Springer

Journal of Mechanical Science and Technology 22 (2008) 1843~1852

Journal of
Mechanical
Science and
Technology

www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-008-0623-2

A dynamic model and numerical study on the liquid balancer used in an automatic washing machine

Chung-Hyo Jung[1,*], Chang-Sub Kim[1] and Yun-Ho Choi[2]

[1]*Samsung Electronics CO., LTD., 416 Maetan-3Dong, Yeongtong-Gu, Suwon, Korea*
[2]*Department of Mechanical Engineering, Ajou University, Suwon, Korea*

(Manuscript Received November 12, 2007; Revised May 1, 2008; Accepted June 11, 2008)

Abstract

The rotational system of an automatic washing machine has two major technical difficulties. One is the collision of the tub against the frame at the beginning of its early spinning stage. A factor that causes such instability of the rotational object can be attributed to the deflection of the liquid inside the liquid balancer due to the unbalance mass. This study proposes the installation of a middle plate to avoid the deflection of the liquid and a CFD method is used to verify its effectiveness. The other is the whirling vibration occurred during the high speed rotation of the basket and it shakes the center of its rotational axis. This is because the balancer cannot generate enough restoration forces during its high speed rotation. It is necessary to employ the multi-race in the balancer in order to increase restoration forces. The analysis for the configuration has been performed using a dynamic model.

Keywords: Automatic washing machine; Liquid balancer; Unbalance mass; Restoration force; Dynamic model; Computational fluid dynamics

1. Introduction

Automatic washing machines have been demonstrated a steady demand in worldwide markets. It can be regarded as a cash-cow that has been traditionally contributed to conduct the accumulation of analog technologies in electronic businesses. However, their market growing has been stagnated due to the great leap of technical evolution in drum washing machines. Accordingly, it is necessary to pursuit high-grade products and low production costs through quality improvements in order to guarantee continual needs.

Automatic washing machines can be largely classified as seven different systems according to their functions (Fig. 1), such as outer case, electric control, tub and basket, washing/spinning, water supply, drain, and display. This study focuses solely on the performance improvement of an unbalance control system, which is a subsystem of the third system. The design factor of the unbalance control system includes balancer height, gap between the tub and the outer case, rinse injector shape, internal liquid of the balancer, balancer type, and damper height. In order to improve the performance of a balancer system, these design factors require an optimization process. The priority of this study is the design shape of the balancer, which is one of core parts in automatic washing machines.

Let us first consider the role of this balancer. When washing and rinsing processes are finished, the basket of an automatic washing machine rapidly rotates to extract water from laundries. In an extraction process, laundries may cause unbalance mass due to the deflection of laundries on one side of the bottom of the basket. Although the centrifugal force generated in this process is deflected due to the unbalance mass, the liquid flow inside the balancer suppresses the degree of such deflection.

Automatic washing machines in the present days

*Corresponding author. Tel.: +82 31 277 7178, Fax.: +82 31 277 7733
E-mail address: chunghyo.jung@samsung.com

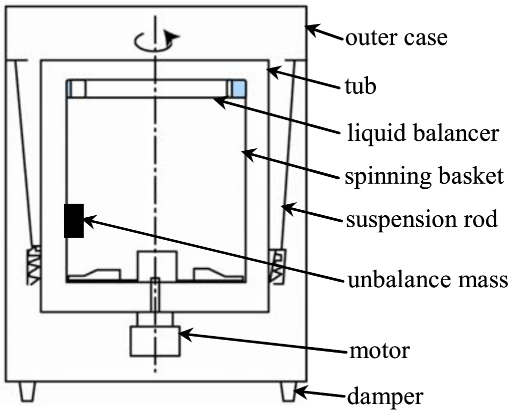


Fig. 1. Schematic diagram of an automatic washing machine system.

have two technical difficulties in the aspect of its balance. One is the collision of the tub against the frame in a spinning mode at the beginning of its rotation due to the unbalance mass. In general, the test focused on this issue was performed under the load conditions of 50% and 80% of the proper set load. Based on these conditions, the reliability requirement does not allow the collision of the tub against the frame even once while the tested washing machine is horizontally set. The other is the center of the basket may present whirling vibration [1] at high RPM condition. This unbalanced rotation may cause the abrasion of parts and noise.

An excessive deflection in the internal fluid of the balancer that becomes the major cause of the collision of the tub against the frame has been considered in previous studies. In recent years, some ideas have been proposed to improve this deflection and applied to certain designs [2, 3]. One of these ideas proposed that the flow can be controlled using an extended plate from the outer side of the case to the inner side in order to suppress the initial fluid deflection (refer to Fig. 2). The plate was arranged at outer periphery of the balancer and it was also determined as a finite length to the inner side in order to avoid the deflection of liquid and allow the free motion of liquid to the circumferential direction at high RPM range. Also, a method in which two vertical plates are installed along the circumference for dividing the chamber space into three races was proposed [4].

In addition, Bae et al. [5] proposed a mathematical model that is able to explain the unbalance force of laundries and restoration force using a dynamic model. This model can be used to find a method that

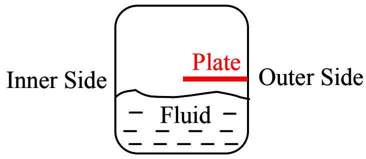


Fig. 2. Cross section of the balancer (Publicized Utility Model 1999-002708).

reduces the centrifugal force occurred by the unbalance mass under steady state conditions.

The objective of this study can be largely divided into two issues. One is the establishment of a dynamic model for the balancer based on the theory proposed by Bae et al [5]. The results obtained by this model can be used as a design guide. The other is the visualization of the excessive deflection of liquid generated inside the balancer in the early stage of the spinning mode through applying a CFD method. Also we propose a method that controls such deflection and a simulation will be performed.

2. Investigation using experiments and dynamic model

2.1 Test for transient state (2 races)

Several methods that control the fluid deflection in transient state have been considered and analyzed as follows: a method that applies two races for the chamber of the balancer instead of one race, a method that reduces the width of the balancer around the rinse injector, a method that injects rinse, the type of liquid applied inside the balancer, the structure of the rib inside the balancer, and the height of the damper. This study will describe the motion of liquid according to the change of number of races from one to two. In the case of one race, there existed small vibrations at high RPM range. Also, it represents a limitation in the control of the early fluid deflection. Thus, two races were used to solve such a problem.

Experiments were performed using a 6 kg automatic washing machine. The liquid inside the balancer was dyed with black paint and the unbalance masses of 0.5 kg and 1 kg were installed in the upper and lower chambers of the basket, respectively. The cover of the balancer was made using a transparent acryl plate to observe the motion of internal fluid. Also the behavior of the liquid was recorded using a high-speed camera that is able to take pictures at a rate of 3000 frames per second while the basket is rotating with an unbalance in a spinning mode. The

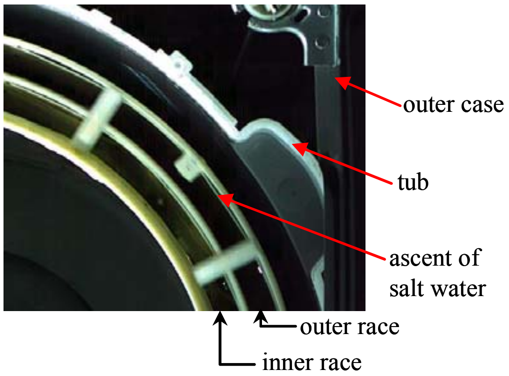


Fig. 3. Recorded image before the collision between the frame and the tub (1 kg unbalance mass).

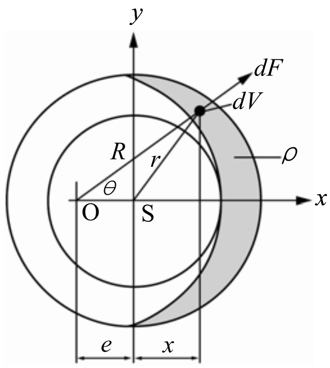


Fig. 4. Centrifugal force of the liquid in the liquid balancer.

time that we start for taking pictures was 6 seconds after the beginning of its spinning in which the basket approached the rotational speed of 300~400 RPM. This point is still not in steady state but in transient state.

Fig. 3 illustrates the behavior of the liquid inside the balancer when the unbalance mass was 1 kg. The role of balancer is to generate restoration forces, which compensate the unbalance generated by the centrifugal force in steady state. In these experimental results, the behavior of the liquid was regarded as a state just before the restoration force was fully activated. Also, this moment can be considered as a time that is dominated by the unbalance force.

In addition, it was evident that water ascended when the tub approached to the right side of the frame (+X direction). Fig. 3 shows the image just before the collision of the tub against the frame. It was also seen that the body of the ascent water was dropping after the collision (not shown here). This motion was observed as repetitive behaviors in the early transient state (within 6 seconds after the tub was started to spin). Based on these behaviors, it was clear that the liquid balancer with two races could not perfectly solve the early fluid deflection. In a theoretical consideration, however, it was verified that the balancer with two races increased the restoration force by extending its linear range compared to that with one race in steady state (high RPM range). The next section demonstrates more details on this result.

2.2 Study with a dynamic model

Several studies on the vibration caused by the unbalance mass in a spinning stage have been reported [1, 5, 6]. These studies analyzed the three dimensional rigid body motion, dynamic roles of the balancer, and vibration mode of the suspension rod and tub through applying a dynamic model. Based on these results, the liquid balancer could reduce the amount of such unbalance force by generating restoration force in the transient and steady state. Also the entire rotational objects including the liquid inside the balancer represent balancing effects in a specific range that exceeds the natural frequency of the system. Let us investigate a dynamic model that describes this mechanism as follows:

Fig. 4 illustrates the dynamic model of the liquid balancer. The centrifugal force generated by the liquid in steady state can be described as follows:

Eq. (1) shows the centrifugal force, dF, generated at an incremental volume of dV.

$$dF = \rho R \omega^2 dV \tag{1}$$

where ρ, R, and ω are the density of the liquid, distance from the rotation axis, and angular velocity, respectively. Here we assumed the density of the liquid and angular velocity as constant values. The centrifugal force acting on the centroid of the liquid can be determined as

$$F = \int dF \cos\theta = \rho\omega^2 e \int dV + \rho\omega^2 \int x dV \tag{2}$$

Using the formula of $c=(\int xdV)/(\int dV)$ and $m=\rho\int dV$, we can rewrite Eq. (2) as

$$F = \rho\omega^2 (e+c) \int dV = m\omega^2 (e+c) \tag{3}$$

The centrifugal force noted in Eq. (3) depends on the eccentricity e of the unbalance mass and the centroidal distance c of the liquid. Here the centroidal distance can be determined based on the magnitude of the eccentricity e. In the case of the small value of e (Fig. 5), the centroidal distance can be expressed as

$$c = \frac{R_s^2 e}{R_o^2 - R_s^2} = \left(\frac{R_o^2 + R_i^2}{R_o^2 - R_i^2} \right) e, \quad e \le \bar{e} \tag{4}$$

where $\bar{e} = R_s - R_i$ or $\bar{e} = R_o - R_s$. Also, in the case of very large value of e (Fig. 6), the centroidal distance c^* can be expressed as

$$c = \frac{2}{3} \frac{R_o^2 + R_o R_i + R_i^2}{R_o + R_i} \frac{\sin(q\pi)}{q\pi} = c^* \tag{5}$$

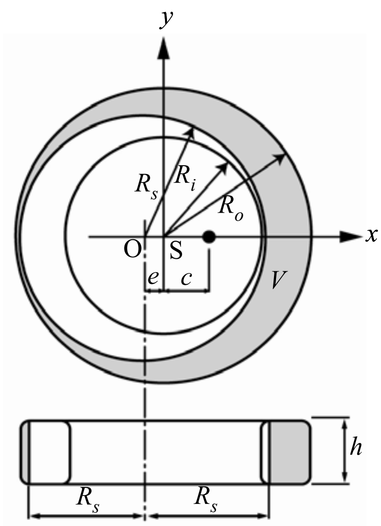


Fig. 5. Centroid of the fluid for $e < \bar{e}$.

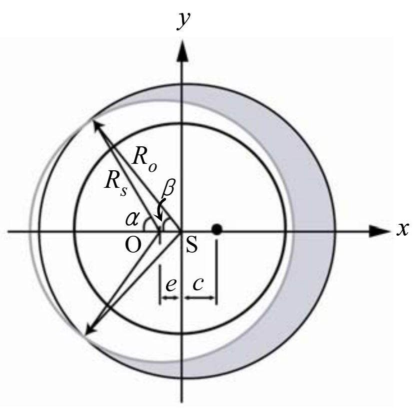


Fig. 6. Centroid of the fluid for $e \ge \bar{e}$.

By combining Eq. (4) and (5), we obtain the following equation.

$$c = \begin{cases} se, & e < \bar{e} \\ s\bar{e} + \dfrac{2}{\pi}(c^* - s\bar{e}) \arctan\left(\dfrac{\pi(e - \bar{e})}{2(c^* - s\bar{e})} \right), & e \ge \bar{e} \end{cases} \tag{6}$$

where $s = R_s^2 / (R_o^2 - R_s^2)$. Eq. (6) shows that the centroidal distance c becomes a linear function of the eccentricity e when the eccentricity is small, whereas the value of c represents a constant value as the eccentricity grows extremely large.

Next, the center of rotation for the system including the tub, unbalance mass, and liquid mass can be determined as

$$e = \frac{rm_1 - cm_2}{M + m_1 + m_2} \tag{7}$$

where r, M, m_1, and m_2 represent the distance from the center of the basket (S) to the unbalance mass, the total mass of the tub and basket, the unbalance mass, and the mass of salt water, respectively. Because the value of c can be determined as $c = se$ for $e < \bar{e}$. Eq. (7) can be rewritten as

$$e = \frac{rm_1}{M + m_1 + (1+s)m_2} \tag{8}$$

Also for $e \ge \bar{e}$, it can be expressed as

$$e = \frac{rm_1 - c^* m_2}{M + m_1 + m_2} \tag{9}$$

Thus, the restoration force of the liquid can be obtained by calculating the eccentricity.

The restoration force F_o in a linear range ($e < \bar{e}$) can be derived as

$$F_o / \omega^2 = \rho \pi R_o^2 he \tag{10}$$

where ρ and h represent the liquid density and the height of the balancer. As shown in Eq. (10), the restoration force is linearly proportional to the eccentricity. The restoration force in a nonlinear range ($e \ge \bar{e}$) can be expressed as

$$F_o/\omega^2 = \rho\{\pi - (\beta - \sin\beta\cos\beta)\} R_o^2 he \tag{11}$$

where,

$$\cos\beta = \frac{R_o^2 - R_s^2 + e^2}{2eR_o},$$

$$\sin\beta = \sqrt{1-\cos^2\beta}\ ,\ 0 < \beta < \pi\ .$$

Here Eq. (11) becomes equivalent to Eq. (10) when the value of β is 0.

Based on these theoretical backgrounds, we wrote a calculation program. The results obtained with this program can be summarized as follows:

The calculation program was designed to obtain the restoration force from the eccentricity. The movement of the liquid is generated by the eccentricity. The restoration force is proportional to the eccentricity when a linear relationship between the values of e and c exists as shown in Eq. (6). However, it will represent nonlinear behaviors if the value of e exceeds its linearity ($e \geq \bar{e}$). This means that the balancer loses its balancing capability for the nonlinear region (refer to Fig. 7).

Thus, the important design point is to maximally extend the range of its linearity. For this, it was proven that R_s - R_i and $R_o - R_s$ should be equal. In an actual calculation, the smaller one of these two values corresponds to the linearity limit.

Also, as previously mentioned above, small amplitudes in steady state have been considered as an issue. It has been known that the basket in a washing machine represents whirling motion in steady state in which the rotation axis of the system shakes due to fine vibrations. Thus, a multi-race method for the liquid balancer has been suggested to suppress such whirling vibration. In the present study, the calculations for a liquid balancer with one and two races have been performed. Table 1 shows the conditions used for this calculation. The parameters V_l and V_l/V_{all} represent the volume of salt water and the volume ratio, respectively. The spin radius was 0.233 m, and the density of salt water was 1.18 g/mL (concentration = 23%).

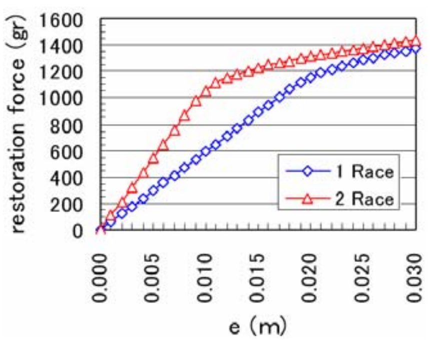


Fig. 7. Comparison of the restoration forces between one race and two races.

Fig. 7 shows the calculation results. It can be seen that the linear limits of the case of one race and two races are 0.019 m and 0.01 m, respectively. Although the case of two races demonstrates smaller linear limit than that of one race, it represents an increase in the restoration force due to the increase in its gradient. This means that the case of two races is more effective in suppressing the small amplitude in steady state than other cases. Thus, the multi-race for the liquid balancer can be considered as a design method that may minimize vibrations in steady state.

Also, this study showed that the increase of the amount of the liquid in the liquid balancer with one race was of advantage in transient state due to the increase of a linear region. However, the restoration force decreased due to the decrease in the gradient of such a linear region. Considering that the maximum eccentricity was 10 mm in the actual design, it was found that the optimum amount of the applied liquid was 50% of the total volume (refer to Fig. 8). This

Table 1. Parameters used in the calculation (m).

Type		R_o	R_i	R_s	V_l(cc)	V_l/V_{all}
1 Race		0.21	0.17	0.19	1900.00	50.31 %
2 Races	Outer	0.21	0.19	0.20	1013.00	51.16 %
	Inner	0.19	0.17	0.18	921.00	51.27 %

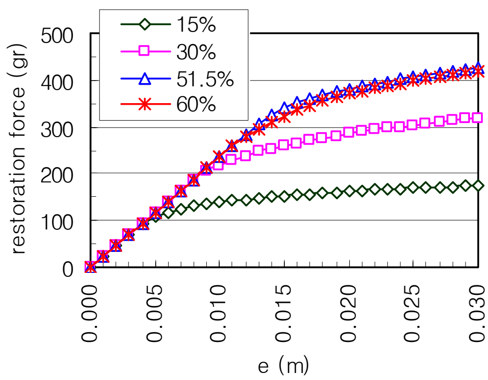


Fig. 8. Comparison of the restoration forces for the volume ratio (V_l/V_{all}).

means that the problem of the early transient state (within 6 seconds) cannot be solved by controlling the amount of the liquid. On the contrary, the increase of the amount of the liquid causes the early transient state only to amplify because the liquid leans to the side where the unbalance mass is located. Thus, there is a limit to the solution of the early fluid deflection through the multi-race method or the control of the amount of the liquid. The next section describes a method that suppresses the initial fluid deflection.

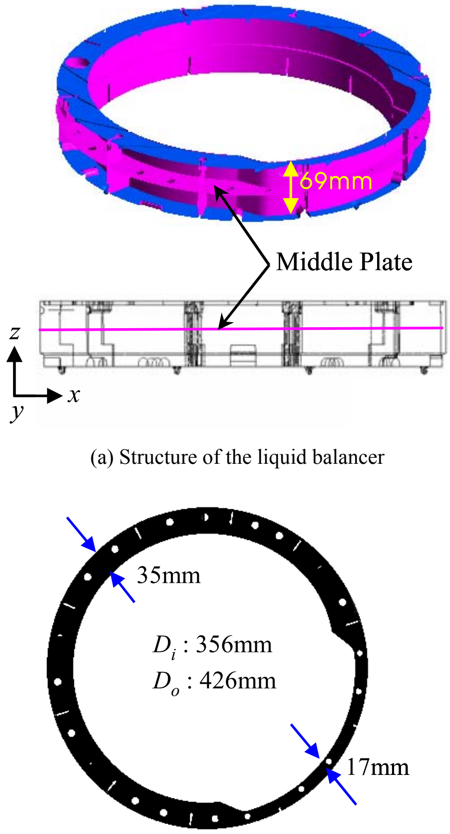
3. Results and discussion

3.1 Fluid analysis model of the balancer

The collision between the tub and the frame in the early spinning stage may cause a decrease in the service life of parts, lowering of performances, noises, and psychological discomfort for customers. Researchers who studied on this problem based on dynamic models have reported several useful results [1, 5, 7]. For their calculations they assumed the liquid as a rigid body. However, there are few studies on the visualization of the internal liquid of the balancer using Computational Fluid Dynamics(CFD). In the present study, a numerical simulation was performed using a CFX code[8] for the two phase flow of air and water. This study will investigate a possibility that overcomes the problems in the early transient state.

Fig. 9 illustrates the analysis model for the liquid balancer with one race applied to a 6 kg automatic washing machine. The thin section on the right side corresponds to the injector of cleaner and rinse. Also the middle plate proposed in this study was installed at halfway along the z direction to avoid the deflection of fluid. The inner and outer diameters of the middle plate are 356 mm and 426 mm, respectively. The widths of the narrow and wide regions are 17 mm and 35 mm, respectively. Fig. 9(b) shows the structure of the middle plate in which holes were arranged along the circumferential direction. Five holes of 8 mm diameter and fifteen holes of 10 mm diameter were provided at the narrow and wide regions of the middle plate, respectively. Also there is a preinstalled L-shaped rib along the circumferential direction.

For present numerical simulation, three dimensional Reynolds-Averaged Navier-Stokes equations with a standard $k-\varepsilon$ turbulence model are considered. In addition, a homogeneous model was used as an option of multiphase in which a free surface model was configured as a standard way. Tetrahedral elements were used for the most part of the domain and prism elements were used near the wall surface to capture the boundary layer. The total number of elements is 1,517,402. The volume ratio of water was set as 50%. The boundary condition used at the wall was no slip.



(a) Structure of the liquid balancer

(b) Geometry of the middle plate

Fig. 9. Simulation model of the liquid balancer with one race.

The analysis conditions depend on the existence of a middle plate and eccentricity. For the case without a middle plate, 750 RPM was used for the steady state calculation, while for the case with a middle plate, 10 and 750 RPM were used to perform the steady and unsteady state calculations. The eccentricity of the basket was determined as 30 mm based on the dynamic model described in Section 2.2. The computational results with and without eccentricity were compared under the same conditions.

Fig. 10 illustrates the air volume region contacted with the outer wall of the balancer. This region is divided into 10 sectors.

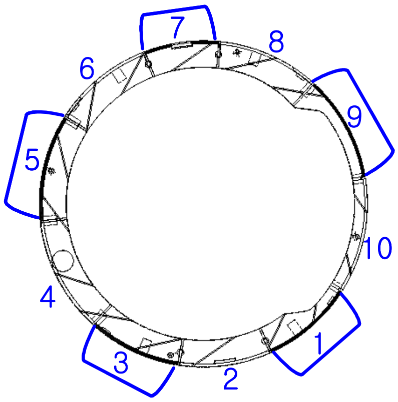
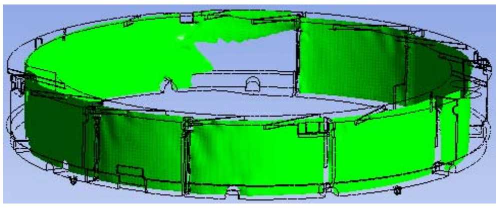
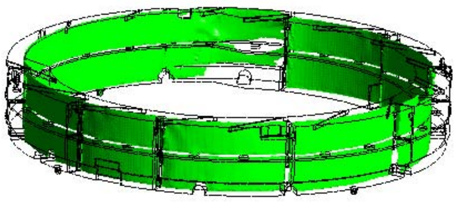


Fig. 10. The air volume region contacted with the outer wall of the balancer. The numbers from 1 to 10 denote the division of the wall zone.



(a) Without middle plate



(b) With middle plate

Fig. 11. Comparison of the isosurface of volume fraction (750 RPM, e=0).

3.2 Analysis results

First we consider the case of the balancer which does not represent eccentricity and is rotating with 750 RPM. Fig. 11 shows the isosurface of the volume fraction of the region where air and water contact each other. It is evident that the calculation was performed in a manner that clearly indicates the boundary between air and water due to the smooth isosurface. However, the discontinuous region of the isosurface can be observed at sector 1.

As shown in Fig. 11(a), this region represents a difficulty in water passing through the sector. It is due to the fact that the width of the race, which corresponds

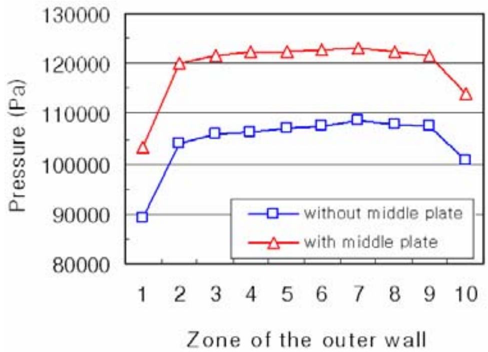


Fig. 12. Pressure on the outer wall of the balancer in the steady state (750 RPM, e=0).

to sector 10, is relatively narrow. However, when the middle plate is installed as illustrated in Fig. 11(b), we can see that there is a large amount of water in sector 1. It is due to the fact that the middle plate causes the liquid inside the balancer to flow more smoothly.

Fig. 12 shows the static pressure acting on the wall of each sector. We can see that the pressure is low in a region filled with a small amount of water. Such unbalance pressure occurred in steady state makes the restoration force of the liquid have locally different values. This may cause certain vibrations and it is difficult to expect reasonable balancing effects. Thus, it is necessary to redesign the balancer for solving these problems.

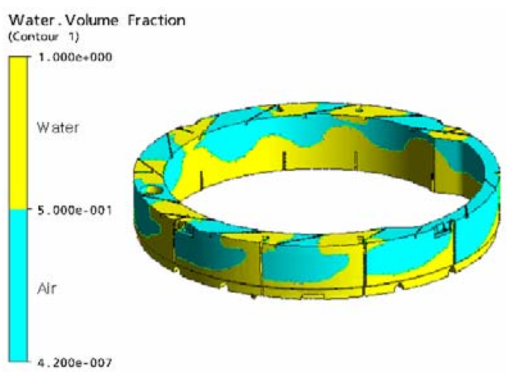
In addition, in the presence of the middle plate, the average static pressure in the entire region increased by 14,907 Pa. It means that this condition is advantageous to the increase of restoration force of the water. Meanwhile, it is proved that the case of two races generates large restoration forces than the case of one race in a linear region as shown in Fig. 7. This is because that the multiple division of the space filled with the liquid avoids the excessive deflection of the liquid caused by the eccentricity. Therefore, the installation of the middle plate can be regarded as a similar method to divide such space. However, it is not enough to fully explain the role of the middle plate. As illustrated in Fig. 12, a high pressure is acting on the wall in the presence of middle plate. It can be understood that a certain scale of large force will act on the water due to the large contact areas between the solid and the water.

Next, let us investigate the results for fluid flow in a low RPM range. The maximum allowed eccentricity in an actual design is 10 mm, but an excessive value of 30 mm is used in this study. The reason for using this value is that the restoration force remains nearly constant for *e* >30 mm as shown in Fig. 7. However, the restoration force of water can be only generated in steady state. Thus, the rotational force of the basket acting on the unbalance mass generates the excessive eccentricity of the balancer in the early spinning stage. Also, the water inside the balancer represents an excessive deflection towards the unbalance mass. Such a deflection in water is a factor to make eccentricity increase. In order to solve this problem, a middle plate was installed inside the balancer.

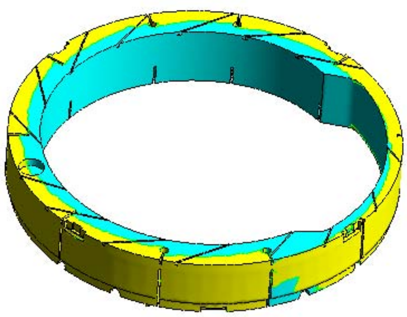
Fig. 13 shows the behavior of water when the middle plate is not used. It passes through an unstable state to approach a steady state of 10 RPM (refer to Fig. 13(a)) and Fig. 13(b)). In this case, the air and water show an unstable behavior due to the rotational and eccentric forces. However when the middle plate is used in the same condition, we can see that the fluctuation of the liquid does not exist as shown in Fig. 14(a). It means that the fluid deflection caused by the unbalance mass in the early spinning stage can be suppressed by the middle plate. The suppression of this fluid flow is also verified in Table 2. However when the middle plate is not used, there are significant variations in the pressure acting on each sector as shown in Fig. 15. Furthermore, it is possible to significantly increase the unbalance of such rotational objects due to the locally generated high-pressure. Based on these facts, the tub shows a collision against the frame in the early spinning stage. Since the pressure of water in the bottom part of the middle plate increases as the RPM increases, the water will be naturally extracted through the hole. As the rotational speed of the basket approaches to a 750 RPM, the

Table 2. Comparison of the average values in sector 8.

Parameter of air and water	10 RPM, *e*= 30 mm	
	Without middle plate	With middle plate
Pressure (Pa)	151606	101414
Rotation velocity (m/s)	16.70	0.21
Velocity (m/s)	7.03	0.04
Force (N)	0.36	5.20e-4
Mass flow of water (Kg/s)	0.01	1.44e-4

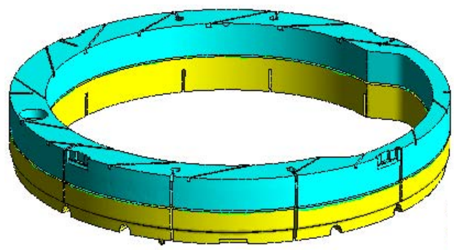


(a) The unstable state of water

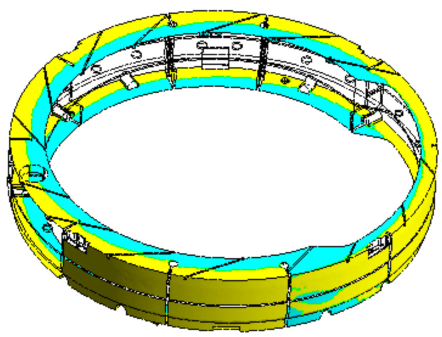


(b) The stable state of water

Fig. 13. Liquid distribution inside the balancer without the middle plate (10 RPM, *e*=30 mm).



(a) 10 RPM, *e*=30 mm



(b) 750 RPM, *e*=0

Fig. 14. Liquid distribution inside the balancer with the middle plate.

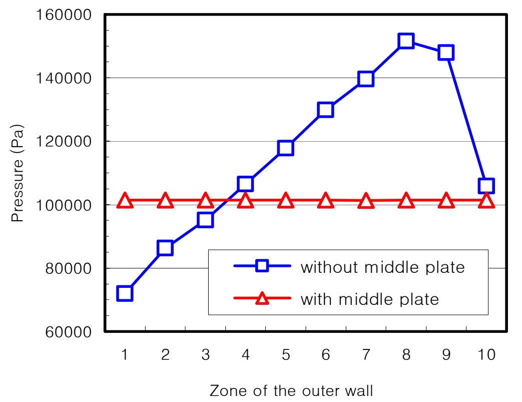


Fig. 15. Pressure on the outer wall of the balancer (10 RPM, *e*=30 mm).

fluid flow inside the balancer will approach the state as shown in Fig. 14(b). Therefore the behavior of the water caused by the middle plate can be summarized as the following three states.

(1) a state in which the water is detained in the bottom part of the middle plate in the early spinning stage

(2) a state in which the water is extracted through the hole with the RPM increase

(3) a state in which a large restoration force is generated when the rotational speed of the basket approaches 750 RPM

Table 2 shows the physical quantities obtained from Sector 8 shown in Fig. 10. The data was obtained in an unsteady state where flow has not been sufficiently developed yet. By comparing physical quantities, we can see that the fluid deflection in the early spinning stage can be suppressed by the middle plate. Thus, it is evident that the middle plate significantly contributes the stability of the system in the early spinning stage.

4. Conclusions

The spinning mode in an automatic washing machine represents two major technical difficulties. One is the collision of the tub against the frame when the basket is started to rotate, and the other is the shaking of the rotation axis due to the whirling vibration at a steady RPM range. In order to analyze these two problems, a CFD method and a dynamic model were used for the former and latter issues, respectively. The results of the application of these two methods can be summarized as follows:

(1) The balancer with two races was installed to a washing machine and the early spinning transient state was recorded using a high-speed camera. From the experiment, fluid deflection was observed during the collision process. Although the balancer with two races could not resolve the problem of early fluid deflection, the analysis with dynamic model showed that restoration forces could be increased by extending its linear range in steady state compared to that of the case of one race.

(2) The dynamic model represented some useful information as follows. The optimum linear range was obtained when the value of R_s - R_i is equal to the value of R_o - R_s . However the smaller one of these two values corresponds to the linear limit in actual calculations. In addition, it was found that the balancer with two races was more effective in suppressing the small amplitude in steady state. Thus the multi-race is a design method that may minimize the possible vibration in steady state.

(3) The CFD model suggests that the average static pressure on the outer wall of the balancer increased to 14907 pa at 750 RPM when the middle plate was installed inside the balancer. This means that the middle plate is useful in increasing the restoration force of water in steady state.

(4) If the middle plate was not used, the static pressure on the outer wall of the balancer showed locally significant variations in unsteady state at 10 RPM. This caused the rotation of the system in the early spinning stage more unstable. This behavior could be mitigated through the use of a middle plate by suppressing the local variation of the pressures caused by the deflection of water.

(5) This study showed that the increase of the amount of the liquid in the liquid balancer with one race was of advantage in transient state due to the increase of a linear region. However, the restoration force decreases due to decrease in the gradient of such a linear region. Considering that the maximum eccentricity was 10 mm in the actual design, it was found that the optimum amount of the applied liquid was 50% of the total volume.

Nomenclature

c : Centroid of the fluid mass (m)
c^* : Limit value of the centroid (m)
e : Eccentricity (m)
F : Centrifugal force (N)

H : Height of the balancer (m)
M : Total mass of the tub and basket (kg)
m_1 : Unbalance mass (kg)
m_2 : Mass of salt water (kg)
R_i, R_o : Inner and outer radii of the balancer (m)
R_s : Distance from the rotating axis to the inner surface of the fluid (m)
V_l : Volume of the fluid (m^3)
V_{all} : Total volume of the balancer (m^3)
ρ : Density of the fluid (kg/m^3)
ω : Rotation speed (RPM)

References

[1] H. J. Oh and U. S. Lee, Dynamic Modeling and Analysis of the Washing Machine System with an Automatic Balancer, *Transactions of the KSME* Series A, (in Korean), 28 (8) (2004) 1212-1200.

[2] H. K. Chae, Publicized Utility Model 1999-002708.

[3] B. S. Yang, J. M. Lee, W. C. Kim and J. K. Koh, A Study on Balancing Performance of Self-Compensating Liquid Balancer, *Journal of the Korean Society of Marine Engineering*, (in Korean), 22 (1) (1998), 35-41.

[4] C. M. Song, Registration Patent 10-0474247.

[5] S. Bae, J. M. Lee, Y. J. Kang, J. S. Kang and J. R. Yun, Dynamic Analysis of an Automatic Washing Machine with a Hydraulic Balancer, *Journal of Sound and Vibration,* 257 (1) (2002) 3-18.

[6] V. Royzman and I. Drach, Improving Theory for automatic Balancing of Rotating Rotors with Liquid Self Balancers, *Mechanika,* 4 (54) (2005) 38-43.

[7] J. Y. Lee, S. O. Jo, T. S. Kim and Y. S. Park, Modeling and Dynamic Analysis of Front Loaded Washing Machine with Ball Type Automatic Balancer, *Journal of the Korean Society for Noise and Vibration Engineering*, (in Korean), 8 (4) (1998) 670-682.

[8] F. R. Menter, P. F. Galpin, T. Esch, M. Kuntz and C. Berner, CFD Simulations of Aerodynamic Flows with a Pressure-Based Method, *24th International Congress of the Aeronautical Sciences,* (2004).