

## Analysis of eccentricity in the ball bar measurement <sup>†</sup>

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### Abstract

The ball bar is widely used in machine tool measurement because it is convenient and cost effective. Eccentricity exists due to machine errors and installation errors during a ball bar test, and the coordinate transformation method is used to remove eccentricity. However, distortion of data (i.e., ovalization) becomes large when eccentricity relative to the radius of a circle cannot be ignored. The number of sampled data for the total angle of circular path is used to estimate the actual rotating angle of the ball bar during measurement. To prevent distortion of data, actual rotating angles of the ball bar must be estimated exactly. In this paper, geometric relations of the ball bar measurement model are described based on the poses of two reference coordinate systems (ball bar and nominal coordinate system). The proposed algorithm calculates the actual rotating angle of the ball bar using geometric conditions and removes eccentricity through coordinate transformation.

**Keywords:** Ball bar; Coordinate transformation; Eccentricity; Circular test; Ovalization

### 1. Introduction

Machine errors directly influence both surface finish and geometric shape of a finished work piece. Hence, it is important to measure machine errors and compensate for them. Among various measuring instruments that measure geometric errors of a machine tool, the ball bar is widely used because measurement is convenient and cost effective [1-2]. Diagnosis and analysis of machine tool errors using the ball bar system has been researched for several decades [3-4].

The ball bar system evaluates machine errors through the circular test. This circular test is mentioned in ISO 230-1 [5]. When the ball bar test is performed, eccentricity exists due to machine and installation errors. Removing eccentricity is necessary to analyze machine errors. Coordinate transformation method (CTM) is often used to remove eccentricity. However, distortion of data (i.e., ovalization) becomes serious when eccentricity relative to the radius of the circle becomes too big to be ignored. Some commercial ball bar systems also warned of the distortion of data when eccentricity is over 100  $\mu\text{m}$  [6]. Even though it is necessary to remove eccentricity carefully, related research on it is very rare [7].

In this paper, a method dealing with eccentricity is proposed. In dealing with eccentricity, the actual rotating angle of the

ball bar is calculated by geometric relations of the ball bar measurement. Data are then centered by calculation through the coordinate transformation method.

### 2. Distortion of data in removing eccentricity

In the ball bar test, a circular path is used as shown in Fig. 1. When the ball bar test is performed, the measured radius of ball bar,  $R_i^B$  with respect to the coordinate system,  $\{B\}$ , is read directly, and the rotating angle of the ball bar is estimated by Eq. (1).

$$\theta_{Ei} = \frac{i}{n} \times \theta_{Total} \quad (1)$$

where  $\theta_{Ei}$  is  $i^{th}$  estimated rotating angle of the ball bar,  $n$  is the number of total sampled data, and  $\theta_{Total}$  is the total moved angle.

$R_i^B$  and  $\theta_{Ei}$  are used to estimate the coordinate point  $P_{Ei}$ , which is written in Eq. (2).

$$P_{Ei} = R_i^B [\cos(\theta_{Ei}) \ \sin(\theta_{Ei})]^T \quad (2)$$

The path of  $P_{Ei}$  has eccentricity. Eccentricity ( $u, v$ ) can be obtained by the circular linear least squares method and can be removed by the simple coordinate transformation method (SCTM). SCTM changes the center of the trajectory; it is expressed in Eq. (3).

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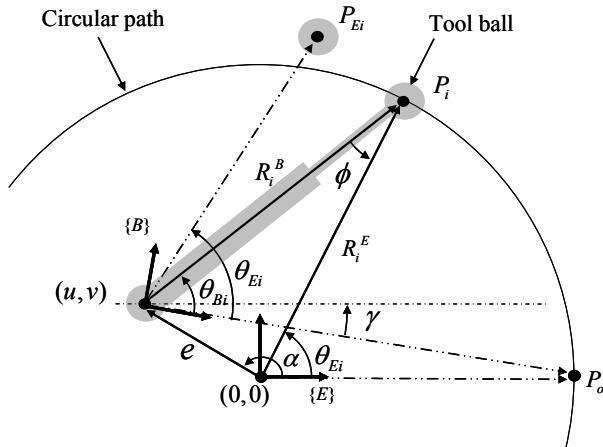


Fig. 1. Geometric relations of ball bar measurement.

$$P_{Ei}' = T \cdot P_{Ei} \quad (3)$$

where  $P_{Ei}'$  is the centered point by simple coordinate transformation method, and  $T$  is the  $3 \times 3$  homogeneous transformation matrix represented by  $(u, v)$ .

$$T = \begin{bmatrix} 1 & 0 & u \\ 0 & 1 & v \\ 0 & 0 & 1 \end{bmatrix}$$

However, with SCTM, the bigger the eccentricity relative to the radius of the circle, the more serious the distortion of the centered data becomes. This results from using  $\theta_{Ei}$  instead of the actual rotating angle of the ball bar  $\theta_{Bi}$ . When the perfect circle path of a 150 mm radius with eccentricity of (0.7, 0.7) mm is simulated, as shown in Fig. 2, distorted trajectory is obtained using SCTM, as shown in Fig. 3.

The distortion of data can be evaluated by the root mean square of errors (RMSE), which shows how centered-data reflect original data. RMSE is written in Eq. (4).

$$RMSE = \sqrt{\frac{\sum_{i=1}^n \|P_{i\_centered} - P_{i\_original}\|^2}{n-1}} \quad (4)$$

where  $P_{i\_centered}$  is a centered point by removing eccentricity,  $P_{i\_original}$  is a point on the original trajectory to be compared, and  $P_{i\_original}$  is a point on the perfect circle.

In the case of SCTM in Fig. 3, RMSE is 0.99365 mm. Considering the resolution of the ball bar system (Renishaw PLC)  $10^{-4}$  mm, the RMSE of SCTM is too big and is not enough to be accepted.

### 3. Calculation of the rotating angle of the ball bar

To reduce distortion of data, the actual rotating angle of the ball bar  $\theta_{Bi}$  must be applied to the coordinate transformation method.

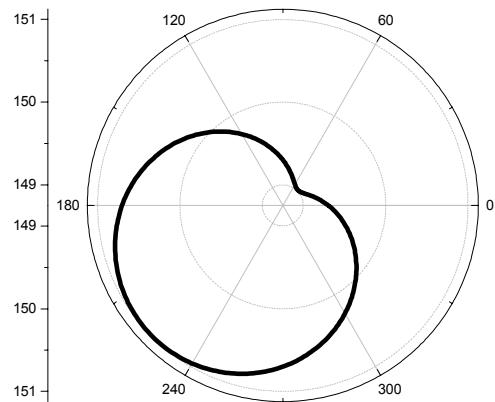


Fig. 2. Simulated perfect circle path with eccentricity.

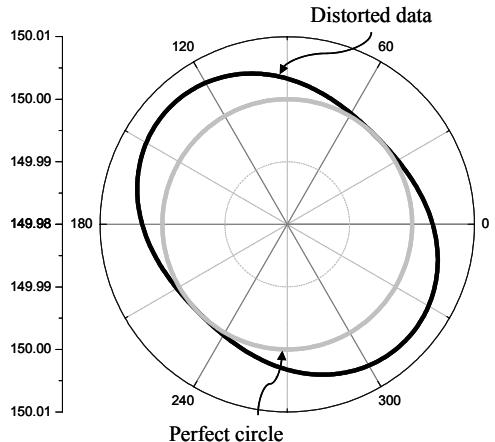


Fig. 3. Distortion of centered data by SCTM.

$\theta_{Bi}$  is derived from the geometric relations of the ball bar measurement as seen in Fig. 1. Geometric relations are based on the nominal coordinate system,  $\{E\}$ , and the ball bar coordinate system,  $\{B\}$ , and are determined by  $(u, v)$  and  $\gamma$ .  $(u, v)$  represents the translation between two coordinate systems and is obtained by the circular least squares method as mentioned previously.  $\gamma$  represents the pose relation between two coordinate systems and is calculated from Eqs. (5) and (6).

$$\alpha = \tan^{-1}\left(\frac{u}{v}\right) \quad (5)$$

$$\gamma = \sin^{-1}\left(\frac{e \sin(\alpha)}{R_0^B}\right) \quad (6)$$

where  $R_0^B$  is the initial value of  $R_i^B$ , and  $e$  is distance from  $(0, 0)$  to  $(u, v)$  in the nominal coordinate system.

$\theta_{Bi}$  is analogized from  $\theta_{Ei}$  and is calculated by the laws of sines in Eq. (7).

$$\frac{e}{\sin(\theta_{Ei} + \gamma - \theta_{Bi})} = \frac{R_i^B}{\sin(\alpha - \theta_{Ei})}$$

$$\therefore \theta_{Bi} = \theta_{Ei} + \gamma - \sin^{-1}\left(\frac{e \sin(\alpha - \theta_{Ei})}{R_i^B}\right) \quad (7)$$

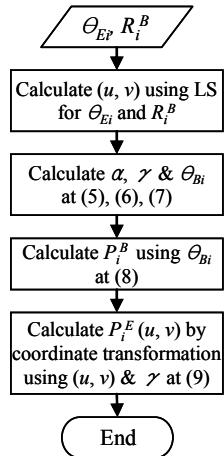


Fig. 4. Procedure of removing eccentricity using  $\theta_{Bi}$ .

#### 4. Procedure of removing eccentricity

The procedure to remove eccentricity using  $\theta_{Bi}$ (PCTM) is proposed and is illustrated in Fig. 4. The process of calculating  $\alpha$ ,  $\gamma$ , and  $\theta_{Bi}$  is mentioned in the previous section, and  $P_i^B$  is calculated by Eq. (8).

$$P_i^B = R_i^B [\cos(\theta_{Bi}) \ \sin(\theta_{Bi})]^T \quad (8)$$

where  $P_i^B$  is a point  $P_i$  with respect to the coordinate system,  $\{B\}$ , as seen in Fig. 1.

Using  $P_i^B$ , the coordinate transformation method is performed to remove eccentricity. It is written as follows:

$$P_i^E = T \cdot P_i^B \quad (9)$$

$$T = \begin{bmatrix} \cos(-\gamma) & -\sin(-\gamma) & u \\ \sin(-\gamma) & \cos(-\gamma) & v \\ 0 & 0 & 1 \end{bmatrix}$$

where  $P_i^E$  is a point  $P_i$  with respect to the coordinate system  $\{E\}$ , and  $T_B^E$  is the coordinate transformation operator.

To evaluate this procedure, a 150 mm perfect circular path with an eccentricity of (0.7, 0.7) is simulated, and simulated data are centered by PCTM as shown in Fig. 5. RMSE evaluation is performed to determine the RMSE trend for changing eccentricity in Fig. 6. According to the results, the centered data show a very small difference in perfect circular path graphically (Fig. 5). RMSE comparison also shows slight difference in a perfect circle in Fig. 6. Considering the resolution of the ball bar system (Renishaw PLC)  $10^{-4}$  mm, the RMSE of PCTM is enough to be accepted.

#### 5. Evaluation for experimented data

It is evaluated that the proposed procedure is effective in removing eccentricity for a perfect circle trajectory. However, the proposed procedure must evaluate the effectiveness of

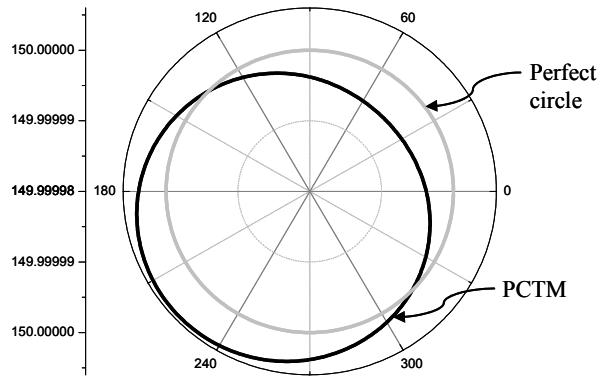


Fig. 5. Centered trajectory by PCTM.

RMSE (mm)

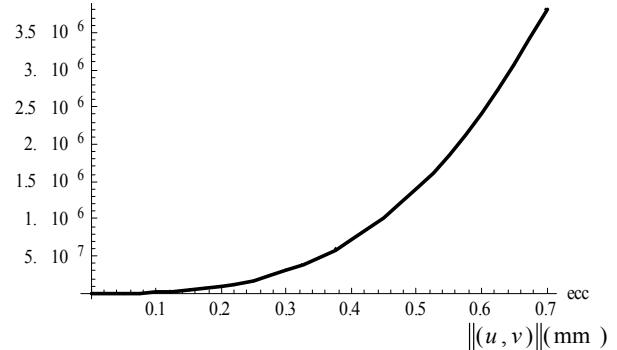


Fig. 6. RMSE of centered data by PCTM.

removing eccentricity for actual experimental data to apply for a real ball bar test. Therefore, this research intends to compare SCTM and PCTM to evaluate the effectiveness for actual ball bar data. As a method of effectiveness comparison between SCTM and PCTM, randomly generated data similar to actual ball bar data, which is a simulated circular path of 150 mm radius with randomized machine errors, is used in the previous verification of experimental data.

The evaluation is then achieved for experimental data from the ball bar test of 150 mm radius on a rotary table and has eccentricity of (0.7032, 0.6993) mm.

Fig. 7 shows that the trajectory of the proposed procedure has a gap with the trajectory of SCTM, and the trend of the centered trajectory is similar to the perfect circle case in Fig. 5. Fig. 8 verifies that the proposed procedure is more effective than SCTM through RMSE comparison.

Fig. 9 also shows that the trajectory of the proposed procedure has a gap with the trajectory of SCTM, and the trend of each centered trajectory is similar to the perfect circle case in Fig. 5. Fig. 9 shows that the proposed procedure is more effective than SCTM through RMSE comparison.

#### 6. Conclusions

In this research, a new method was proposed and evaluated to reduce the distortion of data in removing eccentricity during

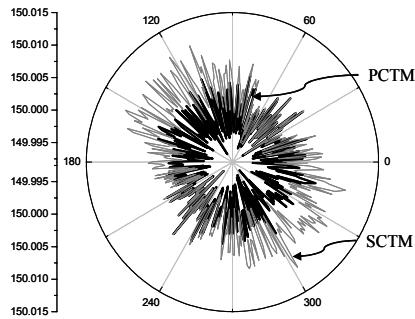


Fig. 7. Centered trajectory comparison between SCTM and PCTM in a randomly generated trajectory.

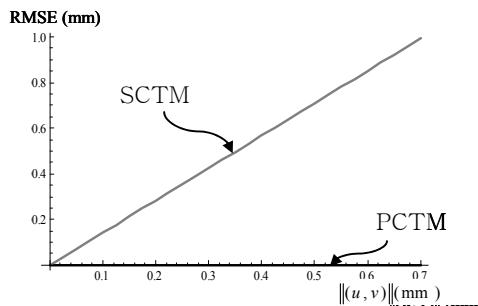


Fig. 8. RMSE comparison between PCTM and SCTM in a randomly generated trajectory.

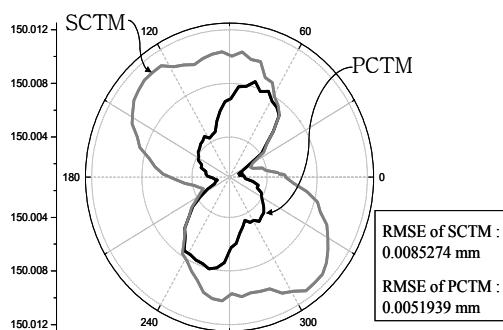


Fig. 9. Centered trajectory comparison and RMSE comparison between SCTM and PCTM in the experimented data condition.

a ball bar test. The results are summarized as follows.

- (1) The actual rotating angle of the ball bar is calculated from the geometric relations of the ball bar measurement.
- (2) The procedure to remove eccentricity using an actual rotating angle of ball bar is proposed.
- (3) The proposed procedure is performed in a perfect circular path. The effectiveness of the proposed procedure is evaluated through centered trajectory and RMSE.
- (4) The proposed procedure is compared with SCTM through a centered trajectory and RMSE to evaluate effectiveness in a randomly generated error condition and experimental ball bar data. It has been verified that the proposed procedure is more effective than SCTM.

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