Springer

KSME

Journal of Mechanical Science and Technology 26 (5) (2012) 1409~1417
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-011-1028-1

Static/dynamic contact FEA and experimental study for tooth profile modification of helical gears †

Yong-jun Wu, Jian-jun Wang* and Qin-kai Han

School of Jet Propulsion, Beijing University of Aeronautics and Astronautics, HaiDian District, Beijing, China

(Manuscript Received March 8, 2011; Revised September 13, 2011; Accepted October 8, 2011)

Abstract

With the development of high-performance computers, the contact finite element analysis (FEA) method has become more and more popular for studying both the static and dynamic behaviors of gear drives. In this paper, a precise tooth profile modification (TPM) approach of the helical gear pairs is presented first. The type and amount of the TPM are accurately determined by the static contact FEA results. Then dynamic contact simulations for the helical gear pairs with and without TPM are, respectively, carried out to evaluate the effect of the presented TPM approach on vibration reduction. No additional assumptions and simplifications are required for the static and dynamic contact analysis models. Vibration comparison experiments are also carried out on an open power flow test rig. Both the simulated and experimental results show that the presented precise TPM of helical gears is effective on vibration reduction around the working load, and the dynamic contact simulation is effective in estimating the effect of the TPM on vibration reduction in the designing stage.

Keywords: Helical gears; Tooth profile modification (TPM); Static/Dynamic contact analysis; Finite element analysis (FEA); Vibration reduction

1. Introduction

Gear train, as a critical part of the power transmission systems, provides the needed torque and rotational speed. Its performance directly determines the performance of the power transmission system. Nowadays, gear trains are widely used in automotive, industrial, and aerospace applications. With the increasing demands of high speed, heavy transmission, light weight and quiet running in modern gear systems, the reduction of vibration and noise becomes increasingly important.

Considering the variation of meshing positions and the elastic deflections of loaded teeth, the gear vibration and noise are mainly caused by the dynamic meshing excitations, including the time-varying meshing stiffness and the meshing impact. The tooth profile modification (TPM) [1, 2], as an important approach to reduce the dynamic excitation of meshing teeth, has been widely used for vibration and noise reduction of gear drives, especially for high-speed and heavy-load gear drives.

In the process of TPM design, the determination of the maximum modification amount is one of the most important parts for the TPM. In 1938, Walker [4] first proposed a tip relief of the spur gears considering the tooth deflections in the evaluation of the tooth load. Since then, many theoretical and experimental researches of the TPM have been made by Ref. [5-10]. Y. Terauchi [5] calculated the tooth deflections and made a profile modification on the spur gear teeth. Literatures [6-9] took the static transmission error (STE) as the objective functions of optimal design and calculated the modified amount of the TPM for spur gears. Moreover, there were many other empirical formulas to estimate the modification amount [2], like the ISO formulas from ISO6336 and the H. Sigg formulas. But most of them require additional simplifications and assumptions to determine the modified amounts. Hence, the conventional methods cannot make a precise TPM for gears, especially for helical gears. It is more complicated for helical gear pairs than spur gears in the meshing process, and it is hard to obtain the accurate deformations of helical gear pairs in the past. So a new method is needed to calculate the deformations of helical gears accurately, and make a precise TPM for helical gears.

Another important part of TPM is to evaluate the effect of the TPM on vibration reduction. Interesting experiments were performed by Kahraman [10], where the influence of the tip relief of spur gear on vibration reduction was analyzed by comparing the tested dynamic transmission error (DTE) and the forced response of gear drives. Experimental study is the most effective method to evaluate the effect of TPM on vibra-

*Corresponding author. Tel.: +86 1082317411, Fax.: +86 1082317432
E-mail address: wangjianjun@buaa.edu.cn
† Recommended by Editor Yeon June Kang.

tion reduction. But it costs too much, and is unavailable in designing stage. Kahraman [11] used a nonlinear finite element contact mechanics model of helical gear pair to study the effect of the tooth profile modifications on durability of helical gears. Song He [12] analyzed the effect of sliding friction on the dynamics of spur gear with realistic time-varying stiffness, where the realistic stiffness, including the effect of tooth profile modifications, was computed by static contact FEA code. Li [13] presented a three-dimensional finite element method to analyze the contact strength and bending strength of spur gears with machining errors, assembly errors and tooth modifications. Parker [14] did a series of studies on the dynamic behaviors of gear systems. The nonlinear dynamic response of spur gear pairs [14] and planetary gears [15] was investigated using a finite element/contact mechanics model. The effect of tooth profile modification on multimesh gears vibration was studied using the nonlinear dynamic model [17]. The effects of the TPM on gears dynamic performances were studied indirectly for some aspects, but few literatures studied the effect of TPM on vibration reduction directly, especially for helical gear pairs. Hence, a new approach is also needed to evaluate the effect of the TPM on vibration reduction in designing stage.

With the development of high performance computers, the contact finite element analysis (FEA) has become more and more popular for studying both the static and dynamic behaviors of gear drives. The deformations or time varying stiffness of the tooth pairs could be computed accurately using the static contact FEA [12]. Based on literature [18], Lin [19] presented a finite element method for 3D static and dynamic contact impact analysis of gear drives, where the mesh stiffness and the impact characteristics were obtained. Hu [20] researched the transient meshing performance of gears with different modification coefficients and helical angles using explicit dynamic FEA software—ANSYS/LS-DYNA. These studies could provide an approach to study the dynamics of gear meshing process, although they did not study the effect of TPM on vibration reduction. It can be seen that there are many studies using contact FEA on normal gears, but a few on modified gears. The work of this paper will fulfill this need.

Hence, the objective of this paper is to present a precise approach for TPM of the helical gears, and investigate a new dynamic simulation method to evaluate the effect of the TPM on vibration reduction in designing stage. The precise approach for TPM of the helical gears is presented according to the basic mechanism of the TPM in section two. The type and amount of the TPM are accurately determined by the static contact FEA results. In the third section, the dynamic contact simulation is introduced to evaluate the effect of the TPM on vibration reduction in designing stage. A vibration comparison experiment is carried out on an open power flow test rig in section four. Both the modification design and dynamic simulation of the TPM are validated by the experimental results in this section. Some conclusions of this study are drawn in the last section.

Table 1. Geometric parameters of helical gear pairs.

Geometric parameters	Gear	Pinion
Normal module, m_n	2.5 mm	2.5 mm
Number of teeth, Z	101	36
Normal pressure angle, α_n	20°	20°
Helix angle, β	20°	-20°
Face width, b	25 mm	18 mm
Modification coefficient, x	0	0.5152

2. TPM of helical gears using static contact FEA

The tooth profile modification of gears has three key points, which contains the maximum modification amount, the modification length, and the modification curve types [2]. And the maximum modification amount of the TPM is the most important factor of all, which is determined by the elastic deformations of gear pairs. So the key point of the TPM is to calculate the deformations of gear pairs accurately. As mentioned above, the static contact FEA of gear pairs will be introduced to calculate the deformations in this section.

According to the formation theory of the involute and helix [21], the modeling codes are written using ANSYS parametric design language (APDL), and the parametric models of the helical gear are directly established in ANSYS. Then the precise meshing model of helical gears can be generated based on the meshing principle and the parametric gear models. In this study, the geometric parameters of the helical gear pairs are shown in Table 1. The working speed of the gear is 1500 r/min, and the torque of the pinion is 27.3 N.m.

Considering the overlap contact ratio of helical gears, the meshing process of helical gears is more complicated than spur gears. According to the geometric parameters of the helical gears, the transverse contact ratio is 1.5123, and the overlap contact ratio is 0.7839, that is, the total contact ratio is 2.2962. This means that there are three teeth pairs in mesh about 29.62% of the time and two teeth pairs in mesh about 70.38% of the time. The meshing impact will occur in the alternation process of the meshing teeth pairs, which means from two pairs change to three pairs contact or from three pairs change to two pairs contact. So the deflections of the teeth pairs in the alternation process are needed to calculate accurately for determining the amount of the TPM in the next.

The meshing process of the helical gear is shown in Fig. 1. The numbers 1 to 4 mean four meshing states in different contact positions. The superscripts mean different meshing tooth pairs of the same state. When the first tooth pair begins to mesh in position 1-1, the second tooth pair meshes in position $1'$-$1'$ and the third one meshes in position $1''$-$1''$. In the whole sections with shadow, there are three teeth pairs engaged in all. While the first tooth pair engages in position 2-2, the third tooth pair disengages in position $2''$-$2''$. Then the gears engage with 2 teeth pairs in the blank sections. So the alternation of the engaged teeth pairs occurs at the meshing position ① and ②, respectively, which are the two

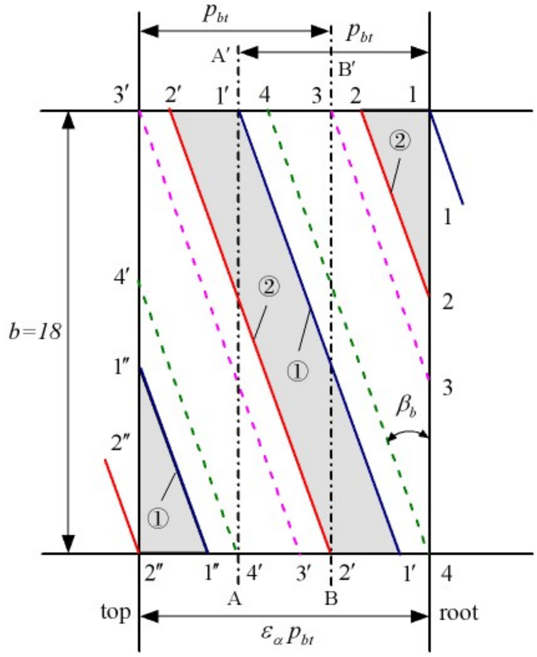

Fig. 1. Meshing process of helical gears.

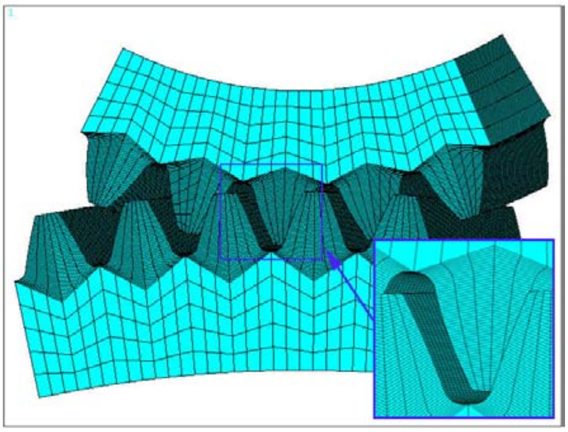

Fig. 2. Meshed finite element model for static analysis.

meshing states that need to be calculated in this paper.

According to Saint-Venant's principle [22], the five teeth pairs static contact FEA model, with two teeth pairs in meshing, is established and shown in Fig. 2. The five teeth pair model is established for reducing the influence of the boundary and loaded conditions on the results, when there are three teeth pairs in meshing at most. The defined contact teeth pair models for two alternation positions are shown in Fig. 3 and Fig. 4 respectively. The friction coefficient of the contact models is 0.1 [23].

For the static contact analysis of gear pairs, the fixed displacement constraint is applied on both the bottom and lateral boundary surfaces of the gear, the radial and axial constraints are defined on both the bottom and lateral boundary surfaces of the pinion. A distributed tangential force is applied uniformly on the nodes of the bottom surface of the pinion as the equivalent torque. The distributed forces q applied on the pinion can be obtained by

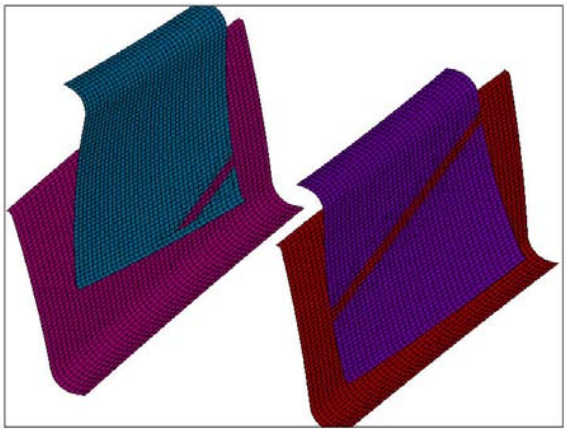

Fig. 3. Contact teeth pairs for the lowest point of double tooth contact model.

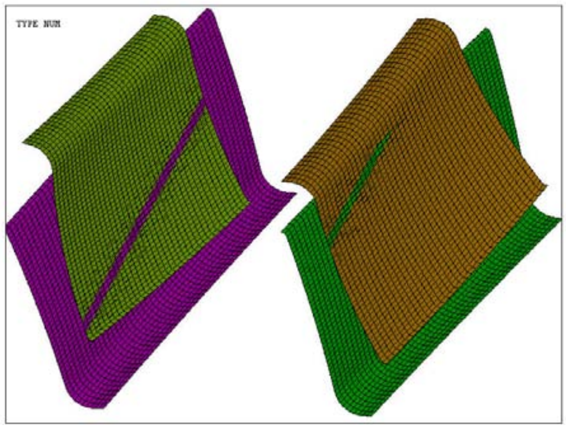

Fig. 4. Contact teeth pairs for the highest point of double tooth contact model.

$$q = \frac{T}{Nr} \tag{1}$$

where T is the torque on the pinion, N is the node amount for the bottom surface of the pinion, r is the radius of the bottom surface of the pinion.

According to the radical principle of the TPM, gear tooth profile modification is introduced to optimize the contact patterns, to compensate for the deflections and manufacturing errors of the gear pairs, and to reduce the gear dynamic excitations. Neglecting the manufacturing errors and assembly errors, the maximum amounts of the TPM at tip and root mainly depend on the integrative elastic deformations of gear pairs at alternant meshing positions, respectively. So only the deformations of the highest and lowest meshing point of double teeth contact are computed for the TPM in this study.

The exact deformations of gear pairs on the meshing line

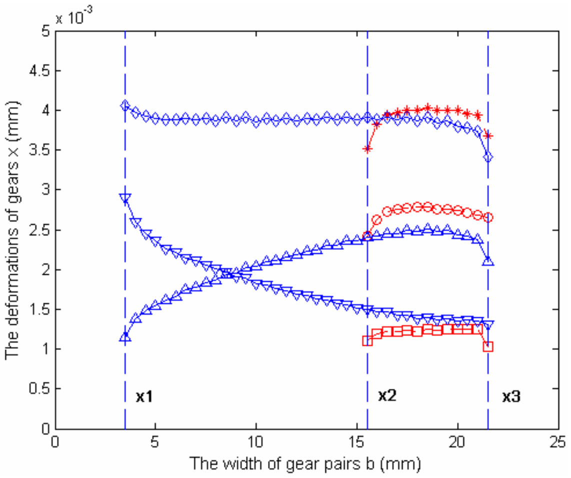

Fig. 5. Deformations of gear pairs on meshing line for lowest point of double tooth contact model, for gear of pairs 1 (○), pinion of pairs 1 (□), integrate of pairs 1 (*), for gear of pairs 2 (△), pinion of pairs 2 (▽), integrate of pairs 2 (◇).

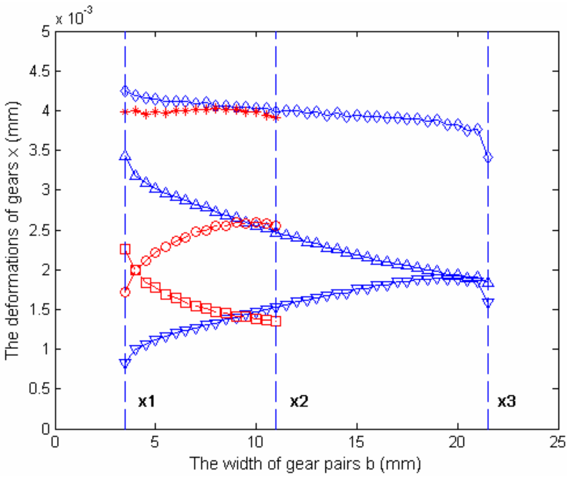

Fig. 6. Deformations of gear pairs on meshing line for highest point of double tooth contact model , for gear of pairs 1 (△), pinion of pairs 1 (▽), integrate of pairs 1 (◇), for gear of pairs 2 (○), pinion of pairs 2 (□), integrate of pairs 2 (*).

are extracted from the results of the static contact FEA. Then the integrative deformations of gear pairs are obtained by superimposing the elastic deformations of gear and pinion at the same meshing point. The deformations of gear pairs on meshing line for lowest and highest meshing points are shown in Fig. 5 and Fig. 6, respectively. In both of the figures, there are two contacted tooth pairs in meshing as shown in Fig. 3 and Fig. 4. The width of the pinion is x1-x3. The tooth pair 1 for the lowest point is partly meshed from x2 to x3, the tooth pair 2 for the highest point is partly meshed from x1 to x2, and the tooth pair 2 for the lowest point and the tooth pair 1 for the highest point are meshed in the whole width x1 to x3.

It can be seen from Figs. 5 and 6 that the deformations are not with the same amplitude along the meshing line. Just because the meshing line of helical gears is not along the direction of the tooth width but slope on the tooth surface, so the integrative deformation curve is slanted too. That is why the lead crowning is needed for avoiding the corner contact. In this study, only the TPM is considered because the amount of slope is tiny. But, the static contact FEA method is also suitable to calculate the amount of the lead crowning.

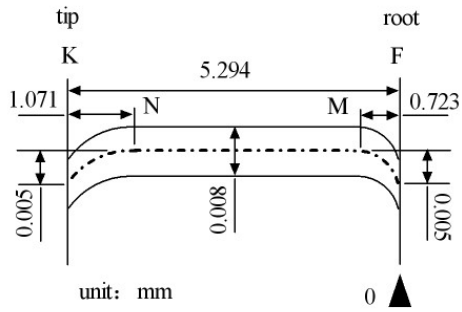

Fig. 7. Design curve of tooth profile modification.

Taking the average of the integrative deformations as the deformations for each meshing positions, the integrative elastic deformations of gear drives are 0.0039 mm at the lowest double teeth contact position and 0.0040 mm at the highest double teeth contact position. To compensate for the deflections of gear pairs and adapt to the realistic machining conditions and precision levels, the maximum modification amount is determined as 0.005 mm for both the tip and root, a little bigger than the half width of the tolerance zone (0.004 mm).

The parabolic modification curve with short form is introduced in this paper. The formulas of the modification curve at tip and root are given as

$$\Delta_{\mathrm{Tip}} = 0.005 \left(\frac{x}{1.071}\right)^2, \tag{2}$$

$$\Delta_{\mathrm{Root}} = 0.005 \left(\frac{x}{0.723}\right)^2 \tag{3}$$

where Δ_{Tip} and Δ_{Root} is the modification curves for the tip and root respectively, 0.005 is the maximum modification amount, x is the relative coordinates of the meshing point, 1.071 is the length of the modified tooth profile on the tip, and 0.723 is the length of the modified tooth profile on the root, as shown in Fig. 7.

The design curve of the TPM according Eqs. (2) and (3) is shown in Fig. 7, where M and N are the origins of the TPM at root and tip, and F and K are the end points of the TPM, respectively. So from N to K is the modification region of the tip, N to M is the unmodified region for double teeth contact, and M to F is the modified region of the root. In Fig. 7, the bold dash-dotted line is the designing modification curve of gear, and the two solid lines are the tolerance line of the modification curve. The width of the tolerance zone is 0.008 mm according to 6 level precision of the gear pairs.

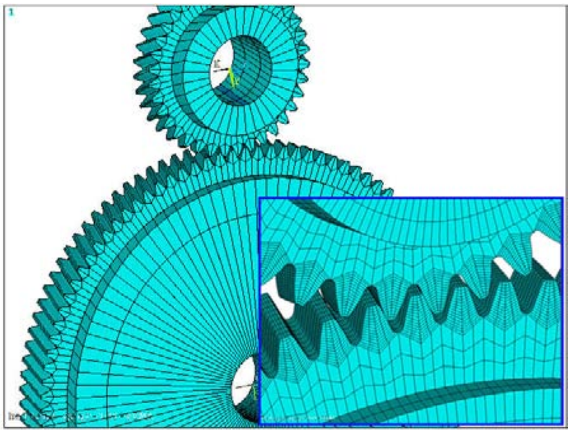

Fig. 8. Dynamic contact model of helical gear pairs.

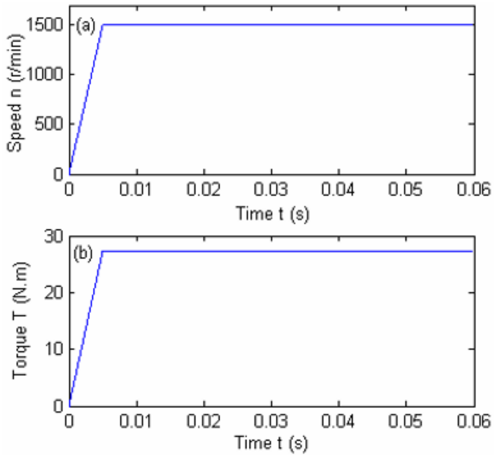

Fig. 9. Load and speed curves for simulations: (a) Speed curve for gear; (b) Load curve of torque for pinion.

In this study, only the left profile of the pinion is modified; the tooth profiles of the gear and the right side of the pinion remain unmodified to minimize the influence of the machining errors and the assembly errors in experiments. The contrast of modification is carried out by making the gears rotate forward and backward for a certain load. The comparison will be made using simulation and experiment in the next two sections.

3. Dynamic simulations of the modified gears

The realistic meshing process of gear pairs contains both the dynamic contact and impact exciting, which are considered as the main inner exciting source of the gear drives' vibration and noise. The TPM is such an approach for vibration and noise reduction by decreasing the dynamic exciting of the meshing process. But the realistic mesh process of gears cannot be simulated effectively using the conventional methods in designing stage in the past. A new approach is required to make a dynamic contact simulation of the modified gear pairs for evaluating the influence of the TPM on vibration reduction.

The explicit dynamic analysis software ANSYS/LS-DYNA [24], with the significant advantages for the contact and impact problems, is introduced for the dynamic contact simulation of the modified and unmodified gears in this paper, which requires no additional assumptions and simplifications for the dynamic contact analysis. The vibration for modified and unmodified gear pairs are compared based on the dynamic simulation results.

First, the normal involute tooth profile is established in the same way with the static models. Then the left profile of the pinion is modified according to the proposed design curve in Fig. 7. The modified gear model is built on the basis of the modified profile at last. The dynamic contact model of helical gear pairs is shown in Fig. 8, where the large figure is the solid model of gear pairs and the small one is the magnified finite element model of gears.

The dynamic contact model of the gear pairs was defined by a means of the so-called auto surface-to-surface contact method. There is no need to define specifically contact teeth pairs as the static contact analysis, because the contact teeth pairs of the gears vary with the rotation of the gears in the dynamic meshing process. In this simulation, the gear and pinion are defined as a separate part first, then the contact model is defined between the gear part and the pinion part, and the contact state is searched automatically in the simulation. Hence, no additional simplifications and assumptions are required for the contact model of the gears. The sliding friction of the gear pairs was considered, and the friction coefficient was also 0.1 [23].

The comparison of the helical gear pairs with and without modification is carried out by applying a positive or negative angular velocity on the gear, and a corresponding torque on the pinion. In this simulation, the angular velocity is applied to the gear directly, and the torque is applied to the pinion. The simulation speed of the gear is ±1500 r/min, where the sign "±" means the direction of rotation, forward with "+" means unmodified tooth pairs in meshing and backward with "-" means modified in meshing. The torque of the pinion is 5, 10, 20, 27.3, 35 Nm with the corresponding direction of the gear, respectively. The speed and torque increase gradually from zero to the maximum in a certain period for avoiding the impact of the sudden loaded. The computing time is set to 0.06s for ensuring that the gear rotates at least one lap according to the speed of the gear. Hence, the load and speed curve for simulation is shown in Fig. 9.

Considering the deformations of gear pairs and the variation of the meshing teeth pairs, there is a periodic dynamic exciting on the meshing teeth. Hence the simulated angular velocity of the pinion will oscillate periodically around the theoretic value. The comparison of the simulated and theoretical angular velocity of the pinion is shown in Fig. 10. It is shown that the simulated angular velocity oscillates severely around the theoretical value in the beginning, but the oscillation decreases gradually until a steady vibration at last. The vibration after 0.03 second is almost steady. So the analyzing time in the next

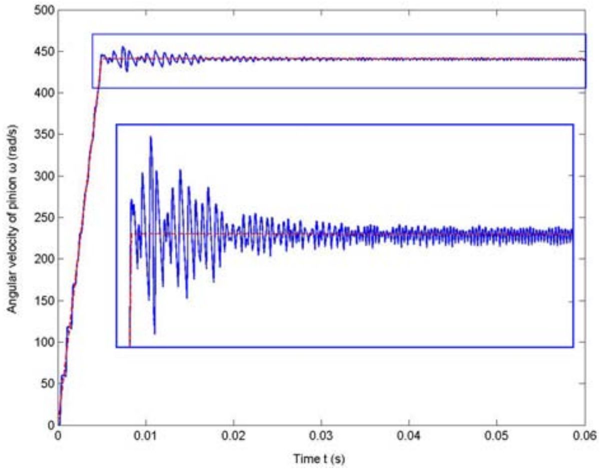

Fig. 10. Simulated angular velocity of pinion for theoretical value (···), simulated result (—).

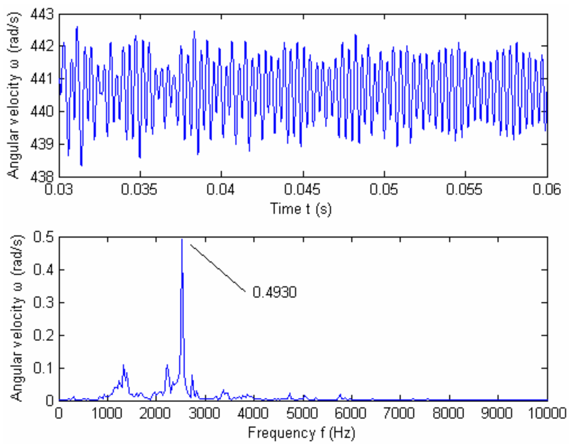

Fig. 11. Simulated angular velocity signal and its spectrum for unmodified pinion.

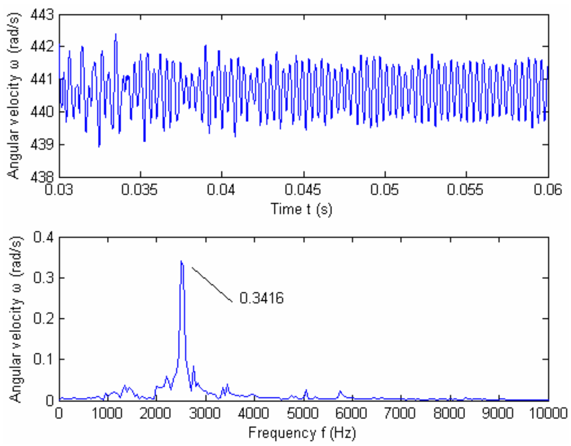

Fig. 12. Simulated angular velocity signal and its spectrum for modified pinion.

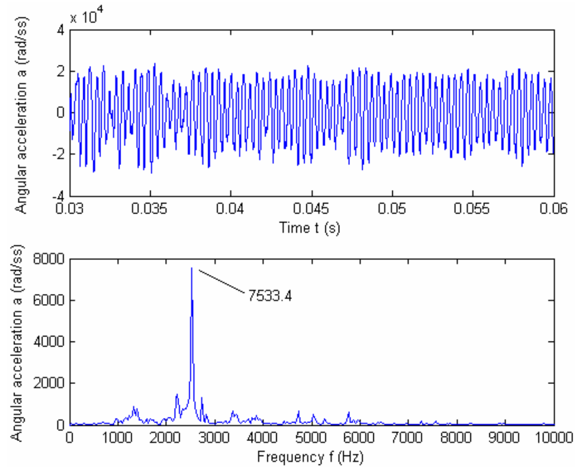

Fig. 13. Simulated angular acceleration signal and its spectrum for unmodified pinion.

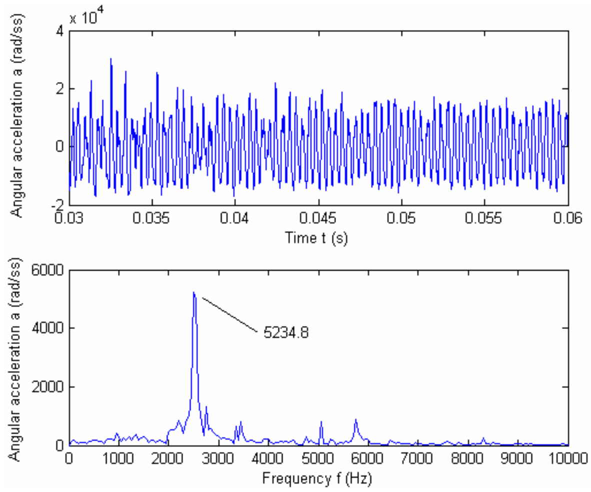

Fig. 14. Simulated angular acceleration signal and its spectrum for modified pinion.

is chosen as 0.03~0.06 second for a better reliability.

For a better understanding of the effect of the TPM on vibration reduction, the simulated angular velocity and angular acceleration will be compared, respectively, in the next. Taking the torque T equal to 27.3 Nm for example, the angular velocity and angular acceleration of the inner ring of the pinion are compared. The simulated angular velocity and angular acceleration signals and their spectra are shown in Figs. 11-14.

According to the comparison of the simulated results of the modified and unmodified gear pairs, the vibration amplitudes of angular velocity and acceleration for modified gear pairs are significantly lower than the unmodified at the mesh frequency of 2525 Hz, which means the proposed TPM of the helical gear pairs is effective in decreasing the vibration of the meshing process.

For assessing the effect of the TPM on vibration reduction quantificationally, we take the vibration level of the velocity or acceleration at the mesh frequency as index. The simulated vibration level reduction of the TPM is calculated as

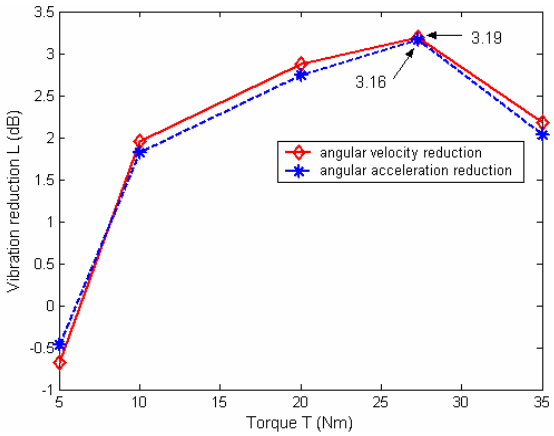

Fig. 15. Simulated vibration reduction of the TPM.

$$L = L_{v1} - L_{v2} = 20\lg\frac{A}{A_0} - 20\lg\frac{B}{A_0} = 20\lg\frac{A}{B} \quad (4)$$

where L is the vibration level reduction of the TPM at mesh frequency (dB), L_{v1} and L_{v2} is the vibration level for the unmodified and modified gear pairs, respectively (dB), A and B are the velocity or acceleration amplitude of the unmodified and modified gear pairs at the same point at mesh frequency, A_0=10^{-6} is the reference value.

Extracting the angular velocity and angular acceleration amplitude of the pinion with and without modification at the mesh frequency, the vibration level reduction for different load conditions is calculated by Eq. (4) and shown in Fig. 15. It can be seen that the TPM is not effective on vibration reduction for light load. But the vibration level reduction increases with the increase of the load until the maximum value. Then the vibration reduction decreases gradually with the continuous increase of the load. The maximum amount of vibration level reduction is obtained in the working condition (27.3 Nm).

The results of Fig. 15 show that the dynamic contact simulation of modified and unmodified gear pairs could provide an approach to evaluate the effects of the TPM on vibration reduction in the design stage. A further comparison will be made with the experimental results in the next section for validating the results of the dynamic simulation.

4. Experimental validation of the TPM

The simulated effects of the TPM on vibration reduction have been obtained in section 3. To make a comprehensive study of the realistic effects of the TPM on vibration reduction and validate the results of the dynamic simulations qualitatively, a comparison experiment is carried out in different working conditions. The experimental speed of the gear is set as ±500, ±1000, and ±1500 r/min, and the experimental torque of the pinion is set as 5, 10, 15, 20, 24, 26, 28, 30 Nm with the corresponding direction of the gear, respectively.

Fig. 16. Open power flow experimental system.

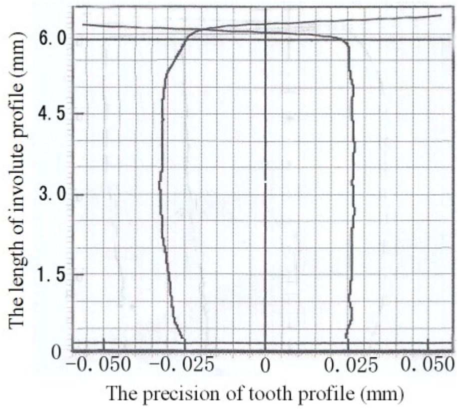

Fig. 17. Detection curve of modified tooth profile.

The open power flow experimental system is shown in Fig. 16. Where, the driving speed is provided by a servomotor, the loaded torque is provided by an electric eddy current loader, and the vibration signals are obtained using six accelerometers fixed on the bearing seat of the gear box. This is because one of the primary causes of the gears noise is the inner dynamic excitation of gear pairs, which generates vibration. Then the vibration is transmitted to the surrounding structures. So the structure-borne noise level of the gear box mainly depends on its vibration acceleration amplitude. The accelerometers were chosen to obtain these vibration acceleration signals from the gear box.

The detection curves of the actual tooth profile are given in Fig. 17. The left curve in the figure is the modified profile, and the right one is unmodified. We can see that the actual modified tooth profile coincides with the design curve well. That means the machining precision of the actual gear is acceptable, and actual modified gear is qualified.

Taking the condition with speed 1500 r/min and torque 20 Nm, for example, the spectra of the measured acceleration signals for the modified and unmodified gear pairs are compared in Fig. 18. The acceleration amplitude of the modified gear pairs is significantly lower than the unmodified gears at the mesh frequency of 2525 Hz. The ratio of their amplitude is

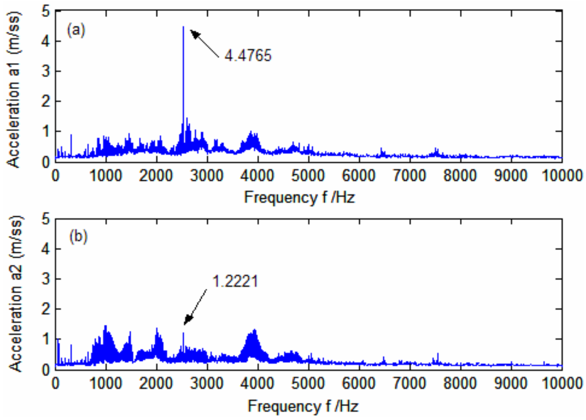

Fig. 18. Spectrum of acceleration for T=20 N.m: (a) unmodified; (b) modified.

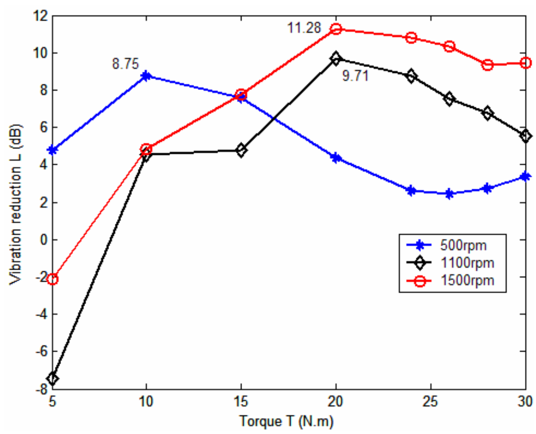

Fig. 19. Experimental vibration reduction of the TPM.

up to 3.66, which means the TPM is effective on vibration reduction at this load condition.

To analyze the actual effect of the TPM on vibration reduction, the vibration level reduction for different working conditions is calculated by Eq. (4) and shown in Fig. 19. It is shown that the effect of the presented TPM is not independent with the working conditions. With the increase of the torque, the effect of the TPM on vibration reduction is different for each speed, but the trends of the vibration reduction are almost the same for different speed. For a certain speed, the presented TPM is not effective at light load, but the effect of the TPM on vibration reduction becomes more and more obvious with the increase of the load until the maximum. With the continuous increase of the load, the vibration reduction decreases gradually. The maximum vibration level reduction is 8.75, 9.71, 11.28 dB, respectively, for different speed. This means that the presented TPM of helical gears is effective in vibration reduction for different speed, and the best effect is obtained in the designed speed for 1500 r/min.

For a better validation of the dynamic contact simulation, the simulated and experimental results of the TPM on vibration reduction are compared in Fig. 20. Where, the line L1 and L2 are the simulated and experimental results of the vibration level reduction, respectively, which are calculated by Eq. (4). It is shown that the simulated results are qualitatively coincident with the experimental results. The simulated and experimental results vary in the same trend with the increasing of the working load. And both of the results show that the presented TPM is effective on vibration reduction around the working condition (1500 r/min, 27.3 Nm).

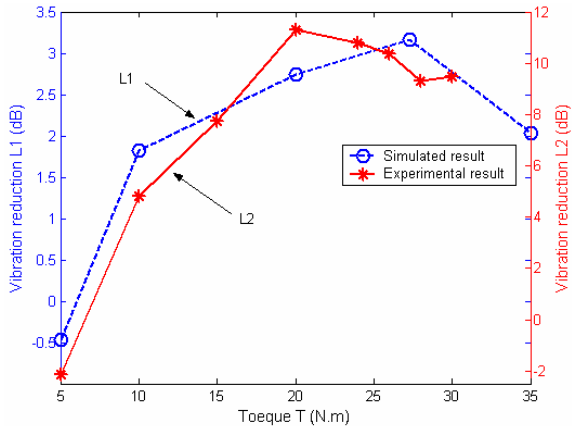

Fig. 20. Simulated and experimental vibration reduction of TPM.

The only difference between the simulated and experimental results is the amplitude of the vibration level reduction. The main reason is that the source of the signals is quite different. The signal of simulation is the angular acceleration of the inner ring of the pinion, but the experimental signal is the acceleration of the bearing seat. The quantitative comparison will be made in our further work.

5. Conclusions

A precise TPM for helical gears is proposed, and the dynamic simulation of the modified and unmodified gears is carried out based on the dynamic contact FEA. The results of the simulations and experiments are compared for validation. Some conclusions can be drawn as follows:

(1) Using the static contact FEA of gear pairs, the accurate deformations of the helical gear pairs on the meshing line are obtained, and a precise TPM of the helical gears is presented based on the accurate deformations.

(2) The dynamic contact simulation of the modified helical gears is carried out to evaluate the effect of the TPM on vibration reduction using the dynamic contact FEA, which requires no additional assumptions and simplifications for the dynamic contact analysis model. The simulated results of the TPM on vibration reduction are similar in trend with the experimental results, which means that the dynamic simulation of the modified gears is effective in evaluating the influence of the TPM in the designing stage.

(3) Both the simulated and experimental results show that the presented precise approach of the TPM is effective in vibration reduction around the working condition. And the ef-

fects of the vibration reduction under different load conditions of the proposed TPM are investigated, which may provide a reference to the design and application of the TPM.

References

[1] R. Li. J.n Wang, *Geared system dynamic: Vibration, shock and noise.* Science Press, Beijing, China (1997).

[2] R. Li, *Modification method and stiffness analysis of gear drives*. Chongqing: Chongqing University Press, China (1998).

[3] General editorial board of design manual of aircraft engine, *The thirteenth of the design manual of aircraft engine-redactor*, Aviation Industry Press, China (2001).

[4] H. Walker, Gear tooth deflections and profile modifications. *The Engineer*, 166 (1938) 409-412.

[5] Y. Terauchi and K. Nakamura, On tooth deflection calculation and profile modification of spur gear teeth. *Int. Sym. Gearing and Power Transmissions*, Tokyo (1981).

[6] M. S. Tavakoli and D. R. Houser, Optimum profile modifications for the minimization of static transmission errors of spur gears. *Journal of Mechanism, Transmissions, and Automation in Design*, 108 (1986) 86-95.

[7] R. G. Munro, N. Yildirim and D. M. Hall, Optimum profile relief and transmission error in spur gears. *Proceedings of Institution of Mechanical Engineers C 404/013* (1990) 35-42.

[8] Y. Cai and T. Hayashi, The optimum modification of tooth profile for a pair of spur gears to make its rotational vibration equal to zero. *ASME Proceedings of Power Transmission and Gearing Conference DE*, 43 (2) (1992) 453-460.

[9] G. Bonori, M. Barbieri and F. Pellicano, Optimum profile modification of spur gears by means of genetic algorithms. *Journal of Sound and Vibration*, 313 (2008) 603-616.

[10] A. Kahraman and G. W. Blankenship, Effect of involute tip relief on dynamic response of spur gear pairs. *Journal of Mechanical Design*, 121 (1999) 313-315.

[11] P. Wagaj and A. Kahraman, Influence of the tooth profile modification on helical gear durability. *Journal of Mechanical Design*, 123 (2002) 501-510.

[12] S. He, R. Gunda and R. Singh, Effect of sliding friction on the dynamics of spur gear pair with realistic time-varying stiffness. *Journal of Sound and Vibration*, 301 (2007) 927-949.

[13] S. Li, Finite element analysis for contact strength and bending strength of a pair of spur gear with machining errors, assembly errors and tooth modifications. *Mechanism and Machine Theory*, 42 (2007) 88-114.

[14] R. G. Parker, S. M. Vijayakar and T. Imajo, Non-linear dynamic response of a spur gear pair: modelling and experimental comparisons. *Journal of Sound and Vibration*, 237 (3) (2000) 435-455.

[15] R. G. Parker, V. Agashe and S. M. Vijayakar, Dynamic response of a planetary gear system using a finite element/ contact mechanics model. *Transactions of the ASME,* 122 (2000) 304-310.

[16] V. K. Ambarisha and R. G. Parker, Nonlinear dynamics of planetary gears using analytical and finite element method. *Journal of Sound and Vibration,* 302 (2007) 577-595.

[17] G. Liu and R. G. Parker, Dynamic modeling and analysis of tooth profile modification for multimesh gear vibration. *Journal of Mechanical Design*, 130 (2008) 1-13.

[18] T. Lin, R. Li, B. Chen and Q. Tang, Nonlinear dynamic contact analysis of meshing gears, *Proc. Int. Conf. Mech.Transmission* (2001) 248-251.

[19] T. Lin, H. Ou and R. Li, A finite element method for 3D static and dynamic contact/impact analysis of gear drives, *Computer Methods in Applied Mechanics and Engineering,* 196 (2007) 1716-1728.

[20] Y. Hu, Y. Shao, Z. Chen and M. J. Zuo, Transient meshing performance of gears with different modification coefficients and helical angles using explicit dynamic FEA, *Mechanical System and Signal,* 25 (5) (2011) 1786-1802.

[21] H. Sun and Z. Chen, *Mechanical theory*. Sixth Edition, Beijing: Higher Education Press, China (2003).

[22] R. V. Mises, On saint-venant's principle, *Bull. AMS,* 51 (1945) 555-562.

[23] G. Liu and R. G. Parker, Impact of tooth friction and its bending effect on gear dynamics, *Journal of Sound and Vibration,* 320 (2009) 1039-1063.

[24] J. O. Hallquist, *LS-DYNA theory manual*, Livermore Software Technology Corporation (2006).

Yongjun Wu is a Ph.D student at the school of Jet Propulsion at Beijing University of Aeronautics and Astronautics. His research interests are in vibration and vibration control for mechanical transmission systems, structural strength and vibration for mechanical structures.

Jianjun Wang is a professor in the School of Jet Propulsion at Beijing University of Aeronautics and Astronautics. He received an MS in Mechanical Engineering at Chongqing University in 1983, and a Ph.D in Solid Mechanics at Tsinghua University in 1998. His research interests are in vibration for mechanical transmission systems, FE simulation for various solid and fluid problems, structural strength and vibration for turbine engines, and active vibration control of engineering structures. He has more than 70 technical papers published on journals at home and abroad, and is the author or coauthor of several books.