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Experimental study on hysteresis phenomena of shock wave structure in an over-expanded axisymmetric jet †

Shigeru Matsuo[1,*], Toshiaki Setoguchi[2], Junji Nagao[3], Miah Md. Ashraful Alam[3] and Heuy Dong Kim[4]

[1]*Department of Advanced Technology Fusion, Saga University, 1, Honjo-machi, Saga-shi, Saga 840-8502, Japan*
[2]*Institute of Ocean Energy, Saga University, 1, Honjo-machi, Saga-shi, Saga 840-8502, Japan*
[3]*Graduate School of Science and Engineering, Saga University, 1, Honjo-machi, Saga-shi, Saga 840-8502, Japan*
[4]*School of Mechanical Engineering, Andong National University, 388, Songcheon-dong, Andong760-749, Korea*

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Abstract

In recent studies on two-dimensional supersonic jets, it is reported that the hysteresis phenomenon for the reflection type of shock waves in the jet flow field is occurred under the quasi-steady flow condition and this phenomenon is affected by the transitional pressure ratio between the regular reflection and Mach reflection. However, so far, there are few researches on the hysteresis phenomenon for the transition of shock waves between regular and Mach reflection in over-expanded supersonic jets and the phenomenon has not been investigated satisfactorily. Therefore, the purpose of this study is to clarify the hysteresis phenomena for the reflection type of shock wave in the over-expanded axi-symmetric supersonic jet experimentally, and to discuss the relationship between hysteresis phenomenon and rate of the change of pressure ratio with time. Furthermore, the effect of Mach number at the nozzle exit on hysteresis loop was investigated for two kinds of nozzle.

Keywords: Compressible flow; Hysteresis; Over-expanded jet; Shock wave; Supersonic

1. Introduction

In this universe, every process is irreversible. If a system has any reversed process, the system and surrounding would not return to their original conditions. Hysteresis phenomenon is itself an example of irreversible process. Hysteresis phenomena commonly occur in magnetic and ferromagnetic materials, as well as in the elastic, electric, and magnetic behavior of materials, in which a lag occurs between the application and the removal of a force or field and its subsequent effect. The term ‘hysteresis’ is sometimes also used in other fields, such as economics or biology; where it describes a memory, or lagging effect, in which the order of previous events can influence the order of subsequent events. Hysteresis was initially seen as problematic matter, but it is now thought to be of great importance in technology.

In recent years, hysteresis phenomena in fluid flow systems draw attention of many researchers for their great variety of industrial and engineering applications in the range from the design of rocket propulsion systems to industrial areas using high pressure gas. During the formation of flow in a certain condition, the flow will experience a transient state until the state and to return to the original state. In general, even a flow changes under the quasi-steady, the flow characteristics is considered to accompany a hysteresis phenomenon (hysteresis loop). In case of a process of rapid change in the flow, this is distinguished from the phenomena for the delay in response time. Hysteresis phenomenon is well known for external flow such as reflection of shock waves in the jet. From some previous researches, it was clarified that when the high-pressure gas was exhausted to atmosphere from the nozzle exit, the expanded supersonic jet with the Mach disk was formed at a specific condition. The jet structure has been known as a fundamental phenomenon of the supersonic fluid mechanics. This type of jet is very important for some industrial devices [1, 2]. Teshima [3] has suggested the possibility of occurring hysteresis in the supersonic jet formed by the rectangular orifice. Chpoun and Ben-Dor [4] numerically confirmed the hysteresis phenomenon in transition from the regular to the Mach reflections in steady flows. After that, many researchers have proven that the hysteresis phenomenon exists on the interference form of the shock wave in the flow.

When the high-pressure gas is exhausted from the supersonic nozzle, the over-expanded supersonic jet is formed at specific

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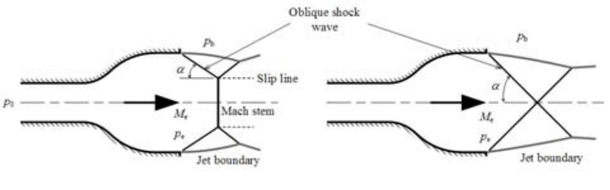
* Corresponding author. Tel.: +82 (0) 952 28 8606, Fax.: +82 (0) 952 28 8587
E-mail address: matsuo@me.saga-u.ac.jp



(a) (b)

Fig. 1. Schematics of jet structure of over-expanded supersonic jet.

condition. Under highly over-expanded conditions, the shock wave generated from the corner at the nozzle exit cannot be regularly reflected from the centerline of symmetry and Mach reflection generally occurs. The configuration of the shock wave changes from regular into Mach reflections with an increment in the ratio of the back pressure to the static pressure at the nozzle exit. The structure of over-expanded supersonic jet exhausted from the nozzle is schematically shown in Fig. 1. In Fig. 1(a), the oblique shock formed at nozzle corner intersects with Mach stem. It is well known that this reflection type is a Mach reflection (MR). On the other hand, the oblique shock formed at nozzle corner intersects with each other at jet axis and this reflection type is a regular reflection (RR), as shown in Fig. 1(b). As shown in these figures, the symbol α shows the incident angle of oblique shock, D_e is the nozzle exit diameter, M_e is the nozzle exit Mach number and p_0, p_e, p_b mean the reservoir, nozzle exit and back pressures, respectively.

Over the past few years, the problem associating with the transition between the regular reflections (RR) and Mach reflection (MR) of shock waves in the supersonic jets under the quasi-steady flow condition have been considered and discussed by several investigators [5-8]. The study of a shock wave transition from regular into Mach reflections i.e. the hysteresis phenomena of the shock structure is important for determination of nozzle flow patterns associated with the development of rocket nozzles and scramjet engines.

Recently, Irie et al. [7] observed that a hysteresis phenomenon of the under-expanded dry air jet is produced during the transient processes of jet pressure ratio, in which the jet flow obtained in the startup transient is different from that in the shutdown transient. As in most of the engineering applications of supersonics jets, the working gas is usually steam or moist air and the moist air jets are qualitatively similar to dry air, but non-equilibrium condensation in moist air jet affects the shock configuration, reported by Baek et al. [9] and Otobe et al. [10]. Meanwhile, Kim et al. [11] and Otobe et al. [12] have investigated the effect of non-equilibrium condensation on the hysteresis phenomenon of under-expanded supersonic jets and reported that the moist air jet reduced the hysteretic phenomenon compared with the dry air jet, and the non-equilibrium condensation that occurs in the under-expanded moist air jet is responsible for these findings. However, only a few works have, to date, been devoted to the hysteresis phenomena for the reflection type of shock wave and for instance, the transitional pressure ratio between the regular reflection (RR) and Mach reflection (MR) under the over-expanded axisymmetric jet, and its physical reasoning to cause the phenomenon is still not satisfactorily known. Among them, Chow et al. [13] performed the detailed calculation about the size and location of the Mach stem and Mach disk in over-expanded jets with variations in back pressure and Mach number at the nozzle exit. Also, Hornung et al. [14, 15] described the mechanism that the relations between quantities in the entrance plane of the nozzle and the nozzle throat control the position and the size of Mach stem in over-expanded jet. They did not clarify the criterion of a shock wave transition from regular into Mach reflections on the over-expanded supersonic jet and there is no clear conception on the relation between the rates of change of jet pressure ratio and hysteresis phenomenon.

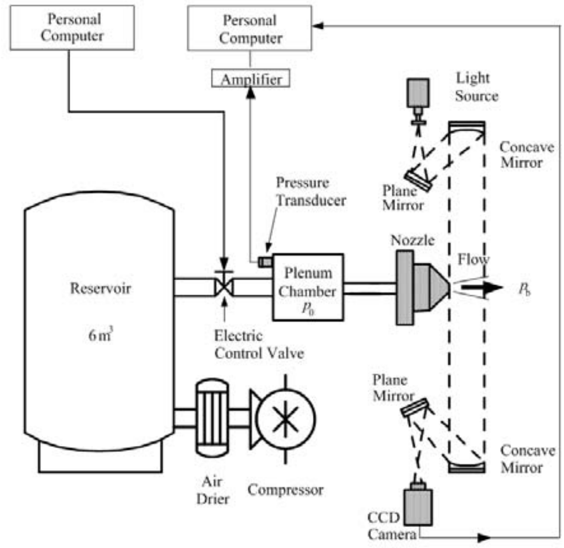


Fig. 2. Experimental apparatus.

The purpose of the present experimental study is to clarify the hysteresis phenomena of a shock wave in an over-expanded axisymmetric supersonic jet. Flow visualizations were performed in order to investigate the flow field in the over-expanded jet. We attempted to clarify the relationship between the rates of change of jet pressure ratio and hysteresis phenomena of shock wave transitions from regular to Mach reflection in over-expanded supersonic jets. Moreover, hysteresis phenomena for the location and diameter of Mach disk in the flow field of over-expanded supersonic jets were investigated experimentally. Furthermore, the effect of Mach number at the nozzle exit on hysteresis loop was investigated for two kinds of nozzle.

2. Experimental setup

Fig. 2 shows the schematic diagram of the experimental apparatus used in the present study. The apparatus was consisted of compressor, air drier, air reservoir, electronic control valve, plenum chamber and nozzle. Plenum chamber was placed

Table 1. Configurations of the nozzle (Unit: mm).

Nozzle type	D_t	D	L
Nozzle A ($M_e = 2.0$)	5.95	8	53
Nozzle B ($M_e = 2.3$)	5.21		55

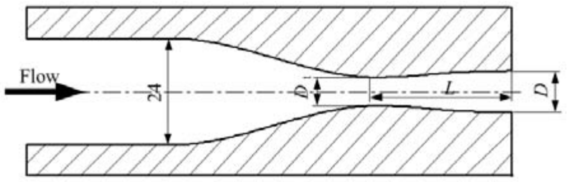


Fig. 3. Details of nozzle (Unit: mm).

upstream of the supersonic nozzle. The compressed air flows into the plenum chamber, whereby the flow rate of the air was adjusted by an electric control valve. After the air stagnated in the plenum chamber, it was discharged from the nozzle exit. The test section was placed downstream of the nozzle exit. In the present study, two nozzles were designed by the method of characteristics. The design Mach numbers of Nozzles A and B are M_e= 2.0 and 2.3, respectively. The details for the nozzle configurations are shown in Fig. 3 and Table 1.

3. Experimental setup

In the present experiments, pressure ratio ($\phi = p_0/p_b$) was continuously changed with time using the electronic control valve. The symbol, p_0 and p_b represent the stagnation pressure of the plenum chamber and back pressure (atmospheric pressure) as shown in Fig. 2, respectively. For shock transition from regular to Mach reflection for Nozzle A, the range of ϕ is from 4.35 to 9.02 and it is from 5.56 to 10.0 for Nozzle B. The rate of the change of pressure ratio with time $\Delta\dot{\phi}$ is from 0.262 (1/s) to 0.514 (1/s) for Nozzle A and from 0.281 (1/s) to 0.716 (1/s) for Nozzle B. The experiments were performed three times for one value of $\Delta\dot{\phi}$ to obtain the averaged diameter and position of Mach disk and as a result repeatability of the experiments was confirmed.

The flow field was investigated by a shadowgraph method. Visualization and measurement of pressure ratio were conducted simultaneously. In the present experimental conditions, it was found that the time-lag exists for response of change of flow to the change of pressure ratio at $\Delta\dot{\phi} \geq 0.514$ (1/s) for Nozzle A and $\Delta\dot{\phi} \geq 0.716$ (1/s) for Nozzle B, as indicated in Table 2. As described later, these results were determined from the characteristics of the hysteresis loop.

4. Results and discussion

The typical shadowgraph pictures for Nozzle A and B are shown in Figs. 4 and 5, respectively. In both figures, the left side sequence represents the decreasing process of pressure ratio ϕ and the right side for the increasing process of pressure ratio ϕ. The symbol $\phi_{tr}^{<d>}$ indicates the pressure ratio just before formation of Mach disk and $\phi_{tr}^{<u>}$ for the pressure ratio just after disappearance of Mach disk.

Fig. 4 shows the results in case of Nozzle A ($\Delta\dot{\phi} = 0.369$ (1/s)). In Figs. 4(a) and (b), the regular reflection (RR) is observed. In Fig. 4(c), the transition of shock wave occurs at $\phi_{tr}^{<d>}=6.61$. The Mach reflection (MR) is observed in the range of $6.61 > \phi > 4.35$ as shown in Figs. 4(d) and (e). However, it is remarkable in case of an increase of ϕ that the reflection type transforms from MR to RR at $\phi_{tr}^{<u>}=6.40$ and there are some differences for the transitional pressure ratio between the increasing and the decreasing processes of ϕ. This means the existence of the hysteresis phenomena.

Table 2. Experimental conditopns.

(a) Nozzle A

Nozzle type	Hysteresis or Time-lag	$\Delta\dot{\phi}$ (1/sec)
Nozzle A ($M_e = 2.0$)	Time-lag	1.994
		1.503
		1.201
		1.007
		0.849
		0.744
		0.665
		0.604
		0.554
		0.528
	Hysteresis	0.514
		0.498
		0.437
		0.369
		0.330
		0.295
		0.262

(b) Nozzle B

Nozzle type	Hysteresis or Time-lag	$\Delta\dot{\phi}$ (1/sec)
Nozzle B ($M_e = 2.3$)	Time-lag	1.951
		1.603
		1.211
		1.015
		0.926
		0.816
	Hysteresis	0.716
		0.641
		0.524
		0.444
		0.377
		0.345
		0.309
		0.281

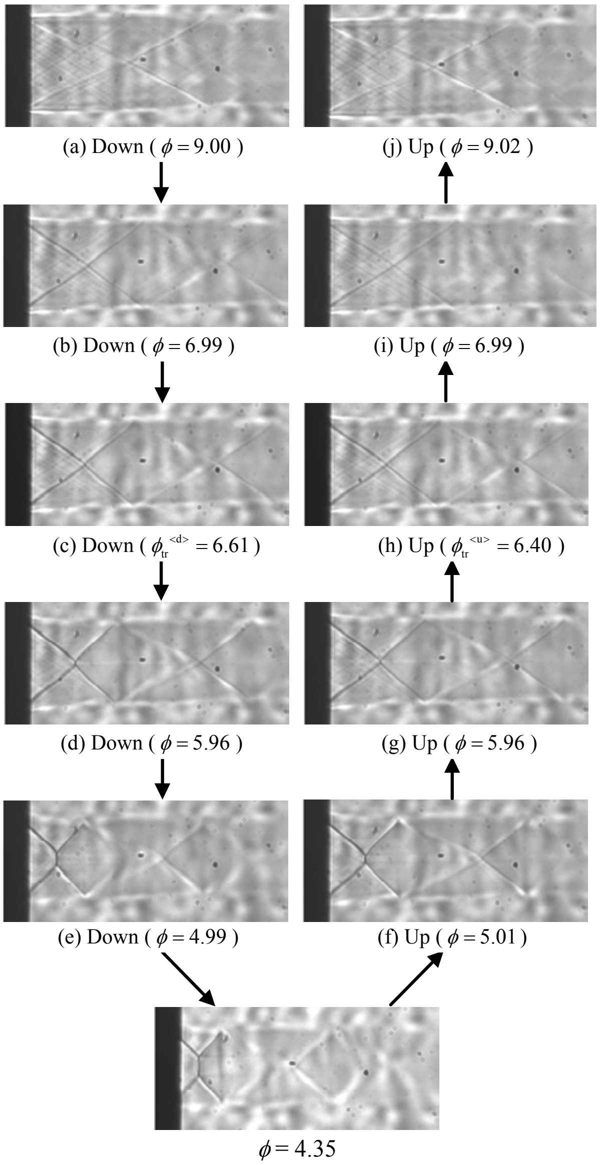


Fig. 4. Shadowgraph pictures (Nozzle A, $\Delta\dot{\phi}$ = 0.369(1/s)).

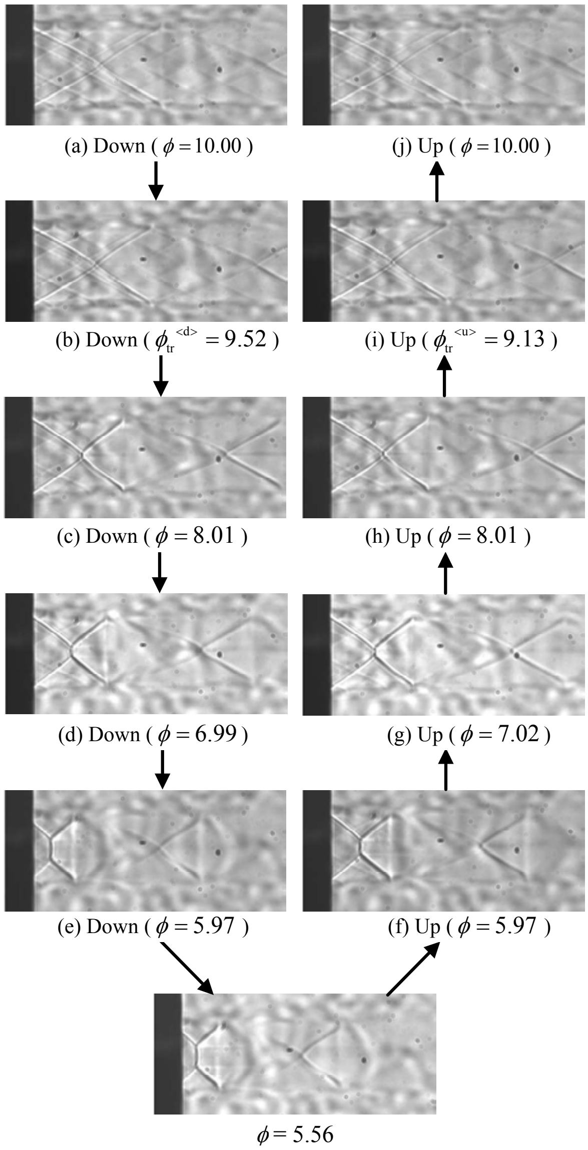
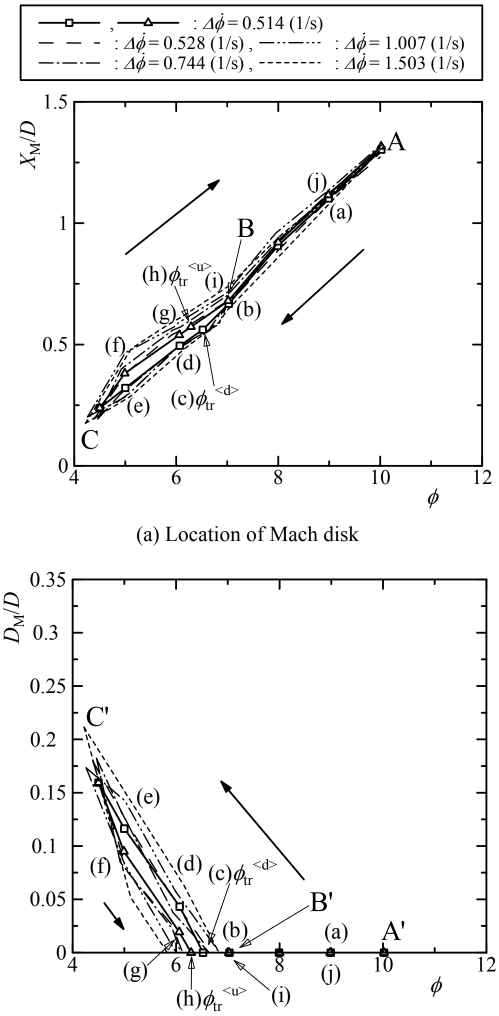


Fig. 5. Shadowgraph pictures (Nozzle B, $\Delta\dot{\phi}$ = 0.377(1/s)).

Fig. 5 shows the result for Nozzle B ($\Delta\dot{\phi}$ = 0.377 (1/s)) and the reflection type transforms from RR to MR or MR to RR because of change of pressure ratio ϕ. As is evident from this figure, the values of $\phi_{tr}^{<d>}$ and $\phi_{tr}^{<u>}$ in case of Nozzle B is larger in comparison with the transition states of Nozzle A, and the difference between the pressure ratio at transition states during the processes of decreasing and increasing of jet pressure ratio ($\phi_{tr}^{<d>}$ - $\phi_{tr}^{<u>}$) for Nozzle B is larger than that of Nozzle A. Therefore, it is found that the hysteresis phenomena are affected by nozzle exit Mach number.

From the results of Fig. 4 and Fig. 5, it is remarkable that the hysteresis phenomena occur at the transitional pressure ratio between RR and MR. Moreover, in both cases, there are differences between the location and diameter of Mach disk in the first shock cell even in the same pressure ratio.

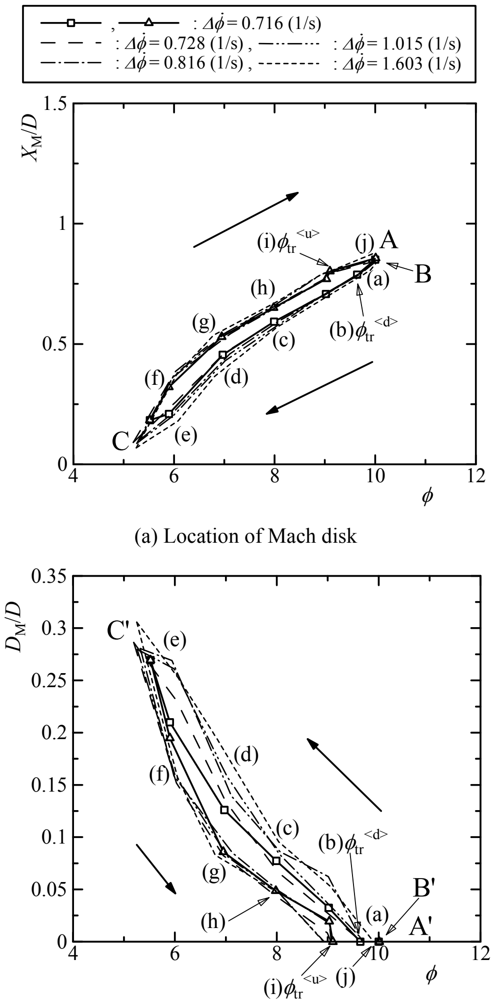
Figs. 6(a) and 6(b) show the effect of $\Delta\dot{\phi}$ on the location X_M and diameter D_M of Mach disk in the range from 0.514 (1/s) to 1.503 (1/s), respectively. Here, the location from nozzle exit and diameter of Mach disk are normalized by the nozzle exit diameter D. As is evident from Fig. 6(a), hysteresis loops exist in the location of Mach disk X_M/D at the course between A, B and C. Processes of variation between A, B and C follow the same course below $\Delta\dot{\phi}$ =0.514 (1/s). The relationship between the pressure ratio and diameter of Mach disk in the range of $\Delta\dot{\phi}$ is shown in Fig. 6(b). That there exist hysteresis loops at the course between A′, B′ and C′ in the same manner as those in Fig. 6(a) and processes of variation between A′, B′ and C′ follow the course below $\Delta\dot{\phi}$ =0.514 (1/s), as like the location of Mach disk. Here, it is mentioned that the symbols (c) and (h) in Figs. 6(a) and (b) represent the transition of reflection type of shock structure from RR to MR and MR to RR, respectively.



(a) Location of Mach disk

(b) Diameter of Mach disk

Fig. 6. Effect of $\Delta\dot{\phi}$ for Nozzle A.



(a) Location of Mach disk

(b) Diameter of Mach disk

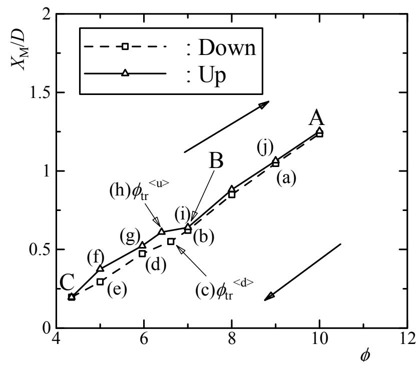
Fig. 7. Effect of $\Delta\dot{\phi}$ for Nozzle B.

Similarly, experimental results for Nozzle B are shown in Fig. 7. Here, $\Delta\dot{\phi}$ is range from 0.716 (1/s) to 1.603 (1/s). In the same manner as results in Figs. 6(a) and 6(b), hysteresis loops exist at the course between A, B and C for the location of Mach disk (Fig. 7(a)), and at the course between A′, B′ and C′ for diameter of Mach disk (Fig. 7(b)). Processes of variation between A, B and C for the location and between A′, B′ and C′ for the diameter of Mach disk follow the same course below $\Delta\dot{\phi}$ =0.716 (1/s) in both cases.

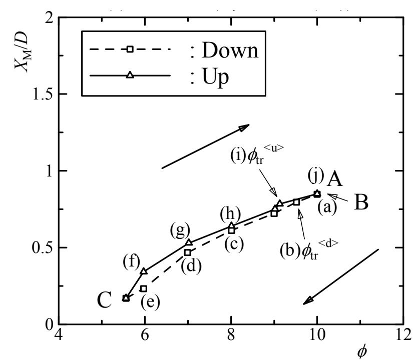
Figs. 8(a) and (b) show the relationships between the location of Mach disk X_M/D and pressure ratio ϕ in case of occurrence of hysteresis phenomena in over-expanded supersonic jets for Nozzles A ($\Delta\dot{\phi}$ =0.369 (1/s)) and B ($\Delta\dot{\phi}$ =0.377 (1/s)), respectively. In both figures, symbols from (a) to (j) correspond to those in Figs. 4 and 5. As is evident from these figures, the existence of two values of X_M/D is confirmed in the ranges of ϕ = 4.35 – 6.61 for Nozzle A and ϕ =5.56 – 9.52 for Nozzle B, respectively.

Similarly, the relationship between the diameter of Mach disk D_M/D and pressure ratio ϕ for Nozzles A ($\Delta\dot{\phi}$ =0.369 (1/s)) and B ($\Delta\dot{\phi}$ =0.377 (1/s)) are shown in Figs. 9(a) and (b), respectively. It is confirmed from this figure that there exist two values of D_M/D in the range of ϕ = 4.35 – 6.61 for Nozzle A and ϕ =5.56 – 9.52 for Nozzle B, respectively.

In our previous researches on the supersonic jet, the maximum mass flow rate in the jet within a certain pressure ratio changes with pressure ratio and this condition agrees with the range of occurrence of the hysteresis phenomenon. From this result, it is considered that the mass flow rate is related to the generation mechanism of hysteresis loop. Further, as the flow has a nonlinear nature, the hysteresis phenomenon in the flow field depends on the mass flux and/or variation of upstream and downstream flow conditions of the nozzle. As a result, upstream and downstream flow conditions of the nozzle should be considered in order to understand the phenomenon in the internal flow field with the interaction between boundary layer and shock wave. Investigation for cause of hysteresis phenomenon is the subject for a future study.

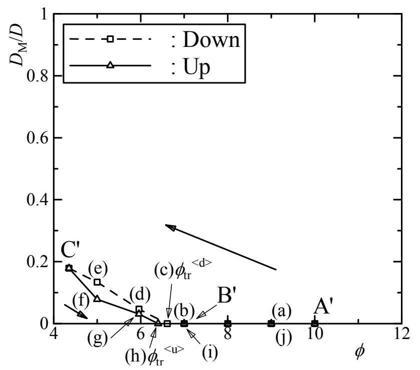


(a) Nozzle A ($\Delta\dot{\phi}$ = 0.369 (1/s))

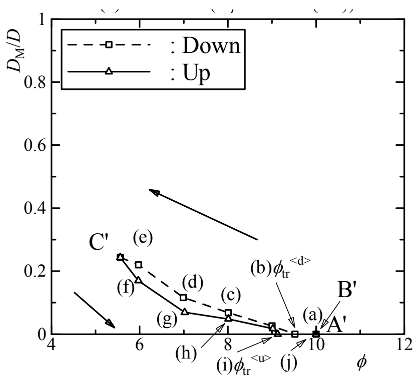


(b) Nozzle B ($\Delta\dot{\phi}$ = 0.377 (1/s))

Fig. 8. Hysteresis diagrams for location of Mach disk.



(a) Nozzle A ($\Delta\dot{\phi}$ = 0.369 (1/s))



(b) Nozzle B ($\Delta\dot{\phi}$ = 0.377 (1/s))

Fig. 9. Hysteresis diagrams for diameter of Mach disk.

5. Conclusions

The experimental study was conducted to investigate the hysteretic behavior of refection type from regular reflection (RR) to Mach reflection (MR) or from Mach reflection (MR) to regular reflection (RR) in over-expanded supersonic jets. As the results, hysteresis phenomenon for the location and diameter of Mach disk was confirmed at a certain specific condition. The relationship between hysteresis phenomenon and the range of the rate of change of pressure ratio with time, both for the location and diameter of Mach disk, was shown experimentally. Moreover, the effect of nozzle exit Mach number on the transition of reflection type of shock structure from RR to MR or MR to RR was investigated.

Acknowledgment

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Nomenclature

D : Height of nozzle throat
D_M : Diameter of Mach disk
p : Static pressure
p_0 : Total pressure of stagnation point
p_b : Back pressure
X_M : Distance from nozzle throat to Mach disk
ϕ : The ratio of back-pressure and total pressure of stagnation point

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Shigeru Matsuo has been working in the fluid engineering area as a Professor at Saga University, Japan, since 1994. His research interests include High-Speed Flows, Shock Tube and Technology, Shock Wave Dynamics, Flow Measurement, and Supersonic Wind Tunnels.