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Tunnel Engineering

A Theoretical Solution of Deformation and Stress Calculation of the Underlying Tunnel Caused by Foundation Pit Excavation

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ABSTRACT

In the methods for mitigating the effect of foundation pit excavation on underlying tunnel, the conventional analytical calculation method adopted the elastic foundation beam to derive the tunnel deformation and internal forces. However, this traditional approach assumes that the released stress is loaded on the excavation face and subsequently affects the tunnel stress, which can be obtained by Mindlin's solution. The limitations of this method are analyzed, and the soil deformation caused by excavation is introduced as an external load in the model. A differential equation of the elastic foundation beam is established and used to derive the analytic solution of the deformation and internal force of the tunnel by analysis of the empirical formula of soil deformation under foundation pit. Additionally, the weighted residual solution is introduced due to the difficulty in calculation of analytic solution. Considering that the project in this study is a three-dimensional problem?the torque is derived for a complete elastic foundation beam solution.

1. Introduction

Excavation of foundation pit will inevitably result in uplift and rebound deformation of the soil (Liu et al., 2000). As the foundation pit crosses the existing tunnel structure, the tunnel structure will produce additional stress and deformation as well (Chang et al., 2001; Byun et al., 2006). At present, there are three methods to calculate the stress and deformation of tunnels from excavation: empirical methods, numerical simulation analysis, and analytical (semi-analytical) theoretical formula method (Marta, 2001; Sharma et al., 2001). The advantages of the empirical method are simple and convenient, but its lack of theoretical foundation necessitates verification of its applicability. Numerical simulation analysis enable accurately evaluation and calculation of the tunnel deformation. However, the modeling process generally needs many time and energy, resulting in a lengthy cycle and complex calculations. The analytical theoretical formula calculation method can integrate the advantages of both aforementioned methods, which is conducive to the actual project application and provide significance reference value (Chen and Li, 2005; Klar et al., 2007; Zhang et al., 2015; Liang et al., 2017).

Faghidian et al. (2016, 2017) studied weighted residual methods and proposed an analytical analysis and the stress function approach to predict the deformation of tunnel.

Based on the existing research utilizing analytical theoretical formula methods, the calculation of the additional deformation and stress in tunnels caused by foundation pit unloading can be roughly categorized into the following two categories:

1. Yamaguchi et al. (1998) assumed that the rheological properties of the actual soil layer are in contact with the incomplete adhesion between the tunnel and the soil, which leads to the error of the hypothetical calculation method.
2. Chen and Li (2005), Klar et al. (2007), Vorster et al. (2005) and Zhang and Huang (2009) proposed a two-stage calculation method based on Mindlin solution (Mindlin Raymond, 1936) to calculate the additional stress of soil around the tunnel (pipeline) from excavation. Faghidian (2014, 2015) developed a smoothed inverse eigenstrain method from limited strain measurements. The reliability of the method is verified by centrifuge tests and engineering examples.

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However, the calculation of this method through multiple integrals poses a significant challenge, thereby impeding its practical application in engineering. At present, there is a lack of a reasonable simplified calculation formula to solve the tunnel vertical deformation from excavation.

In summary, this study employs the two-stage calculation method of additional stress to carry out theoretical analysis and calculation on the stress and deformation of the existing tunnel under foundation pit excavation.

2. Method for Calculating Additional Stress Caused by Uniformly Distributed Load

In the first stage of conventional calculation of tunnel displacement, additional stress can be calculated by the Flamant solution for plane problems and the Mindlin classical solution for three-dimensional problems.

2.1 Additional Stress Calculation Based on Flamant Solution

The vertical uniformly distributed load acting symmetrically on the tunnel structure can be represented by several small concentrated force sections, as shown in Fig. 1. The stress induced at location (x, y) by the concentrated force F at the origin can be determined using the following equations:

$$\sigma_x = -\frac{2F}{\pi}\frac{x^2 y}{\left(x^2+y^2\right)^2}, \tag{1}$$

$$\sigma_y = -\frac{2F}{\pi}\frac{y^3}{\left(x^2+y^2\right)^2}, \tag{2}$$

$$\tau_{xy} = \tau_{yx} = -\frac{2F}{\pi}\frac{xy^2}{\left(x^2+y^2\right)^2}. \tag{3}$$

Then, several small concentrated forces are superimposed, that is, by integrating the Flamant solution of the stress components of Eqs. (1) – (3) (Powrie, 2015), the additional stress solution under vertical uniform load q can be obtained as follows:

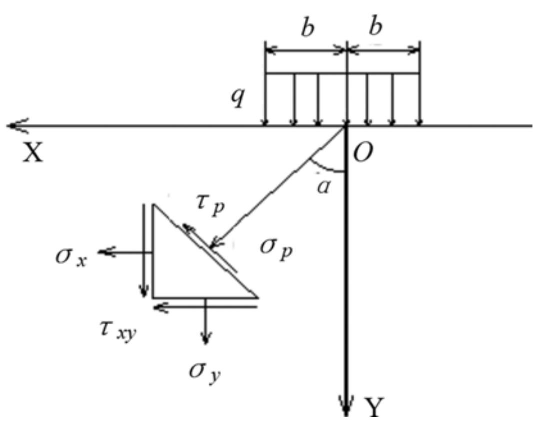

Fig. 1. Calculation Diagram of Flament

$$\sigma_x = -\frac{q}{\pi}\left[\begin{array}{l}\arctan\dfrac{x+b}{y} - \arctan\dfrac{x-b}{y} \\ -\dfrac{y(x+b)}{y^2+(x+b)^2} + \dfrac{y(x-b)}{y^2+(x-b)^2}\end{array}\right], \tag{4}$$

$$\sigma_y = -\frac{q}{\pi}\left[\begin{array}{l}\arctan\dfrac{x+b}{y} - \arctan\dfrac{x-b}{y} \\ +\dfrac{y(x+b)}{y^2+(x+b)^2} - \dfrac{y(x-b)}{y^2+(x-b)^2}\end{array}\right], \tag{5}$$

$$\tau_{xy} = \tau_{yx} = \frac{q}{\pi}\left[\frac{y^2}{y^2+(x+b)^2} - \frac{y^2}{y^2+(x-b)^2}\right]. \tag{6}$$

2.2 Additional Stress Calculation Based on Mindlin Solution

The change of stress from the excavation above the existing tunnel (see Fig. 2). The area of the foundation pit is A, with

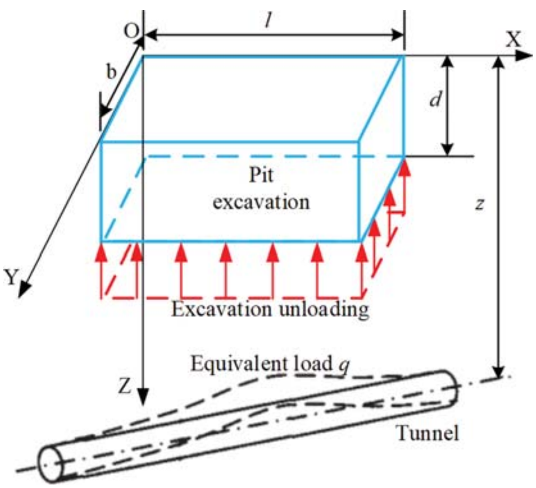

Fig. 2. Schematic Diagram of the Upper-Level Crossing Tunnel for Excavation and Unloading of Foundation Pit

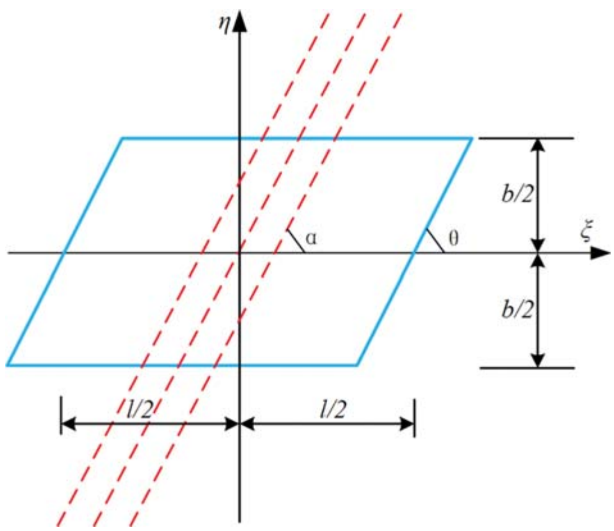

Fig. 3. Plane Projection of Foundation Pit and Tunnel Location

dimension of $l \times b$ and a depth of excavation d. The existing tunnel is buried at a depth of z, and its longitudinal axis is parallel to the surface where the uniformly distributed load acts, forming an angle of α with the load axis, as shown in Fig. 3. It is assumed that the unloading effect of foundation pit excavation is equivalent to a quadrilateral uniformly distributed load (Zhang and Huang, 2009).

According to the Mindlin stress solution, when any point is under the action of concentrated force Q:

$$\sigma_z = \frac{Q}{8\pi(1-\nu)}\begin{bmatrix} \frac{(1-2\nu)(z-d)}{R_1^3}+\frac{3(z-d)^3}{R_1^5} \\ -\frac{(1-2\nu)(z-d)}{R_2^3}+\frac{30dz(z+d)^3}{R_2^7} \\ +\frac{3(3-4\nu)z(z+d)^2}{R_2^5}-\frac{3d(z+d)(5z-d)}{R_2^5} \end{bmatrix} \tag{7}$$

where Q is the vertical concentrated force at (0, 0, d) and υ is Poisson's ratio of the soil layer.

According to Eq. (7), when any point (x, y, z) of the tunnel below the foundation pit is subjected to the force $pd\xi d\eta$ at a certain point (ξ, η), the additional stress in the Z direction of the tunnel is followed:

$$\begin{aligned}\sigma_z = \frac{p}{8\pi(1-\nu)}\Big\{&(1-2\nu)(z-d)\iint_D \frac{d\xi d\eta}{R_1^3} \\ &+3(z-d)^3\iint_A \frac{d\xi d\eta}{R_1^5}+\big[3(3-4\nu)z(z+d)^2 \\ &-3d(z+d)(5z-d)\big]\iint_A \frac{d\xi d\eta}{R_2^5} \\ &-(1-2\nu)(z-d)\iint_D \frac{d\xi d\eta}{R_2^3} \\ &+30dz(z+d)^3\iint_A \frac{d\xi d\eta}{R_2^7}\Big\},\end{aligned} \tag{8}$$

where

$$R_1 = \sqrt{(x-\xi)^2+(y-\eta)^2+(z_0-d)^2}\ ;$$

$$R_2 = \sqrt{(x-\xi)^2+(y-\eta)^2+(z_0+d)^2}\ ;$$

Equation (8) is applicable solely for resolving the additional stress generated by the soil at the tunnel depth of Z. The calculation of the tunnel structural stress caused by the additional soil stress will be presented in the subsequent section.

2.3 Calculation of Tunnel Structure Stress Based on the Galerkin Method

It is assumed that the vertical additional load caused by excavation of the foundation pit above the tunnel, as shown in Fig. 4. The change in water pressure around the tunnel during the excavation of the foundation pit is ignored (Wang, 1990). The separation of elastic resistance is not considered in the upper 45° range of both sides of the tunnel, and the range of the separation area is

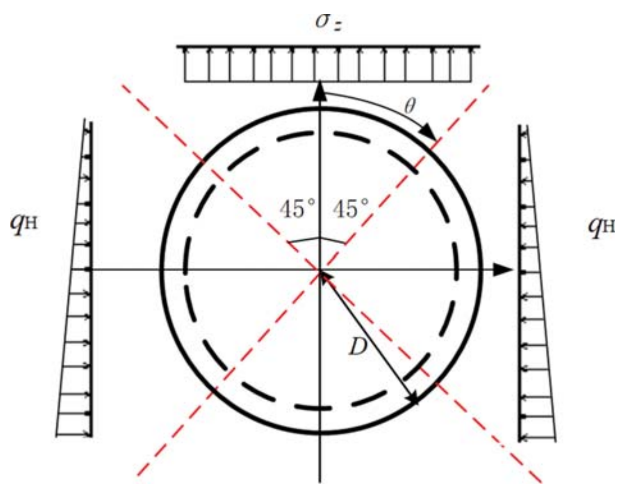

Fig. 4. Structural Stress Calculation Diagram of the Unit Width Tunnel

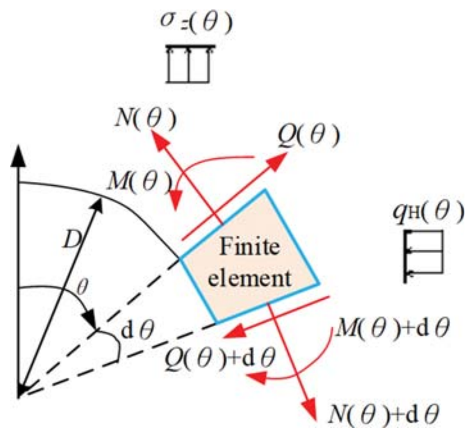

Fig. 5. Calculation Diagram of Micro Elements at Any Part of the Tunnel Structure

recommended as [-45°, 45°] according to the reference (Wang, 1990). The micro element diagram of any section in the tunnel structure reveals that $Dd\theta$, $w(\theta)$ and $v(\theta)$ are normal displacement and tangential displacement; $k_v v(\theta)$ and $k_w w(\theta)$ are tangential and normal elastic resistance; k_v and k_w are tangent and normal subgrade coefficients of foundation soil; axial force, shear force and bending moment caused by additional load of tunnel micro element are $N(\theta)$, $Q(\theta)$ and $M(\theta)$ respectively, as shown in Fig. 5.

According to the static equilibrium equation $\Sigma X = 0$, $\Sigma Y = 0$, $\Sigma M = 0$, the following results are obtained:

$$\sum X = \frac{dQ(\theta)}{d\theta}+N(\theta)-Dk_w w(\theta)-\sigma_z(\theta)D\cos^2\theta+D\sin^2\theta q_H(\theta)=0, \tag{9}$$

$$\sum Y = \frac{dN(\theta)}{d\theta}-Q(\theta)-Dk_V v(\theta)-0.5D(\sigma_z(\theta)+q_H(\theta))\sin 2\theta=0, \tag{10}$$

$$\sum M = \frac{dM(\theta)}{d\theta}+DQ(\theta)=0\ , \tag{11}$$

Substituting Eqs. (9) and (10) into Eq. (11), the following results is obtained:

$$\frac{\mathrm{d}^3 M(\theta)}{\mathrm{d}\theta^3}+\frac{\mathrm{d}M(\theta)}{\mathrm{d}\theta}-D^2 k_{\mathrm{V}} v(\theta)-D^2 k_{\mathrm{w}}\frac{\mathrm{d}w(\theta)}{\mathrm{d}\theta} =\frac{3}{2}D^2(q_{\mathrm{H}}(\theta)+\sigma_z(\theta))\sin 2\theta \cdot \tag{12}$$

The tangential strain of the tunnel caused by the additional axial force is ignored:

$$\varepsilon(\theta)=\frac{\mathrm{d}v(\theta)}{D\mathrm{d}\theta}+\frac{w(\theta)}{D}=0\ . \tag{13}$$

By integrating Eq. (13), the following results is obtained:

$$v(\theta)=\int w(\theta)d\theta+C\,. \tag{14}$$

Combined with the theoretical force balance formula and Eq. (14) of the circular beam, the following formula can be obtained:

$$M(\theta)=\frac{EI}{D^2}\left(-w(\theta)-\frac{\mathrm{d}^2 w(\theta)}{\mathrm{d}^2\theta^2}\right), \tag{15}$$

$$Q(\theta)=\frac{EI}{D^3}\left(\frac{\mathrm{d}^2 w(\theta)}{\mathrm{d}\theta^3}+\frac{\mathrm{d}w(\theta)}{\mathrm{d}\theta}\right), \tag{16}$$

$$N(\theta)=-\frac{EI}{D^3}\left(\frac{\mathrm{d}^4 w(\theta)}{\mathrm{d}\theta^4}+\frac{\mathrm{d}^2 w(\theta)}{\mathrm{d}\theta^2}\right). \tag{17}$$

After substituting Eqs. (15) – (17) into Eq. (12) and simplifying it, the following results is obtained:

$$\frac{\mathrm{d}^5 w(\theta)}{\mathrm{d}\theta^5}+2\frac{d^3 w(\theta)}{\mathrm{d}\theta^3}+\left(1+k_w\frac{EI}{D^4}\right)\frac{\mathrm{d}w(\theta)}{\mathrm{d}\theta} -k_V\frac{EI}{D^4}\left(\int w(\theta)d(\theta+C)\right)=-\frac{3EI}{2D^4}(\sigma_z(\theta)+q_H(\theta)\sin 2\theta) \tag{18}$$

The stress distribution of the circular tunnel is also axisymmetric due to its inherent nature. In addition, the displacement and stress boundary conditions should be satisfied at $\theta = 0$ and $\theta = \pi$:

$$w(\theta)\Big|_{\theta=\pi}^{\theta=0}\neq 0\ , \tag{19}$$

$$v(\theta)\Big|_{\theta=\pi}^{\theta=0}=-\left(\int w(\theta)d\theta+C\right)\Big|_{\theta=\pi}^{\theta=0}=0\ , \tag{20}$$

$$M(\theta)\Big|_{\theta=\pi}^{\theta=0}=-\frac{EI}{D^2}\left(\frac{\mathrm{d}^2 w(\theta)}{\mathrm{d}\theta^2}+w(\theta)\right)\Big|_{\theta=\pi}^{\theta=0}\neq 0\ , \tag{21}$$

$$Q(\theta)\Big|_{\theta=\pi}^{\theta=0}=\frac{EI}{D^3}\left(\frac{\mathrm{d}^3 w(\theta)}{\mathrm{d}\theta^3}+\frac{\mathrm{d}w(\theta)}{\mathrm{d}\theta}\right)\Big|_{\theta=\pi}^{\theta=0}=0\ , \tag{22}$$

$$N(\theta)\Big|_{\theta=0}=-\frac{EI}{D^3}\left(\frac{\mathrm{d}^4 w(\theta)}{\mathrm{d}\theta^4}+\frac{\mathrm{d}^2 w(\theta)}{\mathrm{d}\theta^2}\right)\Big|_{\theta=0}-Dq_{\mathrm{V}}(\theta)\neq 0\ , \tag{23}$$

$$N(\theta)\Big|_{\theta=\pi}=-\frac{EI}{D^3}\left(\frac{\mathrm{d}^4 w(\theta)}{\mathrm{d}\theta^4}+\frac{\mathrm{d}^2 w(\theta)}{\mathrm{d}\theta^2}\right)\Big|_{\theta=\pi}-Dk_{\mathrm{w}}w(\theta)\neq 0\,. \tag{24}$$

Concrete's compressive performance is highly reliable in practical engineering applications as it can handle the stress of bending moment and shear force in tunnel structures. Considering the calculation formula of the additional bending moment and shear force of the tunnel structure by the Galerkin method, the boundary conditions of Eqs. (19) – (24) are satisfied:

$$w(\theta)=\sum_{m=1}^{n}A_m\cos(m\theta)\ , \tag{25}$$

when $\theta = \pi/4$, the elastic resistance has discontinuity; when $\theta = \pi/2$, the vertical additional load σ_z also has discontinuity. By substituting Eq. (25) into Eq. (18), the residual value functions of different intervals are obtained:

at $0 < \theta < \pi/4$,

$$R_{11}=\sum_{m=1}^{n}(m^5-2m^3+m)\sin(m\theta)A_m -\frac{3EI}{2D^4}[q_{\mathrm{H}}(\theta)-\sigma_z(\theta)]\sin(2\theta) \tag{26}$$

at $\pi/4 < \theta < \pi/2$,

$$R_{12}=\sum_{m=1}^{n}\left[m^5-2m^3+m+\frac{mEI}{D^4}\left(k_W+\frac{k_V}{m^2}\right)\right]\sin(m\theta)A_m -\frac{3EI}{2D^4}[q_{\mathrm{H}}(\theta)-\sigma_z(\theta)]\sin(2\theta) \tag{27}$$

at $\pi/2 < \theta < \pi$,

$$R_{13}=\sum_{m=1}^{n}\left[m^5-2m^3+m+\frac{mEI}{D^4}\left(k_W+\frac{k_V}{m^2}\right)\right]\sin(m\theta)A_m -\frac{3EI}{2D^4}q_{\mathrm{H}}(\theta)\sin(2\theta) \tag{28}$$

Let the weight function $W_k(\theta) = \cos(k\theta)$, (where $k = 1, 2, 3,\ldots, n$). The global weighted residual function equation of the Galerkin method can be obtained:

$$\int_0^{\pi}W_k R_1 \mathrm{d}\theta=\int_0^{\pi/4}W_k R_{11}\mathrm{d}\theta+\int_{\pi/4}^{\pi/2}W_k R_{11}\mathrm{d}\theta+\int_{\pi/2}^{\pi}W_k R_{11}\mathrm{d}\theta=0\ \cdot \tag{29}$$

In addition, a simplified calculation is considered, in which $q_{\mathrm{H}}(\theta)$ and $\sigma_z(\theta)$ approximately take the additional stress at the center of the tunnel as follows:

$$q_{\mathrm{H}}(\theta)=\sigma_{\mathrm{H}}=k_0\sigma_z=(1-\sin\varphi)\sigma_z\ ,\quad \sigma_z(\theta)=\sigma_z\ , \tag{30}$$

where φ is the interior friction angle and k_0 is the lateral pressure coefficient, which is equal to 1-sinφ (Jaky, 1944). By substituting Eqs. (26) – (28) and Eq. (30) into Eq. (29), the calculation formula of the global Galerkin weight residual function equation A_m is obtained:

$$A_m=\frac{\left[((1-\sin\varphi)\sigma_z)\int_0^{\pi}\sin(2\theta)\cos(k\theta)\mathrm{d}\theta-\sigma_z\int_0^{\pi/2}\sin(2\theta)\cos(k\theta)\mathrm{d}\theta\right]}{\frac{2D^4}{3EI}\sum_{m=1}^{n}\left\{(m^5-2m^3+m)\int_0^{\pi/4}\sin(m\theta)\cos(k\theta)\mathrm{d}\theta+\left[m^5-2m^3+m+\frac{mEI}{D^4}\left(k_W+\frac{k_V}{m^2}\right)\right]\int_{\pi/4}^{\pi}\sin(m\theta)\cos(k\theta)\mathrm{d}\theta\right\}}, \tag{31}$$

where,

$$\int_{\theta_1}^{\theta_2}\sin(m\theta)\cos(k\theta)\,d\theta = \begin{cases} \left[-\dfrac{\cos(m+k)\theta}{2(m+k)} - \dfrac{\cos(m-k)\theta}{2(m-k)}\right]\Bigg|_{\theta_1}^{\theta_2} (k \neq m) \\ \left[\sin^2(m\theta)/2m\right]\Big|_{\theta_1}^{\theta_2} (k = m) \end{cases}. \tag{32}$$

According to Wang's (1990) study, utilizing a value of $k = m = 8$ meets the accuracy standards required for engineering design. By substituting Eq. (31) into Eqs. (17) and (18), respectively, and converting them into matrix form, the calculation formula of the bending moment and shear force of the tunnel structure at any point can be obtained:

$$M(\theta)=\frac{EI}{D^2}\begin{bmatrix}(1^2\text{-}1)\cos(1\theta)\\(2^2\text{-}1)\cos(2\theta)\\(3^2\text{-}1)\cos(3\theta)\\(4^2\text{-}1)\cos(4\theta)\\(5^2\text{-}1)\cos(5\theta)\\(6^2\text{-}1)\cos(6\theta)\\(7^2\text{-}1)\cos(7\theta)\\(8^2\text{-}1)\cos(8\theta)\end{bmatrix}^T\begin{bmatrix}A_1\\A_2\\A_3\\A_4\\A_5\\A_6\\A_7\\A_8\end{bmatrix}=\frac{EI}{D^2}\begin{bmatrix}0\\3\cos(2\theta)\\8\cos(3\theta)\\15\cos(4\theta)\\24\cos(5\theta)\\35\cos(6\theta)\\48\cos(7\theta)\\63\cos(8\theta)\end{bmatrix}^T\begin{bmatrix}A_1\\A_2\\A_3\\A_4\\A_5\\A_6\\A_7\\A_8\end{bmatrix} \tag{33}$$

$$=\frac{EI}{D^2}\begin{bmatrix}3A_2\cos(2\theta)+8A_3\cos(3\theta)+15A_4\cos(4\theta)+24A_5\cos(5\theta)\\+35A_6\cos(6\theta)+48A_7\cos(7\theta)+63A_8\cos(8\theta)\end{bmatrix},$$

$$Q(\theta)=\frac{EI}{D^3}\begin{bmatrix}(1^3\text{-}1)\sin(1\theta)\\(2^3\text{-}2)\sin(2\theta)\\(3^3\text{-}3)\sin(3\theta)\\(4^3\text{-}4)\sin(4\theta)\\(5^3\text{-}5)\sin(5\theta)\\(6^3\text{-}6)\sin(6\theta)\\(7^3\text{-}7)\sin(7\theta)\\(8^3\text{-}8)\sin(8\theta)\end{bmatrix}^T\begin{bmatrix}A_1\\A_2\\A_3\\A_4\\A_5\\A_6\\A_7\\A_8\end{bmatrix}=\frac{EI}{D^3}\begin{bmatrix}0\\6\sin(2\theta)\\24\sin(3\theta)\\60\sin(4\theta)\\120\sin(5\theta)\\210\sin(6\theta)\\336\sin(7\theta)\\504\sin(8\theta)\end{bmatrix}^T\begin{bmatrix}A_1\\A_2\\A_3\\A_4\\A_5\\A_6\\A_7\\A_8\end{bmatrix} \tag{34}$$

$$=\frac{EI}{D^3}[6A_2\sin(2\theta)+24A_3\sin(3\theta)+60A_4\sin(4\theta)+$$
$$120A_5\sin(5\theta)+210A_6\sin(6\theta)+336A_7\sin(7\theta)+504A_8\sin(8\theta)$$

3. Method for Calculating Tunnel Additional Displacement Based on Soil Rebound Deformation

Studies conducted by Loganathan (1998), Attewell et al. (1986) and Jacobsz (2003) have examined the effects of tunnel excavation on the displacement of the surrounding soil. These studies consider the uneven movement of the soil and the rate of stratum loss in the tunnel area. As a result, they proposed the theoretical formula for the settlement deformation of existing adjacent tunnels caused by tunnel excavation, which can be expressed as follows:

$$U(x)=\frac{\omega_0 D^2}{4}\left\{-\frac{z-h}{x^2+(z-h)^2}+(3-4\nu)\cdot\frac{z+h}{x^2+(z+h)^2}-\frac{2z\left[x^2-(z+h)^2\right]}{\left[x^2+(z+h)^2\right]^2}\right\}\cdot\exp\left\{-\frac{1.38x^2}{(D/2+h)^2}-\frac{0.69z^2}{h^2}\right\}, \tag{35}$$

where x is the distance from the calculation point to the center of the tunnel; υ is Poisson's ratio of the soil layer; ω_0 is the average ground loss ratio; D is the diameter of the tunnel; h is the buried depth of the new tunnel center point; and z is the buried depth of the existing tunnel center point.

However, the application of Eq. (35) to calculate the impact of excavation unloading on tunnel accessories displacement is challenging since the stiffness of the tunnel structure lies between fully flexible and absolutely rigid foundations, making it highly complex to solve for additional stress redistribution caused by excavation unloading. The Winkler foundation model is defined as below.

$$p = ks, \tag{36}$$

where k is the coefficient of subgrade reaction (commonly used soil layer value reference in Gao, 2003), that is, the pressure strength required for unit deformation of foundation, which is affected by the distribution of base pressure, soil compressibility and thickness (Vesic and Johnson, 1964; Susan and Powrie, 2000), measured in kN/m^3; p is the pressure strength at any point on the foundation, measured in kN/m^2; and s is the foundation deformation at the action point, with a unit of meters.

The additional stress in the soil layer of the tunnel is calculated using the Winkler foundation beam model (see Fig. 6). This additional stress reacts to the subway tunnel, and the tunnel displacement is solved through the longitudinal deformation compatibility equation.

According to the solution of the Winkler foundation model, the differential equation of vertical displacement of the tunnel structure (Chen and Wen, 2003; Biot, 1937) is as follows:

$$EI\frac{d^4S(x)}{dx^4}+kDS(x)=kDq(x)\ , \tag{37}$$

where $S(x)$ is the vertical displacement of the tunnel; $q(x)$ is the vertical displacement of the soil; k is the coefficient of the subgrade bed; and EI is the bending rigidity of the tunnel structure. According to the research of Shiba and Kawashima (1988), the calculation formula of the equivalent elastic bending stiffness of the tunnel structure is as follows:

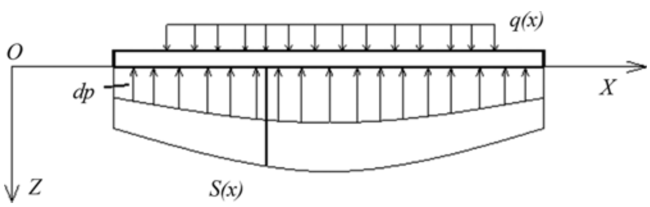

Fig. 6. Calculation Diagram of Infinite Beam on Winkler Foundation

$$EI = K_\varphi E_C I_C = \frac{\cos^3\varphi}{\cos\varphi + \left(\frac{\pi}{2}+\varphi\right)\sin\varphi} E_C I_C . \tag{38}$$

The solution of Eq. (37) can be obtained (Zhang and Huang, 2009):

$$\begin{aligned} s_1(x') &= e^{\alpha x'}[A_1\cos(\alpha x') + A_2\sin(\alpha x')] + \\ &e^{-\alpha x'}[(A_3\cos(\alpha x') + A_4\sin(\alpha x'))] + s_1'(x'), \end{aligned} \tag{39}$$

where $\alpha = \sqrt[4]{\frac{kD}{4EI}}$; A1, A2, A3, and A4 are the correction factors; $s_1'(x')$ is a special solution of Eq. (37) obtained by solving the tunnel boundary conditions.

To simplify the calculation, the combination of the additional deformation method and weighted residual method is considered to circumvent the need for numerous integration operations in the traditional Mindlin stress solution.

According to Peck (1969) and O'Reilly and New (1982), the deformation compatibility equation of foundation soil is proposed:

$$q(x) = q_{\max} \exp\left(\frac{-x^2}{2\omega^2 l_v^{\,2}}\right), \tag{40}$$

where $q_{\max}$ is the maximum uplift deformation value of the pit bottom; x is the horizontal distance from the center of the foundation pit to the calculation point; l_V is the vertical distance between the tunnel and foundation; and ω is the width correction parameter (0.7 for soft soil, 0.2 – 0.3 for sand, and 0.4 – 0.5 for clay).

Substituting Eq. (41) into Eq. (37), the following results can be obtained:

$$\frac{EI}{kD}\cdot\frac{d^4S(x)}{dx^4} + S(x) = q_{\max}\exp\left(\frac{-x^2}{2w^2 l_V^{\,2}}\right) . \tag{41}$$

Since the maximum vertical uplift deformation of the tunnel structure occurs at the center of the foundation pit, the displacement of the tunnel at infinity distance from the center can be assumed to be 0, so the boundary conditions of Eq. (41) are as follows:

$$\begin{cases} S(0) = S_{\max} \\ S(+\infty) = S(-\infty) = 0 \end{cases}, \tag{42}$$

where $S_{\max}$ is the maximum vertical displacement of the top of the tunnel structure, which can be evaluated according to the results of numerical analysis or the following formula:

$$S_{\max} = \frac{\omega\mu d q_{\max}}{d + l_v}, \tag{43}$$

where μ is the additional stress coefficient of soil on the tunnel top.

The $\frac{EI}{kD}\cdot\frac{d^4S(x)}{dx^4}$ fourth-order integral term in Eq. (42) is too small to be ignored. The approximate solution of the differential equation is obtained by substituting Eq. (42) into Eq. (41).

$$S(x) = S_{\max}\exp(-Tx^2),\ T > 0 \tag{44}$$

Then, the approximate solution Eq. (44) is substituted into Eq. (41) to obtain the residual value function of the differential equation:

$$\begin{aligned} R(x) &= 4EIS_{\max}T^2(3 - 12Tx^2 + 4T^2x^2) - q_{\max}kD\exp\left(\frac{-x^2}{2\omega^2 l_V^2}\right) \\ &+ S_{\max}kD\exp(-Tx^2), T > 0 \cdot \end{aligned} \tag{45}$$

The weighted function $W_i = \delta(x - x_i)$, when $x \neq x_i$, $W_i = 0$. Let $x_i = 0$, and the weighted integral equation can be obtained:

$$R(0) = \int_{-\infty}^{+\infty}\delta(x-0)Rdx = 12EIS_{\max}T^2 + kD(S_{\max} - q_{\max}) = 0 \cdot \tag{46}$$

The boundary condition of Eq. (44) is $T > 0$, we can acquire:

$$T = \sqrt{\frac{kD}{12EI}\cdot\left(\frac{q_{\max}}{S_{\max}} - 1\right)} . \tag{47}$$

By substituting Eq. (47) into Eq. (44), the weighted residual method differential equation solution of the Winkler foundation model is obtained (i.e., the vertical deformation of the tunnel at any point along the longitudinal x direction of the tunnel).

$$S(x) = \frac{S_{\max}}{\exp\left(\sqrt{\frac{kD}{12EI}\cdot\left(\frac{q_{\max}}{S_{\max}} - 1\right)}\cdot x^2\right)} , \tag{48}$$

where the maximum uplift deformation displacement $q_{\max}$ is calculated according to the empirical fitting formula proposed by Hou et al. (1992).

$$q_{\max} = -29.17 - 0.0167\gamma H' + 12.5\left(\frac{L}{H}\right)^{-0.5} + 0.637\gamma c^{-0.04}(\tan\varphi)^{-0.54}, \tag{49}$$

where H'. is the equivalent depth of the foundation pit, $H' = H + p/\gamma$; p is the overload of the foundation pit top; H is the actual depth of the foundation pit; γ is the weighted average bulk density of the excavated soil; L is the embedded depth of the supporting pile (wall); c is the weighted average cohesive force of the soil layer at the bottom of the pit; and φ is the weighted average internal friction angle of the soil layer at the bottom of the pit.

4. Method for Calculating the Tunnel Bending Moment and Shear Force Based on Soil Rebound Deformation

According to the differential relationship formula of tunnel displacement, shear force, and bending moment can be obtained:

$$Q(x) = -EI\frac{dM(x)}{dx} = -EI\frac{d^3S(x)}{dx^3} . \tag{50}$$

By substituting Eq. (50) into Eq. (37), the calculation formula of the tunnel and shear force can be obtained:

$$M(x) = -EI\frac{d^2S(x)}{dx^2} = -EI\frac{S'^2(x)}{dx^2} - 2\alpha^2 EI\cdot\cos(\alpha x)\cdot\begin{bmatrix} A_1\cosh(\alpha x) - A_2\cosh(\alpha x)\tan(\alpha x) \\ +A_3\sinh(\alpha x) - A_4\sinh(\alpha x)\tan(\alpha x) \end{bmatrix}, \tag{51}$$

$$Q(x) = -EI\frac{d^3S(x)}{dx^3} = -EI\frac{S'^3(x)}{dx^3} - 2\alpha^3 EI \cdot \begin{bmatrix} (A_1 - A_4)\sinh(\alpha x)\cos(\alpha x) \\ -(A_2 + A_3)\sinh(\alpha x)\sin(\alpha x) \\ -(A_1 + A_4)\cosh(\alpha x)\sin(\alpha x) \\ -(A_2 - A_3)\cosh(\alpha x)\cos(\alpha x) \end{bmatrix}. \tag{52}$$

The direct solution of Eqs. (51) and (52) pose a significant challenge. Alternatively, the combined approach of additional deformation and the method of weighted residual method can simplify the calculation process. The moment of the tunnel can be obtained by substituting Eq. (48) into Eq. (51):

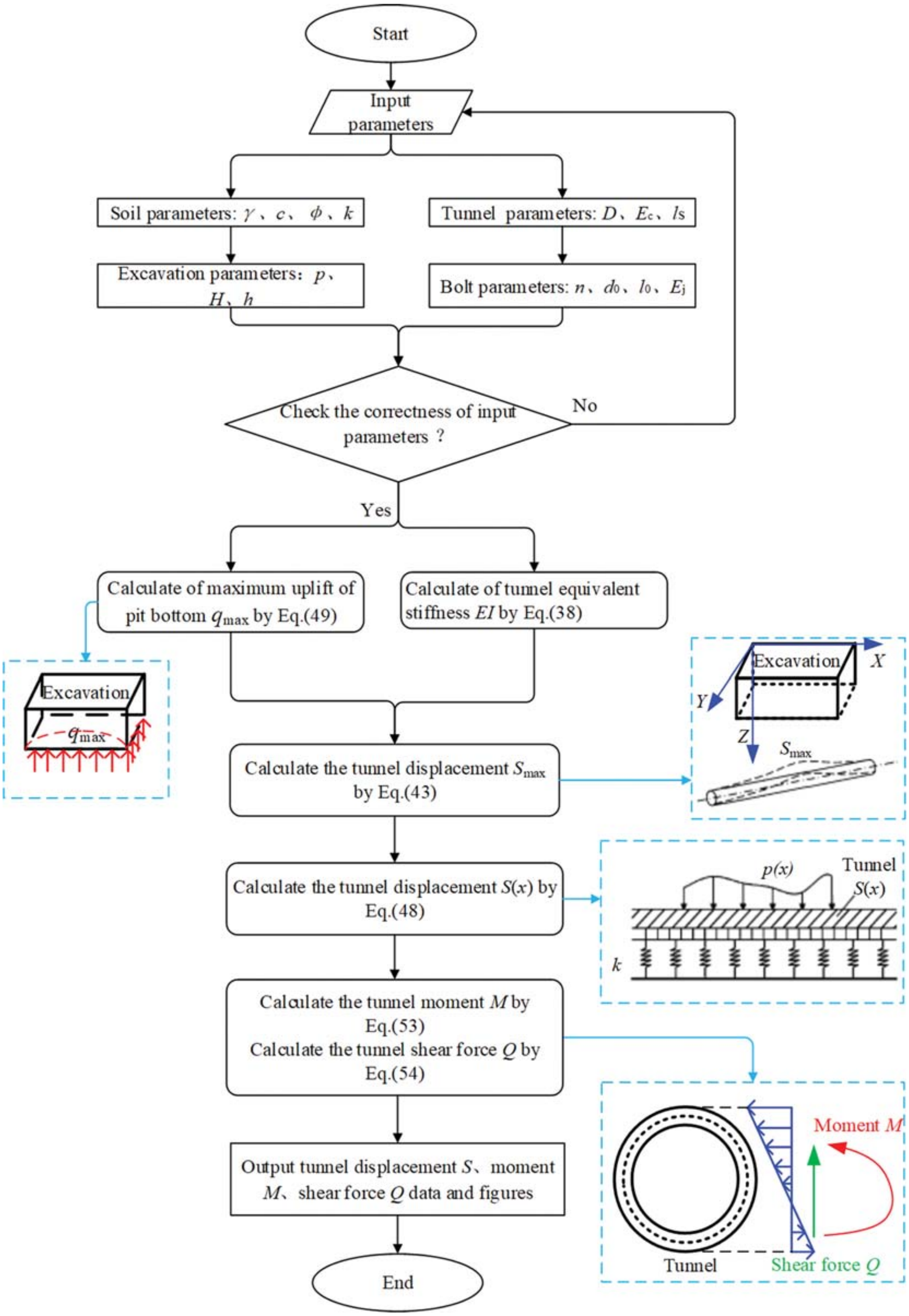

Fig. 7. Flow Chart of the Calculation and Analysis System for the Effect of Excavation Unloading on the Tunnel

$$M(x) = -EI\frac{d^2S(x)}{dx^2} = 2S_{\max}EIT(1-2Tx^2)\exp(-Tx^2)\,. \quad (53)$$

The shear force of the tunnel can be obtained by substituting Eq. (52) into Eq. (53):

$$Q(x) = -EI\frac{d^3S(x)}{dx^3} = 4S_{\max}EI(T^2x)(2Tx^2-3)\exp(-Tx^2)\,. \quad (54)$$

5. Assessment of the Influence of the Foundation Pit Excavation on the Tunnel Stability

5.1 Calculation and Analysis Program Realization

Figure 7 shows the calculation and analysis system for assessing the influence of excavation and unloading on the tunnel is written in Python, which a visual programming language.

5.2 Case Study on the Influence of Tunnel Excavation

Vorster et al. (2005) studied the deformation and bending moment of three different materials caused by tunnel excavation, in which the outer diameter D of the pipeline is 0.4 m, and the longitudinal equivalent bending stiffness of the pipeline EI is 2.625 × 10^4 kN·m^2, 1.05 × 10^5 kN·m^2, 4.20 × 10^5 kN·m^2, respectively. The buried depth of the pipe is 1.1 m, with subgrade coefficients k of is 6,955 kN/m^3, 6,196 kN/m^3 and 5,520 kN/m^3. Fig. 8 shows that the vertical displacement values of pipes with different bending stiffnesses obtained by Eq. (48) and the elastic theory method of Vorster et al. (2005) are consistent; Fig. 9 shows that the results of Eq. (53) are consistent with the bending moments of pipes with different bending stiffnesses obtained by the elastic theory method of Vorster et al. (2005), which shows the reliability of the calculation method in this paper. However, with increasing longitudinal bending stiffness, there is a large gap between them. The main reason is that the larger the longitudinal bending stiffness is, the worse the coordination between the deformation of the surrounding soil and the larger the difference in the deformation results.

5.3 Case Study on the Influence of Foundation Pit Excavation

Chen and Li (2005) studied the uplift deformation of the underlying Metro Line 2 caused by the excavation of a foundation pit in Pudong District, Shanghai. The nearest distance between the top of the tunnel and the bottom of the foundation pit is approximately 2.8 m. The size of the foundation pit is: length × width = 26 m × 18 m, with a depth of 6.5 m. The outer diameter D of the tunnel is 6.2 m, and its top is buried at a depth of 9.3 m. The longitudinal equivalent bending stiffness of the tunnel is 1.087 × 10^5 MN·m^2, and the subgrade coefficient k = 1.0 × 10^4 kN/m^3. Figs. 10 and 11 show the comparison and analysis among the calculation results of Eq. (48) and the measured deformation values, the stress appllication method of left line and right line tunnels in research of Chen and Li (2005) and Zhang and Huang (2009). The Fig. 10

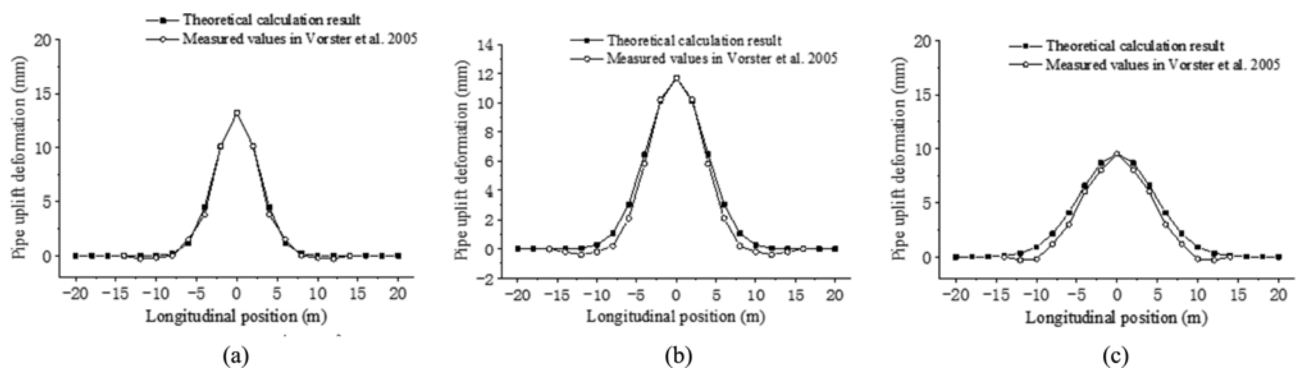

Fig. 8. Comparison of Vertical Deformation Results of Pipelines with Different Longitudinal Bending Stiffnesses: (a) EI = 2.625 × 10^4 kN·m^2, (b) EI = 1.05 × 10^5 kN·m^2, (c) EI = 4.20 × 10^5 kN·m^2

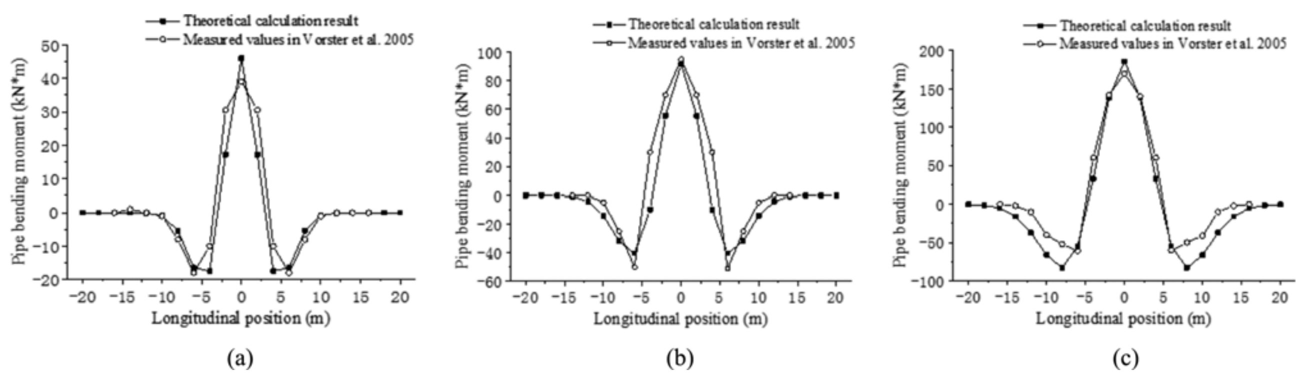

Fig. 9. Comparison of Vertical Bending Moment Results of Pipelines with Different Longitudinal Bending Stiffnesses: (a) EI = 2.625 × 10^4 kN·m^2, (b) EI = 1.05 × 10^5 kN·m^2, (c) EI = 4.20 × 10^5 kN·m^2

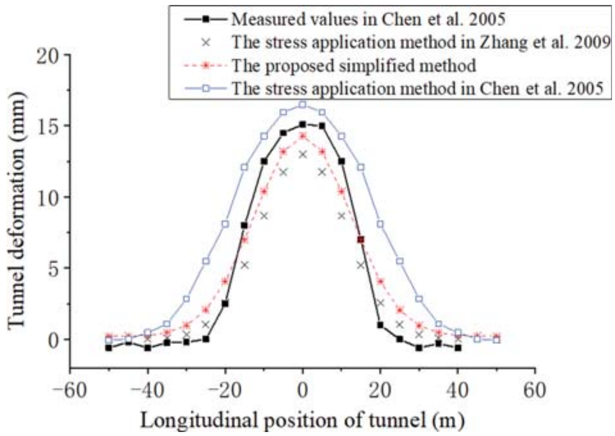

Fig. 10. Comparison of Vertical Uplift Deformation Results of the Tunnel left Line

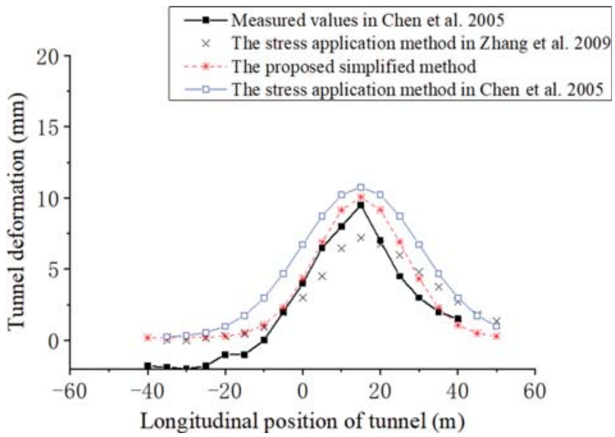

Fig. 11. Comparison of Vertical Uplift Deformation Results of the Tunnel right Line

shows that the calculated solution of the theoretical formula for tunnel deformation in this paper is consistent with the measured longitudinal uplift curve of the tunnel. However, the influences range and longitudinal uplift deformation observed in the measurements are slightly smaller than those predicted by the theoretical formula. This can be attributed to construction measures implemented during actual construction, such as soil reinforcement and partition excavation, which have a time-space effect. These construction measures effectively control and reduce the scope of tunnel uplift deformation.

6. Conclusions

Calculating the integral formula for tunnel deformation itself can be relatively complex. To simplify the traditional Mindlin stress solution calculation of deformation and avoid numerous numerical integral operations, we consider combining the additional deformation method with weighted residual method. The solution is derived by calculating the weighted residual value of the internal force and deformation of the tunnel. The following conclusions are obtained:

1. Integral operations can be avoided by utilizing theoretical formula combining the additional deformation method and the weighted residual method.
2. The weighted residual value solution of the theoretical formula of tunnel deformation in this paper is consistent with the trend of the measured tunnel longitudinal uplift curve.
3. Deformation of the tunnel measured results are slightly smaller than those predicted by the theoretical formula. The main reason is that time-space effect construction measures, such as soil reinforcement and partition excavation, are adopted in the actual project. These construction measures effectively control and reduce the scope of tunnel uplift deformation.

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