

A modified model reference adaptive control with application to MEMS gyroscope[†]

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Abstract

Micro electro-mechanical systems (MEMS) are increasingly being used in measurement and control problems due to their small size, low cost, and low power consumption. The vibrating gyroscope is a MEMS device that will have a significant impact on stability control systems in the transportation industry. This paper investigates the application of a modified model reference adaptive control for MEMS gyroscope. Using this adaptive control algorithm, an estimation of the angular velocity and the damping and stiffness coefficients in real time is easily computable. Changing the conventional model reference input makes it feasible to utilize a low pass filter to remove unwanted oscillations caused by high adaptation gain. This new adaptive control technique enables quick compensation for large changes in the system dynamics, providing consistent estimation of gyroscope parameters including angular velocity and large robustness to parameter variations and external disturbances. The asymptotic stability of the mentioned adaptive controller is guaranteed using the Lyapunov direct method. Numerical simulation is presented to verify the effectiveness of the proposed control scheme.

Keywords: Adaptive control; MEMS gyroscope; Lyapunov direct method; Low-pass filter

1. Introduction

Since the 1980s, MEMS (Micro-Electro-Mechanical Systems) gyroscopes have been developed with the development of silicon surface micro-machining technology. MEMS gyroscopes have many benefits such as low cost, small size, and negligible weight compared to conventional mechanical gyroscopes [1-3]. Furthermore, they have a wide range of applications including navigation and guidance systems, automobiles, and consumer electronics, which has lead many researchers to focus on them over the last few decades. However, some drawbacks of the MEMS gyroscopes, such as low precision due to a small vibrating amplitude, very narrow bandwidth, and imperfections in fabrication are challenges that remain to be solved.

When the gyroscope is subject to a rotation rate, the response of the sensing mode provides the information of the rotation rate. With the advancement of MEMS technology, MEMS gyroscopes have been applied to automobiles for rollover sensors and skid control, consumer electronics (e.g. camera stabilizations), GPS-assisted inertial navigation, industry, aerospace, etc. [4]. However, fabrication imperfections and environmental variations produce undesirable coupling terms, unknown disturbances, input and measurement noises, frequency mismatch between two vibrating modes, and parameter variations which degrade the performance of the gyroscopes. There are also certain factors which may affect the performance of gyroscopes, and these are known as the coupling parameters. Coupling parameters are the terms that exist in the drive and sense motion equations in the form of stiffness and damping terms multiplied by position and velocity states, thereby relating the drive and sense motion equations. Coupled damping parameters are usually small and are not as important as the coupled stiffness terms. As a consequence, a control system is essential to improve the performance and stability of MEMS gyroscopes. Advanced control technologies should focus on exploiting the inherent structures of the vibratory MEMS gyroscopes, so as to achieve disturbance attenuation and performance robustness against modeling uncertainties. There are two major control problems associated with vibrational MEMS gyroscopes: controlling two vibrating axes (or modes) of the gyroscope, and to estimating a time-varying rotation rate [5].

Model reference adaptive control (MRAC) is one of the main approaches to adaptive control. This architecture was originally developed to control linear systems in the presence of parametric uncertainties. This architecture has been facilitated by the Lyapunov stability theory, which gives sufficient

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conditions for stable performance without characterizing the frequency properties of the resulting controller. The basic structure of an MRAC scheme is shown in Fig. 1. The major drawback utilizing the MRAC algorithms with large learning rates is its sensitivity to time-delay. Refs. [6, 7] addressed this challenge and introduced a new paradigm for design of adaptive controllers. The resulting control architecture adapts quickly, leading to desired transient performance with analytically computable performance bounds. Moreover, as demonstrated in Refs. [8-10], unlike the conventional MRAC algorithms, the introduced adaptive control architectures have guaranteed time-delay margins in the instance of rapid adaptation.

Astrom [11], Ioannou and Sun [12], and Tao [13] described the model reference adaptive control. In the last few years, many applications have been developed for the improvement of the performance of adaptive controllers, and it has been addressed from various perspectives in numerous efforts [6, 7, 14-17]. An adaptive control law and merging parameter identification were proposed and analytically studied by Fradkov and Andrievsky [18]. Cao and Hovakimian [10, 19, 20] developed a novel adaptive control architecture that ensures that the input and output of an uncertain linear system track the input and output of a desired linear system during the transient phase, in addition to asymptotic tracking. Zareh et al. [21, 22] utilized this modified MRAC for some special flight control systems. Some control algorithms had been proposed to control the MEMS gyroscope. Fei and Batur [2, 3] derived an adaptive sliding mode control for MEMS gyroscope to identify its angular velocity. Batur et al. [1] developed a sliding mode control for a MEMS gyroscope system combined with a force balancing control strategy to identify angular velocity. Leland [4] presented an adaptive controller for tuning the natural frequency of the drive axis of a vibratory gyroscope. An adaptive controller that drives both axes of vibration and controls the entire operation of the gyroscope for a MEMS gyroscope is reported in Ref. [4].

The major contribution of this paper is to design a modified model reference adaptive controller to estimate angular velocity and all unknown gyroscope parameters and to prove the ability of the mentioned controller to be used in such systems. Due to the fast compensation behavior resulting from a suitable selection of the controller parameters, a faster and more precise estimation of angular velocity is expected.

The present paper consists of 5 sections. After a brief introduction in the present section (Section 1) the mathematical model of MEMS gyroscope will be introduced. In Section 3, the modified model reference adaptive controller is developed. Simulation results are depicted in Section 4. Conclusions are provided in Section 5.

2. Dynamics of MEMS vibrational gyroscope

A z-axis MEMS gyroscope is depicted in Fig. 1. The typical MEMS vibratory gyroscope includes a proof mass suspended



Fig. 1. Block diagram of an MRAC.

by springs, an electrostatic actuation and sensing mechanisms for forcing oscillatory motion and sensing the position and velocity of the proof mass. We assume that the table where the proof mass is mounted moves with a constant velocity and the gyroscope rotates at a slowly changing angular velocity. The centrifugal forces m2z x, m2z y are assumed to be negligible due to small displacements. The Coriolis force is generated in a direction perpendicular to the drive and rotational axes. Following Ref. [1] and considering the assumptions stated above, the dynamics of gyroscope become:

$$m\ddot{x}+c_{xx}\dot{x}+c_{xy}\dot{y}+k_{xx}x+k_{xy}y=u_{x}(t)+2m\Omega^{*}\dot{y}$$
(1)

$$m\ddot{y}+c_{yy}\dot{y}+c_{xy}\dot{x}+k_{yy}y+k_{xy}x=u_{y}(t)+2m\Omega^{*}\dot{x}$$
(2)

where m is the mass of the proof mass, x and y are the coordinates of the proof mass relative to the table frame, and k_{xx} , k_{yy} , k_{xy} are the stiffness matrix parameters. The parameters C_{xx} , C_{yy} , C_{xy} represent the damping, $u_x(t)$ and $u_y(t)$ are the electrostatic driving forces, Ω^* is the unknown angular velocity that is to be determined, and $2m\Omega^*\dot{y}$ and $-2m\Omega^*\dot{x}$ are the coupling forces caused by the Coriolis effect. The following assumptions were made to derive the lumped dynamic models in Eqs. (1) and (2):

$$\Omega_x^{*\,2} \approx \Omega_y^{*\,2} \approx \Omega_z^{*\,2} \cong 0, \qquad \dot{\Omega}_z^* = \dot{\Omega}^* \cong 0.$$
(3)

Friedland and Hutton [23] and Painter and Shkel [24] comprehensively analyzed the behavior of this coupled system. In order to minimize the coupling between the actuation (x) and sense modes (y), four degrees of freedom (two in actuation and two in sensing) have been proposed by Acar and Shkel [25]. The initial development of MEMS gyroscopes and the controller designs can be traced to the work of Shkel et al. [26].

Using non-dimensional time $\tau = \omega_0 t$, and dividing both sides of Eq. (3) by $m\omega_0^2$ with ω_0 being the reference frequency, gives the final form of the non-dimensional equation of motion as:

$$\ddot{q} + \frac{C^* \dot{q}}{m\omega_0} + \frac{K_a q}{m\omega_0^2} = \frac{u^*(t)}{m\omega_0^2} - \frac{2\Omega^* \dot{q}}{\omega_0}.$$
(4)

The parameters in Eq. (4) are defined as follows:



Fig. 2. A schematic view of dynamic model of MEMS z-axis gyroscope.

$$q = \begin{bmatrix} \frac{x}{q_0} \\ \frac{y}{q_0} \end{bmatrix}, \quad C^* = \begin{bmatrix} c_{xx} & c_{xy} \\ c_{xy} & c_{yy} \end{bmatrix}, \\ K_a = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}, \quad u^* = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \\ and \quad \Omega^* = \begin{bmatrix} 0 & -\Omega^* \\ \Omega^* & 0 \end{bmatrix}$$
(5)

where q_0 is the reference length. Now define a set of new parameters as follows:

$$C = \frac{C^*}{m\omega_0}, \quad K_b = \frac{K_a}{m\omega_0^2}$$
$$u = \frac{u^*}{m\omega_0^2}, \quad \Omega = \frac{\Omega^*}{\omega_0}.$$
(6)

Consequently, the non-dimensional representation of Eqs. (1) and (2) becomes

$$\ddot{q} + C\dot{q} + K_b q = u(t) - 2\Omega \dot{q}.$$
(7)

To achieve a compacted gyroscope model, Eq. (7) is written in state space form as

$$\dot{X}(t) = AX(t) + Bu(t)$$
(8)

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_x^2 & -c_{xx} & -\omega_{xy} & -(c_{xy} - 2\Omega^*) \\ 0 & 0 & 0 & 1 \\ -\omega_{xy} & -(c_{xy} + 2\Omega^*) & -\omega_y^2 & -c_{yy} \end{bmatrix}, \quad (9)$$
$$B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}, \quad X = [x \ \dot{x} \ y \ \dot{y}]^{\mathrm{T}}$$

in which

$$\omega_x = \sqrt{\frac{k_{xx}}{m\omega_0^2}}, \quad \omega_y = \sqrt{\frac{k_{yy}}{m\omega_0^2}}, \quad \omega_{xy} = \frac{k_{xy}}{m\omega_0^2}. \tag{10}$$

3. Modified model reference adaptive controller

In this section, as in Refs. [6] and [7], a modified model reference adaptive controller is designed to control MEMS gyroscopes. The control target for a MEMS gyroscope is to design an adaptive controller so that the trajectory of the driving axes can track the state of a reference model while it permits complete transient characterization for a system's input and output signals. The purpose of the control is to establish conditions that the unknown angular velocity can be consistently estimated.

To achieve a continuous oscillating proof mass, the following reference model is selected:

$$\ddot{\mathbf{q}}_{\mathrm{m}} + \mathbf{K}_{\mathrm{m}} \mathbf{q}_{\mathrm{m}} = \mathbf{0} \tag{11}$$

where $K_m = diag\{\omega_1, \omega_2\}$. Then the desired trajectory is achieved as $x_m = A_1 \sin(\omega_1 t)$ and $y_m = A_2 \sin(\omega_2 t)$ in the x and y-directions, respectively. The reference model in Eq. (11) can be written in state space form as

$$\dot{\mathbf{X}}_{m} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_{1}^{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_{2}^{2} & 0 \end{bmatrix} \mathbf{X}_{m} = \mathbf{A}_{m} \mathbf{X}_{m}.$$
(12)

Eq. (12) can also be rewritten as

$$\dot{X}_{m} = A_{m}X_{m} + Bu_{c} \tag{13}$$

in which u_c equals zero. The constant Hurwitz matrix $A_m \in \mathbb{R}^{m \times n}$ should be selected such that (A_m, B) is controllable. To obtain a modified model reference adaptive controller introduced in Ref. [9], a control architecture is presented as follows:

$$\begin{split} \dot{X}_{m}(t) &= A_{m}X_{m} + B(u(t) - K(t)X(t) \\ &- K_{e}(X(t) - X_{m}(t))) \\ &+ \hat{f}_{d}(t), \quad X_{m}(0) = X_{0} \end{split}$$
(14)

where K(t) is the adaptation parameter. The choice of K_e needs to ensure that $A_c = A_m + BK_e$ is Hurwitz or, equivalently, that $H_c(s) = (sI - A_c)^{-1}B$ is stable. One obvious choice is $K_e = 0$. $\hat{f}_d(t)$ is an estimation of external disturbances and will be introduced in the next section.

Following Ref. [2], in this section we consider the system in Eq. (8) with parametric uncertainties ΔA and external disturbance f as

$$\dot{X}(t) = (A + \Delta A)X(t) + Bu(t) + f(t)$$
⁽¹⁵⁾

where ΔA is the unknown uncertainty of the matrix A and f(t) is an uncertain external disturbance.

Assumption 1: ΔA and f(t) have matched and unmatched terms. There exist unknown matrices of appropriate dimension D, G such that $\Delta A = BD(t) + \Delta \tilde{A}$ and $f(t) = BG(t) + \Delta \tilde{f}(t)$, where BD(t) is a matched uncertainty and $\Delta \tilde{A}$ is an unmatched uncertainty. BG(t) is a matched disturbance and $\Delta \tilde{f}(t)$ is an unmatched disturbance. Therefore, Eq. (15) can be rewritten as

$$\dot{X}(t) = AX(t) + Bu(t) + f_d$$
(16)

in which $f_d = Bf_m + f_u$. Here, $Bf_m(t, X)$ represents the system lumped matched uncertainty and disturbance, which is given by

$$f_m(t, X) = DX(t) + G.$$
 (17)

The term $f_u(t, X)$ represents the lumped unmatched uncertainty and disturbance, which takes the form

$$f_{u}(t, X) = \Delta \widetilde{A}(t)X(t) + \widetilde{f}(t).$$
⁽¹⁸⁾

Assumption 2: The external disturbances and system uncertainties have a small derivation with respect to time.

Assumption 3: The matched and unmatched lumped uncertainty and external disturbance f_m and f_u are bounded such as $||f_m(t, X)|| \le \alpha_m$, and $||f_u(t, X)|| \le \alpha_u$, where α_m and α_u are known positive constants, $||\cdot||$ is the Euclidean norm. The tracking error is defined as

$$e(t) = X(t) - X_m(t).$$
 (19)

Differentiating and using Eqs. (14) and (16) gives

$$\dot{e} = A_c e + (A - A_m)X + KX + f_d - \hat{f}_d.$$
 (20)

Assumption 4: There exists a constant matrix of ideal parameters K^* such that the following matching condition $A + BK^* = A_m$ can always be satisfied.

Considering above assumption, Eq. (20) can now be rewritten as

$$\dot{\mathbf{e}} = \mathbf{A}_{\mathbf{c}}\mathbf{e} + \mathbf{B}\widetilde{\mathbf{K}}\mathbf{X} + \widetilde{\mathbf{f}}_{\mathbf{d}} \tag{21}$$

where $\tilde{K} = K - K^*$ and $\tilde{f}_d = f_d - \tilde{f}_d$.

The reference model in Eq. (14) can be viewed as a low-pass system with u(t) as the control signal and K(t)X(t) as a time varying disturbance, which is not prevented from having highfrequency oscillations. Instead of conventional control input for MRAC, a low-pass filtered adaptive version of the control algorithm is proposed for Eq. (15) as:

$$u(s) = F_{LP}(s) (KX(s) + u_c(s))$$
⁽²²⁾

where u(s), X(s) and $u_c(s)$ are the Laplace transformations of u(t), X(t) and $u_c(t)$, respectively. $F_{LP}(s)$ is a low-pass filter with low-pass gain 1. The design procedure for the low-pass filter has been addressed in Refs. [6] and [27]. It has also denoted that the small gain requirement can always be met by increasing the bandwidth of the low pass filter $F_{LP}(s)$.

Consider the following Lyapunov function candidate:

$$V(\mathbf{e}, \widetilde{K}) = \frac{1}{2} \mathbf{e}^{\mathrm{T}}(\mathbf{t}) \mathrm{P}\mathbf{e}(\mathbf{t}) + \frac{1}{2} \mathrm{tr}\left(\widetilde{K}^{\mathrm{T}}(\mathbf{t})\mu^{-1}\widetilde{K}(\mathbf{t})\right) + \frac{1}{2} \widetilde{\mathbf{f}}_{\mathrm{d}}^{\mathrm{T}}(\mathbf{t}) \widetilde{\mathbf{f}}_{\mathrm{d}}(\mathbf{t})$$
(23)

Differentiating V with respect to time yields

$$\dot{V} = \frac{1}{2} e^{T} (A_{c}^{T} P + PA_{c}) e + e^{T} PB\widetilde{K}(t) X(t) + tr \left[\dot{K}^{T} \mu^{-1} \widetilde{K} \right] + e^{T} P \widetilde{f}_{d} + \hat{f}_{d}^{T} \widetilde{f}_{d}$$
(24)
$$= -\frac{1}{2} e^{T} Q e + e^{T} PB\widetilde{K}(t) X(t) + tr \left[\dot{K}^{T} \mu^{-1} \widetilde{K} \right] + e^{T} P \widetilde{f}_{d} + \hat{f}_{d}^{T} \widetilde{f}_{d}.$$

To make $\dot{V} \leq 0$, the adaptive law is chosen as

$$\dot{\tilde{K}}(t) = \dot{K}(t) = -\mu B^{T} P e(t) X^{T}(t)$$

$$(25)$$

$$\hat{f}_{4} = -P^{T} e$$

$$(26)$$

in which K(0) and $\hat{f}(0)$ are determined arbitrarily. Using this adaptation law one can achieve

$$\dot{V} = \frac{1}{2} \mathbf{e}^{\mathrm{T}} \mathbf{Q} \mathbf{e}.$$
(27)

Assuming $Q = -I_{nxn}$, \dot{V} becomes a negative definite matrix and Lyapunov stability conditions are met.

It has been shown that if $\omega_1 \neq \omega_2$, i.e., the excitation frequencies on the x and y axes are different, there would be persistent excitment [2]. Since $\tilde{K} \rightarrow 0$, then the unknown angular velocity as well as all other unknown parameters can be determined from $A + BK^T = A_m$.

To design the low-pass filter, let:

$$F_{LP}(s) = \omega (I_{m \times m} + D(s)\omega)^{-1}D(s).$$
 (28)

The choice of D(s) and K needs to ensure that [9]:

(i) $F_{LP}(s)$ is strictly proper and stable with $F(0) = I_{m \times m}$;

(ii) $F_{LP}(s)H_c^{-1}(s)$ is proper and stable;

(iii) $\|\overline{G}(s)\| L < 1$, (small gain theory)

where

$$L = \max_{i} \left(\sum_{j} |k_{ij}| \right) \tag{29}$$

$$\bar{G}(s) = H_c(s)(I_{m \times m} - F_{LP}(s)).$$
(30)

In which I_{mom} denotes the identity matrix. For simplicity, one can consider the following most common choice of D(s) as $D(s) = (1/s)I_{mom}$, which leads to

$$F_{LP}(s) = \omega (sI_{m \times m} + \omega)^{-1}. \tag{31}$$

т	0.57e - 8kg;
$K_{_{xx}}$	80.98 <i>N</i> / <i>m</i>
$C_{_{XX}}$	0.429e - 6Ns/m
K_{yy}	71.62 <i>N</i> / <i>m</i>
${\cal C}_{yy}$	0.687e - 3Ns / m
$K_{_{xy}}$	5 <i>N / m</i>
C_{xy}	0.0429e - 6Ns/m
$\omega_{_0}$	1kHz
q_{0}	1e-6m

Table 1. Parameter of the MEMS gyroscope.

Lemma 1 For a stable proper MIMO system H(s) with input $r(t) \in \mathbb{R}^m$ and output $x(t) \in \mathbb{R}^n$ we have $||x(t)||_{\infty} \le ||H(s)|| ||r(t)||_{\infty}$, $\forall t \ge 0$.

It follows from Lemma 1 that

$$\|\bar{G}(s)\|_{1} \le \|H_{c}(s)\|_{1} \|I_{m \times m} - F_{LP}(s)\|_{1}.$$
(32)

Since $||H_c(s)||_1$ and *L* are constant parameters, requirement (iii) can be met only by changing ω . To satisfy the requirement (iii), $-\omega$ must be a Hurwitz matrix.

It is worthy to summarize the whole design method as follows:

(1) Selecting the desired reference system behavior (ω_1, ω_2) or a related A_m (Eq. (12)) and K_n .

(2) Selecting adaption gain (μ).

(3) Finding an appropriate ω to ensure Eq. (32).

4. Simulation results

This section gives simulation results for the proposed adaptive controller derived in the previous section. The adaptive controller was performed on a MEMS gyroscope model in MATLAB/SIMULINK. The parameters of the lumped model are given in Table 1. Its response is obtained by simulating the lumped model 3 and 4 under the controller 9 operating with the estimation algorithm 18.

The unknown angular velocity Ω is assumed to be a sinusoidal function as $\Omega = \sin 100t$ and the initial condition on K matrix is $K(0) = 0:95K^*$. K_e is selected as a zero matrix, and, therefore, $A_c = A_m$. The desired motion trajectories are $x_m = \sin(\omega_1 t)$ and $y_m = 1.2\sin(\omega_2 t)$, where $\omega_1 = 4.16$ kHz and $\omega_2 = 5.11$ kHz. Letting $\omega = 10$, it can be verified numerically that the condition (iii) is satisfied for all possible $1 < \omega < 10$. The simulation results are shown in Figs. 3 to 5. As shown in Fig. 5, the proposed adaptive controller has a fast compensating behavior which makes it possible to achieve a faster response.

5. Conclusion

A modified model reference adaptive controller for MEMS gyroscope, considering its multi input-multi output dynamics,



Fig. 3. Control Effort in x-direction.



Fig. 4. Control Effort in y-direction.



Fig. 5. The real input and Estimated angular velocity.

has been achieved in this paper to estimate the unknown angular velocity. It is assumed that all states of the system are measurable. There is an inevitable cross coupling between the orthogonal axes of a vibrating MEMS gyroscope. The proposed adaptive controller structure can compensate fast for large uncertainties and disturbances. Simulation results demonstrate that the use of the proposed modified adaptive control technique is effective in estimating the gyroscope parameters and angular velocity and provides a sufficient condition of robustness against high adaptation gain, system uncertainties and external disturbances.

Nomenclature

m : Proof mass

- x, y: Proof mass coordinates relative to the table frame
- k_{ii} : Stiffness matrix elements
- c_{ij} : Damping matrix elements
- u_i : Control effort in direction *i*
- Ω^* : Angular velocity
- ω_0 : Reference frequency

q : Extended coordinate

- K_a : Stiffness matrix
- C^* : Damping matrix
- *f* : External disturbances

 $F_{LP}(s)$: Low-pass filter

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