Springer

KSME

Journal of Mechanical Science and Technology 26 (2) (2012) 537~543

www.springerlink.com/content/1738-494x

DOI 10.1007/s12206-011-1213-2

Static behavior of nano/micromirrors under the effect of Casimir force, an analytical approach †

Hamid Moeenfard[1], Ali Darvishian[2] and Mohammad Taghi Ahmaidan[1,*]

[1]Center of Excellence in Design, Robotics and Automation, School of Mechanical Engineering, Sharif University of Technology, Tehran, Iran

[2]School of Mechanical Engineering, Sharif University of Technology, Tehran, Iran

(Manuscript Received April 16, 2011; Revised September 24, 2011; Accepted October 18, 2011)

Abstract

In this paper, static behavior of nano/micromirrors under Casimir force is studied. At the first, the equilibrium equation governing the statical behavior of nano/micromirrors is obtained. Then energy method is employed to investigate statical stability of nano/micromirrors equilibrium points and a useful equation is suggested for successful and stable design of nano/micromirrors under Casimir force. Then, equilibrium angle of nano/micromirrors is calculated both numerically and analytically using the homotopy perturbation method (HPM). It is observed that with increasing the instability number defined in the paper, the rotation angle of the mirror is increased and suddenly, pull-in occurs. Since analytical results well follow the numerical ones, the presented analytical method in this paper can be used as a fast, precise and stable design tool in nano/micromirrors under Casimir force.

Keywords: Casimir force; Homotopy perturbation method (HPM); Nano/micromirror; Pull-in; Stability

---

1. Introduction

Technology of N/MEMS has experienced a lot of progress in testing and fabricating new devices recently. Their low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, makes them even more attractive [1, 2].

Successful MEMS devices rely not only on well developed fabrication technologies, but also on the knowledge of device behavior, based on which a favorable structure of the device can be forged [3].

The fact that MEMS devices play important roles in optical systems, caused the development of a new class of systems called micro-opto-electro-mechanical systems (MOEM). MOEMS include a wide variety of devices including digital nano/micromirror devices (DMD) [4], optical switches [5], micro scanning mirrors [6], optical cross connects [7, 8], and etc. Micromirrors and torsional actuators play an important role in MOEMS systems [9]. Nano/micromirrors are very small, highly reflective mirrors, often supported on torsion beams, used in a variety of applications. States of these devices are controlled by applying a voltage between the two electrodes around the mirror arrays. Based on their motion types, nano/micromirrors can be classified into four categories: deformable mirror [10], movable mirror [11], piston mirror [12], and torsional mirror [13]. These four types of nano/micromirrors have been widely studied in recent years with the torsional nano/micromirror being the most interesting among them [9]. Torsional nano/micromirrors have been extensively used for applications because of theirs good dynamic response, for instance in digital projection displays [13], spatial light modulators [14] and optical crossbar switches [15].

Many researches have been done on torsional micro actuators. For example Degani et al. [16] presented a novel displacement iteration pull-in extraction (DIPIE) scheme for the problem of electrostatic torsion micro-actuators. They [16] showed that their presented method converges 100 times faster than the voltage iteration scheme. Zhang et al. [17] describes the static characteristics of an electrostatically actuated torsional micromirror based on parallel plate capacitor model. They extensively studied the snap down phenomenon in micromirrors. They used numerical approach for their simulations. Switching response of torsional micromirrors have been modelled by Bhaskar et al. [18]. Khatami and rezazadeh [19] studied dynamic response of a torsional micromirror to electrostatic force and mechanical shock.

In addition to electrostatic force, intermolecular surface forces, play an important role in MOEMS design. The most important intermolecular surface forces are Casimir force and vdW force [20]. VdW force is the interaction force between

† This paper was recommended for publication in revised form by Editor Maenghyo Cho

*Corresponding author. Tel.: +982166165503, Fax.: +982166000021
E-mail address: ahmadian@mech.sharif.edu

neutral atoms and it differs from covalent and ionic bondings in that it is caused by correlations in the fluctuating polarizations of nearby particles [21]. Casimir force is understood as the longer distances range analog of the van der Waals force, resulting from the propagation of retarded electromagnetic waves, whose distance ranges from a few nanometers up to a few micrometers [22]. When the size of a structure is sufficiently small, Casimir force plays an important role and in the case of poor design, can lead to the collapse of the structure. So investigating the effect of Casimir force on nano/micromirrors is extremely important in their design.

Tahami et al. [23] discussed pull-in phenomena and dynamic response of a capacitive nano-beam switch by considering Casimir effect**.** Casimir effect on the pull-in parameters of nano-meter switches has been studied by Lin and Zhao [24]. They also studied nonlinear behavior of nano-scale electrostatic actuators with Casimir force [25]. Ramezani et al. [26, 27] investigated the two point boundary value problem of the deflection of nano-cantilever subjected to Casimir and electrostatic forces using analytical and numerical methods to obtain the instability point of the nanobeam. Modelling and simulation of electrostatically actuated nano-switches under the effect of Casimir forces have been studied by mojahedi et al [28]. Sirvent et al. [29] theoretically studied pull-in control in capacitive microswitches actuated by Casimir forces using external magnetic fields. Effect of the Casimir force on the static deflection and stiction of membrane strips in MEMS have been studied by Serry et al. [30]. But the statical behavior and stability of torsional nano/micromirrors under Casimir force has not been yet presented. So in this paper, the effect of Casimir force on the tilting angle and stability of torsional nano/micromirror is studied. In this study, HPM is used as a perturbational based analytical tool.

Perturbation methods have been used to analytically solve the nonlinear problems in MEMS. Younis and Nayfeh [31] have investigated the response of a resonant microbeam to an electric actuation using the multiple-scale perturbation method. Abdel-Rahman and Nayfeh [32] used the same method to model secondary resonances in electrically actuated microbeams. Since perturbation methods are based upon the assumption that there is a small parameter in the equations, they have some limitations in problems without involvement of small parameters. In order to overcome this limitation a new perturbational based method, namely HPM was developed by He et al. [33]. His new method takes full advantages of the traditional perturbation methods and homotopy techniques. Homotopy perturbation method has also been used for solving the nonlinear problems encountered in N/MEMS. For example, Moeenfard et al. [34] used HPM to model the nonlinear vibrational behavior of Timoshenko micobeams. Mojahedi et al. [35] applied the HPM method to simulate the static response of nano-switches to electrostatic actuation and intermolecular surface forces. But so far no analytic solution has been presented to model the behavior of nano/micromirrors.

In the current paper, the equation governing the statical behavior of a rectangular nano/micromirror is obtained using Newton's first law of motion and also using the minimum potential energy principle. Then energy method is used to investigate the instability of nano/micromirror equilibrium points. At the end, tilting angle of a nano/micromirror under Casimir force is calculated both numerically and analytically using HPM.

2. Problem formulation

The nano/micromirror shown in Fig. 1 is considered.

The differential Casimir force exerted to the differential surface element of the mirror with width W and infinitesimal length dx shown in Fig. 1 is [36]:

$$dF_{Cas} = \frac{\pi^2 \hbar c}{240 z^4} W dx \tag{1}$$

where c is speed of light, $\hbar$ is Plank's constant divided by 2π, and z is the distance between the mentioned surface element and the substrate. So the torque due to Casimir force that is applied to nano/micromirror is

$$M_{Cas} = \int_0^L \frac{\pi^2 \hbar c}{240(h - x\sin\varphi)^4} W x dx \tag{2}$$

where h is the initial distance between the mirror and the substrate, L is the length of the nano/micromirror and φ is its tilting angle as illustrated in Fig. 1.

The tilting angle in nano/micromirrors are usually small and $\sin\varphi \approx \varphi$. Therefore Eq. (2) can be restated as:

$$M_{Cas} = \int_0^L \frac{\pi^2 \hbar c}{240(h - x\varphi)^4} W x dx\ . \tag{3}$$

The maximum rotation angle is $\varphi_0 \approx \sin\varphi_0 = h/L$, so the normalized rotation angle is introduced as $\beta = \varphi/\varphi_0$ which leads to:

$$M_{Cas} = \frac{\pi^2 \hbar c W L^2}{240 h^4 \beta^2}\left(\frac{1}{6} - \frac{3\beta - 1}{6(\beta - 1)^3}\right). \tag{4}$$

The mechanical restoring torque applied to the micromirror is obtained using the following relation:

$$M_{Mech} = -K\varphi \tag{5}$$

where

$$K = \frac{2GI_p}{l} . \tag{6}$$

In this equation, G is the shear modulus of the torsion beams, l is the length of each torsion beam and I_p is the polar momentum of inertia of the rectangular cross section expressed as [9]:

$$I_p = \frac{1}{3}tw^3 - \frac{64}{\pi^5}w^4\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}\tanh\frac{(2n-1)\pi t}{2w} \tag{7}$$

where t and w are the length and width of the beam's cross section respectively. So using Eq. (6), Eq. (5) is simplified as Eq. (8).

$$M_{Mech} = -\frac{2GhI_p}{lL}\beta \tag{8}$$

At equilibrium point, the net torque applied to the nano/micromirror is zero.

$$M_{Cas} + M_{Mech} = 0 \tag{9}$$

Using Eqs. (4) and (8), Eq. (9) is simplified to the following equation:

$$\frac{\lambda}{\beta^2}\left(\frac{1}{6} - \frac{3\beta-1}{6(\beta-1)^3}\right) - \beta = 0 \tag{10}$$

where

$$\lambda = \frac{\pi^2\hbar cWL^3}{240h^5K} . \tag{11}$$

In Eqs. (10) and (11), λ is called the instability number.

In order to investigate the matter of stability, the total potential energy principal [37] is utilized and the energy of the system is divided into two parts: the potential strain energy of the torsion beams, U and the potential energy of applied loads, V which is equal with minus of work done by external forces, W_e.

$$\prod = U + V = U - W_e \tag{12}$$

where $\prod$ is the total potential energy. At equilibrium points, $\prod$ has no variation.

$$\delta\prod = \delta U - \delta W_e = 0 \tag{13}$$

The potential strain energy in the torsion beams is calculated as:

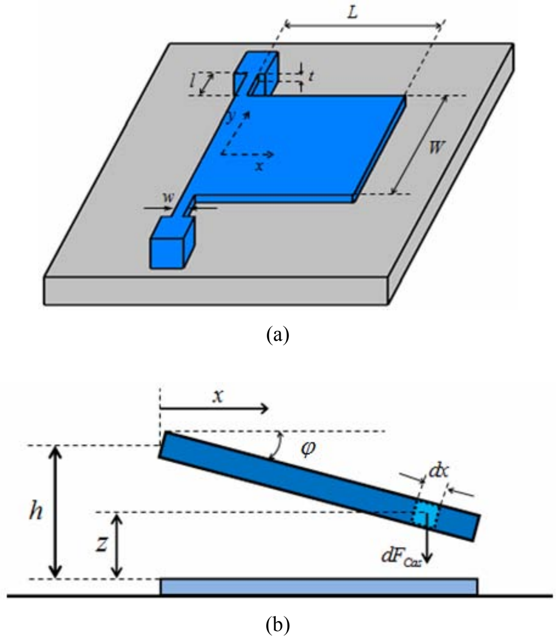

Fig. 1. Schematic: (a) isometric; (b) side view of a nano/micromirror under the effect of Casimir force.

$$U = \frac{1}{2}K\varphi^2 \tag{14}$$

which leads to Eq. (15) for the variation of the potential strain energy.

$$\delta U = K\varphi\delta\varphi \tag{15}$$

The work done by the Casimir forces to deflect the mirror from angle φ to angle $\varphi + \delta\varphi$ is computed using Eq. (16).

$$\begin{aligned}\delta W_e &= \int_0^L (dF_{Cas})(x\delta\varphi) \\ &= \int_0^L \left(\frac{\pi^2\hbar c}{240(h - x\sin\varphi)^4}Wdx\right)(x\delta\varphi)\end{aligned} \tag{16}$$

So at equilibrium points, the following equation is satisfied:

$$K\varphi(\delta\varphi) = \int_0^L \left(\frac{\pi^2\hbar c}{240(h - x\sin\varphi)^4}\right)Wdx(x\delta\varphi) . \tag{17}$$

By performing the integration and using dimensionless variable β, Eq. (18) is obtained.

$$\frac{\lambda}{\beta^2}\left(\frac{1}{6} - \frac{3\beta-1}{6(\beta-1)^3}\right) - \beta = 0 \tag{18}$$

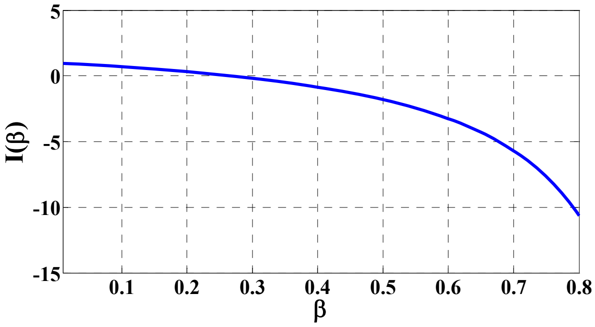

Fig. 2. Stability function $I(\beta)$ at equilibrium points versus β .

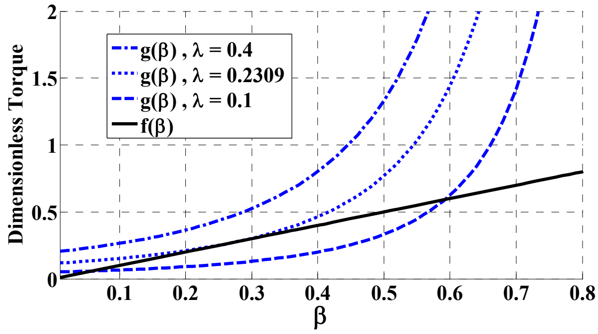

Fig. 3. Dimensionless elastic restoring torque and Casimir torque (at different values of λ) applied to the nano/micromirror versus its rotation angle.

This equation is the same as Eq. (10) for the equilibrium equation. Performing the second variation operator on the Eq. (13), and using equilibrium Eq. (18), one can conclude:

$$\delta^2\Pi = \frac{(\delta\beta)^2 h^2}{L^2}\left(K + \frac{\pi^2\hbar c W L^3}{240\beta^3 h^5}\left(\frac{8}{3}\frac{1}{(1-\beta)^3} - \frac{2}{(1-\beta)^2} - \frac{1}{(1-\beta)^4} + \frac{1}{3}\right)\right). \quad (19)$$

According to minimum total potential energy principle an equilibrium point is stable when $\delta^2\Pi > 0$ and is unstable when $\delta^2\Pi < 0$. So the stability condition is reduced to:

$$I(\beta) = 1 + \frac{\lambda}{\beta^3}\left(\frac{8}{3}\frac{1}{(1-\beta)^3} - \frac{2}{(1-\beta)^2} - \frac{1}{(1-\beta)^4} + \frac{1}{3}\right) > 0 \quad (20)$$

where $I(\beta)$ is called the stability function. Using equilibrium Eq. (18), instability number, λ can be obtained in terms of dimensionless angle β as follows:

$$\lambda = \frac{\beta^3}{\left(\frac{1}{6} - \frac{3\beta - 1}{6(\beta-1)^3}\right)}. \quad (21)$$

Substituting Eq. (21) into Eq. (20) leads to the following equation for the stability of the equilibrium points:

$$I(\beta) = 1 + \frac{\left(\frac{8}{3}\frac{1}{(1-\beta)^3} - \frac{2}{(1-\beta)^2} - \frac{1}{(1-\beta)^4} + \frac{1}{3}\right)}{\left(\frac{1}{6} - \frac{3\beta - 1}{6(\beta-1)^3}\right)} > 0. \quad (22)$$

Fig. 2 shows the stability function $I(\beta)$ versus β .

It is observed that when β is less than 0.2679, the resulting equilibrium point is stable and if β is larger than 0.2679, the resulting equilibrium point is unstable and finally If β equals 0.2679, the equilibrium point corresponds to the pull-in state. The value of λ at this point is obtained by Eq. (23):

$$\lambda_p = \frac{{\beta_p}^3}{\left(\frac{1}{6} - \frac{3\beta_p - 1}{6(\beta_p-1)^3}\right)} = 0.2309 \quad (23)$$

where λ_P and β_p are the values of λ and β at pull-in, respectively. In Fig. 3, the dimensionless torque functions, $f(\beta) = \beta$ and $g(\beta) = \lambda\left(1/6 - (3\beta-1)/6(\beta-1)^3\right)/\beta^2$ are plotted versus β at various values of λ. It is observed that at low values of λ, there exist two roots for Eq. (18) where the smaller one (which is smaller than the critical value of $\beta_P = 0.2679$) is the stable equilibrium point and the larger one (which is larger than the critical value of $\beta_P = 0.2679$) is the unstable equilibrium point. At $\lambda = \lambda_P$, Eq. (18) has just one root. For λ larger than λ_P, Eq. (18) does not have any root and equilibrium can't be established.

The interesting point is that the β_P which is the root of $I(\beta)$ is also the root of $d\lambda/d\beta = 0$ which means at pull-in, the derivative of λ with respect to β is zero. So in designing rectangular nano/micromirrors shown in Fig. 1, Eq. (24) has to be satisfied, otherwise the structure would be unstable and pull-in would happen.

$$0.7082\sqrt[5]{\frac{\hbar c W L^3}{K}} < h \quad (24)$$

Eq. (24) can be used for a safe design of nano/micromirrors under Casimir force.

3. Analytical solutions for equilibrium equation

In this section, it is tried to find the value of the rotation angle of the nano/micromirror analytically in terms of λ. For this purpose, the analytical tool, HPM is utilized.

The linear part of Eq. (10) can be found by using Taylor series expansion of the equilibrium Eq. (10) as follows:

$$L(\beta,\lambda) = \frac{\lambda}{2} + \left(\frac{4\lambda}{3} - 1\right)\beta \quad (25)$$

where $L(\beta,\lambda)$ is the linear part of Eq. (10). Obviously the nonlinear part of equilibrium equation is obtained by subtracting $L(\beta,\lambda)$ from Eq. (25).

$$N(\beta,\lambda)=\frac{\lambda}{\beta^2}\left(\frac{1}{6}-\frac{3\beta-1}{6(\beta-1)^3}\right)-\frac{\lambda}{2}-\frac{4\lambda}{3}\beta \tag{26}$$

Now, the homotopy form is constructed as follows:

$$\Im(\beta,\lambda,P)=L(\beta,\lambda)+P.N(\beta,\lambda)=0. \tag{27}$$

In Eq. (27), $\Im(\beta,\lambda,P)$ is the homotopy form and P is an embedding parameter which serves as perturbation parameter. When $P=1$, the homotopy form would be the same as the equilibrium equation and when $P=0$, homotopy form would be the linear part of equilibrium equation. The value of the dimensionless rotation angle β can also be expanded in terms of the embedded parameter P.

$$\beta=\beta_0+P\beta_1+P^2\beta_2+P^3\beta_3+\ldots \tag{28}$$

Substituting Eq. (28) into homotopy form yields:

$$\begin{aligned}\Im(\beta,\lambda,P)&=L\left(\beta_0+P\beta_1+P^2\beta_2+\ldots,\lambda\right)\\&+P.N\left(\beta_0+P\beta_1+P^2\beta_2+\ldots,\lambda\right)=0.\end{aligned} \tag{29}$$

The Taylor series expansion of right hand side of Eq. (29) in terms of P would be as

$$\begin{aligned}\Im(\beta,\lambda,P)&=L(\beta_0,\lambda)+\left(\beta_1\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}+N(\beta_0,\lambda)\right)P\\&+\left(\beta_2\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}+\beta_1\frac{\partial N(\beta_0,\lambda)}{\partial\beta_0}\right)P^2\\&+\left(\beta_3\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}+\beta_2\frac{\partial N(\beta_0,\lambda)}{\partial\beta_0}\right.\\&\left.+\frac{1}{2}\beta_1^2\frac{\partial^2N(\beta_0,\lambda)}{\partial\beta_0^2}\right)P^3+\ldots=0.\end{aligned} \tag{30}$$

Since the homotopy form must be unified with zero, the coefficients of all powers of P must be zero. This, leads to the following equations:

$$L(\beta_0,\lambda)=0 \tag{31}$$

$$\beta_1\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}+N(\beta_0,\lambda)=0 \tag{32}$$

$$\beta_2\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}+\beta_1\frac{\partial N(\beta_0,\lambda)}{\partial\beta_0}=0 \tag{33}$$

$$\beta_3\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}+\beta_2\frac{\partial N(\beta_0,\lambda)}{\partial\beta_0}+\frac{1}{2}\beta_1^2\frac{\partial^2N(\beta_0,\lambda)}{\partial\beta_0^2}=0. \tag{34}$$

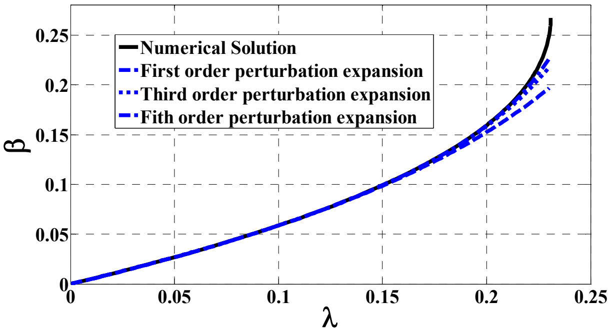

Fig. 4. Equilibrium behavior of the mirror shown in Fig. 1, comparison of analytical HPM results with numerical results.

With solving Eqs. (31) to (34) , the parameters β_i $0\le i\le 3$ can be found as follows:

$$\beta_0=\frac{3\lambda}{6-8\lambda} \tag{35}$$

$$\beta_1=-N(\beta_0,\lambda)\Big/\left(\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}\right) \tag{36}$$

$$\beta_2=-\beta_1\left(\frac{\partial N(\beta_0,\lambda)}{\partial\beta_0}\right)\Big/\left(\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}\right) \tag{37}$$

$$\beta_3=-\left(\beta_2\frac{\partial N(\beta_0,\lambda)}{\partial\beta_0}+\frac{1}{2}\beta_1^2\frac{\partial^2N(\beta_0,\lambda)}{\partial\beta_0^2}\right)\Big/\left(\frac{\partial L(\beta_0,\lambda)}{\partial\beta_0}\right). \tag{38}$$

The value of β is found by substituting β_i $0\le i\le 3$ and $P=1$ in Eq. (28). In Fig. 4 the results of the numerical simulations is compared with those of analytical HPM results. It is observed that with increasing the value of λ, the rotation angle increased and pull-in occurs. This figure also shows that the presented analytical solution closely follows the numerical one. Furthermore, it is observed that with increasing the degree of the perturbation approximation the accuracy of the analytical solution is increased, but by increasing the mentioned number more than 5, the accuracy of the obtained solution does not improve appreciably. So a fifth order perturbation approximation can be used for an accurate design.

4. Conclusions

The statical behavior of nano/micromirrors under the effect of Casimir force was studied. First, the equation governing the statical behavior of a nano/micromirror was obtained. Then energy method was utilized to investigate the statical stability of nano/micromirror equilibrium points. It was shown that when there exist two equilibrium points, the smaller one is stable and the larger one is unstable. Furthermore, a useful equation was suggested for the successful design of nano/micromirrors in the stable operational range.

The tilting angle of nano/micromirrors due to Casimir force was also investigated both numerically and analytically using HPM. It was observed that with increasing the instability number defined in the paper, the rotation angle of the nano/micromirror is increased and suddenly the pull-in occurs. The analytical results well followed the numerical ones and so the presented analytical method can be used as a fast and accurate design tool for nano/micromirrors design optimization.

Nomenclature

G : Shear modulus of elasticity of the beam's material
h : Initial distance between the mirror and the substrate
l : Length of each torsion beam
L : Length of mirror
U : Potential strain energy of the torsion beams
V : Potential energy of applied loads
W : Width of mirror

References

[1] C. N. Maluf and K. Williams, *An introduction to microelectromechanical systems engineering*, Second Edition, MicroelectromechanicalSystems (MEMS) Series, Boston, London, Artech House Inc. (1999).
[2] M. I. Younis, *Modeling and simulation of microelectromechanical systems in multi-physics fields*, PhD thesis, Virginia Polytechnic Institute and State University (2004).
[3] P. C. P. Chao, C. W. Chiu and C. Y. Tsai, A novel method to predict the pull-in voltage in a closed form for micro-plates actuated by a distributed electrostatic force, *Journal of Micromechanics and Microengineering*, 16 (2006) 986-998.
[4] L. J. Hornbeck, *Spatial light modulator and method*, US Patent 5,061,049 (1991).
[5] J. E. Ford, V. A. Aksyuk, D. J. Bishop and J. A. Walker, Wavelenghth add-drop swiching using tilting micromirrors, *Journal of Lightwave Technology*, 17 (1999) 904-911.
[6] D. L. Dickensheets and G. S. Kino, Silicon-micromachined scanning confocal optical microscope, *Journal of Microelectromechanical Systems*, 7 (1998) 38-47.
[7] P. M. Zavracky, S. Majumber and E. McGruer, Micromechanical switches fabricated using nickel surface micromachining, *Journal of Microelectromechanical Systems*, 6 (1997) 3-9.
[8] H. Toshiyoshi and H. Fujita, Electrostatic micro torsion mirrors for an optical switch matrix, *Journal of Microelectromechanical Systems*, 5 (1996) 231-237.
[9] J. M. Huang, A. Q. Liu, Z. L. Deng, Q. X. Zhang, J. Ahn, and A. Asundi, 2004, An approach to the coupling effect between torsion and bending for electrostatic torsional micromirrors, *Sensors and Actuator A*, 115 (2004) 159-167.
[10] L. J. Hornbeck, 128×128 deformable mirror devices, *IEEE Trans. Electron Devices ED*, 30 (1983) 539-545.
[11] R. S. Muller and K. Y. Lau, Surface-micromachined micro-optical elements and systems, *Proceedings IEEE*, 86 (1998) 1705-1720.
[12] T.-H. Lin, Implementation and characterization of a flexure-beam micromechanical spatial light modulator, *Optical Engineering*, 33 (1994) 3643-3648.
[13] P. F. Van Kessel, L. J. Hornbeck, R. E. Meier and M. R. Douglass, MEMS-based projection display, *Proceedings. IEEE,* 86 (1998) 1687-1704.
[14] S. Kurth, R. Hahn, C. Kanfmann, K. Kehr, J. Mehner, V. Wollmann, W. Dotzel and T. Gessner, Silicon mirrors and micromirror arrays for spatial laser beam modulation, *Sensors and Actuators A*, 66 (1998) 76-82.
[15] R. W. Cohn, Link analysis of a deformable mirror device based optical crossbar switch, *Optical Engineering*, 31 (1992) 134-140.
[16] O. Bochobza-Degani, D. Elata and Y. Nemirovsky, An efficient DIPIE algorithm for CAD of electrostatically actuated MEMS devices, *Journal of Microelectromechanical Systems*, 11 (2002) 612-620.
[17] X. M. Zhang, F. S. Chau, C. Quan, Y. L. Lam and A. Q. Liu, A study of the static characteristics of a torsional micromirror, *Sensors and Actuators A*, 90 (2001) 73-81.
[18] A. K. Bhaskar, M. Packirisamy and R. B. Bhat, Modeling switching response of torsional micromirrors for optical microsystems, *Mechanism and Machine Theory*, 39 (2004) 1399-1410.
[19] F. Khatami and G. Rezazadeh, Dynamic response of a torsional micromirror to electrostatic force and mechanical shock, *Microsystem Technologies*, 15 (2009) 535-545.
[20] S. A. Zhou, On forces in microelectromechanical systems, *International Journal of Engineering Science*, 41 (2003) 313-335.
[21] G. Xie, J. Ding, S. Liu, W. Xued and J. Luob, Interfacial properties for real rough MEMS/NEMS surfaces by incorporating the electrostatic and Casimir effects–a theoretical study, *Surface and Interface Analysis.*, 41 (2009) 338-346.
[22] A. Gusso and G. J. Delben, Influence of the Casimir force on the pull-in parameters of silicon based electrostatic torsional actuators, *Sensors and Actuators A*, 135 (2007) 792-800.
[23] F. V. Tahami, H. Mobki, A. A. K. Janbahan and G. Rezazadeh, Pull-in Phenomena and Dynamic Response of a Capacitive Nano-beam Switch, *Sensors & Transducers Journal*, 110 (2009) 26-37.
[24] W. H. Lin and Y. P. Zhao, Casimir effect on the pull-in parameters of nanometer switches, *Microsystem Technologies,* 11 (2005) 80-85.
[25] W. H. Lin and Y. P. Zhao, Nonlinear behavior for nanoscale electrostatic actuators with Casimir force, *Chaos, Solitons and Fractals*, 23 (2005) 1777-1785.
[26] A. Ramezani, A. Alasty and J. Akbari, Pull-in parameters of cantilever type nanomechanical switches in presence of Casimir force, *Nonlinear Analysis: Hybrid Systems*, 1 (2007) 364-382.
[27] A. Ramezani, A. Alasty and J. Akbari, Analytical investigation and numerical verification of Casimir effect on electrostatic nano-cantilevers, *Microsystem Technologies.*, 14

(2008) 145-157.

[28] M. Mojahedi, H. Moeenfard and M. T. Ahmadian, A new efficient approach for modeling and simulation of nano-switches under combined effects of intermolecular surface forces and electrostatic actuation, *International Journal of Applied Mechanics*, 1 (2009) 349-365.

[29] R. E. Sirvent, M. A. Palomino-Ovando and G. H. Cocoletzi, Pull-in control due to Casimir forces using external magnetic fields, *Applied Physics Letters*, 95 (2009) 051909.

[30] F. M. Serry, D. Walliser and G. J. Maclay, The role of the Casimir effect in the static deflection and stiction of membrane strips in microelectromechanical systems (MEMS), *Journal of Applied Physics*, 84 (1998) 2501-2506.

[31] M. I. Younis and A. H. Nayfeh, A study of the nonlinear response of a resonant microbeam to an electric actuation, *Nonlinear Dynamics*, 31 (2003) 91-117.

[32] E. M. Abdel-Rahman and A. H. Nayfeh, Secondary resonances of electrically actuated resonant microsensors, *Journal of Micromechanics and Microengineering* , 13 (2003) 491-501.

[33] J. H. He, A coupling method of a homotopy technique and a perturbation technique for non-linear problems, *International Journal of Non-Linear Mechanics*, 35 (2000) 37-43.

[34] H. Moeenfard, M. Mojahedi and M. T. Ahmadian, A homotopy perturbation analysis of nonlinear free vibration of timoshenko microbeams, *Journal of mechanical science and technolegy*, 25 (2011) 279-285.

[35] M. Mojahedi, H. Moeenfard and M. T. Ahmadian, Analytical solutions for the static instability of nano-switches under the effect of Casimir force and electrostatic actuation, *ASME International Mechanical Engineering Congress and Exposition, Proceedings*, (2010) 63-69.

[36] H. Liu, S. Gao, S. Niu and L. Jin, Analysis on the adhesion of microcomb structure in MEMS, *International Journal of Applied Electromagnetics and Mechanics*, 33 (2010) 979-984.

[37] S. S. Rao, *Vibration of Continuous Systems*, John Wiley & Sons, New Jersey (2007).

Hamid Moeenfard received his M.Sc degree in mechanical engineering from Sharif University of technology, Tehran, Iran, 2008. He is currently working toward a PhD in mechanical engineering at Sharif University of technology. His main interests are nonlinear vibration, N/MEMS and MOEMS, perturbation theory, Kantorovich method and fuzzy logic and control. His researches are mainly about modeling and analysis of static and dynamic pull-in in electrostatically actuated microbeams/plates using analytical models. His current research is the mechanical modeling of nonlinear vibration and static and dynamic pull-in of electrostatically actuated tosional micromirrors considering squeeze film damping and nonlinear electrical and mechanical nonlinearities.

Ali Darvishian received his B.S. degrees in mechanical engineering from the Sharif University of Technology, Tehran, Iran, in 2010. He is currently working on instability of micro/nano mirrors under effect of various forces for his M.Sc. degree in Mechanical engineering from the Sharif University. His main research interests include modeling, nonlinear dynamics and vibrational analysis of nano/micro electromechanical systems.

M. T. Ahmadian received his B.S. & M.S. degrees in Physics 1972 from Shiraz University, Shiraz, Iran and completed the requirements for B.S. & M.S. degrees in Mechanical Engineering in 1980 from University of Kansas in Lawrence. At the same time he completed his PhD in Physics and PhD in Mechanical Engineering in 1981 and 1986, respectively, from University of Kansas. His research interests are micro and nano mechanics as well as bioengineering.