

# Propagation behavior of SH waves in layered piezoelectric plates<sup>†</sup>

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## Abstract

In this study, the propagation of SH waves in a coupled plate consisting of a piezoelectric layer and an elastic layer is analytically investigated. The piezoelectric material is polarized in the z-axis direction and perfectly bonded to an elastic layer. The mathematical model of the SH wave propagation in this plate is based on the type of surface wave solution. Dispersion relations with respect to phase velocity are obtained for electrically open and mechanically free. Numerical examples are provided to illustrate graphically and compare the variations of the phase and group velocities versus the wave number for the different layers. The thickness ratio and the properties of the two layers have a significant effect on the propagation of SH waves. The conclusions are meaningful both theoretically and practically for the design of high-performance surface acoustic wave (SAW) devices.

*Keywords:* SH waves; Piezoelectric material; Dispersion relation; Phase velocity

## 1. Introduction

The most important property of piezoelectric materials is their strong coupling between electric and mechanical constitutive behavior. This coupling nature makes these materials useful for many applications in science and technology such as in displacement transducers, micropositioners, rotary actuators, and sensors. In recent years, piezoelectric materials have been integrated with structural materials to form a class of smart structures. In these applications, piezoelectric shear horizontal type (SH) surface acoustic waves (SAWs) have been adopted in order to achieve high performance. Love [1] investigated the SH-SAWs in isotropic composite structures. These SH-SAWs, known as the Love waves refer to shear waves that are horizontally polarized and propagate at the surface of a piezoelectric material. Curtis and Redwood [2] proposed a solution for the dispersion characteristics of the SH wave in a piezoelectric material and the conditions for the existence of various modes. In recent years, many papers have addressed the propagation of surface electro-elastic Love waves in piezoelectric layered structures [3, 4]. Liu et al. [5, 6], Jin et al. [7] and Qian et al. [8] studied the effect of initial stress on the propagation behavior of Love wave in an elastic substrate with a piezoelectric layer and a piezoelectric sub-

strate with an elastic layer. Yang [9] investigated the propagation of Love waves on a piezoelectric half-medium of polarized ceramic carrying an elastic layer from the three-dimensional equations with full electromagnetic coupling. The magnetoelectric effect of piezoelectric/piezomagnetic composites has been studied by many researchers [10-12]. With the development of the material technology, a new kind of material called functionally graded material (FGM) has been manufactured to improve the efficiency in many applications. Liu et al. [13, 14], Li et al. [15], Du et al. [16] and Qian et al. [17-23] considered the Love wave propagation in a layered structure with graded materials and investigated the influence of gradient coefficients on the phase velocity and electromechanical coupling factors, respectively. However, very few studies have considered the influence of the thickness of the substrate.

In this paper, we analytically investigate SH waves in a layered structure with a piezoelectric layer on a finite elastic substrate and the thickness ratio of the layered structure. The piezoelectric material is polarized in the z-axis direction and the analytical solution of dispersion relations with respect to phase velocity can be obtained for electrically open and mechanically free. The numerical results are presented and discussed.

## 2. Formulation of the problem

Consider an elastic layer of thickness  $h_2$  with its surface bonded perfectly by a layer of piezoelectric material with thickness  $h_1$ , as illustrated in Fig. 1. The piezoelectric material

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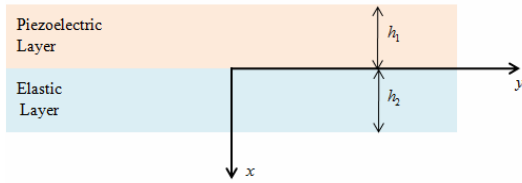


Fig. 1. Geometry of the layered piezoelectric plates.

is polarized along the z-axis direction, perpendicular to the x-y plane. Taking transverse surface wave propagation in such a layered piezoelectric structure into account, it is assumed that the wave propagation is in the y-axis direction, so the out-of-plane displacement and the electric potential can be expressed as

$$u(x, y) = 0, v(x, y) = 0, w = w(x, y, t), \phi = \phi(x, y, t), \quad (1)$$

where  $u$ ,  $v$ , and  $w$  are the mechanical displacement in the x, y, and z direction, respectively, and  $\phi$  is the electric potential.

Generally, the constitutive equations of the piezoelectric medium can be written as

$$\begin{aligned} \sigma_{ij} &= c_{ijkl} S_{kl} - e_{kij} E_k \\ D_j &= e_{jkl} S_{kl} + \varepsilon_{jk} E_k, \end{aligned} \quad (2)$$

where  $\sigma_{ij}$ ,  $S_{kl}$ ,  $D_j$ , and  $E_k$  are the stress tensor, strain tensor, electric displacement and intensity of the electric field, respectively, and  $c_{ijkl}$ ,  $e_{kij}$ , and  $\varepsilon_{jk}$  are the elastic, piezoelectric and dielectric constants, respectively.

The general forms of the motion differential equations of the piezoelectric medium can be expressed as

$$\begin{aligned} \sigma_{ij,j} &= \rho \ddot{u}_i, \\ D_{i,i} &= 0, \end{aligned} \quad (3)$$

where  $u_i$  is the mechanical displacement components,  $\rho$  is the mass density, and the dot on top of the displacement components is the time differentiation.

From Eqs. (1) and (2), the constitutive equations of the piezoelectric materials can be written as

$$\begin{aligned} \tau_{yz} &= c_{44} \frac{\partial w}{\partial y} + e_{15} \frac{\partial \phi}{\partial y} \\ \tau_{zx} &= c_{44} \frac{\partial w}{\partial x} + e_{15} \frac{\partial \phi}{\partial x} \\ D_x &= e_{15} \frac{\partial w}{\partial x} - \varepsilon_{11} \frac{\partial \phi}{\partial x} \\ D_y &= e_{15} \frac{\partial w}{\partial y} - \varepsilon_{11} \frac{\partial \phi}{\partial y}. \end{aligned} \quad (4)$$

The general forms of the motion differential equations of the piezoelectric medium can be obtained through Eqs. (3) and (4):

$$c_{44} \nabla^2 w + e_{15} \nabla^2 \phi = \rho \frac{\partial^2 w}{\partial t^2} \quad (5)$$

$$e_{15} \nabla^2 w - \varepsilon_{11} \nabla^2 \phi = 0.$$

The wave equations of the shear waves in the elastic layer are given as:

$$c_{44}^m \nabla^2 w^m = \rho^m \frac{\partial^2 w^m}{\partial t^2} \quad (6)$$

$$\nabla^2 \phi^m = 0,$$

where  $c_{44}^m = E/(1+\nu)$  is the shear modulus,  $\rho^m$  is the mass density,  $\nu$  is the Poisson ratio,  $E$  is the Young's modulus,  $w^m$  is the displacement of the host medium in the z-axis direction, and  $\nabla^2$  is the Laplacian operator given by  $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2)$ . The superscript m indicates the properties in the substrate. The shear stress can be expressed as

$$\begin{aligned} \tau_{xz}^m &= c_{44}^m \frac{\partial w^m}{\partial x} \\ \tau_{yz}^m &= c_{44}^m \frac{\partial w^m}{\partial y}. \end{aligned} \quad (7)$$

The surfaces of the piezoelectric layer are assumed to be mechanically free and electrically open. The boundary conditions can be described as follows:

at  $x = -h_1$ :

$$\begin{aligned} \tau_{xz} &= 0 \\ D_x &= 0 \end{aligned} \quad (8)$$

at  $x = h_2$ :

$$\begin{aligned} \tau_{xz}^m &= 0 \\ D_x^m &= 0. \end{aligned} \quad (9)$$

The continuity conditions at the interface between the piezoelectric layer and the substrate are written as follows:

at  $x = 0$ :

$$\begin{aligned} w &= w^m \\ \tau_{xz} &= \tau_{xz}^m \\ \phi &= \phi^m \\ D_x &= D_x^m. \end{aligned} \quad (10)$$

### 3. Formulation of the problem

The solution of  $w^m$  and  $\phi^m$  for the wave propagation in the y-axis direction can be written as:

$$\begin{aligned} w^m &= f^m(x)e^{ik(y-ct)} \\ \phi^m &= g^m(x)e^{ik(y-ct)}. \end{aligned} \tag{11}$$

Substituting Eq. (11) into Eq. (6), the solutions of  $w^m$  and  $\phi^m$  are obtained by

$$\begin{aligned} w^m &= (D_1e^{-\lambda_1x} + D_2e^{\lambda_1x})e^{ik(y-ct)} \\ \phi^m &= (C_1e^{-kx} + C_2e^{kx})e^{ik(y-ct)}, \end{aligned} \tag{12}$$

where  $\lambda_1 = k\sqrt{1 - c^2/(c_{sh}^m)^2}$  and  $c_{sh}^m = \sqrt{c_{44}^m/\rho^m}$  are the velocity of the shear waves in the elastic substrate under the assumption that  $c < c_{sh}^m$ . When  $c > c_{sh}^m$ , such waves represent refracted waves carrying energy from the layer. Such a wave system loses its energy and is not of significance at any distance.

Eq. (5) can be decoupled by introducing the Bleustein [24] function, given by  $\psi = \phi - (e_{15}/\epsilon_{11})w$ , which reduces Eq. (5) to

$$\begin{aligned} \overline{c_{44}}\nabla^2 w &= \rho \frac{\partial^2 w}{\partial t^2} \\ \nabla^2 \psi &= 0, \end{aligned} \tag{13}$$

where  $\overline{c_{44}} = c_{44} + e_{15}^2/\epsilon_{11}$ , which includes the piezoelectric effect. When  $c > c_{sh}$ , the solution of Eq. (13) is:

$$\begin{aligned} w &= (A_1 \cos \lambda_2 x + A_2 \sin \lambda_2 x)e^{ik(y-ct)} \\ \psi &= (B_1 e^{-kx} + B_2 e^{kx})e^{ik(y-ct)}, \end{aligned} \tag{14}$$

where  $\lambda_2 = k\sqrt{c^2/(c_{sh}^m)^2 - 1}$  and  $c_{sh} = \sqrt{c_{44}/\rho}$  is a speed of the piezoelectrically stiffened bulk shear wave. The electric potential also can be obtained as

$$\phi = [(B_1 e^{-kx} + B_2 e^{kx}) + \frac{e_{15}}{\epsilon_{11}}(A_1 \cos \lambda_2 x + A_2 \sin \lambda_2 x)]e^{ik(y-ct)}. \tag{15}$$

Substituting Eq. (12) into Eqs. (4) and (7), the shear stresses and the electric displacement of the substrate can be expressed as

$$\begin{aligned} \tau_{xz}^m &= -c_{44}^m \lambda_1 (D_1 e^{-\lambda_1 x} - D_2 e^{\lambda_1 x})e^{ik(y-ct)} \\ D_x^m &= \epsilon_{11}^m k (C_1 e^{-kx} - C_2 e^{kx})e^{ik(y-ct)}. \end{aligned} \tag{16}$$

The shear stresses and the electric displacement of the piezoelectric layer can be obtained by substituting Eqs. (14) and (15) into Eq. (4):

$$\begin{aligned} \tau_{xz} &= [(-\lambda_2)\overline{c_{44}}(A_1 \sin \lambda_2 x - A_2 \cos \lambda_2 x) \\ &\quad + (-k)e_{15}(B_1 e^{-kx} - B_2 e^{kx})]e^{ik(y-ct)} \\ D_x &= \epsilon_{11} k (B_1 e^{-kx} - B_2 e^{kx})e^{ik(y-ct)}. \end{aligned} \tag{17}$$

#### 4. Dispersion relation

From the boundary conditions (8) and (9) and the continuity conditions (10), we can obtain the dispersive characteristic equations for the piezoelectric coupled plate to determine the unknown constants  $A_1, A_2, B_1, B_2, C_1, C_2, D_1, D_2$  :

$$\begin{aligned} \lambda_2 \overline{c_{44}} [A_1 \sin \lambda_2 h_1 + A_2 \cos \lambda_2 h_1] - k e_{15} [B_1 e^{kh_1} - B_2 e^{-kh_1}] &= 0 \\ B_1 e^{kh_1} - B_2 e^{-kh_1} &= 0 \\ D_1 e^{-\lambda_2 h_2} - D_2 e^{\lambda_2 h_2} &= 0 \\ C_1 e^{-kh_2} - C_2 e^{kh_2} &= 0 \\ A_1 &= D_1 + D_2 \\ \lambda_2 \overline{c_{44}} A_2 - k e_{15} (B_1 - B_2) &= -c_{44}^m \lambda_1 (D_1 - D_2) \\ B_1 + B_2 + \frac{e_{15}}{\epsilon_{11}} A_1 &= C_1 + C_2 \\ \epsilon_{11} k (B_1 - B_2) &= \epsilon_{11}^m k (C_1 - C_2). \end{aligned} \tag{18}$$

For nontrivial solutions of the unknown constants, the determinant of the coefficient matrix of linear algebraic must equal zero, so we can obtain the dispersive relation

$$\begin{aligned} &[\overline{c_{44}} \sqrt{\frac{c^2}{c_{sh}^2} - 1} \tan(kh_1 \sqrt{\frac{c^2}{c_{sh}^2} - 1}) - c_{44}^m \sqrt{1 - \frac{c^2}{(c_{sh}^m)^2}} \\ &\quad \tanh(kh_2 \sqrt{1 - \frac{c^2}{(c_{sh}^m)^2}})] \\ &= \frac{e_{15}^2 \epsilon_{11}^m \tanh(kh_2) \tanh(kh_1)}{\epsilon_{11} [\epsilon_{11}^m \tanh(kh_2) + \epsilon_{11} \tanh(kh_1)]}. \end{aligned} \tag{19}$$

For example, when  $e_{15}, \epsilon_{11}, \epsilon_{11}^m$  vanish and  $h_2$  approaches infinity, Eq. (19) reduces to

$$c_{44} \sqrt{\frac{c^2}{c_{sh}^2} - 1} \tan(kh_1 \sqrt{\frac{c^2}{c_{sh}^2} - 1}) - c_{44}^m \sqrt{1 - \frac{c^2}{(c_{sh}^m)^2}} = 0. \tag{20}$$

This is the well-known dispersion equation for purely elastic layered half-space [25].

#### 5. Numerical simulations and discussions

An analytical equation of the phase velocity has been obtained for the propagation of SH waves in layered piezoelectric plates. In this section, based on the dispersion relation Eq. (19), numerical examples are plotted to illustrate the dispersion behaviors of SH waves. Values of the material properties

Table 1. Material properties used in the computation.

Material constants	PZT-4	PZT-5H	PZT-7	SiO <sub>2</sub>
$c_{44}(10^9 N/m^2)$	25.6	23	25	31.2
$\rho(10^3 kg/m^3)$	7.5	7.5	7.8	2.2
$\epsilon_{11}(10^{-9} C^2/Nm^2)$	6.46	27.7	17.1	3.36
$e_{15}(C/m^2)$	12.7	17	13.5	0.0
$c_{sh}(m/s)$	2596.6	2111.3	2138.1	3765.9

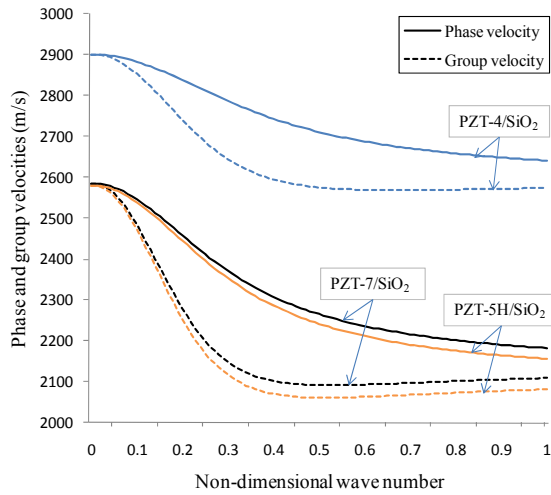


Fig. 2. Phase and group velocities of a three different types of piezoelectric layer, PZT-4, PZT-5H, and PZT-7, on a SiO<sub>2</sub> plate for the first mode.

used in numerical calculation are listed in Table 1, where PZT-4, PZT-5H, and PZT-7 are the piezoelectric materials and SiO<sub>2</sub> is the elastic material.

Also, to illustrate the dispersion relation, the group velocity  $c_g$  which expresses the rate at which energy is transported is introduced. The group velocity can be calculated by the following formula [26]:

$$c_g = \frac{dw}{dk} = c + k \frac{dc}{dk} \tag{21}$$

The non-dimensional wave number is used by  $K = kh_1/2\pi$ . In Fig. 2, the dispersion curves for the first mode of PZT-4/ SiO<sub>2</sub>, PZT-5H/ SiO<sub>2</sub>, and PZT-7/ SiO<sub>2</sub> are shown, respectively. And the thickness ratio of the layered plates is taken as 1. Fig. 2 shows the phase and group velocities approach the bulk shear wave velocity of the piezoelectric material as the wave number increases, and the phase velocity decreases as the wave number increases. However, the group velocity decreases from its maximum value to its minimum value, and then increases toward the bulk shear wave velocity of the piezoelectric material.

Fig. 3(a)-(c) presents the dispersion curves for the first three modes, respectively. The thickness ratio of the layered plates is taken as 1. Fig. 3 shows that the phase velocity approaches

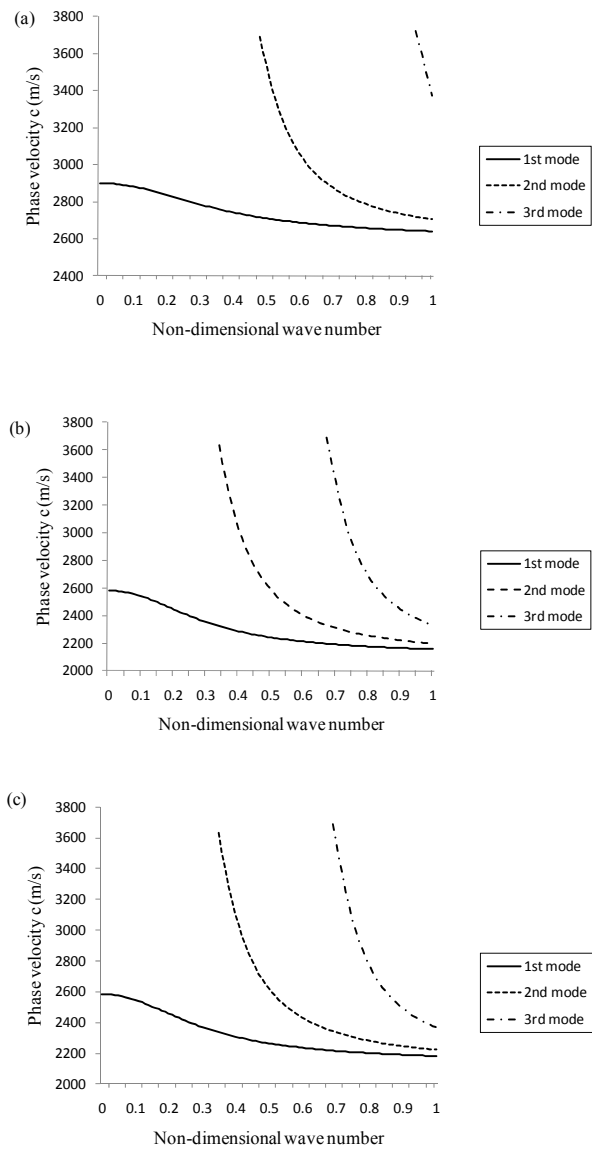


Fig. 3. Dispersion curves of the piezoelectric layered plates for the first three modes. (a) PZT-4/ SiO<sub>2</sub> plate; (b) PZT-5H/ SiO<sub>2</sub> plate; (c) PZT-7/ SiO<sub>2</sub> plate.

the bulk shear wave velocity of the piezoelectric material as the wave number increases for different modes.

In order to provide comparison of the dispersion characteristics, Fig. 4, Fig. 5 and Fig. 6 present the phase and group velocities for the first mode at different ratios of the thickness of the piezoelectric layer to the thickness of the elastic plate. Fig. 4(a), Fig. 5(a) and Fig. 6(a) show that the phase velocity decreases with the decrease in the thickness ratios for the first mode. It is observed that the phase velocities are different for the lower wave number, but when the wave number rises, all the curves converge. This effect is due to the fact that the wave length of the SH wave is comparable to the thickness of the layer at the higher ratios, so the phase velocities vary for different thickness ratios of the layer. Nevertheless, since the

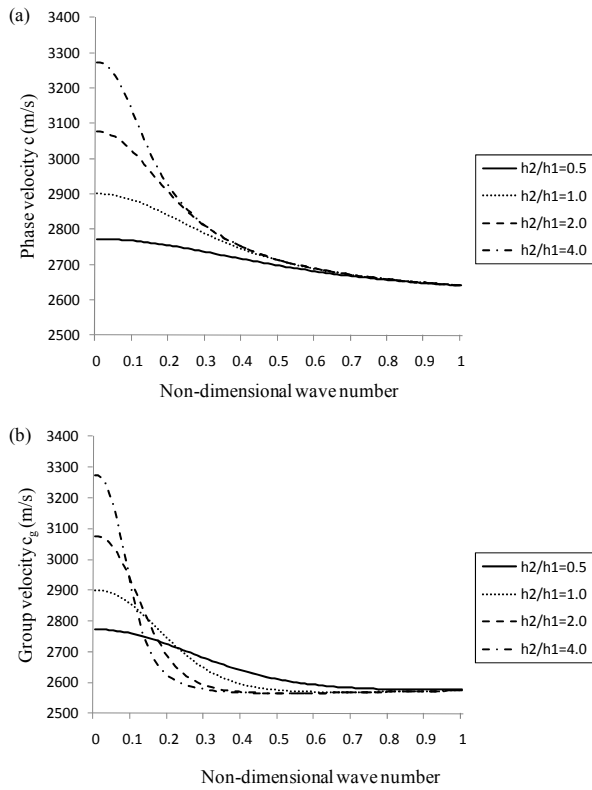


Fig. 4. Dispersion curves of the first mode for different ratios of the thickness of PZT-4/ SiO<sub>2</sub> plate. (a) Phase velocity; (b) Group velocity.

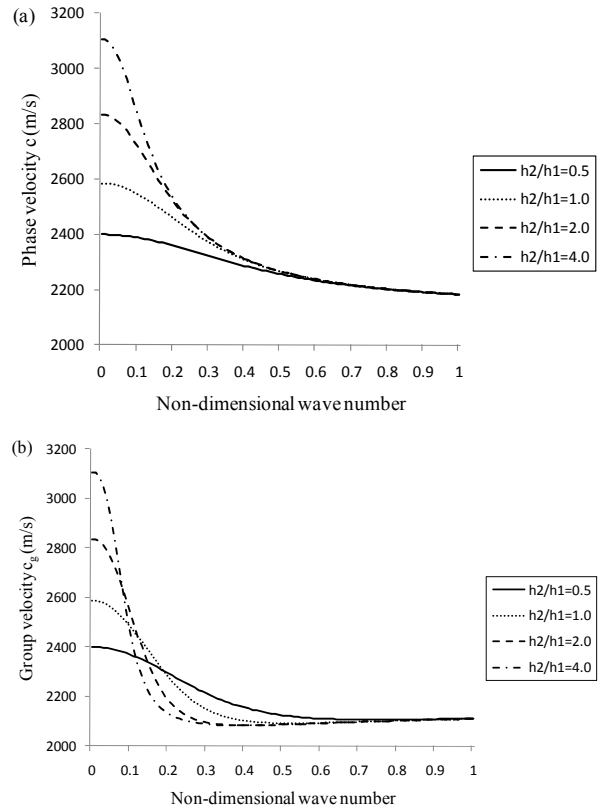


Fig. 6. Dispersion curves of the first mode for different ratios of the thickness of PZT-7/ SiO<sub>2</sub> plate. (a) Phase velocity; (b) Group velocity.

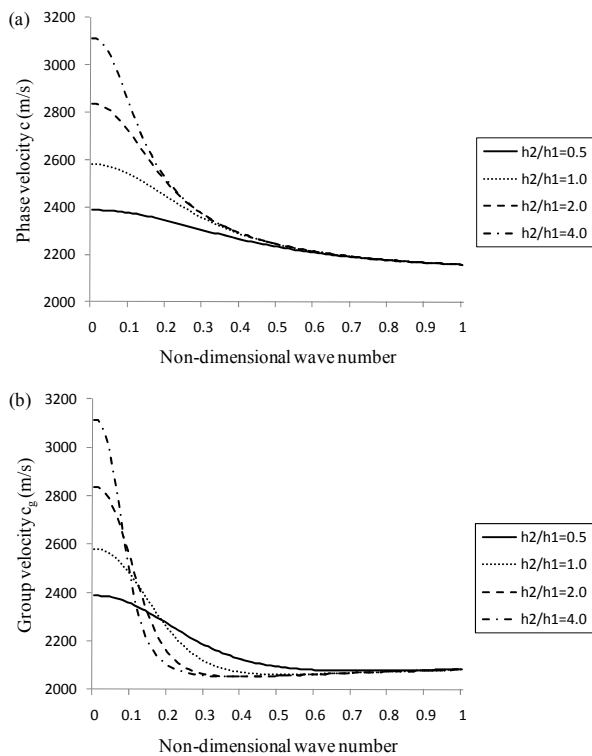


Fig. 5. Dispersion curves of the first mode for different ratios of the thickness of PZT-5H/ SiO<sub>2</sub> plate. (a) Phase velocity; (b) Group velocity.

wavelength of the SH wave is smaller than the thickness of the piezoelectric layer at higher wave number, the piezoelectric layer dominates the characteristics of the SH wave propagation. The group velocity also decreases as the thickness ratios for the first mode decrease and has big discrepancy for the lower wave number, but when the wave number rises, all the curves converge to form Fig. 4(b), Fig. 5(b) and Fig. 6(b). However, the group velocity monotonously decreases at smaller ratios as the wave number increases.

### 6. Conclusions

This paper investigates the propagation behavior of SH waves in layered piezoelectric plates. A general dispersion equation of the wave is derived and the numerical simulations are carried out. The numerical results show that the phase and group velocities approach the shear wave velocity of the piezoelectric layer. The dispersion curves for difference thickness ratios of the piezoelectric layer are also discussed. The thickness ratios have a great effect on the phase and group velocities when the wave number is lower, and all the curves converge as the wave number increases. Group velocity monotonously decreases as the wave number increases at smaller thickness ratios. These results provide a theoretical foundation and a basic model for the design and analysis of surface acoustic wave devices.

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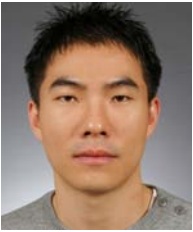
## Nomenclature

$u$	: Mechanical displacement in the x direction
$v$	: Mechanical displacement in the y direction
$w$	: Mechanical displacement in the z direction
$\phi$	: Electric potential
$\sigma$	: Stress
$S$	: Strain
$D$	: Electric displacement
$E$	: Intensity of the electric field
$c$	: Elastic constant
$e$	: Piezoelectric constant
$\varepsilon$	: Dielectric constant
$\rho$	: Mass density
$\nu$	: Possion ratio
$\nabla^2$	: Laplacian operator
$\psi$	: Bleustein function

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