Springer

KSME

Journal of Mechanical Science and Technology 25 (2) (2011) 267~272
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-010-1117-6

Gear fault identification and classification of singular value decomposition based on Hilbert-Huang transform †

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(Manuscript Received July 28, 2009; Revised April 11, 2010; Accepted August 6, 2010)

Abstract

An improved singular value decomposition method of gear fault identification based on Hilbert-Huang transform was proposed to overcome the problem of reconstructing a feature matrix of singular value decomposition. The method includes three steps. First, the instantaneous frequency and amplitude matrices were acquired by Hilbert-Huang transform from faulted gear signals. Second, after the matrices were decomposed by singular value decomposition, the defined distances of singular value vectors and the optimal threshold of the distance for classification were calculated. Third, the fault characteristics of a gearbox were identified and classified by the threshold of the distances. The result demonstrates that the proposed method effectively identifies the gear fault and can realize an automatic gear fault diagnosis.

Keywords: Hilbert-Huang transform; Time-frequency analysis; Singular value decomposition; Gear fault diagnosis

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1. Introduction

Gears and rolling element bearings are critical elements in complex machinery, so predictive maintenance is often applied to them. Signal analysis has been an important and indispensable part of fault diagnosis. Vibration analysis has successfully been applied towards monitoring and diagnosis in many practical areas for three decades. In the application of machine fault diagnosis, vibration signal analysis is used to detect the dynamic characteristics of machines and to extract fault characteristics if a fault occurs and then identify its cause [1, 2]. Fault signals of gearboxes or rolling-element bearings are non-stationary. The signal generated by the inspected machine must be linear and temporally stationary; otherwise, the resulting Fourier spectrum will have little physical sense. The resulting frequency components are not always consistent as the patterns of the acquired signals often change with time. Therefore, the Fourier transform can not fulfill the requirements of fault diagnosis, particularly in real applications. Hitherto, the Fourier transform has dominated in the field of signal analysis because of its prowess and simplicity. However, there are some crucial restrictions on the use of the Fourier transform. Hilbert-Huang transform could analyze non-stationary data well [3, 4]. Although wavelet transform is capable of analyzing nonlinear and non-stationary signals and is deemed suitable for vibration-based machine fault diagnosis, many deficiencies have been reported in the use of wavelet transform [5].

Among those signal analysis, singular value decomposition (SVD) has its own advantages. SVD methods extract algebraic features from stable spectrums. When a small perturbation is added, a large variance of its singular value does not occur. However, for some signals, a different signal may produce the same singular value which makes classification difficult [6-8]. The proposed technique is based on the SVD for reducing noise from a signal's time series using a time–frequency distribution. The feature matrix is associated with the time–frequency representation of the signal. The results from applying the proposed method on synthetic signals indicate superiority of the proposed technique over the existing one in reducing noise from signals [9]. Moreover because of the unavailability of the differential equations of the system for a real vibration signal, some important issues regarding the practical application of the embedding theorem should be carefully considered. First, reconstruction parameters must be determined before reconstructing the state space. Second, noise reduction techniques should be employed to eliminate noise components in a vibration signal while its chaotic components should be kept unmodified. Finally, the phase space

† This paper was recommended for publication in revised form by Associate Editor Eung-Soo Shin

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reconstructed from a vibration signal must be multi-dimensional. To achieve these requirements, the plane of projection of the attractor must be carefully selected [10, 11].

Meanwhile, the feature matrix of SVD can be gotten by a new type of time-frequency analysis called Hilbert-Huang transform (HHT). This transform has been proposed for analyzing nonlinear and non-stationary signals. In comparison to the wavelet transform the EMD, which is the most computationally intensive process in the HHT, does not involve any convolution. Hence, less time is used for computation in EMD and is deemed suitable for analyzing numerous data or signals. The technique of HHT has been used for vibration signal analysis in a number of applications. HHT was used to interpret the nonlinear response of a crack-induced rotor [4, 12]. Traditional methods can not deal with it well. Hilbert-Huang transform is a new way to process non-stationary signals. It is capable of extracting all oscillatory modes present in a signal at different length scales.

This research presents a method of SVD based on Hilbert-Huang transform in order to overcome the traditional problems of SVD. This method includes three main steps: ① The feature matrix for SVD is acquired by computing Hilbert-Huang transform; ② A singular value is obtained using SVD; ③ Characteristics are extracted and faults are identified. In Section 2, the method of singular value decomposition based on HHT is introduced. Then in Section 3, a novel threshold is described from the singular value. Consequently, experiment analysis results are presented in Section 4, followed by discussions and conclusions.

2. Identification methods of singular value decomposition on Hilbert-Huang transform

It is difficult to determine lag time and embedding dimension for phase space reconstruction of SVD, so a method of SVD based on Hilbert-Huang transform is presented. The feature matrix for SVD is acquired by computing Hilbert-Huang transform and the problem of feature matrix reconstruction is avoided.

2.1 Time-frequency distribution on Hilbert-Huang transform

HHT is derived from the principals of empirical mode decomposition and the Hilbert Transform. First, the EMD will decompose the acquired signal into a collection of intrinsic mode functions (IMF). The IMF is a kind of adaptive and nearly orthogonal representation for the analyzed signal. The IMF is almost mono-component, so all the instantaneous frequencies of the IMF can be determined from the nonlinear or non-stationary signals. Second, the amplitude of each instantaneous frequency can be derived through the Hilbert transform. Hilbert spectrum on HHT is an amplitude-frequency-time distribution of the signal.

Each signal $x(t)$ can be decomposed as following Eq. (1) by using EMD method [4].

$$x(t) = \sum_{i=1}^{n} c_i + r_n \tag{1}$$

in which c_i is an IMF. Thus, one can achieve a decomposition of the signal into n IMFs and a residue r_n. The IMFs c_1, c_2, ···, c_n include different frequency bands ranging from high to low. The frequency components contained in each frequency band are different and they change with the variation of the signal $x(t)$. EMD has the following characteristics [4]:

(1) The aim of EMD is to decompose the multi-component signal to a number of signal components.

(2) IMF has a unique local frequency. The good character of narrowband is presented. Then the time-frequency distributing of any IMF can be obtained by Hilbert Transform.

EMD decomposes a time series into components by empirically identifying the physical time scales intrinsic to the data. Each extracted mode, named Intrinsic Mode Function (IMF), is symmetric with respect to zero, and different IMFs do not share the same frequency at the same time.

The Hilbert transform of a function $f(t)$ is defined for all t by

$$\hat{x} = \frac{1}{\pi} \mathrm{P} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} \mathrm{d}\tau \tag{2}$$

where the integral exists.

It is normally not possible to calculate the Hilbert transform as an ordinary improper integral because of the pole at $\tau = t$. However, the P in front of the integral denotes the Cauchy principal value, which expands the class of functions for the integral in Eq. (2).

In EMD and Hilbert Transform, the two key steps of the Hilbert spectrum effectively analyze the data. Hilbert–Huang spectrum can be defined by the following:

$$H(t) = \sum_{j=1}^{\infty} a_j(t) \mathrm{e}^{i\omega_j(t)t} \tag{3}$$

where $a_j(t)$ is the instantaneous amplitude, and the rate of instantaneous phase transformation is the instantaneous frequency $\omega_j(t)$. The relation of the time, frequency and amplitude is represented in the Hilbert spectrum.

2.2 Singluar value decomposition on Hilbert-Huang transform

A singular value decomposition of an $M \times N$ matrix $\boldsymbol{X}$ is of the form

$$\boldsymbol{X} = \boldsymbol{U} \, \Sigma \, \boldsymbol{V}^{\mathrm{T}} \tag{4}$$

where $\boldsymbol{U}$ $(M \times M)$ and $\boldsymbol{V}$ $(N \times N)$ are orthonormal matrices, and Σ is an $M \times N$ diagonal matrix of singular values with components $\sigma_{ij} = 0$ if $i = j$ and $\sigma_{ii} > 0$. Furthermore, it can be shown that there exist nonunique matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ such that σ_{11}, σ_{22}, ···, 0. The columns of the orthonormal matrices $\boldsymbol{U}$ and

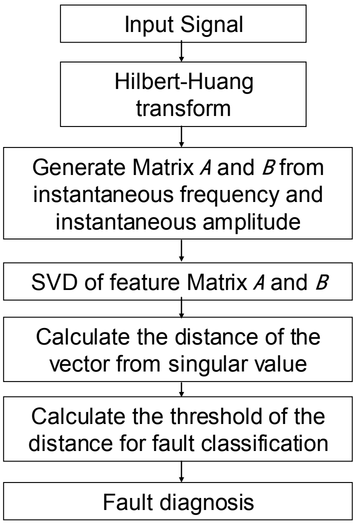

Fig. 1. The scheme for fault identification.

$\boldsymbol{V}$ are called the left and right singular vectors (SVs), respectively. An important property of $\boldsymbol{U}$ and $\boldsymbol{V}$ is that they are mutually orthogonal [10].

The sampling signal is a time series. The precondition of applying SVD is obtaining feature matrix $\boldsymbol{X}$ from the signal. And Hilbert-Huang transform can reconstruct the feature matrix.

3. Proposed scheme for fault identification and its criteria

Characteristics extraction is the key point, and the fault is diagnosed by extracting characteristics. Then, fault identification and classification are realized. The scheme for fault identification follows Fig. 1.

The algorithm of gear fault feature extraction is given as:

Step 1: Calculate Hilbert-Huang transform of the vibration signal.

Step 2: Obtain characteristic matrix $\boldsymbol{A}$ and $\boldsymbol{B}$ from instantaneous frequency and instantaneous amplitude of IMF.

Step 3: Realize SVD of characteristic matrix $\boldsymbol{A}$ and $\boldsymbol{B}$ and obtain singular value.

Step 4: Calculate the distance of singular value vector and the threshold of the distance to classify.

Step 5: Output the classification and realize fault diagnosis.

3.1 The definition of fault characteristics vector

The vibration signal $x(t)$ is acquired from the set-up and the characteristics of Hilbert spectrum are computed as Eq. (3). The instantaneous amplitude a_i and the instantaneous frequency ω_i of each IMF can be obtained. Then the feature matrix $\boldsymbol{A}$ of the instantaneous amplitude a_i of each IMF is obtained as follows:

$$\boldsymbol{A} = [a_1, a_2, \cdots, a_n]. \tag{5}$$

The feature matrix $\boldsymbol{B}$ is made up of the instantaneous frequency ω_i of each IMF as follows:

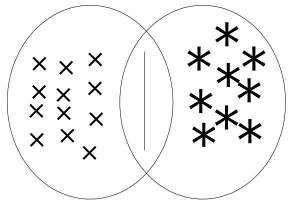

Fig. 2. Data classification to two sets.
× — data of set I
* — data of set II
| — threshold of set I and set II

$$\boldsymbol{B} = [\omega_1, \omega_2, \cdots, \omega_n]. \tag{6}$$

The feature matrix is obtained by computing Hilbert-Huang transform of the sampling signal. Then, the singular value decompostion of matrix $\boldsymbol{A}$ and $\boldsymbol{B}$ can be applied. After computing, the singular values are obtained. The singular value vector of feature matrix $\boldsymbol{A}$ is given as follows:

$$\sigma_1 = <\sigma_{11}, \sigma_{12}, \cdots, \sigma_{1n}>.$$

The singular value vector of feature matrix $\boldsymbol{B}$ is defined as follows:

$$\sigma_2 = <\sigma_{21}, \sigma_{22}, \cdots, \sigma_{2n}>.$$

The relative distance d_i of the singular value vector can be defined as follows:

$$d_i = \frac{\| \sigma_i - \bar{\sigma} \|_2}{\| \bar{\sigma} \|_2} \tag{7}$$

where σ_i —is the singular value vector of the current status, $\bar{\sigma}$ —is the average singular value vector of the normal status. The status will be identified and classified after the characteristics of relative distance d_i are obtained.

3.2 The optimal threshold for classification

If the data can be classified into two sets by the threshold x, the value of variance of set I should be minimal, and the value of variance of set II should be minimal as well. The times series a_n are constructed as follows:

$$a_n = [d_1, d_2, \cdots, d_{n-1}, x]\ . \tag{8}$$

Thus, the value of variance of the time series a_n must be minimum.

Sample variance $f(x)$ of a finite time sample is an unbiased and consistent estimation of data variance. The threshold value can be obtained when sample variance is minimal. The optimal threshold x_{opt} of the distance can be calculated as follows:

Fig. 3. Gearbox set-up.

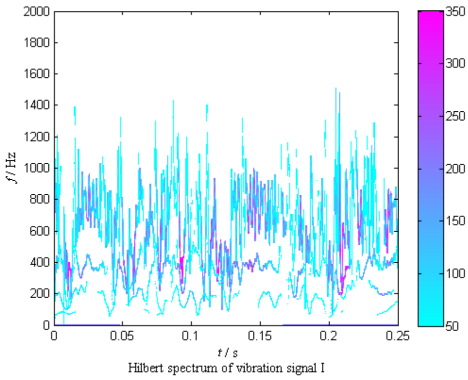

Fig. 4. Hilbert spectrum of vibration signal I.

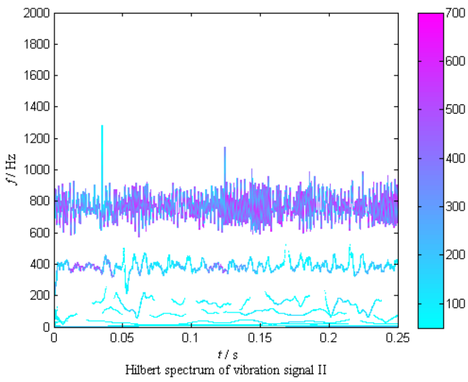

Fig. 5. Hilbert spectrum of vibration signal II.

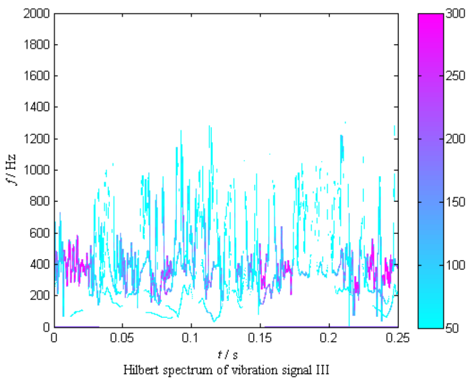

Fig. 6. Hilbert spectrum of vibration signal III.

$$f(x) = S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(a_i - \bar{a})^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(a_i - \frac{(n-1)\bar{d} + x}{n}\right)^2$$

$$\frac{\partial[(n-1)f(x)]}{\partial x} = 2\sum_{i=1}^{n-1}\left(a_i - \frac{(n-1)\bar{d}+x}{n}\right)\left(-\frac{1}{n}\right) +$$
$$2\left(x - \frac{(n-1)\bar{d}+x}{n}\right)\left(1-\frac{1}{n}\right) = \frac{2}{n}\sum_{i=1}^{n-1}\frac{(n-1)\bar{d}+x}{n} - \frac{2}{n}\sum_{i=1}^{n-1}a_i +$$
$$2\left(x - \frac{(n-1)\bar{d}+x}{n}\right)\left(1-\frac{1}{n}\right) =$$
$$\frac{2}{n^2}\left[(n-1)x + (n-1)^2\bar{d} - n(n-1)\bar{d} +\right.$$
$$\frac{2}{n^2}\left\{x\left[(n-1)+(n-1)^2\right] - \left[n(n-1)\right]\bar{d}\right\} =$$
$$\frac{2}{n^2}\left[x(n^2-n) - (n^2-n)\bar{d}\right] \tag{9}$$

$$\left.\frac{\partial f}{\partial x}\right|_{x=x_{opt}} = 0 \tag{10}$$

$$x_{opt} = \bar{d} = \frac{1}{n-1}\sum_{i=1}^{n-1} d_i\ . \tag{11}$$

Through Eq. (11), the optimal threshold x_{opt} of the distance for classification can be obtained. The data can be classified and identified by the threshold of characteristics, and then the fault can be diagnosed.

4. Experiment and discussion

From the above analysis, we know the method of singular value decomposition based on Hilbert-Huang transform is effective especially for multi-component non-stationary signals. Gear fault is common in running machines, and is difficult to diagnose. Gear vibration is a multi-component signal and when fault occurs, non-stationary signals may be shown. In this section, the method proposed above is used to analyze the faulted gear signal.

Experimental data were collected from an induction motor driven gearbox set-up, which is shown in Fig.3. An accelerometer was mounted on the gearbox to acquire the vibration signal from the gear. Local defect gear fault and abrasion wear gear were introduced, and the motor speed was 800 rpm. The signal was sampled from three kinds of gears and each class of data corresponds to the following gear conditions: (i) normal status; (ii) a gear flaw fault; (iii) abrasion wear fault. The sampling frequency is 3838 Hz and the gear mesh frequency is 385 Hz. For every gear condition, 10 group samples were obtained and each had 1024 data. Of these samples, 20 groups are used for training and 10 groups for testing.

Before feature extraction, Hilbert-Huang transform method is used to obtain the Hilbert spectrum and the matrix from the instantaneous frequency and instantaneous amplitude of IMF. First, Fig. 4 shows the Hilbert spectrum of the signal acquired from the gearbox with abrasion wear. The time-frequency distribution is complex, but the main energy of the vibration fluctuates around mesh frequency and double mesh frequency,

Table 1. Comparison of actual and target output of test samples.

Data Number	Fault type	Feature matrix		Classification I (σ_A)	Classification II (σ_B)	Target output		
		Singular value σ_A	Singular value σ_B					
①	Gear flaw fault	<201.5, 86.6, 60.3>		y =+1 (d=0.466)		1	0	0
②	Gear flaw fault	<201.1, 92.1, 56.7>		y=+1 (d=0.469)		1	0	0
③	Gear flaw fault	<200.9, 81.1, 62.6>		y=+1 (d=0.460)		1	0	0
④	Abrasion wear fault	<118.8, 76.2, 47.7>	<219.1, 109.1, 64.8>	y=-1 (d=0.074)	y=+1 (d=0.053)	0	1	0
⑤	Abrasion wear fault	<115.8, 73.1, 41.6>	<221.5, 104.1, 62.2>	y=-1 (d=0.093)	y=+1 (d=0.062)	0	1	0
⑥	Abrasion wear fault	<119.2, 77.9, 44.5>	<217.7, 108.0, 62.7>	y=-1 (d=0.083)	y=+1 (d=0.066)	0	1	0
⑦	Normal status	<130.5, 74.6, 52.3>	<231.0, 100.7, 68.3>	y=-1 (d=0.012)	y=-1 (d=0.016)	0	0	1
⑧	Normal status	<128.6, 72.4, 48.8>	<233.9, 103.3, 63.7>	y=-1 (d=0.017)	y=-1 (d=0.018)	0	0	1
⑨	Normal status	<127.4, 74.3, 50.3>	<234.1, 106.9, 66.7>	y=-1 (d=0.016)	y=-1 (d=0.013)	0	0	1

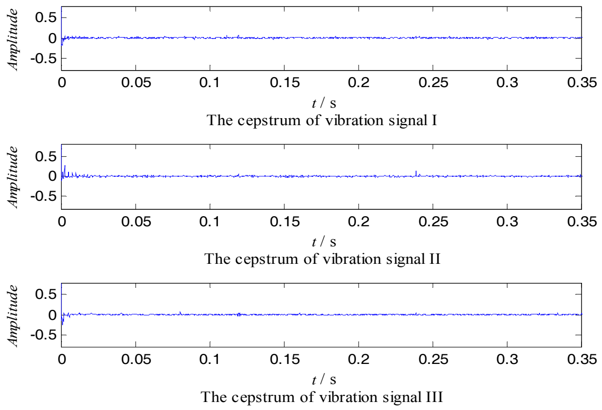

Fig. 7. The cepstrum of the signals.

which is colored magenta in Fig. 4. Second, the Hilbert spectrum of the signal from a flawed gear is represented in Fig. 5. The main energy of vibration is in the band around double mesh frequency. The fault feature is extracted well. Third, Fig. 6 represents the Hilbert spectrum of the vibration from a normal status gear. The main energy of vibration is in the band around mesh frequency.

Despite the tremendous computational load induced by such a scheme in practical application, it is a realistic consideration for automatic fault diagnosis because the method does not need any manual intervention. Most importantly, Hilbert-Huang transform and SVD have been tested extensively using different vibration data, always with acceptable results. By extensive experimentation, 3 group samples were randomly collected from each fault type; in total, 9 group samples for automated fault diagnosis of a gearbox were collected.

According to classification I in Table 1, many sampling groups of gear fault are classified by criteria I and the boundary of classification B_A is 0.188 by Eq. (11). The data can be classified into two sets: One is gear flaw fault, the other is indeterminate. And according classification II in Table 1, the other indeterminate data can be classified to two sets by criteria II, and the boundary of classification B_B is 0.038,. One is abrasion wear fault while the other is normal status gear. Thus, three type faults can be identificatied and classificated.

For all 9 groups of samples, we can obtain 9 group input vectors by feature extraction. The 9 test samples are input into the system of the method and output results show that all test samples are classified correctly. We selected three samples from every kind of gear test signal and listed its actual and target output in Table 1, from which we can see that the output result is very close to target output. Therefore, we conclude that this method could realize accurate classification and the method is proved to effectively identify and classify the gear faults. By comparing the cepstrum of the signals in Fig. 7, the double period of flaw gear can be obtained as 0.23 s, but the abrasion wear fault can not be identified.

This method has the feature extracting merits of both the Hilbert spectrum and singular value decomposition and solves the problem of reconstructing the feature matrix. The method can then be used to realize automatic fault diagnosis.

5. Conclusion

A novel method of singular value decomposition based on Hilbert-Huang transform was proposed. It has the feature extracting merits of both Hilbert-Huang transform and singular value decomposition, and the reconstruction of feature matrix is solved well.

The vector of singular value and the distance of the vector of singular value are defined. The optimal value of distance threshold to classify is derived.

The classification of experimental data from a gearbox set-up demonstrates the method is effective to classify fault and identify gear fault. Thus, the whole algorithm could obtain a reliable result for gear fault diagnosis and the method can be used to realize automatic fault diagnosis.

Acknowledgment

This work supported by National Natural Science Foundation of China (Grant No. 50775035), National Hi-tech Research and Development Program of China (863 Program, Grant No. 2006AA050203 and No. 2007AA04Z421). And thanks a lot for their helpful comments and constructive suggestions of the anonymous reviewers.

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