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# Reliability based optimal design of a helicopter considering annual variation of atmospheric temperature[†]

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## Abstract

This study presents the Reliability Based Design Optimization (RBDO) of a helicopter, used in order to guarantee target performances for a large variation of annual atmospheric temperatures. To this end, analytic methods – statistical and empirical equations of aerodynamics, structure, propulsion and so on – are synthetically coupled within the multidisciplinary design analysis tool of the conceptual helicopter. Additionally, an atmospheric temperature model for annual air temperature variation is constructed in bimodal shape by considering 10 years worth of day-averaged air temperature data provided by the Korea Meteorological Administration. Based on this analysis tool and the annual atmospheric temperature model, the RBDO of a helicopter is performed to minimize maximum takeoff gross weight, with the helicopter rotor configuration parameters as a design variable. A Monte-Carlo simulation is used to accurately evaluate the reliabilities of endurance and range. This RBDO strategy is applied to a 22,000lb class medium utility helicopter, and the results are compared with those of a deterministic design optimization (DO) using constant air temperature and baseline helicopter. Through comparison of the results obtained from RBDO and those from the deterministic design optimization, it can be confirmed that the optimal design of RBDO results in greater improvements in performance over the baseline, for a wide range of operating air temperatures, than baseline helicopter and optimal shape of DO using constant air temperature. Therefore, in designing a helicopter to be operated in temperate climatic regions that show large variation in air temperature, such as Korea, China, the U.S.A, and so on, it is important to perform RBDO with reasonable annual air temperature models constructed from well-known and reliable data.

*Keywords*: Annual variation of atmospheric temperature; Bimodal model of annual atmospheric temperature; Helicopter conceptual design; Reliability based design optimization

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## 1. Introduction

It is common to design a helicopter for a constant atmospheric temperature, such as the average temperature of the primary operational location. However, helicopters are operated under air temperatures ranging from below the freezing point to over 303K. Because the performance of a helicopter rapidly degrades under the high temperatures of summer, it is difficult to achieve designed performances in high temperature regions. This is because the aerodynamic performance of a helicopter is influenced by air density, which is influenced by air temperature. Due to this change, range and endurance fall short of their designed values. These performance parameters are further decreased at high temperatures by greater fuel flow requirements for an engine with limited fuel weight.

Reliability Based Design Optimization (RBDO), which can take into account annual air temperature variation, is necessary because air temperature strongly influences helicopter performance. RBDO is one of the optimization techniques that take into account uncertainties which exist in engineering design analysis [1]. Approximated RBDO methods such as the first order reliability method, second order reliability method, and Bayesian approach [2] are useful in performing efficient reliability calculations or reliability calculations with unknown information. Although annual air temperature varies greatly with unknown distribution, an annual air temperature model is sufficiently predictive based on air temperature data provided by a governmental office such as the Korea Meteorological

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Administration (KMA). It is important to model real air temperature distribution and perform RBDO using this air temperature model to account for its uncertainty.

Many studies on the RBDO of helicopters have been conducted recently, but there is no existing study on the effects of air temperature on helicopter performance. Kang et al. [3] and Choi et al. [4] performed studies on the conventional optimization of helicopters. They conducted minimization of maximum takeoff gross weight using the rotor blade configuration parameters as design variable. They found an optimized minimum gross weight while satisfying design constraints. Kim et al. [5, 6] conducted RBDO of a helicopter that considered icing on the rotor blade and fuel flow changes as uncertainties. Kim et al. also performed minimization of maximum takeoff gross weight using rotor blade configuration as design variable. They showed reliable optimum results despite changes in the aerodynamic performance of the rotor blade due to icing and fuel flow changes in the engine. These studies showed optimized results for given design constraints, but they considered air temperature as a constant. In these cases, the optimized results show good performance at constant temperatures, but there have been no studies investigating the performance of these optimized results under variations in air temperature. The uncertainty factors of the RBDO studies above are assumed to have normal distribution, and analysis of those uncertainties is weak. In the absence of sufficient data, it may be reasonable to assume uncertainty model is a normal distribution model; however, it is important to investigate a model of uncertainty with sufficient data, such as air temperature. The study of RBDO of a helicopter using an air temperature model is crucial component of helicopter design.

Thus, this study considers annual air temperature variation while conducting RBDO of a helicopter. To achieve this, a helicopter conceptual design program constructed for multidisciplinary analysis is used to estimate the performance of the helicopter. A model of annual air temperature variation is composed using air temperature data provided by the KMA. Analysis of the air temperature distribution is conducted to find out the shape of the annual air temperature distribution and create an accurate model. The helicopter configuration with minimum takeoff gross weight is investigated. From this optimization, the optimal configuration of RBDO and the optimal configuration of deterministic design optimization (DO), considering air temperature as a constant, are compared to discuss the benefit of an RBDO including air temperature variation.

## 2. Helicopter multidisciplinary analysis

### *2.1 Analysis modules*

Helicopter multidisciplinary analysis for conceptual design [7-9] uses as analysis modules for reliability based design optimization (RBDO). These analysis modules include environment, aerodynamic analysis, propulsion analysis, weight analysis, performance analysis, and so on. Each module is connected to other modules, as shown in Fig. 1 [10], and each exchanges data with others. The atmosphere module calculates atmospheric conditions using input data, the mission module analyzes a helicopter's mission, and the configuration module determines the configuration data of the helicopter using statistical equations [11]. The aerodynamic analysis modules consist of the main rotor, tail rotor, and fuselage aerodynamic analysis [12]. The main rotor aerodynamic analysis exchanges data with main rotor inflow [13] to calculate main rotor aerodynamic performance accurately. Trim analysis [14] and aerodynamic analysis work together to calculate the trim conditions using trim equations and accurate aerodynamic performance. The propulsion module consists of empirical equations [15] and real engine data [16], and this study uses the real engine data of a UH-60. Weight analysis [17] and cost analysis [18] use statistical equations. From the outputs of the above analyses, the performance module calculates aspects of helicopter performances such as range and endurance.

In the atmosphere module, air density is calculated from air temperature change via Eq. (1), which is derived from the ideal gas law [19].

$$\rho(T) = \frac{p}{RT} \tag{1}$$

In Eq. (1), p is air pressure and R is the universal gas constant.

In the aerodynamic analysis, the main rotor and tail rotor aerodynamic analyses adopt the blade element method (BEM) [20]. The BEM calculates aerodynamic performances using Eqs. (2)-(4) and Fig. 2.

$$dL = \frac{1}{2}\rho U^2 c C_l(\alpha) dy \tag{2a}$$

$$dD = \frac{1}{2}\rho U^2 c C_d(\alpha) dy \tag{2b}$$

$$dF_z = dL\cos\phi - dD\sin\phi \tag{3a}$$

$$dF_x = dL\sin\varphi + dD\cos\varphi \tag{3b}$$

$$dT = N_b dF_z \tag{4a}$$

$$dP = N_b dF_x \Omega y \tag{4b}$$

Eqs. (2a) and (2b) are used to calculate the lift and drag forces of the blade element, Eqs. (3a) and (3b) calculate perpendicular and parallel force of blade element to the rotor disk plane. Thrust and required power of the rotor blade are obtained using Eqs. (4a) and (4b) via the integrals of *dT* and *dP*. Required power is used in the engine analysis to calculate the fuel consumption rate.

In the engine analysis, the fuel consumption rate is calculated using empirical equations and real engine data. Fuel consumption rate is calculated using Eq. (5) [14].

$$\dot{W}_F = C_1 \delta \sqrt{\theta} HP_{MAX} + C_2 HP_R \tag{5}$$

$C_1$ and $C_2$ are engine factors that represent characteristics of

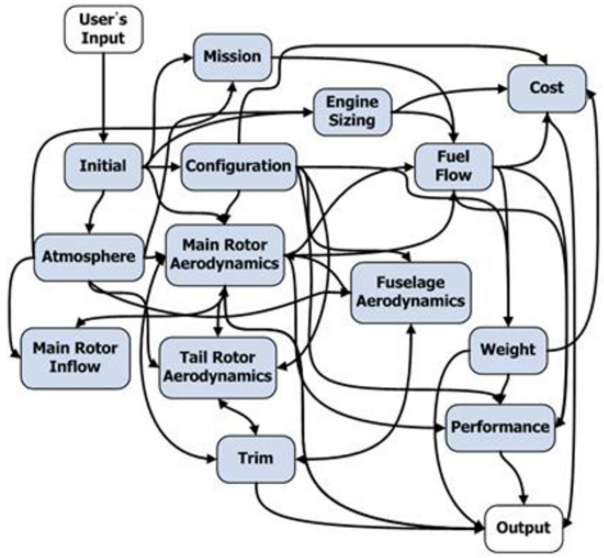

Fig. 1. Flow chart of helicopter multidisciplinary analysis.

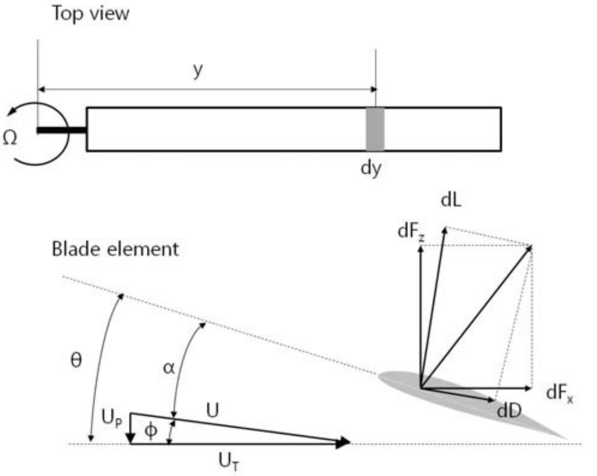

Fig. 2. Velocities and aerodynamic environment at a typical blade element.

each engine, $\delta$ is the ratio of atmospheric pressure at a specific altitude and $HP_{MAX}$ means the maximum power of the engine. In Eq. (5), fuel consumption rate is influenced by the ratio of ambient temperature to the standard day sea level temperature ($\theta$) and by the required power ($HP_R$). Air temperature and required power also influence the fuel consumption rate through the real engine data module.

In the performance analysis, range and endurance are calculated using Eqs. (6) and (7) below:

$$SR = \frac{V}{\dot{W}_F} \tag{6a}$$

$$SE = \frac{1}{\dot{W}_F} \tag{6b}$$

$$Range = \int SRdW \tag{7a}$$

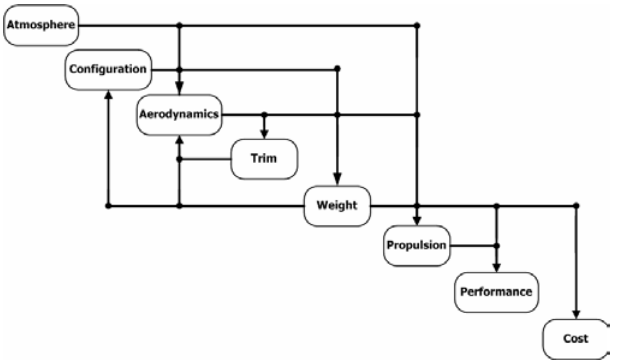

Fig. 3. Design structure matrix of helicopter multidisciplinary analysis.

$$Endurance = \int SEdW \tag{7b}$$

*SR* and *SE* are specific range and specific endurance, i.e. range and endurance values per fuel consumption rate. Range and endurance values are calculated using the integrals of *SR* and *SE*.

Helicopter multidisciplinary analysis contains a number of coupled modules and variables that require more calculation time. To reduce these backward couplings, the design structure matrix is optimized using parallel decomposition. Its result is shown in Fig. 3.

### *2.2 Effect of atmospheric temperature on helicopter performance*

As seen in Fig. 4, air temperature influences the performance of a helicopter because the effect of air temperature propagates from the atmosphere module to the aerodynamic and fuel flow modules. In the atmosphere module, air density $\rho$ is a function of air temperature $T$ as shown in Eq. (1). Eqs. (2)-(4) show the effects of air temperature on aerodynamic analysis. Lift $dL$ and drag force $dD$ are calculated by Eq. (2) with changed air density $\rho$, and parallel force $dF_x$ is influenced by lift and drag forces by Eq. (3b). The required power of the rotor $dP$ is changed through Eq. (4b) and the fuel consumption rate of the fuel flow module is influenced by the change of required power $HP_R$, as shown in Eq. (5).

As is also seen in Fig. 5, air temperature $T$ of the atmosphere module influences the fuel consumption rate of the engine module by affecting the ratio of ambient temperature to standard day sea level temperature $\theta$ in Eq. (5).

Through these variations in the fuel consumption rate, specific range $SR$ and specific endurance $SE$ are changed in Eq. (6). Finally, range and endurance are changed by Eq. (7).

### *2.3 Validation of helicopter multidisciplinary analysis*

A 22,000lb class helicopter is designed using the conceptual design program for a validation. It is compared with a S-70A [21]-civil version of the UH-60-that has a similar gross weight. This class of helicopter is a medium utility helicopter operated

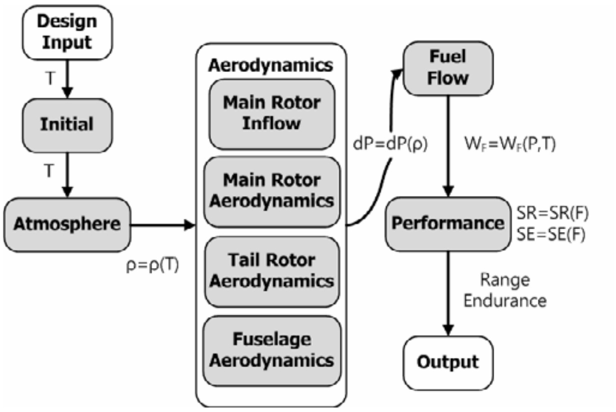

Fig. 4. Effect of air temperature on air density through aerodynamic analysis module.

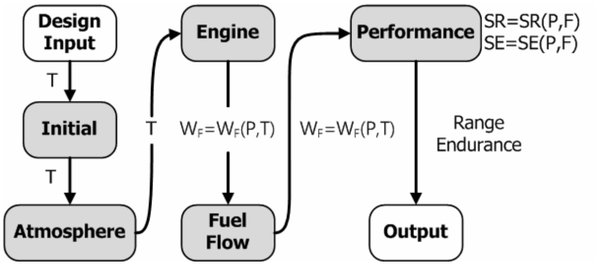

Fig. 5. Effect of air temperature on fuel flow rate through engine analysis module.

as a multirole helicopter worldwide. The designed configuration of a 22,000lb class helicopter is the BASEHEL. The BASEHEL is designed to have the same range and endurance as the S-70A [21] using helicopter conceptual multidisciplinary analysis. The compared results are shown in Table 1.

As shown in Table 1, the configurations of the two helicopters are similar, the error range of the gross weight is 3%, and the fuel weight is 5%. A maximum error of 7% is observed for the endurance.

Table 1. Comparison between BASEHEL and S-70A.

| | | BASEHEL | S-70A [21] |
|---|---|---|---|
| Main rotor | Radius (ft) | 26.12 | 26.84 |
| | Chord (ft) | 1.87 | 1.73 |
| Tail rotor | Radius (ft) | 5.31 | 5.5 |
| | Chord (ft) | 0.73 | 0.81 |
| Fuselage length (ft) | | 51.23 | 50.75 |
| Weight (lbs) | Gross Weight | 21288 | 22000 |
| | Empty Weight | 11761 | 11744 |
| | Fuel Weight | 2461 | 2338 |
| Range (nm) | | 246.5 | 248 |
| Endurance (hr) | | 2.88 | 2.69 |

## 3. Annual variation of air temperature of a temperate climatic region

### *3.1 Annual variation of atmospheric temperature*

The effects of air temperature are significant in areas characterized by large variations in air temperature, such as temperate climatic regions with 4 distinct seasons. In this study, the atmospheric temperature data of Inje, Korea – a temperate climatic region – is analyzed in order to consider annual air temperature variation as uncertainty in RBDO.

Inje is a mountainous area near Seoul, the capital city of Korea. Helicopters are constantly operated in this area for transportation, extinguishing forest fires, etc. For this reason, Inje was selected for this study to investigate annual air temperature variation. All air temperature data used in this study has been provided by the Korea Methodological Administration [22].

Frequency via air temperature is described in Fig. 6 using the air temperature data of Inje from the past 3, 10, and 20 years.

As seen in Fig. 6, all air temperature distributions show similar characteristics. There are 2 local maxima around 275K and 295K and a local minimum around 280-285K. Distributions using more than 3 years' worth of air temperature data show similar shapes. The root mean square error (RMSE) between 3 years and 10 years, and between 10 years and 20 years is 0.026 and 0.012, respectively. The RMSE between 10 years and 20 years is smaller than the others, meaning the 10-year data could be used to represent the 20-year data. From this analysis, the 10 year air temperature distribution is adopted as the annual air temperature variation shape.

Meanwhile, just as air temperature varies by season, there is air temperature variation within a given day. For this reason, day-averaged and hour-averaged air temperature data are collected over 10 years and analyzed to investigate the annual air temperature variation, as depicted in Fig. 7. All distributions in Fig. 7 show similar characteristics to those in Fig. 6. Day-averaged and hour-averaged distributions show almost the same shape. The RMSE between the day-averaged and hour-averaged distributions is 0.025. This analysis indicates that day-averaged air temperature data distribution could be used to represent annual air temperature variation. Therefore, the 10-years data of day-averaged air temperatures are used to construct the annual air temperature variation model.

From these analyses of air temperature distribution, the 10-years day-averaged air temperature distribution is not assumed to be incomplete information because it can accurately describe real air temperature variations. Rather than using the Bayesian approach, which is applied to RBDO when dealing with incomplete information, an annual air temperature model is constructed based on these analyses.

### *3.2 Modeling of annual variation of atmospheric temperature*

It is observed that annual air temperature variation has 2 lo-

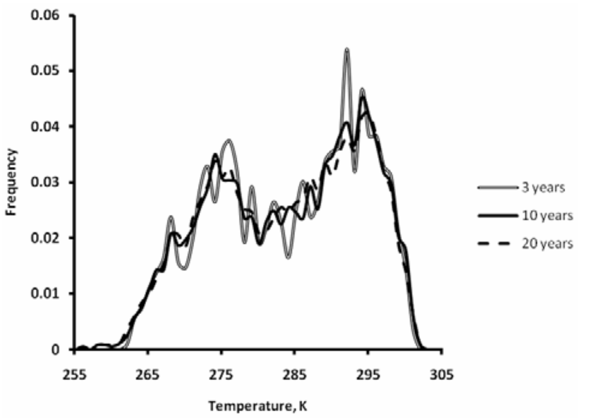

Fig. 6. Annual temperature variations of Inje, Korea using 3, 10 and 20 years of air temperature data.

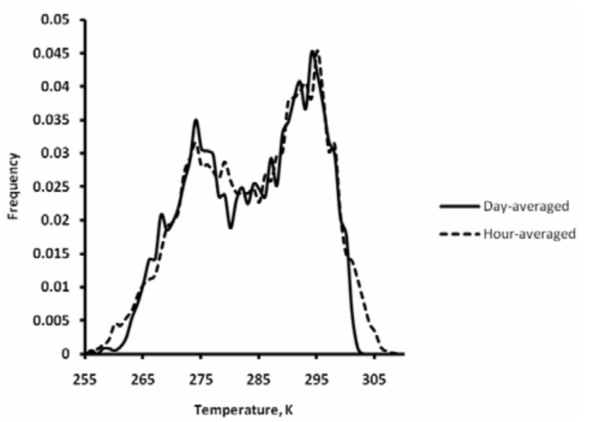

Fig. 7. Annual temperature variations of Inje, Korea using 10 years of month, day and hour-averaged air temperature data.

cal peaks, as seen in Figs. 6 and 7. This distribution could be described as bimodal distribution, which combines 2 normal distribution equations [23] as seen in Eq. (8).

$$F(x) = w\frac{1}{\sqrt{2\pi\sigma_1^2}}\exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right) + (1-w)\frac{1}{\sqrt{2\pi\sigma_2^2}}\exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right) \tag{8}$$

Using Eq. (8), the bimodal annual air temperature model of Inje is constructed, and in Fig. 8 it is compared with 10 years' worth of day-averaged air temperature distribution.

To fit real data to the bimodal distribution, Eq. (8) is used to calculate the coefficients of the bimodal model in such a way as to minimize the real air temperature and value of the bimodal distribution. The results are summarized in Fig. 8. The bimodal model accurately describes the 10 years of day-averaged air temperature data. Like the real air temperature distribution, the bimodal model also shows 2 local maxima at 275K and 293K. The RMSE between the bimodal model and the real air temperature distribution is less than 0.16, and the coefficient of determination ($R^2$) of the bimodal model is more than 0.9. Thus, the bimodal distribution model shows good agreement with the real annual air temperature distribution and it is reasonable to apply the bimodal annual air temperature model to the RBDO of a helicopter.

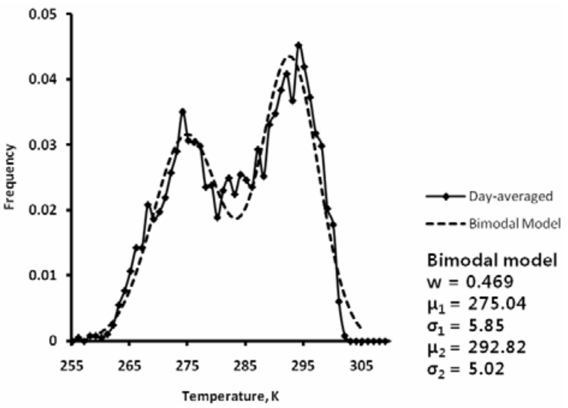

Fig. 8. Comparison between bimodal model and 10 years of day-averaged air temperature distribution of data from Inje, Korea.

## 4. Formulation of a helicopter conceptual design problem

Based on the designed result of the 22,000lb-class helicopter (BASEHEL), two design optimizations were carried out in an attempt to increase the helicopter's reliability under annual air temperature variations. Reliability based design optimization (RBDO) considers annual air temperature variation, and deterministic design optimization (DO) treats the air temperature as a constant. To account for annual air temperature variation, the bimodal model of annual air temperature is considered an uncertainty in the RBDO.

### 4.1 Selection of design variables

Configuration of the rotor blade, which is crucial to the conceptual design of a helicopter, is used as a design variable. The main rotor blade radius, chord, and twist angle and the tail rotor radius and chord are used in the optimization design. These variables significantly influence the gross weight and performance of the helicopter. The upper and lower limits of the design space are 20% from the baseline values of the design variables, as shown in Table 2.

### 4.2 Definition of design problem

Minimization of the maximum takeoff gross weight (MTOW) is identified as the design objective. MTOW is a critical factor in performance and cost due to its effects on required power and production and operational costs. High MTOW helicopters show decreased performance for limited fuel weight because high MTOW helicopters require more power. Low MTOW helicopters, on the other hand, cost

Table 2. Design space of helicopter conceptual design optimization.

| Design variable | Lower | Baseline | Upper |
|---|---|---|---|
| Main rotor radius (ft) | 20.9 | 26.12 | 31.34 |
| Main rotor chord (ft) | 1.5 | 1.87 | 2.24 |
| Main rotor twist angle (°) | -21.6 | -18.0 | -14.4 |
| Tail rotor radius (ft) | 4.25 | 5.31 | 6.37 |
| Tail rotor chord (ft) | 0.58 | 0.73 | 0.87 |

less because production and operational costs are proportional to the MTOW of the helicopter. By minimizing the MTOW, a low cost, high performance helicopter can be obtained.

Range and endurance, the primary performance factors of helicopters, are used as design constraints. Range and endurance are the most important factors in performing a given mission because these represent the radius of action and the duration of flight.

To demonstrate the benefits of RBDO considering annual variation of air temperature, the RBDO results are compared with those of DO, which considers air temperature as a constant. The two optimizations fix the fuel weight at the same baseline (BASEHEL).

The design problem is defined as:

DO

*minimize* Maximum Takeoff Gross Weight (MTOW)

*subject to* $\text{Endurance} \geq \text{Endurance}_{\text{baseline}}$

$$\text{Range} \geq \text{Range}_{\text{baseline}} \tag{9}$$

RBDO

*minimize* Maximum Takeoff Gross Weight (MTOW)

*subject to* $\text{Endurance} \geq \text{Endurance}_{\text{baseline, average temperature}} + k\sigma_{\text{endurance}}$

$$\text{Range} \geq \text{Range}_{\text{baseline, average temperature}} + k\sigma_{\text{range}} \tag{10}$$

where the subscript ‘baseline’ represents the baseline configuration, which is given in Table 1, and the subscript ‘average temperature’ represents the helicopter performance as determined for an annual average temperature. An optimal solution is sought using a progressive quadratic response surface method, and a Monte Carlo simulation is used for a reliability analysis. 100,000 sample data points are used in the Monte Carlo simulation to obtain accurate reliability in the analysis results. It is difficult to get good reliability in air temperatures because helicopter performance is strongly sensitive to air temperature, which can vary by up to 50K. In this study, the reliability factor k is set to 0.9 (probability of success is above than 81%), which is the largest reliability factor capable of obtaining a converged result. This reliability factor was chosen using trial and error, decreasing k from k=2. For the optimization process, PIAnO [24] (a commercial PIDO tool) is adopted.

Table 3. Optimal configuration and performance at mean temperature (286K).

| | | Baseline | DO* | RBDO** |
|---|---|---|---|---|
| Design Variables | Main rotor radius (ft) | 26.12 | 21.74 | 30.72 |
| | Main rotor chord (ft) | 1.87 | 1.5 | 1.51 |
| | Main rotor twist angle (°) | 5.31 | -18.46 | -15.08 |
| | Main rotor radius (ft) | 0.73 | 5.29 | 5.50 |
| | Main rotor chord (ft) | 21288 | 0.58 | 0.597 |
| Objective | MTOW (lbs) | 11761 | 20210.1 | 21761.6 |
| Performance | Endurance (hr) | 2461 | 3.22 | 3.14 |
| | Range (nm) | 246.5 | 270.5 | 262.8 |

* Deterministic design optimization considering air temperature as constant

** Reliability based design optimization considering annual air temperature variation

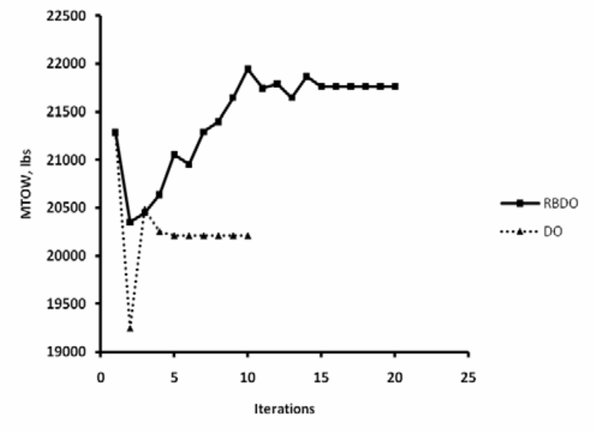

Fig. 9. Optimization history of MTOW.

For the annual air temperature uncertainty model, a bimodal annual air temperature model derived from data collected in Inje, Korea is used to represent the annual variation of air temperature of a temperate climatic region.

## 5. Helicopter conceptual design optimization results and discussion

### 5.1 Optimization history and optimization results

Optimization of medium utility helicopter is performed via reliability based design optimization (RBDO) considering air temperature variation and deterministic design optimization (DO) considering air temperature as constant. The optimization history of the design objective, maximum takeoff gross weight (MTOW), is shown in Fig. 9. As seen in Fig. 9, the RBDO and DO results are converged after 20 and 10 iterations, respectively. RBDO seems to require more iterations than DO because of reliability constraints.

Optimized configurations and their performances at the mean air temperature of Inje are shown in Table 3. Through RBDO and DO, both optimal results show better range and endurance than the baseline. The DO results show 5% improvement in MTOW and 11.8% and 9.7% improved range and endurance, respectively. The DO result includes a smaller rotor configuration to achieve minimization of MTOW. The smaller MTOW requires less power and therefore, DO results in improved performance. The RBDO result shows -2% improvement in MTOW and 9% and 6% improved range and endurance, smaller than the improvements achieved by DO.

However, RBDO results in better performance in high-temperature regions because the optimization process of RBDO considers air temperature variation. Generally, the range and endurance of a helicopter increase as air temperature decreases. For this reason, RBDO results in better performance than the baseline at average temperatures.

### 5.2 Comparison of optimization results

Fig. 10 shows the range and endurance of RBDO, DO, and the baseline as a function of air temperature variation. For range and endurance, the RBDO result shows better performance than the baseline along a 50K variation in air temperature. The RBDO result shows between 2.6% to 5.8% improvement in range and 6.9% to 8.3% improvement in endurance over the baseline. However, the DO result performs below both the baseline and the RBDO result under the hot air temperatures of the summer season, yielding maxima of -5.7% and -11.5% for range and endurance, respectively. Furthermore, the DO result shows rapid performance degradation with air temperature variation. The RBDO result shows better range and endurance above 288K, and it reaches maximum improvements of 9.8% and 17%, respectively, at 305K. The RBDO result shows slightly lower performances in low temperature regions, but the maximum decrease is only 2-4%, which is smaller than the maximum improvement under high temperatures. One of major reasons for this performance variation is the difference between the rotor configurations of RBDO and DO. The better performance of the RBDO result in high-temperature regions is due to a longer main rotor radius. At high air temperatures, air density is degraded according to Eq. (1), and the helicopter rotor rotating speed (Ω) or angle of attack (α) as shown in Eqs. (2) and (4b) must increase in order to maintain the same power. A longer main rotor radius may generate more power by slightly increasing its rotating speed or angle of attack at high air temperatures. However, the DO result has a relatively short main rotor radius, which requires a high rotating speed or high angle of attack in order to generate additional power. Increasing the rotating speed of the rotor blade causes aerodynamic effects such as transonic effect and shock at the tip area of rotor blade, and these effects cause additional drag force. Increased angle of attack increases the power to the helicopter but also causes greater drag force. Because of these increased drag forces, the helicopter requires a higher fuel consumption rate to maintain its power. For this reason, both the RBDO result and the baseline show better performance at high temperatures than does the DO result.

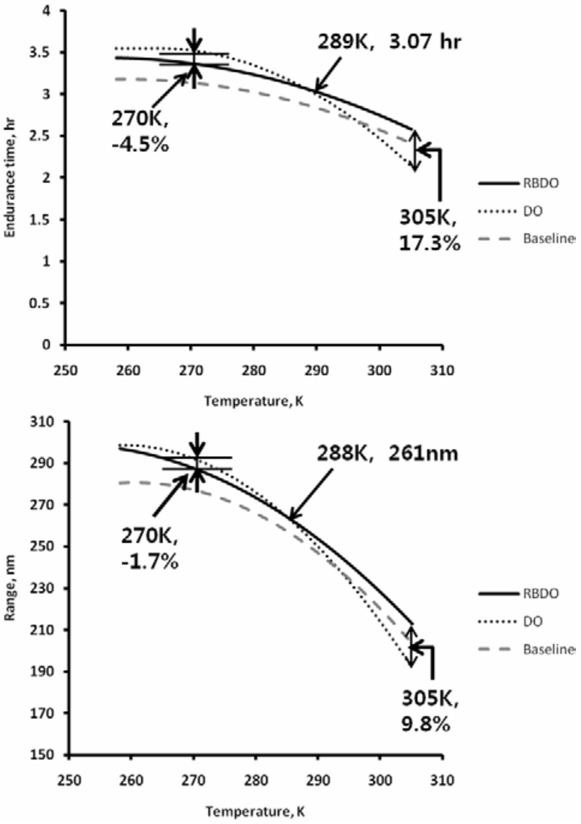

Fig. 10. Performance change as a function of air temperature.

Fig. 11 shows probability density function (PDF) of range and endurance time. Vertical line of each graph of PDF indicates design constraint of each performances and right side of this vertical line is feasible region. RBDO result shows 81.4% and 88.6% reliability in range and endurance time, DO result shows 68.6% and 72.9% reliability in range and endurance time. RBDO result shows almost 13-16% improvement in reliability than DO result while RBDO result shows maximum 10-17% better performance at hot summer air temperature region.

Fig. 11 shows a probability density function (PDF) for range and endurance. The vertical line on each graph of the PDF indicates the design constraint of each performance, and the right side of this vertical line represents the feasible region. The RBDO result shows 81.4% and 88.6% reliability in range and endurance, while the DO result shows 68.6% and 72.9% reliability in range and endurance. The RBDO result shows almost 13-16% improvement in reliability over the DO result, while the RBDO result shows 10-17% better performance in high-temperature regions.

As seen in Fig. 11, the RBDO result shows less variation in performance than does the DO result. The standard deviations of the RBDO result are 26.5 and 0.27 in range and endurance;

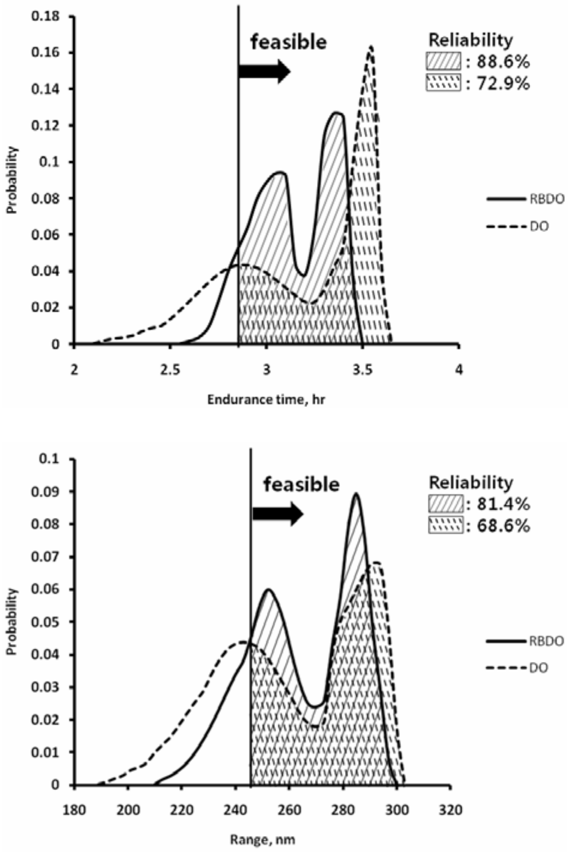

Fig. 11. Probability density function of range and endurance.

however, the DO result shows standard deviations of 34.4 and 0.46 in the same areas. The RBDO result shows 20-40% less standard deviation than does the DO result. Overall, use of RBDO on a helicopter can yield both a more reliable optimal design and a result that is less sensitive to air temperature.

## 6. Conclusions

In this study, Reliability Based Design Optimization (RBDO) of a helicopter was performed, taking annual air temperature variation into consideration. To this purpose, 10 years' worth of day-averaged air temperature data was analyzed to identify annual air temperature distribution. A bimodal model of annual air temperature distribution was then constructed and incorporated into the RBDO as an uncertainty. This RBDO procedure was applied to a 22,000lb-class medium utility helicopter design. Through a deterministic design optimization (DO) in which air temperature was considered a constant, maximum takeoff gross weight was improved 5% and range and endurance achieved 11.8% and 9.7% improvement, respectively, compared to the baseline. With the RBDO, maximum takeoff gross weight was increased by 2% and range and endurance showed 9% and 6% improvement over the baseline. The RBDO result showed approximately 15% improvement in reliability of range and endurance compared to those of the DO result, given an annual air temperature variation. Through comparison of the RBDO result with the DO result, the following conclusions were reached;

(1) RBDO result shows overall improvement performance via annual variation of air temperature. Performance of DO result could not guarantee improvement than baseline at other air temperatures Because DO was performed using a constant air temperature, it resulted in maxima of -5.7% and -11.5% for range and endurance, respectively. However, the performance of the RBDO result showed overall improvement, from a minimum of 2.6% to a maximum of 8.3% over the baseline, and performance variations in the RBDO were smaller than those in the DO. This is because the air temperature variation between 255K and 305K was considered in the RBDO.

(2) Through the RBDO of a helicopter that considers annual air temperature variation, range and endurance improve up to 9.8% and 17% respectively at high temperatures. The maximum takeoff gross weight and performance of the RBDO result at an average air temperature showed less improvement than did the result of DO. However, the RBDO result showed 10-17% performance improvement under temperatures above 288K. One of the major reasons for this performance difference was that the RBDO result had a longer main rotor radius than did the DO result. A helicopter with a longer main rotor radius consumes less fuel than one with a short main rotor blade, because a longer rotor blade requires a lower rotating speed and smaller increase in angle of attack to maintain power at high temperatures. Therefore, the performance of the RBDO result showed greater overall improvement than did the DO result.

(3) Based on 10 years of day-averaged temperature data from Inje, Korea provided by the Korea Meteorological Administration, it was observed that the annual air temperature distribution displayed bimodal characteristics. Using two normal distribution equations, a bimodal annual air temperature distribution model was composed to represent the annual air temperature distribution. This bimodal model obtained a root mean square error (RMSE) of 0.16 and coefficient of determination ($R^2$) of 0.9.

Therefore, realistic and reliable results in helicopter design optimization could be obtained by applying RBDO based on annual air temperature variation, which was represented by a bimodal model.

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## Nomenclature

$c$ : Chord length of airfoil (ft)
$C_d$ : Drag coefficient of airfoil
$C_l$ : Lift coefficient of airfoil
$dD$ : Increment drag force (lbf)
$dF_x$ : Increment parallel force to rotor disk plane (lbf)
$dF_z$ : Increment perpendicular force to rotor disk plane (lbf)
$dL$ : Increment lift force (lbf)
$dP$ : Increment power of rotor (lbf ft/s)
$dT$ : Increment thrust of rotor (lbf)
$dW$ : Increment fuel weight (lb)
$dy$ : Increment length of rotor blade section (ft)
$HP_R$ : Required Power (hp)
$k$ : Reliability index
$N_b$ : Number of rotor blades
$T$ : Air temperature (K)
$U$ : Resultant velocity at blade element (ft/s)
$w$ : Weighting factor
: Fuel consumption rate (lb/hr)
$y$ : Length between rotor blade section and rotating center of rotor (ft)
$\alpha$ : Angle of attack
$\theta$ : Pitch angle, Ratio of ambient temperature to standard day sea level temperature
$\mu$ : Average
$\rho$ : Density (lb/ft3)
$\sigma$ : Standard deviation
$\phi$ : Inflow angle of attack
$\Omega$ : Rotational frequency of rotor

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