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A method for vibration isolation of a vertical axis automatic washing machine with a hydraulic balancer†

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Abstract

This paper discusses a method for vibration isolation of a vertical axis automatic washing machine with a hydraulic balancer. First, a way to isolate vibration through a small amplitude of the suspension rod's axial force is proposed, and a base circle of a cone along which the lower joint of a suspension rod moves is discussed. Based on the circle, a geometric constraint involving the slant angle of the suspension rod, the deflection angle of the washing/spinning assembly, the suspension radius of the tub and the eccentricity of the system at the steady state is derived. Considering that the trace along which the suspension rod moves is also affected by the dynamics of the system, a governing equation satisfying the equilibrium conditions of the centrifugal forces and torques is obtained. Combing the geometric constraint and governing equation together achieves a general governing equation for vibration isolation of the system. Finally, the general governing equation is proven by simulations, and the relations between the optimal installing height and several parameters are discussed.

Keywords: Hydraulic balancer; Suspension rod; Vibration isolation; Washing machine

1. Introduction

Strong vibration often occurs due to clumped clothes in the spin drying process of a vertical axis automatic washing machine. The random distribution of clothes makes vibration suppression difficult. An effective way is to employ a dynamic balancer that counteracts the clothes automatically. Thus, a hydraulic balancer is often used in the vertical axis automatic washing machine.

So far, research on the vibration suppression of the vertical axis automatic washing machine has been focused on the stability and steady-state response of the spin drying process. Employing the continuation and bifurcation software AUTO [1], Ref. [2] analyzed the stability of the spin drying process when the hydraulic balancer is not considered. A mathematical model involving both tangential and axial damping forces of the suspension system was built, and a Hopf bifurcation phenomenon due to a small tangential damping coefficient was identified. Due to the complexity of a hydraulic balancer, rigid balls are often used to simulate the effect of the liquid in the balancer to analyze the transience or stability of the spin drying process when the hydraulic balancer is involved. It has been proven that this method can achieve qualitative results. Ref. [3] used three balls to describe the effect of the hydraulic balancer and presented the similarities between the results of simulations and experiments. Lately, Ref. [4] employed a two-ball model to analyze the stability of the spin drying process of a washing machine with a hydraulic balancer. Two unstable regions denoted by M and N were identified and the unstable region N is discussed in detail with its existence proven by both simulation and experiment. A numerical model for the balancer has to be built to analyze the steady-state response of the spin drying process. Thus, Ref. [5] built a numerical description of the liquid in the balancer and then discussed its influence on the steady-state response of the spin drying process. Employing the same model, Ref. [6] proposed a new approach for analyzing the steady-state response of the spin drying process and discussed a method for obtaining a small deflection angle of the washing/spinning assembly at the steady state.

The structure of a vertical axis automatic washing machine is shown in Fig. 1(a). This consists of a cabinet, a suspension system and a washing/spinning assembly. The assembly is connected to the cabinet through a suspension system with four suspension rods whose structures are shown in Fig. 1(b). As can be seen, each suspension rod contains two spherical joints: a rod and a damping tub. During the spin drying process, the suspension rod can swing and stretch simultaneously.

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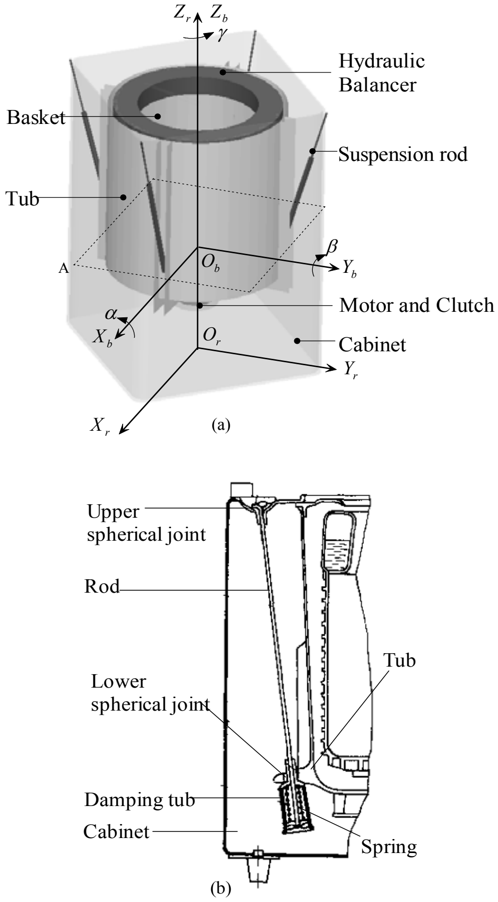

Fig. 1. (a) Structure of a vertical axis automatic washing machine; (b) structure of a suspension rod.

Thus, both tangential and axial forces could be aroused.

Although the hydraulic balancer is effective for vibration suppression at the steady state of the spin drying process, it does not have the capability of complete balance. What's more, for a vertical axis automatic washing machine, the condition is even worse because the hydraulic balancer and the clothes usually lie in different horizontal planes. Thus, complete balance of the system using only one single balance is unlikely. Furthermore, vibrations are often transmitted to the cabinet through the suspension system, which may result in further vibrations of the cabinet. This vibration may arouse serious noise of the cabinet or cause great discomfort when users put their hands on the surface of the cabinet. Today, low noise and low vibration have become two of the most important performance characters of a washing machine. To reduce vibrations of the cabinet, good vibration isolation of the system is required. An effect way to do this is to control the amplitude of the axial forces transmitted to the cabinet. This paper will present such a method.

During the process of designing a hydraulic balancer, designers usually concentrate their efforts on the centrifugal forces it generates but pay little attention to the balancer's installing height. Actually, according to Ref. [6], at the steady state of the spin drying process, there are two equilibrium conditions that should be noted: one is the equilibrium condition of the centrifugal forces and the other is that of the centrifugal torques. To emphasize the importance of the latter condition and stress the effect of the centrifugal torques generated by the hydraulic balancer, mainly the installing height of the balancer will be discussed in this paper.

2. Vibration isolation of a vertical axis automatic washing machine

2.1 Method to control the amplitude of the axial force transmitted by a suspension rod

Axial forces of the suspension system are the primary forces among those transmitted to the cabinet that are responsible for the cabinet's vibration. An effective method for vibration isolation is to control the amplitudes of these forces. If these forces do not vary, there will be constant forces acting on the cabinet, which is ideal for vibration isolation. We now analyze the axial forces. As can be seen from Fig. 1(b), the axial force of a suspension rod is composed of two parts: one is the restoring force of the spring and the other is the axial damping force provided by the damping tub. The axial force of the suspension rod can be described as

$$F_a = K_s\left(l - l_0\right) - C_a\dot{l} \tag{1}$$

where K_s is the stiffness coefficient of the spring, C_a is the axial damping coefficient of the damping tub, l is the distance between the upper and lower spherical joints and l_0 is the initial length of the rod at the equilibrium position. Let $x = l - l_0$, the above equation could be expressed as

$$F_a = K_s x - C_a\dot{x}\,. \tag{2}$$

Considering at the steady state of the spin drying process, the variable x could be expressed as $x = X_0\sin\Omega t$ where X_0 is the vibration amplitude and Ω is the rotation speed. Substituting $x = X_0\sin\Omega t$ into Eq. (2) yields

$$F_a = K_s X_0\sin\Omega t - C_a\Omega X_0\cos\Omega t\,. \tag{3}$$

Based on the above equation, the amplitude of the axial force can be derived as

$$A = \sqrt{\left(K_s X_0\right)^2 + \left(C_a\Omega X_0\right)^2} = X_0\sqrt{K_s^2 + \left(C_a\Omega\right)^2}\,. \tag{4}$$

As can be seen, if the stiffness coefficient K_s ,damping coefficient C_a and rotation speed Ω are set, the vibration amplitude X_0 must be designed to be as small as possible in order to achieve a small amplitude for the axial force A. If X_0 turns zero, the amplitude of the axial force A will be zero as well, which is ideal for vibration isolation. Be-

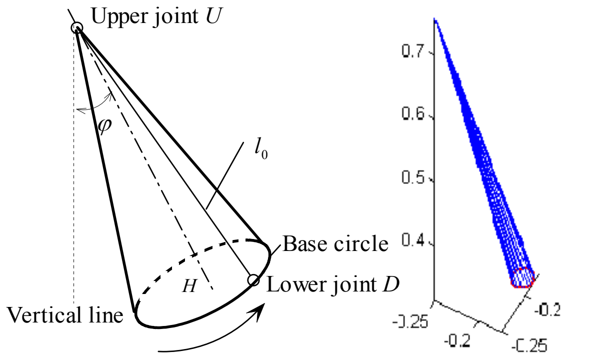

Fig. 2. The traces along which the lower joint moves: (a) base circle of a right circular cone; (b) simulation result.

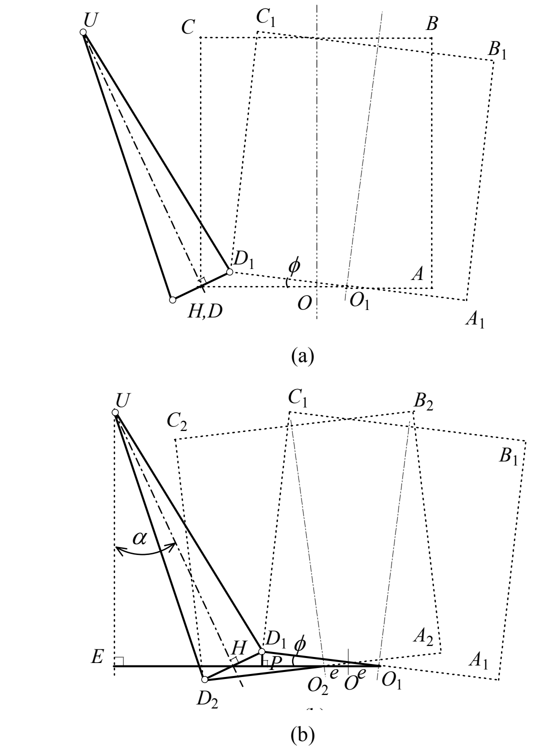

Fig. 3. Traces of the washing/spinning assembly.

cause $x = l - l_0 = X_0 \sin \Omega t$, once X_0 equals zero, the distance between the upper and lower spherical joints l will be equal to the constant length l_0. At this condition, considering that the upper spherical joint of the suspension rod is fixed, in order to keep $l = l_0$, the lower spherical joint must move along a special trajectory, such as the surface of a sphere with the upper joint as the center and l_0 as the radius, or the base circle of a right circular cone as shown in Fig. 2(a) with the upper joint as the vertex and l_0 as the generatrix. At these conditions, because the length of the suspension rod does not change, the axial force of the suspension rod stays constant, which is ideal for vibration isolation.

Fig. 2(b) displays simulations of the traces of a suspension rod at the steady state of the spin drying process when the amplitude of the axial force is small. As can be seen, the trajectory of the suspension rod nearly forms the lateral surface of a right circular cone shown in Fig. 2(a) with the upper joint U as the vertex and l_0 as the generatrix, and the trace along which the lower joint D moves trends to the base circle of a cone whose radius depends on the eccentricity of the system and the deflection angle of the tub. In the figure, UH is the axis of the cone. At the static equilibrium position of the system, the suspension rod lies in the position UH, and the angle between UH and the vertical line is the slant angle of the suspension rod φ.

We now present a governing equation to obtain a small amplitude for the rod's axial force based on the trace displayed in Fig. 2. Assume that the vibration period of the washing/spining assembly is T. As Fig. 3(a) shows, at first ($t = 0$), the washing/spinning assembly lies in the position $A_1B_1C_1D_1$ and the suspension rod lies in UD_1. Suppose that the lower joint D moves exactly along the base circle of the cone shown in Fig. 2(a). After T/4 seconds ($t = T/4$), the washing/spinning assembly will move to the position $ABCD$ and the suspension rod will move to UD. In Fig. 3(a), UH is the axis of the cone and the projections of points H and D coincide. After another T/4 seconds ($t = T/2$), the washing/spinning assembly will move to the position $A_2B_2C_2D_2$ shown in Fig. 3(b) and the suspension rod will move to UD_2. In the figure, the point O is the suspension center of the assembly. As time passes, it first moves from O_1 to O, and then from O to O_2. Note that O_1, O, O_2 and H lie in the same line at the steady state of the spin drying process. The distance from O_1 to O equals that from O to O_2; namely, $\overline{O_1O} = \overline{OO_2} = e$. Here, e denotes the eccentricity of the system at the steady state. In Fig. 3(b), the two angles $\angle D_1O_1E$ and $\angle D_2O_2E$ are identical; namely, $\angle D_1O_1E = \angle D_2O_2E = \phi$. Here, ϕ denotes the deflection angle of the washing/spining assembly at the steady state, which can be described as $\phi = \sqrt{\alpha^2 + \beta^2}$ where α and β are the Cardan's angles in Fig. 1(a) that are used to describe the assembly's posture. In Fig. 3(b), the angle $\angle HUE = \varphi$ is the slant angle of the suspension rod discussed in Fig. 2(a).

Asymmetry of the washing/spinning assembly is neglected and the slant angles of the four suspension rods are assumed to be identical. We now explore the relation between the slant angle of the suspension rod φ and the deflection angle ϕ of the washing/spinning assembly. In Fig. 3(b), a line is drawn through D_1 that is perpendicular to the line OE and intersects it at point P. We now analyze the length of the line HP. As Fig. 3(b) reveals, for the triangle ΔO_1D_1H, based on the Law of Cosines, the following relation exists:

$$\begin{aligned} \overline{D_1H}^2 &= \overline{D_1O_1}^2 + \overline{HO_1}^2 - 2\overline{HO_1}\,\overline{D_1O_1}\cos\phi \\ &= \overline{D_1O_1}^2 + \left(\overline{HO_2} + 2e\right)^2 - 2\left(\overline{HO_2} + 2e\right)\overline{D_1O_1}\cos\phi \end{aligned} \quad (5)$$

Similarly, for the triangle ΔO_2HD_2, the following equation can be derived:

$$\overline{D_2H}^2 = \overline{HO_2}^2 + \overline{D_2O_2}^2 - 2\overline{HO_2}\,\overline{D_2O_2}\cos\phi . \quad (6)$$

In the above equations, because both $\overline{D_1H}$ and $\overline{D_2H}$ are the base radii of the cone, they are identical; $\overline{D_1H} = \overline{D_2H}$. Furthermore, because both $\overline{D_1O_1}$ and $\overline{D_2O_2}$ are suspension radii of the tub, they are also identical; $\overline{D_1O_1} = \overline{D_2O_2}$. Substituting $\overline{D_1H} = \overline{D_2H}$, $\overline{D_1O_1} = \overline{D_2O_2}$ into the above equations and then subtracting Eq. (5) by Eq. (6) yields

$$\overline{D_1O_1}\cos\phi = \overline{HO_2} + e . \quad (7)$$

Based on the triangle ΔD_1O_1P , the item $\overline{D_1O_1}\cos\phi$ in the above equation can also be expressed as

$$\overline{D_1O_1}\cos\phi = \overline{O_1P} = \overline{O_2P} + 2e . \quad (8)$$

Subtracting Eq. (7) by Eq. (8) yields

$$\overline{O_2P} = \overline{HO_2} - e . \quad (9)$$

As can be seen from Fig. 3(b), the item $\overline{O_2P}$ in Eq. (9) can be also described as

$$\overline{O_2P} = \overline{HO_2} - \overline{HP} . \quad (10)$$

Comparing Eqs. (9) and (10), the length of the line *HP* can be achieved as

$$\overline{HP} = e . \quad (11)$$

We now derive the relation between the slant angle of the suspension rod φ and the deflection angle of the washing/spinning assembly ϕ based on $\overline{HP}$. As can be seen from Fig. 3(b), the sum of the two angles $\angle D_2HE$ and $\angle EHU$ is 90°,

$$\angle D_2HE + \angle EHU = 90^o \quad (12)$$

and the sum of the two angles $\angle EUH$ and $\angle EHU$ is also 90°,

$$\angle EUH + \angle EHU = \varphi + \angle EHU = 90^o . \quad (13)$$

Based on Eqs. (12) and (13), the angle $\angle D_2HE$ can be described as

$$\angle D_2HE = \varphi . \quad (14)$$

As Fig. 3(b) reveals, because $\angle D_2HE$ and $\angle D_1HP$ are vertical angles, they are identical,

$$\angle D_1HP = \angle D_2HE = \varphi . \quad (15)$$

Now, the tangent value of $\angle D_1HP$ can be expressed as

$$\tan\angle D_1HP = \tan\varphi = \frac{\overline{D_1P}}{\overline{HP}} = \frac{\overline{D_1O_1}\sin\phi}{\overline{HP}} . \quad (16)$$

Let $\overline{D_1O_1} = r$. Then, substituting $\overline{D_1O_1} = r$ and $\overline{HP} = e$ into the above equation yields the following equation:

$$\frac{\sin\phi}{e} = \frac{\tan\varphi}{r} \quad (17)$$

where ϕ and e are the deflection angle and eccentricity of the washing/spinning assembly at the steady state, respectively. φ is the slant angle of the suspension rod shown in Fig. 2(a) which can be measured at the static equilibrium position of the system. r is the suspension radius of the tub.

Eq. (17) is a geometric constraint that guarantees that the lower joint of the suspension rod moves exactly along the base circle of the cone shown in Figs. 2 and 3. At this condition, because the length of the suspension rod does not vary, the axial force of the suspension rod stays constant, which is ideal for vibration isolation. As can be seen from Eq. (17), once the suspension radius of the tub r and the slant angle of the suspension rod φ are set, the ratio between $\sin\phi$ and e has to be specially designed. Actually, this ratio is influenced by the equilibrium conditions of the centrifugal forces and torques acting on the washing/spinning assembly. We will now discuss these conditions. At the steady state of the spin drying process, the rotation speed of the system is very high, and centrifugal forces acting on the washing/spinning assembly are much larger than the forces provided by the suspension system. Thus, only centrifugal forces are considered in the following sections.

Centrifugal forces acting on the washing/spinning assembly at the steady state are displayed in Fig. 4(a): *U* represents the clumped clothes; *C* is the centroid of the liquid; *P* is mass center of the washing/spinning assembly (excluding the clothes and the liquid in the balancer); Z_r -axis is the axis around which the system rotates at the steady state; *S*-axis is the symmetric axis of the assembly; *A* is the suspension plane of the tub; ϕ is the deflection angle of the washing/spinning assembly which equals to the angle $\angle D_1O_1E$ or $\angle D_2O_2E$ shown in Fig. 3; F_t , F_u and F_h are centrifugal forces acting on the clothes, liquid and washing/spinning assembly, respectively. We will now express F_t , F_u and F_h one by one.

First, we analyze the centrifugal force acting on the washing/spinning assembly F_t shown in Fig. 4(b). As the figure shows, h_t is the height of the washing/spinning assembly's mass center *P* relative to the suspension plane *A*; ϕ is the deflection angle of the washing/spinning assembly; O and O_1 denote the suspension center of the tub; e is the distance from O to O_1 , which is the eccentricity of the system at the

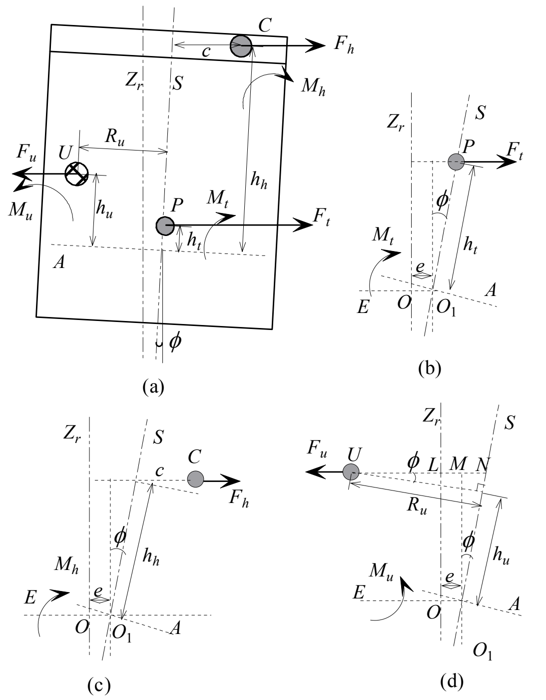

Fig. 4. Centrifugal forces acting on the washing/spinning assembly.

steady state. The centrifugal force F_t in Fig. 4(b) can be described as

$$F_t = m_t\left(e + h_t \sin\phi\right)\Omega^2 \tag{18}$$

where Ω denotes the rotation speed of the system at the steady state. The torque generated by F_t relative to point O can be described as

$$M_t = F_t h_t \cos\phi \, . \tag{19}$$

Second, we analyze the centrifugal force F_h that acts on the liquid in the balancer. Based on the derivation of Eq. (14) in Ref. [6], F_h can be expressed using Fig. 4(c) where h_h is the installing height of the balancer relative to the suspension plane A. As can be seen, F_h can be described as

$$F_h = m_h\left(e + h_h \sin\phi + c\right)\Omega^2 = m_h\left(e + h_h \sin\phi\right)\left(1+\eta\right)\Omega^2 \tag{20}$$

where $\eta = c/\left(e + h_h \sin\phi\right)$. The torque generated by F_h relative to point O can be described as

$$M_h = F_h h_h \cos\phi \, . \tag{21}$$

Third, we discuss the centrifugal force F_u shown in Fig. 4(d) that acts on the clothes U. As the figure reveals, h_u is the height of the clothes relative to the suspension plane A, and R_u is the gyration radius of the clothes. As can be seen from Fig. 4(d), F_u can be described as

$$F_u = m_u \overline{UL}\Omega^2 \tag{22}$$

where

$$\overline{UL} = \overline{UN} - \overline{LM} - \overline{MN} = \frac{R_u}{\cos\phi} - e - \left(h_u + R_u \tan\phi\right)\sin\phi \, . \tag{23}$$

When the deflection angle ϕ is small, the item $R_u \tan\phi$ in Eq. (23) is much smaller than h_u, and thus Eq. (23) can be simplified as

$$\overline{UL} = \frac{R_u}{\cos\phi} - e - h_u \sin\phi \, . \tag{24}$$

In Fig. 4(d), the torque generated by F_u relative to point O can be described as

$$M_u = F_u\left(h_u + R_u \tan\phi\right)\cos\phi \, . \tag{25}$$

Neglecting the item $R_u \tan\phi$, the above equation can be simplified as

$$M_u = F_u h_u \cos\phi \, . \tag{26}$$

We now describe the equilibrium conditions governed by the above forces or torques. The equilibrium condition of the centrifugal forces in Fig. 4(a) can be expressed as

$$F_t + F_h = F_u \, . \tag{27}$$

Submitting Eqs. (18), (20) and (22) into the above equation yields

$$\begin{aligned} & m_t\left(e + h_t \sin\phi\right)\Omega^2 + m_h\left(e + h_h \sin\phi\right)\left(1+\eta\right)\Omega^2 \\ & = m_u\left(\frac{R_u}{\cos\phi} - e - h_u \sin\phi\right)\Omega^2 \end{aligned} \tag{28}$$

from which the following equation can be derived

$$\begin{aligned} & \left[m_t + m_h\left(1+\eta\right) + m_u\right]e \\ & = m_u \frac{R_u}{\cos\phi} - \left[m_u h_u + m_h h_h\left(1+\eta\right) + m_t h_t\right]\sin\phi \end{aligned} \, . \tag{29}$$

We now analyze the equilibrium condition of the torques. As can be seen from Fig. 4(a), the equilibrium condition of the torques can be expressed as

$$M_t + M_h = M_u \, . \tag{30}$$

Substituting Eqs. (19), (21) and (26) in the above equation yields

$$m_t\left(e+h_t\sin\phi\right)\Omega^2 h_t\cos\phi+m_h\left(e+h_h\sin\phi\right)\left(1+\eta\right)\Omega^2 h_h\cos\phi = m_u\left(\frac{R_u}{\cos\phi}-e-h_u\sin\phi\right)\Omega^2 h_u\cos\phi \quad (31)$$

from which the following equation can be derived

$$\left[m_t h_t+m_h\left(1+\eta\right)h_h+m_u h_u\right]e = m_u\frac{R_u}{\cos\phi}h_u-\left[m_u h_u^2+m_t h_t^2+m_h h_h^2\left(1+\eta\right)\right]\sin\phi. \quad (32)$$

Subtracting Eq. (32) by the product of Eq. (29) times h_u, namely, $(32)-(29)\times h_u$, yields

$$\left[m_t\left(h_t-h_u\right)+m_h\left(1+\eta\right)\left(h_h-h_u\right)\right]e = \left[m_t h_t\left(h_u-h_t\right)+m_h h_h\left(1+\eta\right)\left(h_u-h_h\right)\right]\sin\phi. \quad (33)$$

After a further derivation, the following relation between $\sin\phi$ and e can be achieved

$$\frac{\sin\phi}{e}=\frac{m_t\left(h_t-h_u\right)+m_h\left(1+\eta\right)\left(h_h-h_u\right)}{m_t h_t\left(h_u-h_t\right)+m_h h_h\left(1+\eta\right)\left(h_u-h_h\right)}. \quad (34)$$

Eq. (34) is a governing equation of the system based on dynamics that satisfies the equilibrium conditions of centrifugal forces and torques. As can be seen, the ratio between $\sin\phi$ and e is governed by this equation. Recall that the ratio between $\sin\phi$ and e is also governed by the geometric constraint described by Eq. (17) that guarantees a small amplitude of the axial forces. Substituting Eq. (17) into Eq. (34), a general governing equation for vibration isolation of the system can be obtained

$$\frac{\tan\varphi}{r}=\frac{m_t\left(h_t-h_u\right)+m_h\left(1+\eta\right)\left(h_h-h_u\right)}{m_t h_t\left(h_u-h_t\right)+m_h h_h\left(1+\eta\right)\left(h_u-h_h\right)}. \quad (35)$$

Parameters governed by Eq. (35) mean that the lower joint of the suspension rod moves along the base circle of the cone shown in Figs. 2 and 3, which implies a small amplitude of the axial force and, thus, is ideal for vibration isolation of the system. As can be seen from Eq. (35), if the slant angle of the suspension rod φ, the suspension radius of the tub r and the height of the clothes h_u are set, to obtain a small amplitude of the axial force, the mass of the washing/spinning assembly m_t, the height of the assembly's mass center h_t, the mass of the liquid in the hydraulic balancer m_h, the installing height of the balancer h_h and the ratio of the hydraulic balancer η have to be specially designed.

2.2 Discussion

Two equilibrium conditions of the system at the steady state should be noted while designing a hydraulic balancer for the vertical axis automatic washing machine. One is the equilibrium condition of the centrifugal forces described by Eq. (27), and the other is the equilibrium condition of the centrifugal torques governed by Eq. (30). Usually, designers concentrate their efforts on the first condition but pay little attention to the second one. Thus, to stress the later condition, the installing height of the balancer is mainly discussed in this section. Let $\delta=\tan\varphi/r$. After a further derivation of Eq. (35), the optimal installing height of the hydraulic balancer can be achieved as

$$h_h^*=\frac{-\left(1/\delta-h_u\right)+\sqrt{\left(1/\delta-h_u\right)^2+4\Delta}}{2} \quad (36)$$

where

$$\Delta=\frac{h_u}{\delta}+\frac{m_t\left(h_u-h_t\right)\left(1+h_t\delta\right)}{m_h\left(1+\eta\right)\delta}. \quad (37)$$

We now validate this result using numerical simulations. Take a washing machine as an example: the mass of the washing/spinning assembly $m_t=10.615\text{Kg}$, the height of the assembly's mass center $h_t=-0.0075\text{m}$, the outer radius of the hydraulic balancer $R_o=0.2\text{m}$ and the inner radius of the balancer $R_i=0.16\text{m}$. To calculate the optimal installing height h_h^*, the two parameters relating to the balancer m_h and η in Eq. (37) have to be set. To maintain a larger linear region for the hydraulic balancer, the optimal volumetric ratio of the balancer described by Eq. (51) in Ref. [6] is adopted:

$$q^*=\frac{R_o^2-\left(R_o+R_i\right)^2/4}{R_o^2-R_i^2}. \quad (38)$$

Submitting $R_o=0.2\text{m}$ and $R_i=0.16\text{m}$ into the above equation yields $q^*=0.5278$. Assuming the capacity of the balancer is 1.3 Kg, the mass of the liquid in the balancer can be calculated as $m_h=1.3\times0.5278=0.686\text{ Kg}$. In the linear region, the ratio of the balancer η can be described using Eq. (4) in Ref. [6] as

$$\eta=s=\frac{R_s^2}{R_o^2-R_s^2} \quad (39)$$

where $R_s=\sqrt{R_o^2-q\left(R_o^2-R_i^2\right)}$.

Substituting $R_o=0.2\text{ m}$, $R_i=0.16\text{ m}$, $q^*=0.5278$ into R_s and then submitting R_o and R_s into Eq. (39) yields $\eta=s=4.263$. Assume the height of the clothes $h_u=0.1\text{ m}$. To obtain the optimal installing height of the balancer described in Eq. (36), the variable $\delta=\tan\varphi/r$, which depends on the suspension radius of the tub r and the tangent value of the slant angle of the suspension rod φ, has to be calculated. The suspension radius r can be easily measured as

r =0.26 m. The problem is how to obtain $\tan\varphi$. To find $\tan\varphi$, the lengths of the lines $\overline{UE}$ and $\overline{EH}$ in Fig. 3(b), which depend on the static equilibrium position of the system, have to be obtained. It should be noted that in order to let the suspension rod lie exactly in the position $\overline{UH}$ at the static equilibrium position, the clothes should at first lie in the center of the basket. In other words, the gyration radius of the clothes R_u should be zero while solving the static equilibrium position.

The static equilibrium position can be solved by static or dynamic methods. This paper employs a simple dynamic method. Let the mass of the clothes m_u =1 Kg, the gyration radius of the clothes R_u =0m, and rotation speed of the basket Ω =0Hz. Assume the initial position of the system is $\mathbf{q}_0 = \begin{bmatrix} 0 & 0 & 0.33 & 0 & 0 & 0 \end{bmatrix}^T$. Calculations are carried out by applying the ode45 function in MATLAB on the vibration model described by Eq. (21) in Ref. [6]. After 10 seconds, the value of $\mathbf{q}$ stays still and the result shows $\mathbf{q}^* = \begin{bmatrix} 0 & 0 & 0.3125 & 0 & 0 & 0 \end{bmatrix}^T$. Based on the result, the lengths of the lines $\overline{UE}$ and $\overline{EH}$ can be obtained as $\overline{UE}$ =0.4475 and $\overline{EH}$ =0.0933m. Now, the tangent value of the slant angle of the suspension rod φ can be obtained as $\tan\varphi = \overline{EH}/\overline{UE} \approx 0.209$. Considering the radius of the tub $r = 0.26$, the variable δ can be easily calculated as $\delta = \tan\varphi / r \approx 0.802$. Substituting the valve of δ and all the parameter values listed above into Eq. (36), the optimal installing height of the hydraulic balancer can be calculated as $h_h^* \approx 0.346$ m.

It can be seen from the above discussion that the static equilibrium position of the system is influenced by the mass of the clothes m_u , and because the slant angle of the suspension rod φ depends on the static equilibrium position, the optimal installing height of the hydraulic balancer h_h^* is affected by the mass of clothes m_u . However, this influence can be neglected in the interval $m_u \in [0,2]$. Calculations show that the interval of the optimal installing height h_h^* corresponding to $m_u \in [0,2]$ is $h_h^* \in [0.3458, 0.3461]$, which is very small. As can be seen, the value of h_h^* in the interval is about 0.346 meters.

When the parameter values listed above are considered, the optimal installing height of the hydraulic balancer is about 0.346 meters. To validate this result, simulations are carried out based on the vibration model described by Eq. (21) in Ref. [6]. Parameter values listed above are considered and the ode45 function in MATLAB is employed. After each simulation, the generalized coordinates $\mathbf{q}$ and velocities $\dot{\mathbf{q}}$ of the system can be solved, and the solutions are then substituted into Eqs. (30) and (31) in Ref. [2] again to get the axial forces of the suspension rods. Fig. 5 displays the axial forces of the first suspension rod at the steady state when the mass of the clothes m_u =1 Kg and gyration radius of the clothes R_u =0.1 m. To obtain the amplitude of the axial force, the peaks and troughs of the curve are searched. Assuming that the average value of the peaks is F_p and that of troughs is F_t , the amplitude of the axial force can be calculated as $(F_p - F_t)/2$.

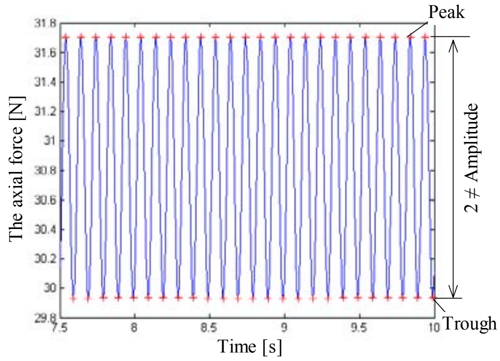

Fig. 5. The axial force of the first suspension rod at the steady state.

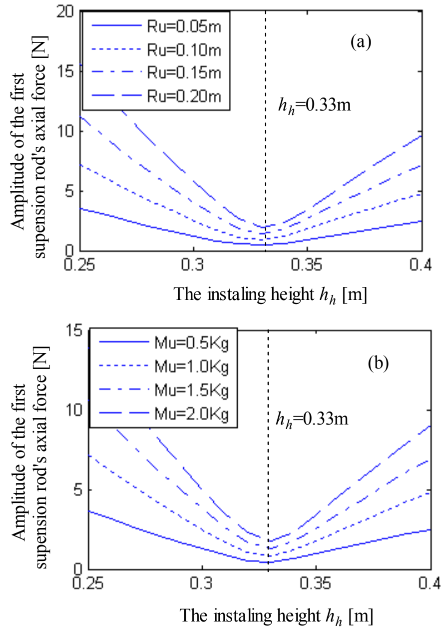

Fig. 6. Changes of the amplitude of the first suspension rod's axial force upon the increase of the installing height of the balancer under different gyration radiuses: (a) and masses; (b) of the clothes when the height of the clothes $h_u = 0.1$m .

Fig. 6(a) explores changes of the amplitude upon increasing the installing height h_h at different gyration radiuses of the clothes. During the simulations, the mass of the clothes is set at $m_u = 1$Kg with four different gyration radii of the clothes; $R_u = 0.05$m, $R_u = 0.1$m, $R_u = 0.15$m and $R_u = 0.2$m are considered. As can be seen, the smallest values corresponding to each curve almost lie in the same installing height h_h =0.33 m. Fig. 6(b) shows similar results. The gyration radius of the clothes is set at $R_u = 0.1$m , and four different masses of clothes at $m_u = 0.5$ Kg , $m_u = 1$ Kg , $m_u = 1.5$ Kg and $m_u = 2$ Kg show the same installing height at which the smallest amplitudes appear. It can be seen from Fig. 6 that the optimal installing height indicated by the simulations is h_h =0.33 m, which is a little smaller than $h_h^* = 0.346$ m predicted by Eq. (36). This may be caused by neglecting the item

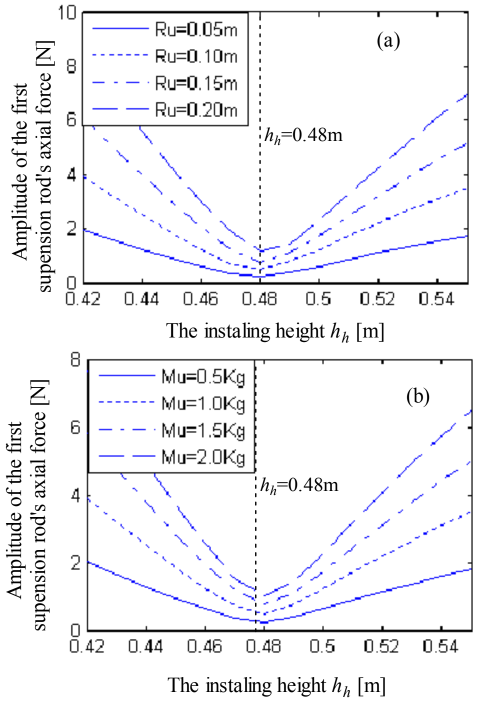

Fig. 7. Changes of the amplitude of the first suspension rod's axial force upon the increase of the installing height of the balancer under different gyration radiuses: (a) and masses; (b) of the clothes when the height of the clothes $h_u = 0.15$ m .

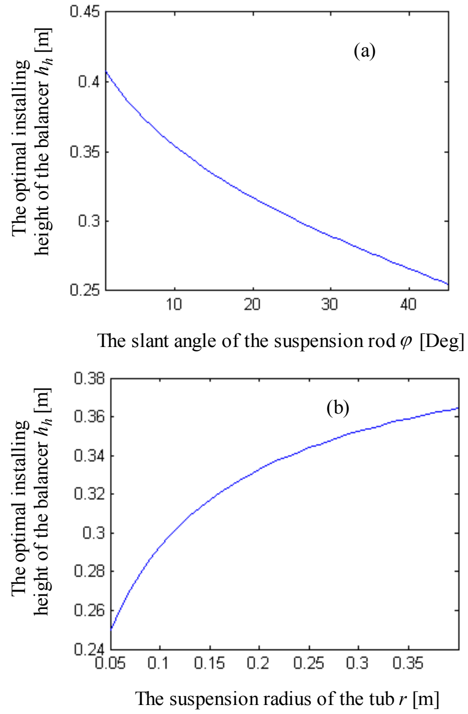

Fig. 8. Changes of the optimal installing height of the balancer h_h^* upon the increases of the slant angle of the suspension rod α (a) and the suspension radius of the tub r (b).

$R_u \tan\phi$ in Eqs. (23) and (25). To prove this, the height of the clothes is changed to $h_u = 0.15$m and the optimal installing height corresponding to it can be calculated using Eq. (36) as $h_h^* = 0.482$ m . Fig. 7(a) shows the simulation results when the mass of the clothes $m_u = 1$ Kg , and Fig. 7(b) shows the results when the gyration radius of the clothes $R_u = 0.1$ m . Both figures show the same installing height $h_h = 0.48$ m at which the smallest amplitudes appear. It can be seen that the height $h_h = 0.48$ m is very close to $h_h^* = 0.482$ m predicted by Eq. (36). Besides, as can be seen from Figs. 6 and 7, the optimal installing heights are hardly affected by the mass or gyration radius of the clothes.

We now make a comparison between the optimal installing height of the hydraulic balancer discussed in this paper and that discussed in Ref. [6]. The optimal installing height discussed in this paper corresponds to a small amplitude of the axial force of the suspension system, which is helpful for vibration isolation, while the installing height discussed in Ref. [6] corresponds to a small deflection angle of the washing/spinning assembly that could leave more space for preventing collisions between the assembly and the cabinet. The two heights are different. Take the following parameter values for example: when the mass of the clothes m_u =1 Kg , the gyration radius of the clothes R_u =0.1 m and the height of the clothes $h_u = 0.1$ m, the optimal installing height for vibration isolation is 0.346 m while that for a small deflection angle is 0.416 m. As can be seen, the former is a little smaller than the later. This means a small deflection angle of the washing/drying assembly is not sufficient enough for vibration isolation of the system.

We now discuss influences of the slant angle φ and the suspension radius of the tub r on the optimal installing height h_h^*. Calculations are carried out based on Eqs. (36) and (37). Fig. 8(a) shows the relation between the installing height h_h^* and the slant angle φ when the suspension radius of the tub $r = 0.26$ m . As can be seen, a larger slant angle φ corresponds to a lower installing height h_h^*. Fig. 8(b) displays changes of the optimal installing height h_h^* upon the variations of the suspension radius of the tub r when the slant angle $\varphi = 11.8$Deg . As can be seen, the optimal installing height of the balancer changes larger when the suspension radius of the tub increases.

We now discuss the influence of the item $m_h(1+\eta)$ in Eq. (37) on the optimal installing height of the balancer h_h^*. According to Eq. (55) in Ref. [6], in the linear region of the hydraulic balancer, the item $m_h(1+\eta)$ can be described as

$$\chi = m_h(1+\eta) = \rho\pi h R_o^2 \tag{40}$$

where ρ is density of the liquid, h is the depth of the balancer and R_o is the outer radius of the balancer. Substituting Eq. (40) into Eq. (37) yields

$$\Delta = \frac{h_u}{\delta} + \frac{m_t(h_u - h_t)(1 + h_t\delta)}{\chi\delta}. \tag{41}$$

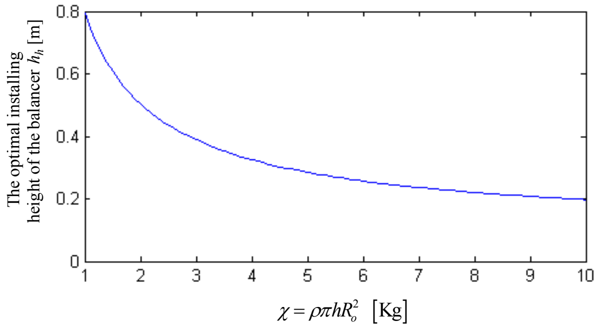

Fig. 9. Changes of the optimal installing height of the balancer h_h^* upon the increase of χ .

Calculations are carried out based on Eqs. (36) and (41). Fig. 9 shows changes of the optimal installing height of the balancer h_h^* upon the variations of the item χ taking into account the following parameter values: the slant angle of the suspension rod $\varphi = 11.8\text{deg}$, the suspension radius of the tub $r = 0.26\text{m}$, the height of clothes $h_u = 0.1\text{m}$. As can be seen, the optimal installing height h_h^* becomes smaller as χ grows larger. However, the slope of the curve changes more slowly which implies that the influence of χ on h_h^* becomes weaker as χ increases.

3. Conclusions

An effective method for vibration isolation of the vertical axis of an automatic washing machine is to control the amplitude of the axial force of the suspension system. Obtaining a small amplitude of the axial force requires the lower joint of the suspension rod to move along a special path, such as the base circle of a cone shown in Figs. 2 and 3. Based on the circle, a geometric constraint of the system was first derived. Considering that the trace along which the lower joint of the suspension rod moves is also affected by the equilibrium conditions of the centrifugal forces and torques, a governing equation based on the dynamics of the system was then derived. Combing the geometric constraint and the governing equation, a general governing equation for vibration isolation of the system was at last obtained.

To validate this general governing equation, simulations were carried out based on the vibration model described in Ref. [6]. As can be seen from the simulation results, in the linear region of the hydraulic balancer, the influence of the mass or gyration radius of the clothes on the optimal installing height h_h^* is small. However, h_h^* becomes smaller when the slant angle of the suspension rod φ or the design parameter of the hydraulic balancer $\chi = \rho\pi h R_o^2$ rises and grows larger when the suspension radius of the tub *r* increases.

The optimal installing height of the hydraulic balancer to obtain a small deflection angle of the washing/spinning assembly discussed in this paper is different from that discussed in Ref. [6]. As can be seen from the comparison, a small deflection angle of the washing/spinning assembly is insufficient for vibration isolation of the system. This is because the installing height discussed in Ref. [6] did not consider the geometric constraint discussed in this paper. For good vibration isolation, both the geometric constraint and equilibrium conditions of the centrifugal forces and torques should be considered.

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References

[1] http://cmvl.cs.concordia.ca/auto/ (Accessed January 20,2011).
[2] H. W. Chen and Q. J. Zhang, Stability analyses of a vertical axis automatic washing machine without balancer, *Journal of Sound and Vibration,* 329 (11) (2010) 2177-2192.
[3] Z. W. Wang and H. M. Wu, Dynamic analysis and simulation of a top loaded washing machine, *China Mechanical Engineering,* 13 (23) (2002) 2033-2035.
[4] H. W. Chen and Q. J. Zhang, Stability analyses of a vertical axis automatic washing machine with a hydraulic balancer, *Mechanism and Machine Theory*, doi:10.1016/j.mechmachtheory.2011.02.007.
[5] S. Bae, J. M. Lee and Y. J. LeeKang, etc., Dynamic analysis of an automatic washing machine with a hydraulic balancer, *Journal of Sound and Vibration,* 257 (1) (2002) 3-18.
[6] H. W. Chen, Q. J. Zhang and S.Y. Fan, Study on steady-state response of a vertical axis automatic washing machine with a hydraulic balancer using a new approach and a method for getting a smaller deflection angle, *Journal of Sound and Vibration* (2010) doi:10.1016/j.jsv.2010.11.006.

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