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Multiobjective optimization for force and moment balance of a four-bar linkage using evolutionary algorithms †

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Abstract

In this study, force and moment balance of a planar four-bar linkage is implemented using evolutionary algorithms . In the current problem, the concepts of inertia counterweights and physical pendulum are utilized to complete balance of all mass effects, independent of input angular velocity. A proposed multiobjective particle swarm optimization, and non-dominated sorting genetic algorithm II are applied to minimize two objective functions subject to some design constraints. The applied algorithms produced a set of feasible solutions called pareto optimal solutions for the design problem. Finally, a fuzzy decision maker is utilized to select the best solution among the obtained pareto solutions. The results show that optimal solutions minimize the weights of applied counterweights and eliminate both shaking forces and moments transmitted to the ground, simultaneously.

Keywords: Force and moment balance; Four-bar linkage; Multiobjective particle swarm optimization; Non-dominated sorting genetic algorithm; Fuzzy decision maker

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1. Introduction

Without considering the interface between a mechanism and its mounting frame, design of that mechanism cannot be completed. During the time that an unbalanced linkage moves, it transmits shaking forces and moments to its surroundings. These transmitted forces and moments may cause some serious and undesirable problems such as vibration, noise, wear, and fatigue. Therefore, several methods are developed to eliminate the shaking forces and shaking moments in planar mechanisms.

Determining of counterweights to minimize frame vibration for a planar four-bar linkage is formulated as a convex optimization problem by Verschuure et al. [1]. The obtained results show a significant reduction of frame vibration during the counterweight design. Arakelian et al. [2] suggested a method based on displacements of the axes of rotation of counterweights connected to the crank and rocker to minimize shaking force and moment of a planar four-bar linkage. Applying this method, they significantly reduce the shaking moments. Arakelian and Dahan [3] also considered the shaking force and moment balancing of planar and spatial linkages considering mechanisms with constant and variable angular velocity for input link. Chaudhary and Saha [4] presented the inertia properties of a planar mechanism by dynamically equivalent systems to identify design variables and constraints. Their formulation leads to an optimization scheme for the mass distribution and balancing of the mechanism. They also applied the same technique for complete balancing of spatial mechanisms in Ref. [5]. Yan and Soong [6] proposed a method for four-bar linkages that finds the optimal design parameters for reaching the trade-off of dynamic balance and satisfying kinematic design requirements.

The problem of complete balance of a mechanism which is used in the present work has been addressed by Berkof [7-10] and Berkof and Lowen [11] in depth. In their approach, two methods, complementing each other, have been developed; permitting elimination of both shaking forces and shaking moments transmitted to the ground. This method redistributes link masses so that time dependent terms of equations of motion for the center of mass become zero [11]. The distribution of mass locations of the centers of mass is defined by deriving a set of linearly independent time dependent vectors, and force balance is achieved such that the center of mass for the entire system remains fixed.

Moment-balance of a linkage is achieved by writing moent-f-momentum equations for the system. The shaking moments for the entire system is vanished when the vector sum of the moments of momentum becomes zero. This task is accom-

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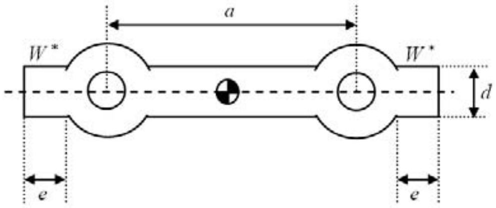

Fig. 1. An augmented link changed to a physical pendulum by addition the masses with the weight of W^*.

plished by adding the inertia counterweights and some constraints on the configuration of linkage. Complete force and moment balancing of an in-line four-bar linkage is accomplished utilizing the concepts of inertia counterweights and the physical pendulum which allow eliminating all mass effects (both linear and rotary, but excluding external loads), independent of input angular velocity.

In this work, two objective functions are considered to be minimized as the thickness of counterweights and the thickness of disks for both input and output links of a planar four-bar. Multiobjective Particle Swarm Optimization (MOPSO) [11, 12] and Non-Dominated Sorting Genetic Algorithm II (NSGA-II) [13, 14] generate a whole set of pareto optimal solutions, that provides a full picture of all possible compromise solutions and thus makes the decision process easier. The theory of fuzzy sets [15, 16] is also applied for decision making of balance problem. The proposed fuzzy decision maker selects the best compromises among the obtained solutions in order to satisfy design criteria.

2. Force and moment balance

Before considering the balance problem, it is needed to define the concepts of physical pendulum and inertia counterweights which are needed to maintain complete force and moment balance for the four-bar linkage.

2.1 Physical pendulum

The physical pendulum is defined as an in-line link with the radius of gyration k. As a physical pendulum has the same amount of radius of gyration when suspended by either end, it is symmetric about its center of mass which is in the midpoint of the line connecting two pivots of the link.

There are different credible shapes satisfy the general requirements of physical pendulum. In this paper, the configuration of the augmented link is used to meet the physical pendulum requirements. In Fig. 1, the configuration of a physical pendulum is illustrated. The physical pendulum contains added masses that weigh W^*. This added masses changed the augmented link to a physical pendulum [9, 17].

2.2 Inertia counterweight

When a linkage moves it causes unbalanced shaking moments which are proportional to angular acceleration. These unbalanced moments can be balanced using inertia counterweights. By adding the inertia counterweights, no net inertia forces are generated. Thus, the force balance of the mechanism is not affected, and the four bar linkage is now completely force and moment balanced. However, the input torque must be increased to drive the linkage with added counterweights. If the linkage transmits an unbalanced shaking moment to its mounting frame, it is needed to have a pair of spur gear to eliminate and some additional mass which is statically balanced about its axis of rotation to eliminate those shaking moments [17, 18].

2.3 Force and moment balance

Berkof and Lowen [11] proposed an approach called "method of linearly independent vectors" in which link masses are rearranged so that time dependent terms of motion equations for the center of mass become zero [17].

For a linkage, total moment acted on ground consists of the ground reaction due to the input torque as well as the moment resulting from the ground bearing forces. Provided that this shaking moment can be reduced to zero, the mechanism can be completely balanced. It is necessary to consider the angular momentum for a four-bar linkage in order to achieve moment balancing. When the angular momentum of a linkage is constant, the mechanism does not transmit a shaking moment to its frame. As discussed in the preceding sections, full shaking moment balance can also be achieved in a force balanced linkage through the use of counterweight inertias and physical pendulum.

The parameters of the unbalanced linkage shown in Fig. 2 are given in Table 1. The links are steel of density γ = 38311 N/m^3. As mentioned in Ref. [17], the following three steps must be passed to achieve complete balance.

Step 1: Convert the coupler to a physical pendulum

As the coupler is an augmented link, some weights must be added to it for meeting the requirements for a physical pendulum. This amount of added weights is related to the amounts of a and d which are shown in Fig. 3. The value of e which is needed for converting the coupler to a physical pendulum is is given in Ref. [17].

According to Table 1, $a_3/d_3 = 8$ and $e_3/d_3 = 1.887$. Thus, e_3 = 9.435 cm, and the added weight is given by (Fig. 1):

$$W^* = 2\gamma e_3 d_3 h_3 . \tag{1}$$

Hence, the added weight is:

$$W^* = 3.6146N . \tag{2}$$

The radius of gyration of the physical pendulum is:

$$k_3{}^2 = r_3 r_3' = \left(\frac{a_3}{2}\right)^2 = 400cm^2 \tag{3}$$

Table 1. Parameters of unbalanced four bar linkage.

Parameter	Link 2	Link 3	Link 4	Link 1
a_i (cm)	10.0	40.0	30.0	30.0
b_i (cm)	5.0	5.0	5.0	-----
h_i (cm)	1.0	1.0	1.0	-----
W_i° (N)	3.85	15.40	11.55	-----
r_i° (cm)	5.0	20.0	15.0	-----
ψ_i° (deg)	0.0	0.0	0.0	-----
k_i° (cm)	1.0305	3.715	2.803	-----

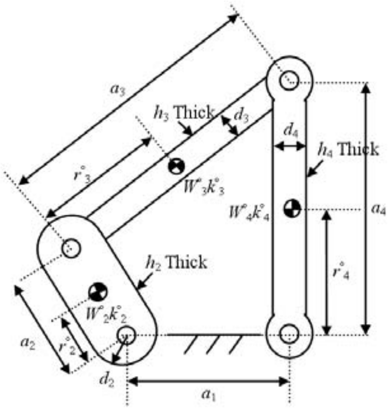

Fig. 2. Unbalanced four bar linkage.

so

$$k_3 = 20cm. \tag{4}$$

To satisfy the conditions of physical pendulum, it is needed:

$$\begin{cases} \psi_3 = 0 \\ \psi'_3 = \pi. \end{cases} \tag{5}$$

Thus, the coupler is converted to the physical pendulum without contributing to the shaking moment.

Step 2: Establish force moment

It is needed to use counterweights for the input and output links alteration to achieve force balance as discussed before. It is also assumed the circular counterweights with radius r_i^* are chosen to add to links 2 and 4. Then,

$$W_i^* = \gamma \pi h_i^* r_i^{*2}. \tag{6}$$

The balance conditions in Ref. [17] and Eq. (6) yield:

$$h_i^* = \frac{1}{\gamma \pi r_i^{*3}} [(W_i r_i)^2 + (W_i^\circ r_i^\circ)^2 - 2 W_i r_i W_i^\circ r_i^\circ \cos(\psi_i - \psi_i^\circ)]^{\frac{1}{2}}. \tag{7}$$

The new position of the center of mass is determined by vector addition that can be shown in following form:

$$r_i e^{i\psi_i} = \frac{W_i^\circ}{W_i} r_i^\circ e^{i\psi_i^\circ} + \frac{W_i^*}{W_i} r_i^* e^{i\psi_i^*} \tag{8}$$

while W_i^* and W_i are the added and total weights of link i.

Step 3: Add inertia counterweights to establish moment balance

As the centers of mass of links 2 and 4 have been changed, it is needed to calculate the new radius of gyration for those links. Because the circular counterweights are used, the contribution from the counterweight is given by:

$$k_i^* = \frac{r_i^*}{\sqrt{2}}. \tag{9}$$

The following equation is derived to determine the radius of a gyration of a counter balanced link, and for an in-line link with counterweight, it is:

$$k_i^2 = \frac{W_i^\circ}{W_i}[k_i^{\circ 2} + (r_i^\circ + r_i)^2 + \frac{W_i^*}{W_i}[k_i^{*2} + (r_i^* - r_i)^2]. \tag{10}$$

The weight moment of inertia of the inertia counterweight using 1:1 gearing:

$$I_i^{**} = \frac{W_i}{g}(k_i^2 + r_i^2 + a_i r_i). \tag{11}$$

And if the counterweight is a circular disk,

$$I_i^{**} = \frac{\pi}{2}(\gamma \rho_i^4 h_i^{**}) \tag{12}$$

or

$$h_i^{**} = \frac{2 I_i^{**}}{\pi \gamma \rho_i^4}. \tag{13}$$

In order to formulate the optimization problem mathematically, it is needed to introduce independent design variables and objective functions. In this work, r_i^* and ρ_i are considered as the design variables that are radius of the counterweight and disk for the input and output links. Objective functions of the problem are the thickness of counterweights and disks to be minimized for link 2 and 4. In other words, the weights of counterweights and disks are to be minimized by minimizing those thicknesses. Therefore, the objective function of the problems can be defined as:

$$f_{i1} = h_i^* = \frac{1}{\gamma\pi r_i^{*3}}[(W_i r_i)^2 + (W_i^\circ r_i^\circ)^2 + 2W_i r_i W_i^\circ r_i^\circ \cos(\psi_i - \psi_i^\circ)]^{\frac{1}{2}} \quad (14)$$

$$f_{i2} = h_i^{**} = \frac{2W_i}{g\gamma\pi x_i^4}(r_i^2 + a_i r_i) \frac{2}{g\pi\gamma\rho_i^4}\left\{W_i^\circ[k_i^{\circ 2} + (r_i^\circ + r_i)^2]\right\} + \frac{2}{g\pi\gamma\rho_i^4}\left\{W_i^*[\frac{r_i^{*2}}{2} + (r_i^* - r_i)^2]\right\} \quad (15)$$

subject to:

$$\begin{cases} 5cm \le r_i^* \le 10cm \\ 3cm \le \rho_i \le 5cm \end{cases} \quad (16)$$

where i is 2 or 4.

Minimizing objective functions, f_{i1} and f_{i2}, for the input and output links, the weight of counterweights and disks are minimized. When these functions are minimized for link 2 and link 4, simultaneously, both force and moment balanced conditions are satisfied for the linkage. There are also some restrictions dictated by environment, design process and/or resources, which are defined by Eq. (16).

3. Multiobjectiove optimization

Unlike single objective optimization methods that a single solution is provided with respect to a single objective function, multiobjective optimization methods result in a set of optimal solutions (pareto optimal solutions) which represent compromises among conflicting objectives. Any one of the pareto solutions can be an acceptable solution and considered optimum in some respect.

The multiobjective design optimization problem can be defined as the problem of finding a vector of n design variables $X = [x_1, x_2, \ldots, x_n]$ which satisfies m equality $h_i(X) = 0$, $i = 1, \ldots, m$ and p inequality $g_j(X) \le 0$, $j = 1, \ldots, p$ constraints and optimize (minimize or maximize) a vector of k objective functions $F(X) = [f_1(X), f_2(X), \ldots, f_k(X)]^T$, simultaneously. For multiobjective problems, each objective function achieves its optimum at different points. Thus, a pareto optimality concept is used to consider this type of problems [18]. A point $x^* \in X$ is called Pareto optimal (non-dominated) if and only if there exists no $x \in X$ such that $f_q(x) \le f_q(x^*)$, for $q = 1, \ldots, k$ and with $f_r(x) \le f_r(x^*)$ for at least one r.

4. Fuzzy decision making

The theory of fuzzy sets is proposed by Zadeh [15] with explicit reference to the vagueness of natural language, when describing quantitative or qualitative goals. Here it is assumed that local criteria, minimum thickness of the counterweight and minimum thickness of the disk can be presented by fuzzy sets. A final decision is defined by the Bellman and Zadeh model [16] as the intersection of all fuzzy criteria and is presented by its membership function $\mu(x)$ as follows:

$$M_C(X) = \mu_{counterweight}(X) \cap \mu_{disk}(X) \\ i = 1,\ldots,k; X \in X_P. \quad (17)$$

The membership function of the objectives, linear or nonlinear, can be chosen depending on concept of the problem. One of possible fuzzy convolution schemes is presented below:
(1) Initial approximation X - vector is chosen. Minimum values for each objective function K_j are established via scalar minimization. Results are denoted as "ideal" points $\{X_j^0, j = 1, \ldots, m\}$.
(2) The matrix table $\{T\}$, where the diagonal elements are "ideal" points, is defined as follows:

$$\{T\} = \begin{bmatrix} K_1(X_1^0) & K_2(X_1^0) & \ldots & K_n(X_1^0) \\ K_1(X_2^0) & K_2(X_2^0) & \ldots & K_n(X_2^0) \\ \vdots & \vdots & \vdots & \vdots \\ K_1(X_m^0) & K_2(X_m^0) & \cdots & K_n(X_m^0) \end{bmatrix}. \quad (18)$$

(3) Maximum and minimum bounds for the objectives are defined:

$$\begin{cases} K_i^{\min} = \min_j K_j(X_j^0) \\ K_i^{\max} = \max_j K_j(X_j^0) \end{cases} \quad (19)$$

for $i = 1, \ldots, n$.
(4) The membership functions are assumed for all fuzzy goals as follows:

$$\mu_{K_i}(X) = \begin{cases} 0, & K_i(X) = K_i^{\max} \\ \dfrac{K_i^{\max} - K_i}{K_i^{\max} - K_i^{\min}}, & K_i^{\min} < K_i \le K_i^{\max} \\ 1, & K_i \le K_i^{\min}. \end{cases} \quad (20)$$

(5) A final decision is determined as the intersection of all fuzzy objectives represented by its membership functions.

5. Results and discussion

Figs. 3 and 4 present the flow diagram of MOPSO and NSGA-II. Indeed, these diagrams state the procedure of obtaining pareto optimal solutions shown in Figs. 5 and 6.

Particle swarm optimization (PSO) is a stochastic and population-based optimization algorithm first proposed by Kennedy and Eberhart [11]. PSO is inspired by the social and cognitive behaviors of a flock of birds or school of fish seeking for food. As shown in Fig. 3, Multiobjective particle swarm optimization (MOPSO) algorithm defines a population

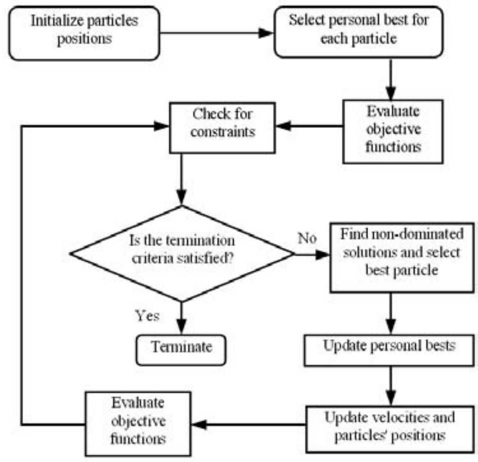

Fig. 3. Flow diagram of MOPSO.

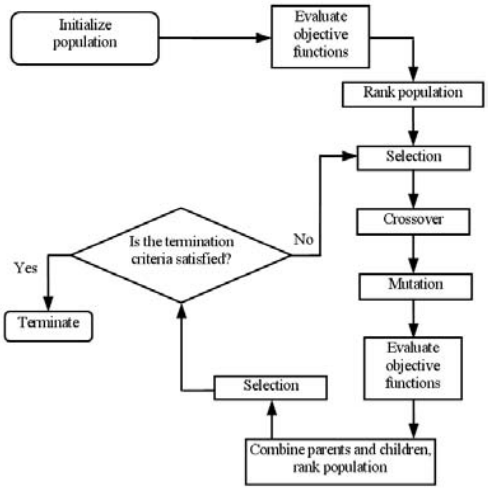

Fig. 4. Flow diagram of NSGA-II.

of particles (swarm) as random guesses in the variables' search space (r_i^* and ρ_i). Then, an iterative process is set by changing the position of particles within the search space to improve the objective functions quality. The movements through the search space are leaded by the social (personal bests) and cognitive (the best particle) experiences of each particle to minimize both objective functions (h_i^* and h_i^{**}), simultaneously. Finally, a set of non-dominated (pareto) solutions are found in each iteration using the concept of pareto optimal. The optimization process stops when the termination criteria (here number of iterations) meets the requirement [12].

Genetic Algorithms (GAs) [13] are search methods which imitate the natural biological evolution. GA utilizes a population of potential solutions and the principle of survival of the fittest to produce better approximations to a solution. Using the process of selecting individuals based on their level of fitness in the problem domain, a new set of solutions is created at each generation. This process guides the populations of solutions to the best values in the environment of the problem. In Non-dominated Sorting Genetic Algorithm-II (NSGA-II),

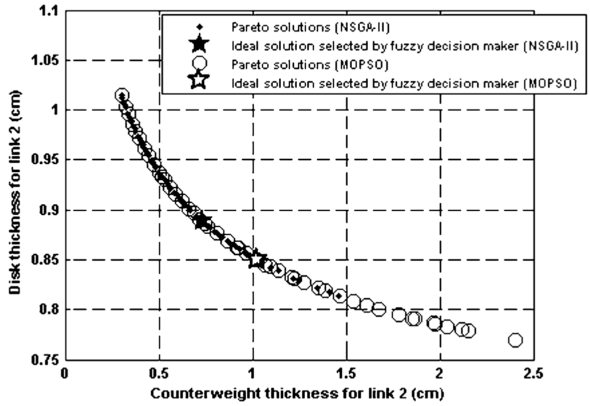

Fig. 5. Pareto fronts and selected solutions for link 2.

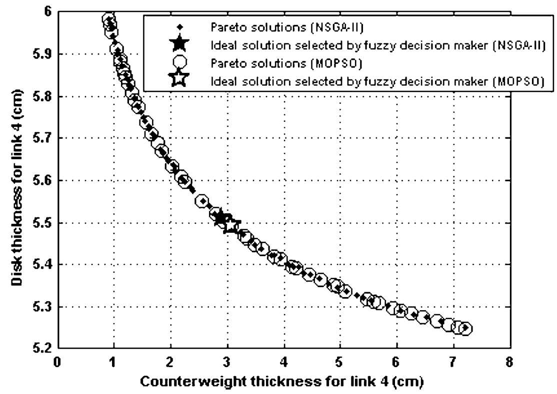

Fig. 6. Pareto front and selected solution for link 4.

shown in Fig. 4, the population of design variables is initialized based on the problem range and constraints. The initialized population is sorted based on non-domination and a fast sort algorithm is used to sort the non-dominated individuals. Once the non-dominated sort is completed, the crowding distance is assigned [14]. Since individuals are selected based on rank and crowding distance values. After sorting the individuals based on non-domination and assigning crowding distance to each of them, the selection is carried out using a crowded comparison operator. Two GA operators (crossover and mutation) are applied and the offspring population is created. Finally, this new created population is combined with the current population and selection is performed to set the individuals of next generation. Again, this process stops when the termination criteria (maximum number of iterations) is satisfied.

In the present paper, MOPSO and NSGA-II are used with a population size of 50 and generation size of 100 for both the input and output links, respectively. As minimization of the counterweight thickness and the disk thickness are conflicting objectives, the obtained solutions show a compromise between those objectives subject to the design constraints. These solutions also satisfy the conditions of fully force and moment balance which is considered in the preceding sections. Because of the non-linear nature of the derived objective func-

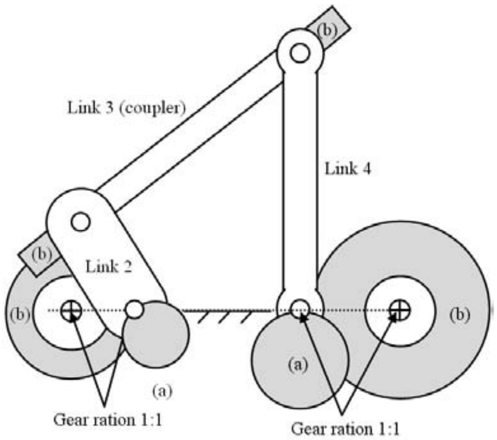

Fig. 7. Fully force and moment balanced in-line four bar linkage. Shaded area is material added to achieve: (a) force; (b) moment balance.

tions (Eqs. (14) and (15)), both algorithms are proper candidates. Regarding Figs. 5 and 6, both methods produce well distributed solutions along pareto front which helps a decision maker to choose final solution without conflicting with other near solutions. However, as it can be seen in Fig. 5, MOPSO produces a better distribution of objective functions for link 2.

Using fuzzy decision maker which regards the restraints of design and manufacturing processes, one can choose the best solutions along the pareto optimal fronts to minimize the weights of the counterweights and disks and also balance the linkage completely. The goals are defined as $K_{f1} = h_i^*$ and $K_{f2} = h_i^{**}$.

Fuzzification of the goals which leads to the membership functions is stated by Eqs. (18)-(20). Combination of the pareto optimality and fuzzy set concepts allows the decision maker to conduct a comprehensive study of obtained results, considering various combinations of two objective functions.

The intersected points maintained by fuzzy decision maker are shown in Figs. 5 and 6 by star points. These points are the best trade-offs to minimize both thickness of the counterweight and disk for link 2 and link 4. In other words, the selected counterweights and disks for link 2 and link 4 along the pareto optimal fronts by using a fuzzy decision maker show the best compromise among the solutions for minimizing weights of the counterweights and disks of the balanced linkage, simultaneously. Fig. 7 depicts the schematic configuration for the force and moment balanced linkage when shaded area is the added material to achieve (a) force and (b) moment balance using fuzzy decision maker. The results of multiobjective force and moment balance using NSGA-II and MOPSO are also presented in Tables 2 and 3. It is shown that both methods result in competitive values of objective functions while satisfying design constraints. It is worth to note that the bound constraints in this work are considered regarding the baseline design variables and could be changed by designers' demands. Therefore, the obtained values of ρ_2 and ρ_4, which are placed on bounds in Tables 2 and 3, are not very critical.

Table 2. Multiobjective force and moment balance results selected by fuzzy decision maker for link 2.

Parameters	$r_2^*(cm)$	$\rho_2\,(cm)$	$h_2^*\,(cm)$	$h_2^{**}\,(cm)$
MOPSO	6.64	5.00	1.03	0.85
NSGA-II	7.43	5.00	0.732	0.89

Table 3. Multiobjective force and moment balance results selected by fuzzy decision maker for link 4.

Parameters	$r_4^*(cm)$	$\rho_4\,(cm)$	$h_4^*\,(cm)$	$h_4^{**}\,(cm)$
MOPSO	6.65	5.00	3.06	5.49
NSGA-II	6.78	5.00	2.89	5.51

6. Conclusions

Force and moment balance of a four-bar linkage is implemented by utilizing two recent evolutionary algorithms named non-dominated sorting genetic algorithm and multiobjective particle swarm optimization. The objective functions are derived from the concepts of inertia counterweights and physical pendulum that permit complete balance of all mass effects both linearly and rotary. In addition, a fuzzy decision maker is applied to select an ideal solution among the obtained optimal solutions considering design criteria. The optimal results show that selected solutions minimize two conflicting objective functions and eliminate both shaking forces and moments transmitted to the ground simultaneously.

Nomenclature

a_i	: Length of link *i*
d	: Width of link *i*
F	: Vector of objective functions
h_i	: Thickness of link *i*
h_i^*	: Thickness of counterweight for link *i*
h_i^{**}	: Thickness of disk for link *i*
I	: Moment of inertia of unbalanced linkage
I_{cwt}	: Moment of inertia of counterweights
k_i	: Radius of gyration of link *i*
K	: Objective function value
M_C	: Intersection value of membership functions
μ	: Membership function value
r_i	: Center of mass position from one pin for link *i*
r_i^*	: Circular counterweights for link *i*
ρ_i	: Radius of disk for link *i*
γ	: Steel density
ψ_i	: Angle between line from pin to pin and line from pin to center of mass for link *i*
W_i°	: Weight of link *i* for the unbalanced linkage
W_i^*	: Weight of counterweight for link *i*
W_i	: Weight link *I* after adding counterweights
X	: Vector of variables

References

[1] M. Verschuure, B. Demeulenaere, J. Swevers and J. De Schutter, Counterweight balancing for machine frame vibration reduction: design and robustness analysis, *Proc. of ISMA, International Conference on Noise and Vibration Engineering*, Leuven, Belgium (2006).

[2] V. H. Arakelian, M. Dahan and M. R. Smith, Complete shaking force and partial shaking moment balancing of planar four-bar linkage, *Proc. of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics* (2001) 215: 31.

[3] V. H. Arakelian and M. Dahan, Partial shaking moment balancing of fully force balanced linkages, *Mechanism and Machine Theory*, 36 (2001) 1241-1252.

[4] H. Chaudhary and S. K. Saha, Balancing of shaking forces and shaking moments for planar mechanisms using the equimomental systems, *Mechanism and Machine Theory*, 43 (2008) 310-334.

[5] H. Chaudhary and A. K. Saha, An optimization technique for the balancing of spatial mechanisms, *Mechanism and Machine Theory,* 43 (2008) 506-522.

[6] H. S. Yan and R. C. Soong, Kinematic and dynamic design of four-bar linkages by links counterweighing with variables input speed, *Mechanism and Machine Theory*, 36 (2001) 1051-1071.

[7] R. S. Berkof, Force balancing of a six-bar linkage, *Proc. of the Fifth World Congress on Theory of Machines and Mechanisms*, New York: American Society of Mechanical Engineers (1979) 1082-85.

[8] R. S. Berkof, The input torque in linkages, *J. Mechanism and Machine Theory*, 14 (1979) 61-73.

[9] R. S. Berkof, Complete force and moment balancing of inline four-bar linkages, *J. Mechanism and Machine Theory*, 8 (3) (1973) 397-410.

[10] R. S. Berkof and G. G. Lowen, A new method for completely force balancing simple linkages, *J. Engineering for Industry*, 93 (1) (1971) 53-60.

[11] J. Kennedy and R. C. Eberhart, *Swarm intelligence*, Morgan Kaufman Publishers, San Francisco, California (2001).

[12] X. Li, Better Spread and Convergence: Particle swarm multiobjective optimization using the maximin fitness function, *GECCO* (2004) 117-128.

[13] D. E. Goldberg, Genetic algorithm in search, optimization, and machine learning, Addison Wesley Longman Inc. (1989).

[14] K. Deb, A. Pratap, S. Agarwal and T. Meyarivan, A fast elitist multi-objective genetic algorithm: NSGA-II, *IEEE Transactions on Evolutionary Computation*, 6 (2) (2002) 182-197.

[15] L. A. Zadeh, Fuzzy sets, *Inform Control*, 8 (1965) 338-353.

[16] R. E. Bellman and L. A. Zadeh, Decision making in a fuzzy environment, *J. Manage Sci.*, 17 (1970) 141-164.

[17] G. N. Sandor and A. G. Erdman, *Advanced mechanism design: Analysis and synthesis vol. I*, Prentice-Hall Inc., Englewood Cliffs, NJ, USA (1984).

[18] V. Pareto, *Cours d'economie politique*, *vol. I and II*, F. Rouge, Lauanne (1896).

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