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TECHNICAL NOTE

# Re-examining the Influence of End Restraint on Mechanical Behaviors of Dense Quartz Sands

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## ABSTRACT

Mechanical behaviors of soils may be effectively observed, relying on precise and accurate experimental techniques adopted. The critical state of soils, therefore, can be determined through a series of conventional triaxial compression tests. However, some studies claim that the critical state may not uniquely exist under some conditions, such as end restraint, specimen size, confining pressure, initial relative densities, etc. The objective of this study is to revisit the end restraint condition where it could be negligible for specimens with a height-to-diameter ratio between 2.0 and 2.5, where the ratio is recommended by ASTM D7181-20. A ratio of 2.46 was adopted in this study to ensure proper comparisons under element testing, minimizing end-platen effects, and meeting the ASTM standards. The other aim of this study is to evaluate and compare the mechanical behaviors of the quartz sand with lubricated or nonlubricated end platens, including stress-strain-volume change and critical states. A series of consolidated-drained triaxial compression tests on quartz sand with an initial relative density of 75% and 85% were conducted. The results indicate that the end effect can be minimized when a height-to-diameter ratio is 2.46. Under the circumstances, the lubricated and nonlubricated end platens have few impacts on peak shear strength of the sands tested but they may affect the volume change behavior and the determination of critical state conditions.

## 1. Introduction

Laboratory tests are used to develop and validate constitutive models of soils, assuming that these are representative of real-world soil behavior. The most popular constitutive behavior is based on the critical state soil mechanics (Schofield and Wroth, 1968). Research on the mechanical behavior under this framework of critical state soil mechanics has been ongoing for decades, where the existence and uniqueness of a critical state condition of sands remain debatable. This critical state describes an ultimate state where plastic shearing can continue indefinitely without a change in volume or effective stress. It can be displayed as a curved line in three-dimensional space defined by effective mean confining pressure $p'$, deviatoric stress $q$, and void ratio $e$. This single curved line can be projected onto $p'$-$q$ space and $p'$-$e$ space. The critical state in the $p'$-$q$ space is reached with an effective stress ratio $q_{cs}/p'_{cs} = \eta_{cs} = M_{cs}$. Typically, the slope of the critical state line is calibrated based on the data obtained from monotonic triaxial tests at a large shearing up to 20 – 25% axial strain $\varepsilon_1$ or an almost constant value of deviator stress $q$. Many constitutive models are based on the determination and validity of the critical state behavior of sands. Current approaches in critical state soil mechanics and most constitutive models are constructed upon the uniqueness of critical states (Schofield and Wroth, 1968; Been et al., 1991; Sadrekarimi and Olson, 2012). However, some evidence shows a non-unique relationship for sands (e.g., Mooney et al., 1998; Yamamuro and Lade, 1998; Finno and Rechenmacher, 2003). Many studies approached the critical state of sands from different perspectives, and their results on the response of sands contain many apparent contradictions concerning such factors (e.g., initial relative density, drainage conditions, level of confining stress, sample preparation method, end restraint type, fabric effect, and effective stress path) governing the strength of sands (e.g., Been et al., 1991; Finno and Rechenmacher, 2003; Sadrekarimi and Olson, 2012; Omar and Sadrekarimi, 2015; Kang et al., 2019). For instance, the friction angle is the property that

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**Table 1.** The Past Investigations on the End Restraint Effect of the Sand, Clay, or Other Geo-Materials

| Authors | Purpose | Type of source | Summary points |
|---|---|---|---|
| Rowe (1962) | One of the goals is to reduce friction at the end-platens for triaxial compression specimens. | Journal article | A method for guaranteeing smooth ends was proposed. Upper and lower end-platens were proposed to be coated with silicon grease and covered by a thin rubber sheet. |
| Rowe and Barden (1964) | Free ends are essential for triaxial testing. The triaxial test is criticized for its non-uniformity of stress and deformation, except for small strains. | Journal article | Using lubricated end platens largely overcomes both the barreling effect and the concentration of dilation in local zones. Besides, short samples with lubricated endplates are effective in triaxial testing. |
| Bishop and Green (1965) | Different end-platen types and ratios of height to diameter were tested to examine the influence of end restraint on the strength and stress-strain-volume change. | Journal article | The results indicate that end restraint increases the apparent strength of the sample. This effect decreases as the height-to-diameter ratio increases and becomes insignificant at about a 2-to-1 ratio. |
| Raju et al. (1972) | The comparison between conventional and lubricated end platens in triaxial tests on sands. | Journal article | Stronger strength values are obtained when conventional end-platens are used. Height-to-diameter ratio of 1:1 is more effective in tests with lubricated ends. |
| Tatsuoka et al. (1984) | Tests were conducted on the lubrication layers (consisting of grease and latex discs) end-platens to observe the strength behavior of the samples. An appropriate lubrication method reduces the apparent friction angle between a sample and a steel plate. | Journal article | During cyclic loading, multiple-layer lubrication layers provide slightly better lubrication than single-layer lubrication for the first loading. However, multiple layers introduce significant extra bedding errors. When compressed, the grease of the lubricating layers should be ensured strong enough to resist squeezing. Oil with a high viscosity was not suitable for lubrication after many cycles. Dow grease provides slightly better lubrication than silicone grease KS63G. For Silicone grease KS63G, one layer of grease filler was always left. |
| Goto et al. (1993) | The quality of reducing the apparent frictional angle of the different lubrication layers on granular material was investigated. | Journal article | For Toyoura sand, if a proposed lubrication layer is used, the apparent frictional angle can be reduced. To test Hime Gravel and Silver Leighton Buzzard Sand, a proper amount of powder should be added to the grease to improve the lubrication quality, that is, to prevent the squeezing of grease from the end. |
| Sheng et al. (1997) | During triaxial testing, end restraint may cause inhomogeneities on specimens. A finite element framework was used to simulate the conventional triaxial tests and investigate how axial strain rates and drainage conditions affect soil inhomogeneities. | Journal article | Based on the simulations, both end restraint in drained and undrained tests and insufficient drainage in drained tests can cause non-uniform barrel-shaped deformation of specimens at large strains. An appropriate position for measuring stress path and stress-strain behavior around a specified clay specimen was suggested by this study. |
| Frost and Yang (2003) | Microstructure evolution in dilatant specimens (Ottawa sands) was influenced by end-platens and quantified in terms of spatial and local void ratio distribution. | Journal article | The end-platen effects (e.g., nonlubricated, thin lubricated, thick lubricated) were analyzed for dense sand specimens under low confining pressure and considering the specimen's slenderness. Sand specimens with a 1:1 height-to-diameter ratio and thick lubricated ends display the most uniform deformed shape and void ratio distributions. End lubrication is recommended to better understand soil behavior, particularly for large strain tests. |
| Omar and Sadrekarimi (2014) | Multiple corrections to the triaxial compression testing of sands, such as volume change during saturation and due to membrane penetration, specimen boundaries, area correction for specimen deformation, bedding error, and membrane resistance. | Journal article | Experimental errors affecting loose sand shearing behavior have been reviewed extensively. The strength of sand is significantly affected by non-uniform deformations at large strains. Results from this study have shown that lubricated and enlarged end platens can minimize the effects of end restraint and allow uniform deformations. |
| Muraro and Jommi (2019) | A finite element numerical approach is used to investigate the effects of end restraints on the stress-dilatancy rule of highly compressible peat. | Journal article | Both frictional and smooth end-platen were used to observe the end-restraint effects. With smooth end-platens, a corrected stress-dilatancy relationship for reconstituted peat can be proposed using experimental data and numerically validated. |
| Da Fonseca et al. (2021) | Lubricated end plates are used to assess critical state locus in cohesionless remolded geomaterials. | Journal article | An embedded top-cap loading ram connection and oversized lubricated end plates in the triaxial apparatus can produce uniform shearing without spurious shear bands, reducing strain softening and stabilizing volumetric strain. The proposed configuration can provide reliable data to determine the critical state line. |
| Mozaffari et al. (2022) | Different specimen preparation techniques and end-platen configurations have been questioned due to strain localization and non-uniformity on sand specimens. | Journal article | A three-dimensional finite element framework considering the constitutive model of NorSand was used to simulate strain localization and non-uniformity in sand, validated by experimental data. The configuration is the most effective for drained triaxial compression tests when specimens of a height-to-diameter ratio of two-to-one on a fixed top end without lubrication. This configuration exhibits less strain localization near the platens. |

controls the strength of sands at large strains, where the value is needed for stability analyses, so finding a reliable friction angle relies on a well-conducted series of laboratory tests. Specimen sizes also affected the undrained and drained shearing behavior (e.g., friction angle) that Park and Jeong (2015) investigated. To evaluate the undrained behavior of sands, they suggested using large specimens. By taking into account the different drainage conditions, Liu et al. (2022) established a framework for predicting the shear behavior of sands. The most important factor, the end-restraint effect, is explored in the following. Table 1 briefly lists some studies on the end restraint effect of sand, clay, or other geo-materials, with their purpose and summary points. The end-restraint effect has been shown in some studies to affect material responses. Stiffness at small shear strain levels may be overestimated (e.g., Nguyen and Koseki, 2005). Under undrained monotonic triaxial tests with frictionless ends, peak strength may be overestimated (Lee, 1978). Changes in failure patterns may affect post-peak behavior; i.e., in triaxial compression tests on dense sand, relative sliding as a rigid body occurs along with a shear band formed, where strain localization takes place within this range and is constrained by nonlubricated end conditions (Frost and Yang, 2003; Mozaffari et al., 2022). Inhomogeneities caused by end restraints, strain rates, and insufficient drainage during element testing have also been discussed by Sheng et al. (1997) using the finite element numerical technique. Even though many factors affect specimen responses, this study only focused on the behavior of sand subjected to end restraints under drained conditions. This includes the strength parameters for both the peak and residual states and the dilatancy of the specimens. Besides, the non-uniformity of the samples is also not considered herein.

Triaxial tests are the standard methods to understand the mechanical behaviors of soils and geomaterials. Based on different stress states during testing, triaxial tests can be divided into true and conventional triaxial tests. With geotechnical engineering development, triaxial devices are improved with more advanced techniques (Bai et al., 2022). True triaxial tests, compared with conventional triaxial tests, are relatively complicated. Because the study aims to investigate the end restraint affecting the shearing behavior of sands, complex stress shearing to meet the actual site condition is unnecessary. As a result, the study utilized the conventional triaxial apparatus for further investigation. Conventional triaxial tests are generally used to evaluate the stress-strain-volume behavior of soils. In these conventional triaxial tests, confining pressures can be applied evenly to the soil specimen, vertical and horizontal pressures are considered principal stresses, and drainage conditions and stress paths can be easily controlled. Since this test configuration does not constrain the shear zone, a shear band can naturally develop on the weak zone. However, the friction between the soil specimen and the end platens may partially limit the lateral deformation of the ends of the specimen during the shearing, which is called the end platen effect. The interaction between the partial shear stress caused by friction and the applied axial stress resulted in a complicated stress state within the specimen. Bishop and Green (1965) indicated that the influence of peak strength increases because the frictional end platen resulted in additional constraints on the specimen. Some studies have addressed the issue of reducing the influence of the end platen effect. Rowe (1962) proposed that the platen be coated inside with silicone grease and covered by a thin rubber sheet to reduce end friction to 1 to 2°. However, a thin rubber attached to the silicone grease is easy to deform, affecting the accuracy of the experimental data. As a result, Lee (1978) and Tatsuoka et al. (1984) have tried to find alternative greases to replace the silicone grease attached to a thin rubber or membrane. Still, their studies showed that rubber with silicone grease is the most effective way to have less friction coefficient. One aspect of decreasing end restraint's influence is increasing the height-to-diameter ratio. The ratio is usually about 2.0, as suggested by Bishop and Green (1965). Suppose a platen is considered to use very efficient lubrication. In that case, a specimen's size (the height-to-diameter) ratio of 1.0 is sufficient to reduce the apparent strength caused by the end platen. The contradictory results were found that even though the height-to-diameter ratio is higher or equals 2.0. For instance, Mozaffari et al. (2022) found that lubrication may lead to higher errors, resulting in strain localization closer to end plates from drained triaxial compression tests on specimens with height-to-diameter ratios of two-to-one. However, Lade (2016) recommended a height-to-diameter ratio of 2.5 or slightly higher for dense soil specimens to see the development of the shear bands so they can develop freely without interfering with the endplates (Lade et al., 1996; Wang and Lade, 2001; Da Fonseca et al., 2021). The strength of the soils is still affected by end platen (e.g., Raju et al., 1972; Lee, 1978; Norris, 1981; Frost and Yang, 2003). According to ASTM D7181-20, the average height-to-average diameter ratio should be between 2 and 2.5.

Few studies have focused on this end-restraint effect over the past few years. In order to clarify the uniqueness of the critical state and factors (such as relative density, level of confining stress, and different end restraints methods) that may influence the constitutive behavior of quartz sand, a testing program, namely the conventional triaxial drained tests with enlarged lubricated and nonlubricated end platens, was re-examined. Tests were performed on quartz sands with initial relative densities of 75% and 85% to identify the impact of the end restraint on dilatancy behavior and strength behavior. Overall, the study investigates the level of confining pressures, relative densities, and restraint effect on the stress–strain–volume change behavior under the consolidated triaxial drained tests for dense to very dense quartz sand.

## 2. Material and Testing Program

### 2.1 Tested Material and Test Condition

The quartz sand material, clean white sand with almost no fines content, was used in this study. Physical tests were carried out to characterize the sand according to the ASTM standards: specific gravity test (ASTM D854-06e1), particle-size analysis test (ASTM D422-63), and maximum index density test (ASTM D4253-00). These properties of the sand are introduced as follows: the specific gravity $G_s = 2.65$, the mean grain size $d_{50} = 0.56$ mm, the effective size $d_{10} = 0.38$ mm, the uniformity coefficient $C_u = 1.60$, the coefficient of gradation $C_d = 0.98$, and the group symbol in the USCS classification system is a poorly graded sand (SP). The grain size distribution curve is shown in Fig. 1. Its maximum dry unit weight ($\gamma_{d,max}$) and minimum dry unit weight ($\gamma_{d,min}$) is 15.78 kN/m$^3$ and 13.49 kN/m$^3$, respectively, and the maximum and minimum void ratios of $e_{max} = 0.927$ and $e_{min} = 0.647$ were further calculated. Most of the grains have a subangular shape.

Twelve drained triaxial tests were conducted with two ranges of relative densities: one range is $0.702 \le D_{r0} \le 0.766$ ($D_{r0} \approx 0.75$;

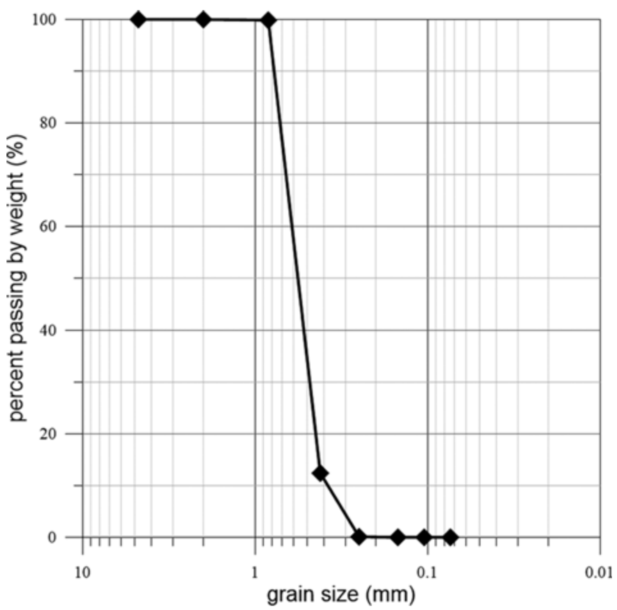


**Fig. 1.** Particle Distribution Curve of the Tested Material

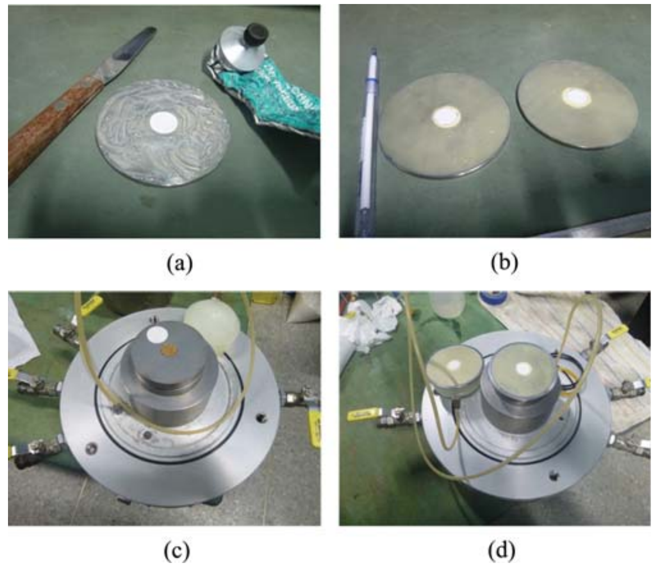


**Fig. 2.** The Process of Making Lubricated End Platen: (a) Apply Silicone Grease to Metal Plate, (b) Affix the Rubber Membrane, (c) Triaxial Base, (d) Lubricated End Platen

dense sand; CD1-CD3 and LCD1-LCD3), and the other is $0.823 \leq D_{r0} \leq 0.861$ ($D_{r0} \approx 0.85$; very dense sand; CD4-CD6 and LCD4-LCD6). Different restraint end effects (nonlubricated and lubricated) have been considered for each group, and at least three different initial effective confining pressures $\sigma'_3 = 100$, 200, and 400 kPa have been applied. The tested program is listed in Table 2.

## 2.2 Specimen Preparation

All the tests in this study were performed on Vietnam silica sand. The test sand was washed, dried in the oven at 100°C for at least 24 hours, and then cooled to room temperature before the specimen was prepared. The tested specimens were prepared directly on the triaxial base plate using a split mold. The dimensions of the specimens were 17.5 cm in height and 7.1 cm in diameter ($H/D = 2.46$). The specimen preparation method is dry-tamping. The dry-tamping method was considered because it can produce denser specimens. The sample preparation steps were to pour oven-dried sand into the mold and tamp the dried sand into five layers, and each layer was compacted to the same target density. The surface of each layer was roughed before the next layer started. After that, the de-aired water was percolated within the specimen until the water overflowed the top vent. Then, the specimens were saturated under cell pressure and back pressures until their B-value of about 0.95 was achieved.

In order to check the effect of end restraint, nonlubricated and enlarged lubricated end platens were used in this study. The process of making an enlarged lubricated end platen is shown in Fig. 2. This polished metal lubricated end platen (40 mm in thickness and 82 mm in diameter) was used with a thin layer of silicone grease and rubber membrane (0.3 mm in thickness), where a small disc of porous stone (15 mm in diameter) was embedded in central. The concept of lubricating material "rubber membrane-silicone grease" and a polished metal plate was by Head (1985). Some studies have also discussed a proper composition of the lubrication layer for experimental element testing (Norris, 1981; Tatsuoka and Haibara, 1985; Goto et al., 1993).

# 3. Test Results and Discussions

Isotropically consolidated triaxial tests were conducted to observe

**Table 2.** Program of CD Triaxial Tests on Nonlubricated and Lubricated End Platens

| Groups | Test no. | $D_{r0}$ [−] | $e_0$ [−] | $p_0$ [kPa] | Test no. | $D_{r0}$ [−] | $e_0$ [−] | $p_0$ [kPa] |
|---|---|---|---|---|---|---|---|---|
| End platen type | Nonlubricated | | | | Lubricated | | | |
| $D_{r0} \approx 0.75$ (Dense sand) | CD1 | 0.761 | 0.714 | 100 | LCD1 | 0.715 | 0.727 | 100 |
| | CD2 | 0.702 | 0.731 | 200 | LCD2 | 0.766 | 0.713 | 200 |
| | CD3 | 0.775 | 0.710 | 400 | LCD3 | 0.754 | 0.716 | 400 |
| $D_{r0} \approx 0.85$ (Very dense sand) | CD4 | 0.851 | 0.689 | 100 | LCD4 | 0.823 | 0.697 | 100 |
| | CD5 | 0.823 | 0.697 | 200 | LCD5 | 0.823 | 0.697 | 200 |
| | CD6 | 0.861 | 0.686 | 400 | LCD6 | 0.859 | 0.686 | 400 |

Note: Void ratios $e_0$ and relative densities $D_{r0}$ measured at initial mean pressure $p_0$ prior to shearing; L means the lubricated restraint has been considered.

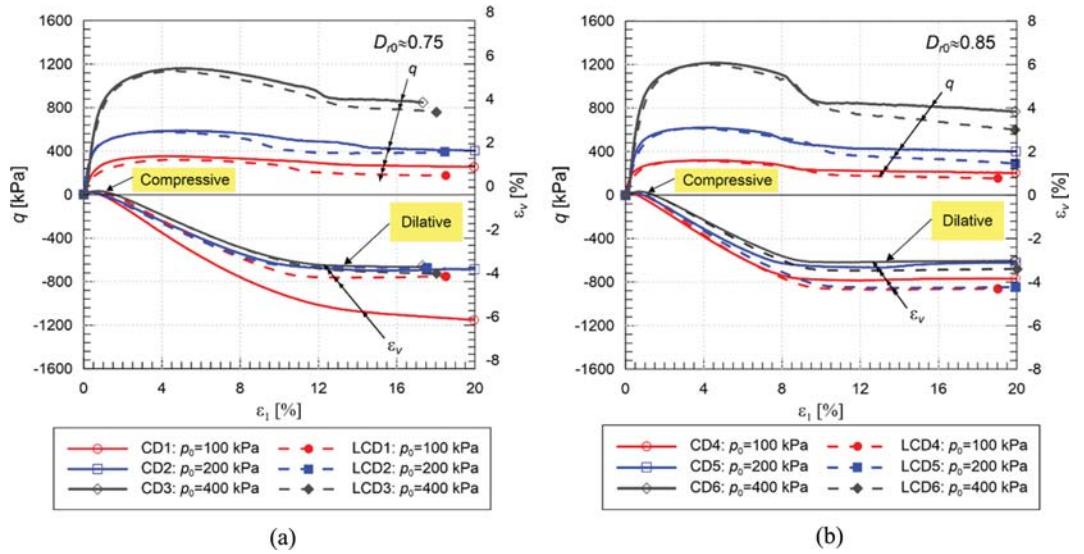


**Fig. 3.** Effect of Nonlubricated and Lubricated end Platens for Dense and Very Dense Sands after Shearing: (a) $D_{r0} \approx 0.75$ (Dense sand), (b) $D_{r0} \approx 0.85$ (Very dense sand)

the shear behavior of sand consistently. In order to observe the effect of end restraints on the strength and dilatancy of sand, drained triaxial tests (CD) were carried out on lubricated and nonlubricated end platens. Two ranges of initial states (i.e., relative densities $D_{r0} \approx 0.75$ and 0.85) of specimens under various confining pressures were tested and compared. The stresses were defined as follows: the mean stress $p = (\sigma_1 + 2\sigma_3)/3$ where $\sigma_1$ is the axial stress and $\sigma_2 = \sigma_3$ is the radial stress for the triaxial loading condition, the mean effective stress $p' = (\sigma_1' + 2\sigma_3')/3$ with deviatoric stress $q = \sigma_1' - \sigma_3'$ where $\sigma_1'$ is the effective axial stress and $\sigma_2' = \sigma_3'$ is the effective radial stress for the triaxial loading condition. The measured strains were: $\varepsilon_1$ is the axial strain, and $\varepsilon_v = (\varepsilon_1 + 2\varepsilon_3)$ is the volumetric strain. The critical state framework proposed by Schofield and Wroth (1968) is adopted in this study, as described in the following two equations:

$$q = M_{cs} p' \tag{1}$$

$$e = e_\Gamma - \lambda \ln p' \tag{2}$$

where $M_{cs}$ is the slope of the line connecting zero stress rates ($\dot{p} = \dot{q} = 0$ kPa), $e_\Gamma$ represents the void ratio at a very low effective mean stress ($p' = 1$ kPa), and $\lambda$ is the model parameter defining the slope of the critical state line.

## 3.1 End Restraint Effects on Strength and Dilatancy

For the dense sand and very dense sand, the observed relationships between $q$ versus $\varepsilon_1$ and $\varepsilon_v$ versus $\varepsilon_1$ were sketched in Fig. 3, where the positive volumetric strain means the compressive behavior and the negative volumetric strain means the dilative behavior. In general, it was found that the shapes of these curves of $q$ versus $\varepsilon_1$ are similar; the feature of the $q$ versus $\varepsilon_1$ curve behaves as strain-softening. The shape of the curves of $\varepsilon_v$ versus $\varepsilon_1$ is also similar. The dilatancy behavior of the specimens contracted initially and then dilated afterward. Besides, the specimens compressed to higher initial mean effective stress values sustain higher shear strength at failure and critical state but lower dilation at a critical state.

In Fig. 3(a), the test results for dense sand show more dilated behavior when using the lubricated end restraint, except for the CD1. For the very dense sand behavior in Fig. 3(b), the tests considering lubricated and nonlubricated end restraint present a similar tendency as the results of dense sand; both restraint conditions seem not to affect the peak strength but the residual strength. Both the dense and very dense sands show that having lubricated end restraints allows sand to behave more dilatively, meaning that greater volumetric dilatation is obtained.

Table 3 shows that the peak friction angles of the nonlubricated end restraint ($D_{r0} \approx 0.75$, $\phi'_p = 36.0°$; $D_{r0} \approx 0.85$, $\phi'_p = 36.6°$) are so close to the results of the lubricated end restraint ($D_{r0} \approx 0.75$, $\phi'_p = 35.6°$; $D_{r0} \approx 0.85$, $\phi'_p = 36.7°$). The residual strengths considering

**Table 3.** Shear Strength and Critical States for Each Group

| Groups | Test no. | $e_{cs}$ [–] | $p'_{cs}$ [kPa] | $q_{cs}$ [kPa] | $\phi'_{cs}$ [°] | $\phi'_p$ [°] |
|---|---|---|---|---|---|---|
| $D_{r0} \approx 0.75$ (Dense sand) | CD1 | 0.816 | 190.6 | 255.7 | 30.5 | 36.0 |
| | CD2 | 0.791 | 340.0 | 403.9 | | |
| | CD3 | 0.767 | 694.2 | 846.6 | | |
| | LCD1 | 0.793 | 166.2 | 176.7 | 28.6 | 35.6 |
| | LCD2 | 0.772 | 343.0 | 393.5 | | |
| | LCD3 | 0.779 | 661.7 | 757.3 | | |
| $D_{r0} \approx 0.85$ (Very dense sand) | CD4 | 0.755 | 183.2 | 202.0 | 28.9 | 36.6 |
| | CD5 | 0.750 | 338.6 | 399.7 | | |
| | CD6 | 0.738 | 666.1 | 765.4 | | |
| | LCD4 | 0.771 | 156.0 | 154.1 | 24.9 | 36.7 |
| | LCD5 | 0.770 | 302.4 | 288.6 | | |
| | LCD6 | 0.745 | 602.1 | 594.2 | | |

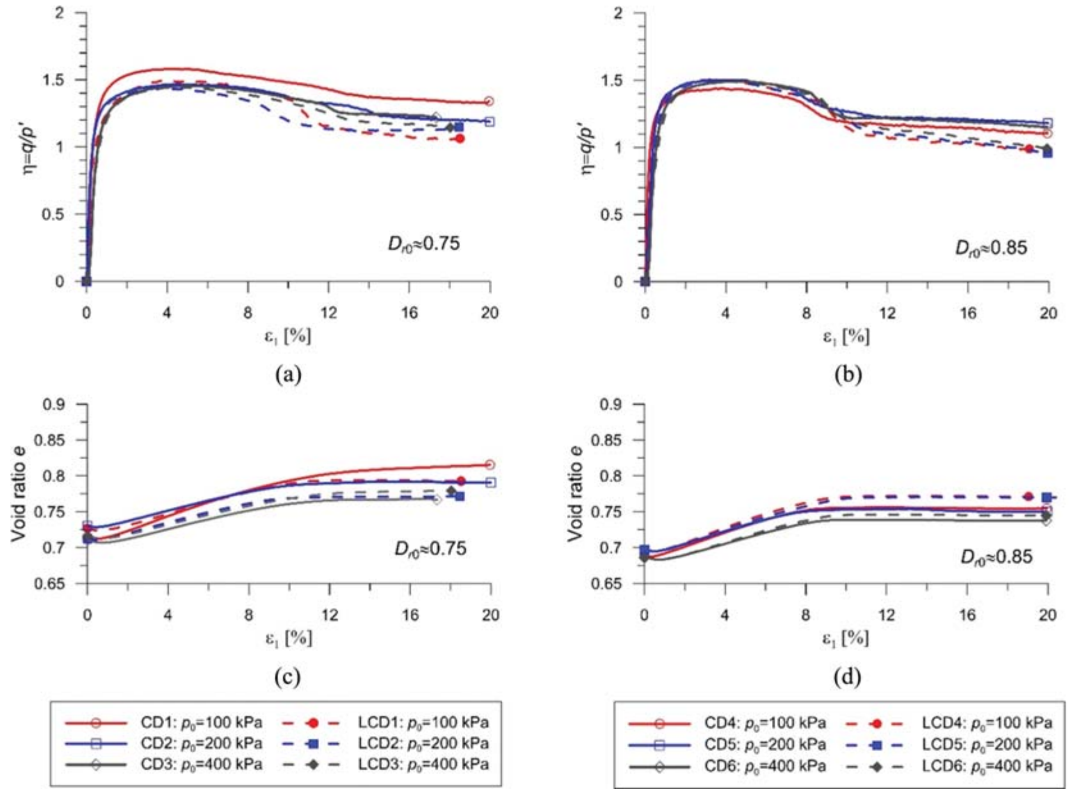


**Fig. 4.** Relationship between Stress Ratio ($\eta$) - Void Ratio (e) - Axial Strain ($\varepsilon_1$): Effect of Nonlubricated and Lubricated End Platens for Dense and Very Dense Sands: (a), (c) $D_{r0} \approx 0.75$, (b), (d) $D_{r0} \approx 0.85$

the lubricated end restraint ($D_{r0} \approx 0.75$, $\phi'_{cs} = 28.6°$; $D_{r0} \approx 0.85$, $\phi'_{cs} = 24.9°$) are smaller than the results on the nonlubricated end restraint ($D_{r0} \approx 0.75$, $\phi'_{cs} = 30.5°$; $D_{r0} \approx 0.85$, $\phi'_{cs} = 28.9°$). Both peak shear and residual shear strengths for nonlubricated end restraints are slightly higher than for lubricated end restraints, as the same observation was reported by (Raju et al., 1972; Lee, 1978). Fig. 3 demonstrates the approximate flat residual strengths and dilatancy of these experiments. After shearing, the dilatancy behavior of the very dense sand, considering the lubricated end restraint, reacted a bit more dilative. Fig. 4 shows the shearing behavior between stress ratio ($\eta$) and axial strain ($\varepsilon_1$) and the void ratio ($e$) versus $\varepsilon_1$, which demonstrates that with lubricated end restraint, the upper-limited shear strength and dilative behavior can be reached. The results reflected here show that the end platen effect can be ignored. Fig. 5 shows the difference of dilatancy behavior between lubricated and non-lubricated ends. It is apparent that lubricated end plates enabled the top end of the specimen to freely develop.

Overall, the type of restraint may have a very slight effect on peak strength (Fig. 6(a)) since the height-to-diameter ratio ($H/D$) for each specimen is 2.46. The ratio adopted meets the ASTM standard of 2.0 to 2.5. The friction of the end restraints did not adversely affect the peak strengths, but the lubricated end restraints can reduce the constraint force on the specimens, resulting in more dilative behavior of the sand.

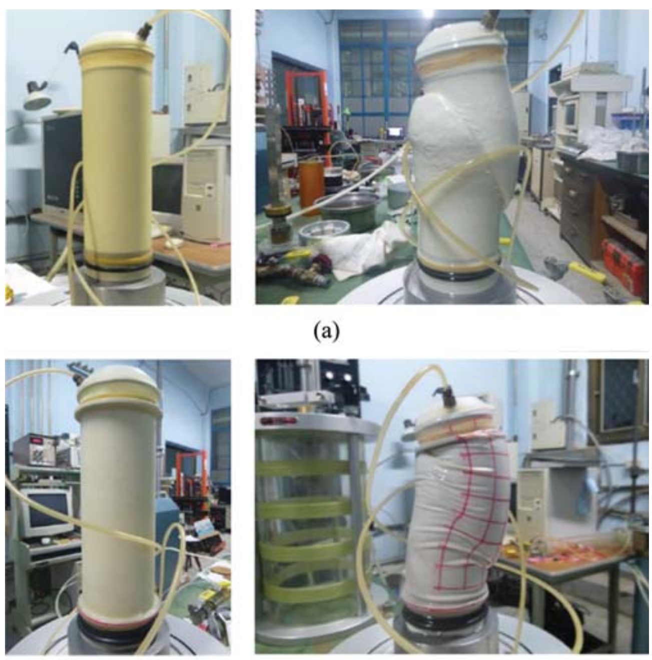


**Fig. 5.** Images of Typical Specimen Shapes after Tests: (a) Non-Lubricated End-Platens, (b) Lubricated End-Platens

## 3.2 End Restraint Effect on the Critical State of Sand

Ideally, the critical state of each soil is unique when applying different stress paths, initial conditions, and configuration methods. Fig. 6(b) shows the critical states for dense and very dense sands for nonlubricated and lubricated end restraints. The slopes of the

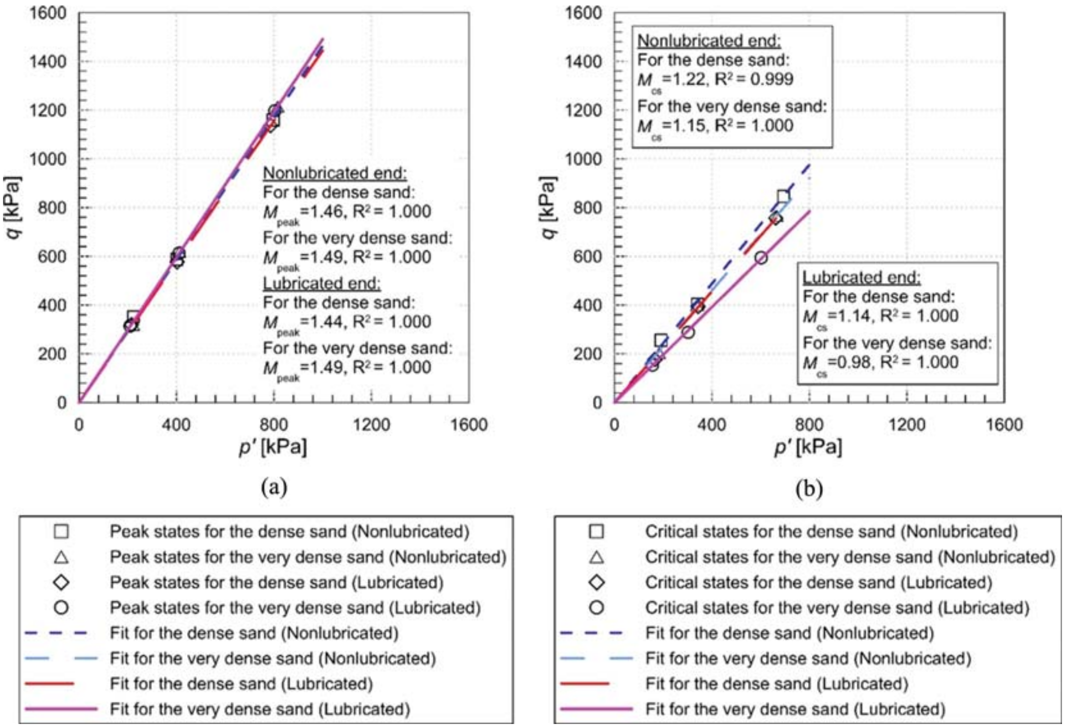


**Fig. 6.** Envelopes of: (a) Peak States, (b) Critical States

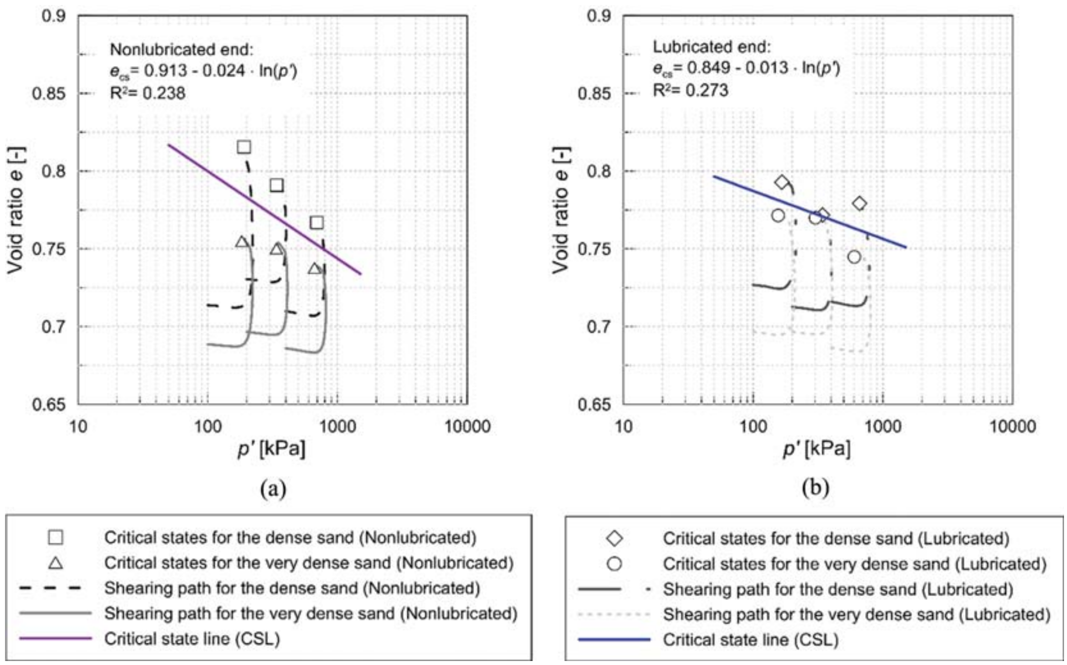


**Fig. 7.** Critical States for: (a) Non-Lubricated End Platen, (b) Lubricated End Platen

critical states of the dense and very dense sand on the nonlubricated end restraint are 1.22 and 1.15, respectively. The slope of the critical state for the whole specimens on the nonlubricated end restraint is 1.19, and its $R^2$ is 0.995. Similarly, the slopes of the critical states of the dense and very dense sand on the lubricated end restraint are 1.14 and 0.98, respectively. According to the regression analysis for all specimens on the lubricated end restraint, the critical state slope is 1.06 with $R^2$ of 0.976. Evidently, the results demonstrate that the critical state of the tested sand in the *p*-*q* space is unique.

However, in Fig. 7(a), in the *e*-*p* space and without considering the lubricated end restraint, the critical state line was shifted downward in the cases of higher initial relative density, and the critical state line is $e = 0.913\text{-}0.024 \cdot \ln(p')$ ($R^2 = 0.238$). In contrast, in Fig. 7(b), when considering the lubricated end restraint, the data set used to regress the critical state has a tendency, showing the uniqueness of the critical state void ratio line at different initial relative densities, and the critical state line is $e = 0.849 - 0.013 \cdot \ln(p')$ ($R^2 = 0.273$). Among these test results, it is observed that as density increases, the critical state points shift towards lower void ratios, especially in drained tests with nonlubricated end restraints. This is possibly due to the development of shear bands (localization of strain), where a lower fraction of sample volume is involved in shearing and dilatancy. This results in a lower average void ratio calculated based on global volume change measurements. When developing advanced constitutive models, part of the essential relationships should be incorporated, e.g., the uniqueness of the critical state. Some researchers indicated that the critical state in the $p'$-$q$ space is unique, but not in the $e$-$p'$ space; the uniqueness of critical states could be affected by fabric-dependent, different stress paths, and so on (e.g., Been et al., 1991; Chu, 1995; Mooney et al., 1998). A growing number of studies claim that the unique critical state line in the $p'$-$q$-$e$ space existed, affirmed by Li and Dafalias (2012). Hence, the use of lubricated end restraints has a potential advantage from the perspective of obtaining the uniqueness of the critical state in the $e$-$p'$ space.

## 4. Conclusions

A series of consolidated drained triaxial tests were conducted to re-visit the influence of two types of end restraints (nonlubricated and lubricated) on the mechanical behaviors of quartz sands, including peak strength, dilatancy, and critical state. These experimental results considering three initial confining pressures ranging from $p_0$ = 100, 200, and 400 kPa and two initial relative densities at around 75% and 85%, were evaluated. For the peak strength (or friction angle), the results indicate that the end platen effect is negligible since the height-to-diameter ratio for each specimen is around 2.46, which is between 2.0 and 2.5, as recommended by ASTM D7181-20. For the dilatancy behavior in volumetric strain versus axial strain space, the results for both types of end restraints achieve constant volume, indicating that the critical state is reached. It was also found that the more dilative behavior of the sand is observed in the lubricated end platen, which may be due to the reduced constraint force on both ends. Furthermore, a unique critical state in the $p'$-$q$ space on the quartz sand can be obtained for both types of end restraint, which proves that if the height-to-diameter ratio of 2.46 is used, the results, including the peak shear strength, peak friction angle, and critical state friction angle, are similar either by nonlubricated end restraint or lubricated end restraint. From the point of view of obtaining the uniqueness of the critical state in the $e$-$p'$ space, the use of lubricated end restraint has more advantages. However, the uniqueness of the critical state in the void ratio versus mean confining pressure space under some circumstances remains questionable.

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## References

Bai J, Diao Y, Jia C, Liu C, Zhang M, Wang C (2022) A review of advances in triaxial tests: instruments, test techniques and prospects. *KSCE Journal of Civil Engineering* 26(8):3325-3341, DOI: 10.1007/s12205-022-1345-1

Been K, Jefferies MG, Hachey J (1991) The critical state of sands. *Geotechnique* 41(3):365-381, DOI: 10.1680/geot.1991.41.3.365

Bishop AW, Green GE (1965) The influence of end restraint on the compression strength of a cohesionless soil. *Geotechnique* 15(3): 243-266, DOI: 10.1680/geot.1965.15.3.243

Chu J (1995) An experimental examination of the critical state and other similar concepts for granular soils. *Canadian Geotechnical Journal* 32(6):1065-1075, DOI: 10.1139/t95-104

Da Fonseca AV, Cordeiro D, Molina-Gómez F (2021) Recommended procedures to assess critical state locus from triaxial tests in cohesionless remoulded samples. *Geotechnics* 1(1):95-127, DOI: 10.3390/geotechnics1010006

Finno RJ, Rechenmacher AL (2003) Effects of consolidation history on critical state of sand. *Journal of Geotechnical and Geoenvironmental Engineering* 129(4):350-360, DOI: 10.1061/(ASCE)1090-0241(2003)129:4(350)

Frost JD, Yang CT (2003) Effect of end platens on microstructure evolution in dilatant specimens. *Soils and Foundations* 43(4):1-11, DOI: 10.3208/sandf.43.4_1

Goto S, Park CS, Tatsuoka F, Molenkamp F (1993) Quality of the lubrication layer used in element tests on granular materials. *Soils and Foundations* 33(2):47-59, DOI: 10.3208/sandf1972.33.2_47

Head KH (1985) Manual of soil laboratory testing. *Effective Stress Tests* (No. 2 ed., Vol. 3): John Wiley and Sons

Kang X, Xia Z, Chen R, Ge L, Liu X (2019) The critical state and steady state of sand: A literature review. *Marine Georesources & Geotechnology* 37(9):1105-1118, DOI: 10.1080/1064119X.2018.1534294

Lade PV (2016) Triaxial equipment. In L. John Wiley & Sons (Ed.), *Triaxial Testing of Soils* (First ed., pp. 99-143). John Wiley & Sons, Ltd, DOI: 10.1002/9781119106616.ch3

Lade PV, Yamamuro JA, Skyers BD (1996) Effects of shear band formation in triaxial extension tests. *Geotechnical Testing Journal* 19(4):398-410, DOI: 10.1520/GTJ10717J

Lee KL (1978) End restraint effects on undrained static triaxial strength of sand. *Journal of the Geotechnical Engineering Division* 104(6): 687-704, DOI: 10.1061/AJGEB6.0000643

Li XS, Dafalias YF (2012) Anisotropic critical state theory: Role of fabric. *Journal of Engineering Mechanics* 138(3):263-275, DOI: 10.1061/(asce)em.1943-7889.0000324

Liu X, Miao X, Qin Z, Huang G, Lan H (2022) Shear behavior of loess: The role of drainage condition. *Engineering Geology,* 309, DOI: 10.1016/j.enggeo.2022.106835

Mooney MA, Finno RJ, Viggiani MG (1998) A unique critical state for sand? *Journal of Geotechnical and Geoenvironmental Engineering* 124(11):1100-1108, DOI: 10.1061/(ASCE)1090-0241(1998)124:11(1100)

Mozaffari M, Liu W, Ghafghazi M (2022) Influence of specimen non-uniformity and end restraint conditions on drained triaxial compression test results in sand. *Canadian Geotechnical Journal* 59(8):1414-1426, DOI: 10.1139/cgj-2021-0505

Muraro S, Jommi C (2019) Implication of end restraint in triaxial tests on the derivation of stress–dilatancy rule for soils having high compressibility. *Canadian Geotechnical Journal* 56(6):840-851, DOI: 10.1139/cgj-2018-0343

Nguyen N, Koseki J (2005) Quasi-elastic deformation properties of Toyoura sand in cyclic triaxial and torsional loadings. *Soils and Foundations* 45:19-38, DOI: 10.3208/sandf.45.5_19

Norris GM (1981) Effect of end membrane thickness on the strength of frictionless cap and base test. *ASTM Special Technical Publications,* 304-314, DOI: 10.1520/STP28759S

Omar T, Sadrekarimi A (2014) Effects of multiple corrections on triaxial compression testing of sands. *Journal of GeoEngineering* 9(2):75-83, DOI: 10.6310/jog.2014.9(2).3

Omar T, Sadrekarimi A (2015) Specimen size effects on behavior of loose sand in triaxial compression tests. *Canadian Geotechnical Journal* 52(6):732-746, DOI: 10.1139/cgj-2014-0234

Park S-S, Jeong SW (2015) Effect of specimen size on undrained and drained shear strength of sand. *Marine Georesources & Geotechnology* 33(4):361-366, DOI: 10.1080/1064119X.2013.879627

Raju VS, Sadasivan SK, Venkataraman M (1972) Use of lubricated and conventional end platens in triaxial tests on sands. *Soils and Foundations* 12(4):35-43

Rowe PW (1962) The stress-dilatancy relation for static equilibrium of an assembly of particles in contact. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences* 269(1339): 500-527, DOI: 10.1098/rspa.1962.0193

Rowe PW, Barden L (1964) Importance of free ends in triaxial testing. *Journal of the Soil Mechanics and Foundations Division* 90(1):1-27, DOI: 10.1061/JSFEAQ.0000586

Sadrekarimi A, Olson SM (2012) Effect of sample-preparation method on critical-state behavior of sands. *Geotechnical Testing Journal* 35(4):548-562, DOI: 10.1520/GTJ104317

Schofield AN, Wroth P (1968) Critical state soil mechanics. McGraw-Hill

Sheng D, Westerberg B, Mattsson H, Axelsson K (1997) Effects of end restraint and strain rate in triaxial tests. *Computers and Geotechnics* 21(3):163-182, DOI: 10.1016/S0266-352X(97)00021-9

Tatsuoka F, Haibara O (1985) Shear resistance between sand and smooth or lubricated surfaces. *Soils and Foundations* 25(1):89-98, DOI: 10.3208/sandf1972.25.89

Tatsuoka F, Molenkamp F, Torii T, Hino T (1984) Behavior of lubrication layers of platens in element tests. *Soils and Foundations* 24(1):113-128, DOI: 10.3208/sandf1972.24.113

Wang Q, Lade PV (2001) Shear banding in true triaxial tests and its effect on failure in sand. *Journal of Engineering Mechanics* 127(8): 754-761, DOI: 10.1061/(ASCE)0733-9399(2001)127:8(754)

Yamamuro JA, Lade PV (1998) Steady-state concepts and static liquefaction of silty sands. *Journal of Geotechnical and Geoenvironmental Engineering* 124(9):868-877, DOI: 10.1061/(Asce)1090-0241(1998)124:9(868)