating crack. It is, however, assumed that the propagation of the crack has prevailed for such a long time that the stress distribution around its tip is time invariant in translating reference frame. Therefore, Eq. (4) can be transformed into

$$\beta^2\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + 2\zeta\frac{\partial w}{\partial y} = 0 \tag{6}$$

where $w(x,y)=W(X,Y,t)$ and $\beta=\sqrt{(1-V^2/C^2)}$. Let a Volterra type screw dislocation with Bergers vector b_z be situated at the origin of the coordinate system with the dislocation line $x>0$. The conditions representing the Volterra-type screw dislocation are

$$\begin{aligned} &w(x,0^+)-w(x,0^-)=b_z H(x)\\ &\sigma_{zy}(x,0^+)=\sigma_{zy}(x,0^-). \end{aligned} \tag{7}$$

Here, $H(x)$ is the Heaviside step-function. The first Eq. (7) shows the multi-valuedness of displacement. Moreover, the second Eq. (7) is continuity of traction on the line of dislocation. It is worth mentioning that the above conditions for screw dislocation were utilized by several investigators, e. g., Weertman and Weertman [20]. In order to derive a solution of Eq. (6), Fourier transform is introduced:

$$f^*(\lambda)=\int_{-\infty}^{+\infty}e^{i\lambda x}f(x)dx\ . \tag{8}$$

In the above equation $i=\sqrt{-1}$, The solution to Eq. (6) is achieved by means of the Fourier transform. The inversion of the complex Fourier transformation yields:

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}e^{-i\lambda x}f^*(\lambda)d\lambda\ . \tag{9}$$

Transform methods are now used to reduce the partial differential Eq. (6) to ordinary differential equation for $w^*(\lambda,y)$. The solution to this equation readily known

$$w^*(\lambda,y)=A(\lambda)e^{(-\zeta-\sqrt{\zeta^2+\beta^2\lambda^2})y}\ . \tag{10}$$

The unknown coefficient may be obtained by taking the Fourier transform of Eq. (8) and applying the resultant expression to Eq. (10). The transformed displacement field becomes

$$w^*(\lambda,y)=\frac{b_z}{2}(\pi\delta(\lambda)+i/\lambda)e^{(-\zeta-\sqrt{\zeta^2+\beta^2\lambda^2})y} \tag{11}$$

where $\delta(\xi)$ is the Dirac delta function. The displacement component in view of Eqs. (9) and (11) leads to

$$\begin{aligned} &w(x,y)=\frac{b_z}{4}e^{-2\zeta y}+\frac{ib_z e^{-\zeta y}}{4\pi}\int_{-\infty}^{+\infty}\frac{e^{-y\sqrt{\zeta^2+\beta^2\lambda^2}-i\lambda x}}{\lambda}d\lambda\\ &w(x,y)=\frac{b_z}{4}e^{-2\zeta y}+\frac{ib_z e^{-\zeta y}}{4\pi}\int_{-\infty}^{+\infty}\frac{e^{-y\sqrt{\zeta^2+\beta^2\lambda^2}-i\lambda x}}{\lambda}d\lambda\ . \end{aligned} \tag{12}$$

The first terms on the right side of Eq. (12) are constant and represent the rigid body motion of plane. Furthermore, the stress components by virtue of Eqs. (1) and (12), are expressed as:

$$\begin{aligned} &\sigma_{zx}(x,y)=\frac{\mu_0 b_z e^{y\xi}}{4\pi}\int_{-\infty}^{+\infty}e^{-y\sqrt{\zeta^2+\beta^2\lambda^2}-i\lambda x}d\lambda\\ &\sigma_{zy}(x,y)=\frac{-\zeta\mu_0 b_z}{2}\\ &\qquad-\frac{i\mu_0 b_z e^{y\zeta}}{4\pi}\int_{-\infty}^{+\infty}\left(\frac{\zeta+\sqrt{\zeta^2+\beta^2\lambda^2}}{\lambda}\right)e^{-y\sqrt{\zeta^2+\beta^2\lambda^2}-i\lambda x}\ . \end{aligned} \tag{13}$$

The integrals in Eq. (13) may be evaluated by employing the procedure described in Duffy [21]. For the sake of brevity, the details of manipulation are not given here. The final results are

$$\begin{aligned} &\sigma_{zx}(x,y)=\frac{\mu_0 y\zeta b_z e^{y\zeta}}{2\pi r}K_1(r\zeta/\beta)\\ &\sigma_{zy}(x,y)=\frac{\mu_0 x\zeta b_z e^{y\zeta}}{2\pi r}\{K_1(r\zeta/\beta)+\int_1^{+\infty}\frac{ue^{-(r\zeta/\beta)u}}{\sqrt{u^2-1}(\beta^2y^2/r^2-u^2)}du\\ &\qquad-\frac{\beta y}{r}\int_1^{+\infty}\frac{e^{-(r\zeta/\beta)u}}{\sqrt{u^2-1}(\beta^2y^2/r^2-u^2)}du\} \end{aligned} \tag{14}$$

where $K_1(x)$ is the modified Bessel function of the second kind, from Eq. (14) it is obvious that stress components are Cauchy singular at dislocation position.

In the particular case of screw dislocation in the isotropic plane, letting $V\to 0$ and $\zeta\to 0$ in Eq. (6), the displacement and stress fields become

$$\begin{aligned} &w(x,y)=\frac{b_z}{2\pi}tg^{-1}(y/x)\\ &\begin{Bmatrix}\sigma_{zx}(x,y)\\ \sigma_{zy}(x,y)\end{Bmatrix}=\frac{\mu b_z}{2\pi}\left(\frac{1}{x^2+y^2}\right)\begin{Bmatrix}-y\\ +x\end{Bmatrix}. \end{aligned} \tag{15}$$

The above solutions are identical to those in Ref. [22].

3. Non-homogeneous plane with several moving cracks

The dislocation solutions accomplished in the preceding section may be employed to analyze functionally graded material with several moving cracks. The distributed dislocation technique is an efficient means to carry out the task, see for instance Ref. [23] The moving cracks configuration may be described in parametric form as

$$\begin{aligned} &x_i=x_{0i}+l_i s\\ &y_i=y_{0i}\qquad i=1,\ 2,\ldots,\qquad N-1\le s\le 1. \end{aligned} \tag{16}$$

We consider local coordinate systems moving on the face of ith crack. The anti-plane traction on the face of the i-th crack in terms of stress components in Cartesian coordinates becomes:

$$\sigma_{nz}(x_i, y_i) = \tau_0 . \tag{17}$$

Suppose dislocations with unknown density $B_{zj}(p)$ are distributed on the infinitesimal segment dl_j located at the face of the jth crack where the parameter $-1 \le p \le 1$ and prime denotes differentiation with respect to the relevant argument. Employing the principal of superposition, the traction components on the face of ith crack caused by dislocations distributed on all N cracks yields:

$$\sigma_{nz}(x_i(s), y_i(s)) = \sum_{j=1}^{N} \int_{-1}^{1} k_{ij}(s,p) l_j B_{zj}(p) dp, \qquad i = 1, 2, ..., N \tag{18}$$

where from Eq. (14), the kernel of integral Eq. (18) becomes:

$$\begin{aligned} k_{ij}(x_i(s), y_i(s), x_j(p), y_j(p)) &= \frac{\mu_0 (x_i(s) - x_j(p)) \zeta l_j e^{(y_i(s) - y_j(p))\zeta}}{2\pi r_{ij}} \\ &\times \{ K_1(r_{ij}\zeta/\beta) + \int_1^{+\infty} \frac{u e^{-(r_{ij}\zeta/\beta)u}}{\sqrt{u^2-1}(\beta^2 [y_i(s) - y_j(p)]^2 / r_{ij}^2 - u^2)} du \\ &- \frac{\beta (y_i(s) - y_j(p))}{r_{ij}} \int_1^{+\infty} \frac{e^{-(r_{ij}\zeta/\beta)u}}{\sqrt{u^2-1}(\beta^2 [y_i(s) - y_j(p)]^2 / r_{ij}^2 - u^2)} du \}. \end{aligned} \tag{19}$$

In Eq. (19), $r_{ij} = \sqrt{[x_i(s) - x_j(p)]^2 + [y_i(s) - y_j(p)]^2}$, it is worth mentioning that kernels in integral Eq. (19) are Cauchy singular for $i = j$ as $s \to p$. We substitute Eqs. (17) and (19) into Eq. (18), becomes:

$$\begin{aligned} &\sum_{j=1}^{N} \int_{-1}^{1} \{ \frac{\mu_0 (x_i(s) - x_j(p)) \zeta l_j e^{(y_i(s) - y_j(p))\zeta}}{2\pi r} \\ &\times [K_1(r_{ij}\zeta/\beta) + \int_1^{+\infty} \frac{u e^{-(r_{ij}\zeta/\beta)u}}{\sqrt{u^2-1}(\beta^2 [y_i(s) - y_j(p)]^2 / r_{ij}^2 - u^2)} du \\ &- \frac{\beta (y_i(s) - y_j(p))}{r_{ij}} \times \\ &\int_1^{+\infty} \frac{e^{-(r_{ij}\zeta/\beta)u}}{\sqrt{u^2-1}(\beta^2 [y_i(s) - y_j(p)]^2 / r_{ij}^2 - u^2)} du] \} B_{zj}(p) l_j dp = \tau_0 . \end{aligned} \tag{20}$$

Employing the definition of the dislocation density function, the equation for the crack opening displacement across the jth crack is

$$w_j^-(s) - w_j^+(s) = \int_{-1}^{s} l_j B_{zj}(p) dp, \qquad j = 1, 2, 3, ..., N. \tag{21}$$

The displacement field is single-valued for the faces of cracks. Thus, the dislocation densities functions are subjected to the following closure requirements

$$l_j \int_{-1}^{1} B_{zj}(p) dp = 0, \qquad j = 1, 2, 3, ..., N. \tag{22}$$

To evaluate the dislocation density on the crack faces, the Cauchy singular integral Eqs. (20) and (22) ought to be solved simultaneously. This is accomplished by means of the Gauss-Chebyshev quadrature scheme developed by Erdogan et al. [24]. The stress fields in the neighborhood of crack tips behave like $1/\sqrt{r}$ where r is the distance from the crack tip. Therefore, the dislocation densities are taken as

$$B_{zj}(p) = \frac{g_{zj}(p)}{\sqrt{1-p^2}}, \qquad -1 \le p \le 1 \qquad j = 1, 2, 3, ..., N. \tag{23}$$

Substituting Eq. (21) into Eqs. (20) and (22) and discretizating of the domain, $-1 \le p \le 1$, by $m+1$ segments, we arrive at the following system of $N \times 2m$ algebraic equations

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix} \begin{bmatrix} g_{z1}(p_n) \\ g_{z2}(p_n) \\ \vdots \\ g_{zN}(p_n) \end{bmatrix} = \begin{bmatrix} q_1(s_r) \\ q_2(s_r) \\ \vdots \\ q_N(s_r) \end{bmatrix} \tag{24}$$

where the collocation points are

$$\begin{cases} s_r = \cos(\dfrac{\pi r}{m}) & r = 1, 2, ..., m-1 \\ p_n = \cos(\dfrac{\pi(2n-1)}{2m}) & n = 1, 2, ..., m \end{cases} . \tag{25}$$

The components of matrix in Eq. (24) are

$$A_{ij} = \frac{\pi}{m} \begin{bmatrix} k_{ij}(s_1, p_1) & k_{ij}(s_1, p_2) & \cdots & k_{ij}(s_1, p_m) \\ k_{ij}(s_2, p_1) & k_{ij}(s_2, p_2) & \cdots & k_{ij}(s_2, p_m) \\ \vdots & \vdots & \ddots & \vdots \\ k_{ij}(s_{m-1}, p_1) & k_{ij}(s_{m-1}, p_2) & \cdots & k_{ij}(s_{m-1}, p_m) \\ \delta_{ij} l_i & \delta_{ij} l_i & \cdots & \delta_{ij} l_i \end{bmatrix}. \tag{26}$$

In Eq. (26), δ_{ij} in the last row of A_{ij} designates the Kronecker delta. The components of vectors in Eq. (24) are

$$\begin{aligned} g_{zj}(p_n) &= \left[g_{zj}(p_1) \quad g_{zj}(p_2) \quad \cdots \quad g_{zj}(p_m) \right]^T, \\ q_j(s_r) &= [\sigma_{yz}(x_j(s_1), y_j(s_1)) \quad \sigma_{yz}(x_j(s_2), y_j(s_2)) \quad \cdots \\ &\qquad \sigma_{yz}(x_j(s_{m-1}), y_j(s_{m-1})) \quad 0]^T \end{aligned} \tag{27}$$

where superscript T stands for the transpose of a vector. The stress intensity factors at the tip of i-th crack in terms of the crack opening displacement reduce to Ref. [25]

$$k_{Li} = \frac{\sqrt{2}}{4}\mu_{Li}(y)\lim_{r_{L_i}\to 0}\frac{w_i^-(s)-w_i^+(s)}{\sqrt{r_{L_i}}}$$
$$k_{Ri} = \frac{\sqrt{2}}{4}\mu_{Ri}(y)\lim_{r_{R_i}\to 0}\frac{w_i^-(s)-w_i^+(s)}{\sqrt{r_{R_i}}} \tag{28}$$

where L and R designate, the left and right tips of a crack, respectively. The geometry of a crack implies

$$r_{Li} = \left[(x_i(s)-x_i(-1))^2+(y_i(s)-y_i(-1))^2\right]^{\frac{1}{2}},$$
$$r_{Ri} = \left[(x_i(s)-x_i(1))^2+(y_i(s)-y_i(1))^2\right]^{\frac{1}{2}}. \tag{29}$$

In order to take the limits for $r_{Li}\to 0$ and $r_{Ri}\to 0$, we should let, in Eq. (29), the parameter $s\to -1$ and $s\to 1$, respectively. The substitution of Eq. (23) into Eq. (21), and the resultant equations and Eq. (29) into Eq. (28) in conjunction with the Taylor series expansion of functions $x_i(s)$ and $y_i(s)$ around the points $s\to \pm 1$ yields

$$k_{Lj} = \frac{\mu(y_{Li})}{2}[(x_j'(-1))^2+(y_j'(-1))^2]^{\frac{1}{4}}g_{zj}(-1),$$
$$k_{Rj} = -\frac{\mu(y_{Ri})}{2}[(x_j'(1))^2+(y_j'(1))^2]^{\frac{1}{4}}g_{zj}(-1), \quad j=1,2,3,...,N. \tag{30}$$

The solutions of Eq. (24) are plugged into Eq. (30) thereby the stress intensity factors are obtained.

4. Discussion on solutions

The analysis developed in the preceding section allows the consideration of a plane made up of FGM with any number of moving straight cracks. We now furnish some examples to demonstrate the applicability of the applied method. In all examples, the plane is under anti-plane moving traction on the surface of the crack with magnitude τ_0. The validity of formulation is examined by obtaining stress intensity factor for a crack situated parallel to the x-axis of an isotropic plane. We obtained numerically k/k_0 which is identical with that in the literatures. Ref. [16] and by setting, $\zeta=0$ and $\beta=1$ excellent agreement is observed with the results presented in Ref. [22].

The first example, deals with FGM material weakened by one moving crack with a length of $2l$ situated in the x-axis, Fig. 2. The crack velocity is V at time $t=0$, in the positive x-direction. The dimensionless stress intensity factors of crack tips, k/k_0, versus the dimensionless crack velocity V/C for different FGM constants are shown in Fig. 2. As it may be observed, the trend of variation remains the same by changing the FGM constant. In general, dynamic stress intensity factors increases as the FGM constant increases. From this figure, it can be found that, the difference of the dynamic stress intensity factors between the different FGM constant become more obvious as the value of ξ increases. It can also be found for

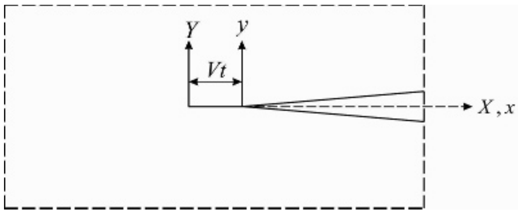

Fig. 1. Schematic view of functionally graded material with a screw dislocation.

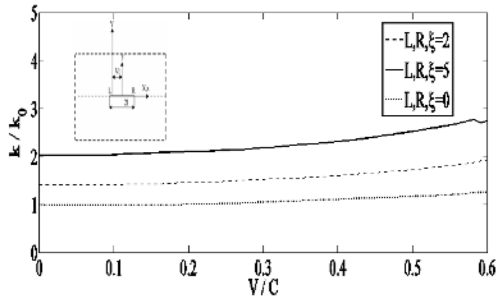

Fig. 2. Normalized stress intensity factor versus the dimensionless crack velocity for different FGM constant.

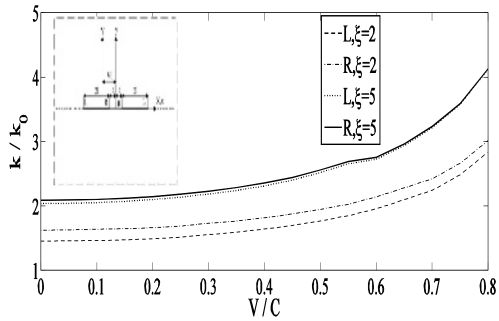

Fig. 3. Variation of the stress intensity factor of two cracks versus the dimensionless crack velocity.

homogeneous material the crack velocity has no effect on the dynamic stress intensity factors.

In the second example, we consider two collinear moving cracks with length $2l$ which are placed on the x-axis. The non-homogeneous properties of materials are chosen $\xi = 2.0, 5.0$. Variation of the normalized stress intensity factors of crack tips k/k_0, against the V/C, is depicted in Fig. 3 where $2a$ is the distance between two crack tips. As it can be observed k/k_0, for both adjacent tips are increased with crack velocity growing. However, when the material properties are graded, a significant increases in dynamic stress intensity factor is observed with an increase in crack velocity and gradient of material.

In the third example, the infinite plane contains two parallel identical moving cracks with lengths $2l$ which are also parallel with the x-axis. The cracks lines are located on the vertical distances $2a$ of each other. The variation of the normalized stress intensity factors versus dimensionless crack velocity V/C for various values of FGM constant ξ are shown in

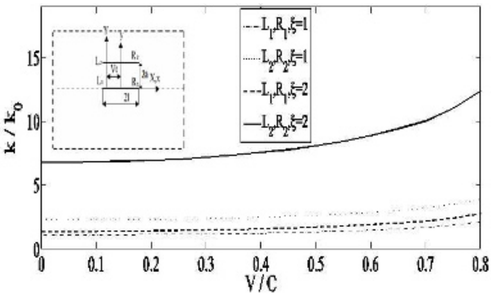

Fig. 4. Variation of Normalized stress intensity factors with V / C.

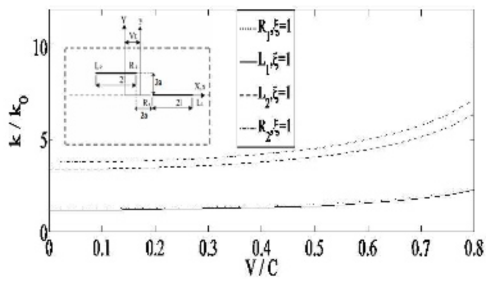

Fig. 5. Normalized stress intensity factors of crack tips versus the dimensionless crack velocity.

Fig. 4. As it was expected the highest k / k_0, occurs where the velocity of the interacting crack is maximum.

In the last example, two equal-length cracks which are parallel to the x-axis are shown in Fig. 5. The distance between crack centers remain fixed while the crack velocity are changing. The dimensionless stress intensity factors verses the dimensionless crack velocity, for value of FGM constant $\xi = 1$ is depicted in Fig. 5. As it might be observed the maximum stress intensity factor for the crack tips occur when the crack velocity is increased.

5. Conclusions

The anti-plane stress analysis of functionally graded material weakened by several moving cracks is carried out in this article. The analysis is based upon an integral transform technique. The present method is applied to illustrate the fundamental behavior of multiple cracks propagating in the functionally graded medium under the anti-plane loading. Furthermore, the effects of the material constants and the speed of crack propagation on dynamic stress intensity factors are investigated. It is found that the dynamic stress fields depend upon the material constants and the speed of crack propagating in functionally graded medium. To show the applicability of the procedure more examples are solved wherein the interaction between moving cracks is investigated.

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