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Future Urban Mobility with MaaS

# Joint Optimization of Zone Area and Headway for Demand Responsive Transit Service under Heterogeneous Environment

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**ABSTRACT**

This paper presents a mathematical model to optimize zonal demand responsive transit (DRT) considering heterogeneous environment (i.e., community boundary, land use, demand distribution, line-haul travel time, etc.) under the advent of Mobility-as-a-Service (MaaS). Since most previous models over-simplified conditions of the DRT service area, we propose a new modeling approach to formulate the operator and user costs. Passengers with varied expectations of vehicle arrival time at a drop-off location are considered. The average cost is minimized through optimizing service zone areas and associated headways subject to practical constraints (i.e., policy headway and vehicle capacity). A real-world region in the City of Calgary, Canada, is applied to demonstrate the applicability of the model. The impact of real-time vehicle arrival information to the optimal solution is assessed. The relationship between system parameters (i.e., line-haul travel time, demand density, vehicle capacity, and passenger composition, etc.) and the optimized solutions (i.e., zone area, headway, and costs) is explored through the sensitivity analysis.

## 1. Introduction

Mobility-as-a-Service (MaaS) is an emerging concept in transportation, which aims to improve passenger mobility services (Hietanen, 2014) by using a single interface to integrate information from different modes of transportation (e.g., public, for hired, ridesharing systems, etc.) via the Internet and mobile apps. Thus, the interconnectivity of real-time information among transportation modes is critical in the success of MaaS (Jittrapirom et al., 2017). It is expected that MaaS can improve transit accessibility and mobility for people who cannot afford private vehicles and improve environmental impacts.

Demand responsive transit (DRT) provides friendlier service (including doorstop pick-ups and drop-offs) to passengers living or working in a low demand area, comparing to fixed transit (FT). The cost of operating DRT is less than that of operating FT especially in low demand areas. However, DRT can still be costly (Goodwill and Carapella, 2008; Mulley et al., 2012) if the service is not well designed, even with the applications of advanced technologies (Palmer et al., 2008), such as MaaS. Hence, a sound model is desirable to optimize the service which minimizes the cost.

Zone-based service (also called zonal service) has been applied as it is an effective way to reduce travel time compared to a non-zoning service (Tsao and Schonfeld, 1983), especially for many-to-one feeder services (Jordan and Turnquist, 1979). Zonal service can be managed easier, reduce operating costs, and promote system productivity and service quality (Furth, 1986).

Passenger's wait-time is an index reflecting the service quality of a transit system. Wait cost is also an essential component of the system's total cost. The methods used to approximate the wait-cost incurred by FT passengers seem inappropriate for DRT's. The average wait time of passengers is driven by bus headway. However, the wait time of DRT is affected by the service reliability depending on traffic conditions and boarding demand. In addition, passenger's in-vehicle time is dependent on route length, number of stops, and pick-up sequence. In general, a passenger picked up earlier will experience longer in-vehicle time than those picked up later. With the application of MaaS, DRT passengers' wait time may be significantly reduced, which shall be considered

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while optimizing the DRT service.

This paper aims to optimize zonal DRT service, including service zone area and headway, for an irregular service region with a heterogeneous environment (i.e., land use, demand density, line-haul travel time, passenger type, etc.). Passengers who will and will not desire a specific arrival time at destinations are considered while formulating the cost function. Subject to a set of practical constraints (i.e., policy headway and vehicle capacity), the objective is to minimize the average cost. A sequential approach is developed to partition the study region in the city of Calgary, considering realistic geographic and demographic conditions. Furthermore, sensitivity analysis is conducted to explore the impact of key system parameters on the optimal solutions.

## 2. Literature Review

In general, public transit systems can be categorized into fixed transit (FT) and demand-responsive transit (DRT). FT is commonly operated in high demand areas, yet costly in low demand areas. Unlike FT, DRT is a user-oriented mode which can accommodate passengers' accessibility and desired departure and arrival times.

Since the 1970s, researchers introduced DRT under the circumstances where FT is not economically viable (Davison et al., 2014). Many studies (Chang and Schonfeld, 1991; Chien and Schonfeld, 1997; Chien and Yang, 2000; Dessouky et al., 2003; Diana et al., 2006; Li and Quadrifoglio, 2010; Chandra and Quadrifoglio, 2013; Yang et al., 2021) have focused on various DRT concepts (e.g., paratransit, dial-a-ride, demand responsive connector). However, cost was always a major concern that might impede the implementation. Thus, several optimization models were developed to promote the DRT performance (Daganzo, 1978; Kikuchi, 1984; Chang and Schonfeld, 1991; Dessouky et al., 2003; Diana et al., 2006; Amirgholy and Gonzales, 2016; Shen et al., 2021).

Wait time of passengers is an essential indicator commonly used to assess transit service. Most DRT studies assumed that the average wait time linearly increases as the headway increases. Chang and Schonfeld (1991) assumed the average wait time as half of the headway for a pre-scheduled subscription bus service. Quadrifoglio and Li (2009) assumed the average wait time as a linear function of headway for a flexible feeder transit service, which was affected by pick-up location, trip direction (inbound vs. outbound) and fleet size. Nourbakhsh and Ouyang (2012) found average wait time at transfer or pick-up locations was half of the headway. Chandra and Quadrifoglio (2013) found that average wait time was affected by vehicle capacity. Under a saturated condition, average wait time was dependent on passenger demand, number of cycles (i.e., terminal-to-terminal) per day, number of requests and trips served per cycle, and cycle length. On the other hand, when the system was not saturated, average wait time was affected by number and type of passengers, cycle length, and travel time per cycle.

Additional wait time may be experienced by passengers because of late vehicle arrivals (Li et al., 2012; Chen et al., 2013; Verbas and Mahmassani, 2013). However, vehicles sometimes wait for late passengers, which likely increase the wait time of downstream passengers. Passengers' wait time can be significantly reduced by means of knowing vehicle arrival information via advanced technologies such as MaaS. Passengers might take an earlier bus and arrive at destinations too early because of limited service. To this end, a more comprehensive formula representing wait cost, considering early arrival penalties, is desirable.

The shape of the service region influences the mode of operation and vehicle travel distance, which is critical in planning transit service, which is commonly irregular. If the region is too large to operate, zone partition is a critical step for optimizing zonal DRT. Most studies did not consider the heterogeneity of the service region and oversimplified environment, which may lead to biased estimation of system performance. Few studies considered heterogeneous environment of the service region while optimizing transit systems (Chien and Schonfeld, 1997; Chien and Yang, 2000; Chien and Qin, 2004; Chien et al., 2004; Wang, 2017; Kim and Roche, 2021).

According to the limitations of previous studies indicated above, we proposed a new modeling approach to optimize service areas and headways of a zonal DRT service considering heterogeneous environment (i.e., community boundary, land use, demand distribution, street network, etc.), which minimizes the average cost.

## 3. Methodology

A mathematical model is proposed here to optimize a zonal DRT service connecting an irregular region and a terminal located outside the region. As shown in Fig. 1, the region shall be partitioned into several zones based on the areas to be optimized later. The terminal is a large trip generator such as a Central Business District (CBD) or a major transfer station. A transit agency dispatches empty vehicles from the terminal, which travel a line-haul distance without stopping and enter a designated zone, pick up passengers within the zone, return to the terminal, and unload the passengers. In each zone, vehicle capacity should be carefully chosen to accommodate passenger demand.

Passenger's wait time defined here is the actual vehicle arrival

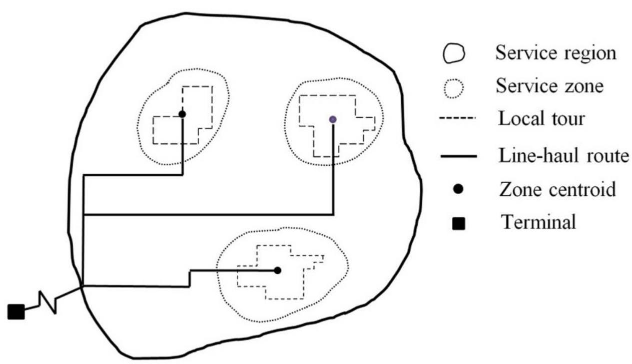



**Fig. 1.** Configuration of a General DRT Service

time less the scheduled arrival time at a stop. As real-time information (e.g., vehicle arrival and/or departure time information) is available via MaaS, wait time is negligible. Hence, the proposed DRT is operated under either pre-time information scenario (PTS) or real-time information scenario (RTS). Under PTS, passengers are notified with a pick-up time window. On the other hand, under RTS, real-time information is available for passengers.

## 3.1 Assumptions

The following assumptions are made for formulating the proposed model:

1. A region in Fig. 1 consists of a set of communities with irregular shapes and different characteristics such as demand density, street pattern, and line-haul travel time.
2. The region can be partitioned into several zones. Each zone may cover one or multiple communities. The demand density of each zone is a weighted average over the communities within the zone.
3. The travel demand pattern is many-to-one (i.e., from any places within the zone to the terminal) and inelastic to service quality. Walk-in passengers are not accepted.
4. The vehicle local travel time within a zone is the sum of vehicle running time and stop delays. The line-haul travel time consists of the round-trip travel time between the zone and the terminal.
5. Passengers who desire a specific arrival time at destination denoted as type 1, and those who do not desire specified arrival time denoted as type 2. The fractions of types 1 and 2 out of total demand are $\alpha$ and (1-$\alpha$), respectively.
6. Average wait time increases linearly as the number of prior stops increases under PTS but remains a constant under RTS.

## 3.2 Model Formulation

The objective function of the proposed model is average cost per passenger of zone $i$ denoted as $C_i$, which consists of wait cost $C_{wi}$, in-vehicle cost $C_{ri}$, and operating cost $C_{oi}$.

### 3.2.1 Wait Cost ($C_{wi}$)

The wait cost per passenger in zone $i$ is the value of wait time $\gamma_w$ multiplied by the expected wait time $t_{wi}$ incurred by type 1 ($t_{1i}$) and type 2 ($t_{2i}$) passengers plus an early arrival penalty $P_i$. Thus,

$$C_{wi} = \gamma_w t_{wi} + P_i \,. \tag{1}$$

Under PTS, $t_{wi}$ can be estimated by Eq. (2) which is a weighted average wait time, while under RTS, $t_{wi}$ is a constant $\beta_i$. Hence,

$$t_{wi} = \begin{cases} a + \frac{1}{2}b(n_i - 1), & \text{PTS} \\ \beta_i, & \text{RTS}, \end{cases} \tag{2}$$

where $a$ is a fixed time interval affected by traffic conditions; and $b$ is a variable time affected by potential delay of passengers boarding from upstream stops.

Vehicle stop delay is affected by number of stops, the product of demand density $q_i$, area $A_i$, and headway $h_i$ of zone $i$. Thus,

$$n_i = q_i A_i h_i \,. \tag{3}$$

$P_i$ is the penalty per passenger is formulated as a weighted average of penalties for types 1 and 2 passengers denoted as $P_{1i}$ and $P_{2i}$, respectively. Thus,

$$P_i = \alpha_i P_{1i} + (1 - \alpha_i) P_{2i} \,, \tag{4}$$

where $\alpha_i$ and 1-$\alpha_i$ represent the fractions of types 1 and 2 passengers, respectively. $P_{1i}$ is the product of average early arrival time (e.g. half of headway) multiplied the value of time. Thus,

$$P_{1i} = \frac{1}{2}\gamma_w h_i \,. \tag{5}$$

Since type 2 passengers do not specify desired arrival time, $P_{2i}$ is equal to 0. Finally, the average wait cost per passenger in zone $i$ can be expressed by Eq. (6):

$$C_{wi} = \begin{cases} \gamma_w\left(a + \frac{1}{2}bq_iA_ih_i + \frac{1}{2}\alpha_i h_i - \frac{1}{2}b\right), & \text{PTS} \\ \frac{1}{2}\gamma_w \alpha_i h_i, & \text{RTS}. \end{cases} \tag{6}$$

### 3.2.2 In-vehicle Cost ($C_{ri}$)

The average in-vehicle cost per passenger in zone $i$ is the value of in-vehicle time $\gamma_r$ multiplied by the average in-vehicle time (= half of vehicle round-trip time $T_i$). Thus,

$$C_{ri} = \gamma_r T_i / 2 \,. \tag{7}$$

$T_i$ consists of line-haul round-trip travel time between the terminal and zone $i$ denoted as $t_{Hi}$, local non-stop travel time denoted as $t_{ci}$, and stop delay time denoted as $t_{Si}$. Thus,

$$T_i = t_{Hi} + t_{Ci} + t_{Si} \,, \tag{8}$$

where $t_{Ci}$ is travel distance $D_i$ within zone $i$ divided by average vehicle speed denoted as $V$. Thus,

$$t_{Ci} = \frac{D_i}{V} \,. \tag{9}$$

Note that $D_i$ can be expressed by Eq. (10) if the demand $n_i$ is uniformly distributed over $A_i$:

$$D_i = k\sqrt{n_i A_i} = kA_i\sqrt{q_i h_i} \,, \tag{10}$$

where $k$ is a constant and equal to 1.15 for a grid street network (Daganzo, 1984).

The stop delay of zone $i$ is the product of $n_i$ and average stop delay denoted as $\tau$. Thus,

$$t_{Si} = \tau q_i A_i h_i \,, \tag{11}$$

where $\tau$ is treated as an exogenous parameter. Finally, the average in-vehicle cost per passenger in zone $i$ can be expressed by Eq. (12):

$$C_{ri}=\frac{1}{2}\gamma_r\left(t_{Hi}+\frac{kA_i\sqrt{q_ih_i}}{V}+\tau q_iA_ih_i\right). \tag{12}$$

### 3.2.3 Operating Cost ($C_{oi}$)

The average operating cost per passenger in zone $i$ is the unit operating cost per vehicle denoted as $\gamma_o$ multiplied by fleet size denoted as $F_i$, and then divided by hourly demand. Thus,

$$C_{oi}=\frac{\gamma_oF_i}{q_iA_i}. \tag{13}$$

Note that $F_i$ is vehicle round-trip time divided by the headway. Thus,

$$F_i=T_i/h_i. \tag{14}$$

Finally, the operating cost per passenger trip in zone $i$ can be expressed by Eq. (15):

$$C_{oi}=\frac{\gamma_o\left(t_{Hi}+\frac{kA_i\sqrt{q_ih_i}}{V_i}+\tau q_iA_ih_i\right)}{q_iA_ih_i}. \tag{15}$$

## 3.3 Objective Function

Considering a DRT service in zone $i$, the objective is to optimize zone areas and headways which minimize the average cost $C_i$, subject to practical constraints (e.g., capacity and headway). Thus,

$$\text{Min } C_i=C_{wi}+C_{ri}+C_{oi}, \tag{16}$$

where $C_{wi}$, $C_{ri}$, and $C_{oi}$ are formulated as Eqs. (6), (12), and (15).

Subject to

$$c_i\geq n_i=q_iA_ih_i, \tag{17}$$

$$h_i\leq\frac{c_i}{q_iA_i}, \tag{18}$$

$$h_i\leq h_{max}, \tag{19}$$

where $h_{max}$ is the maximum headway. Eqs. (17) and (18) ensure that vehicle capacity and headway satisfy the demand, while Eq. (19) ensures that the optimized headway will not exceed the maximum headway.

## 3.4 Optimization

The average cost defined here is the sum of wait, in-vehicle, and operating costs divided by number of served passengers, which can be minimized by optimizing decision variables, including headway $h_i$ and zone area $A_i$ of zone $i$. The optimal $h_i$ and $A_i$ can be derived by setting the first order of objective function to 0 and solving them. Thus,

$$\frac{\partial C_i}{\partial A_i}=0. \tag{20}$$

The optimal zone area $A_i^*$ is derived as

$$A_i^*=\begin{cases}\left[\dfrac{2\gamma_ot_{Hi}}{\frac{k\gamma_r}{V}q_i^{\frac{3}{2}}h_i^{\frac{3}{2}}+(b\gamma_w+\gamma_r\tau)q_i^2h_i^2}\right]^{\frac{1}{2}}, & \text{PTS}\\[2ex] \left[\dfrac{2\gamma_ot_{Hi}}{\frac{k\gamma_r}{V}q_i^{\frac{3}{2}}h_i^{\frac{3}{2}}+\gamma_r\tau q_i^2h_i^2}\right]^{\frac{1}{2}}, & \text{RTS.}\end{cases} \tag{21}$$

Similarly, the optimal headway can be derived from Eq. (22) and solve it for $h_i$. Thus,

$$\frac{\partial C_i}{\partial h_i}=0. \tag{22}$$

By substituting Eq. (21) into Eq. (22), the optimal headway $h_i^*$ can be determined by solving Eq. (23).

$$\begin{cases}\left[\dfrac{2\gamma_ot_{Hi}\gamma_r}{\frac{kq_i^{\frac{1}{2}}h_i^{*\frac{5}{2}}}{V}+\left(\frac{b\gamma_w}{\gamma_r}+\tau\right)q_ih_i^{*3}}\right]^{\frac{1}{2}}+\dfrac{2\gamma_o}{q_i^{\frac{1}{2}}h_i^{*\frac{3}{2}}}=\dfrac{2\alpha_i\gamma_wV}{k}, & \text{PTS}\\[2ex] \left[\dfrac{2\gamma_ot_{Hi}}{\frac{k\gamma_r}{V}q_i^{\frac{3}{2}}h_i^{\frac{3}{2}}+\gamma_r\tau q_i^2h_i^2}\right]^{\frac{1}{2}}+\dfrac{2\gamma_o}{q_i^{\frac{1}{2}}h_i^{*\frac{3}{2}}}=\dfrac{2\alpha_i\gamma_wV}{k}, & \text{RTS.}\end{cases} \tag{23}$$

The optimal zone area $A_i^*$ can be derived by substituting $h_i^*$ into Eq. (21). Finally, the minimum average cost $C_i^*$ can be obtained by substituting Eqs. (21) and (23) into Eq. (16).

## 3.5 Zone Partition

This section introduces a method to partition a region into several zones with the optimal areas suggested by the proposed model, considering the associated geographical conditions. The general step procedures are shown in Fig. 2 and described below.

Step 1: Select a community located adjacent to the region boundary farthest away from the terminal (Wang, 2017), and assign the community as a part of zone $i$ (e.g. $i=1$).

Step 2: Determine line-haul travel time $t_{Hi}$ of the community based on historical traffic data.

Step 3: Compute the optimal zone area $A_i^*$ with Eq. (21) based on $t_{Hi}$ determined in Step 2.

Step 4: Compare $A_i^*$ with the actual area of the community $A_i'$. If $\rho=A_i'/A_i^*<1$, expand zone $i$ by merging the community identified in Step 1 and an adjacent community to create a new pseudo community and go to Step 2. Otherwise, go to Step 5.

Step 5: If $\rho>1$, A portion of the adjacent community shall be attributed to zone $i$ to satisfy $A_i^*$. The rest of the community is used as a new pseudo community, and then go to Step 2. If $\rho\approx 1$, the community can be set as a zone $i$ and go to Step 6.

Step 6: If $\Sigma A_i'=A_\sigma$, where $A_\sigma$ is the area of the region, iteration

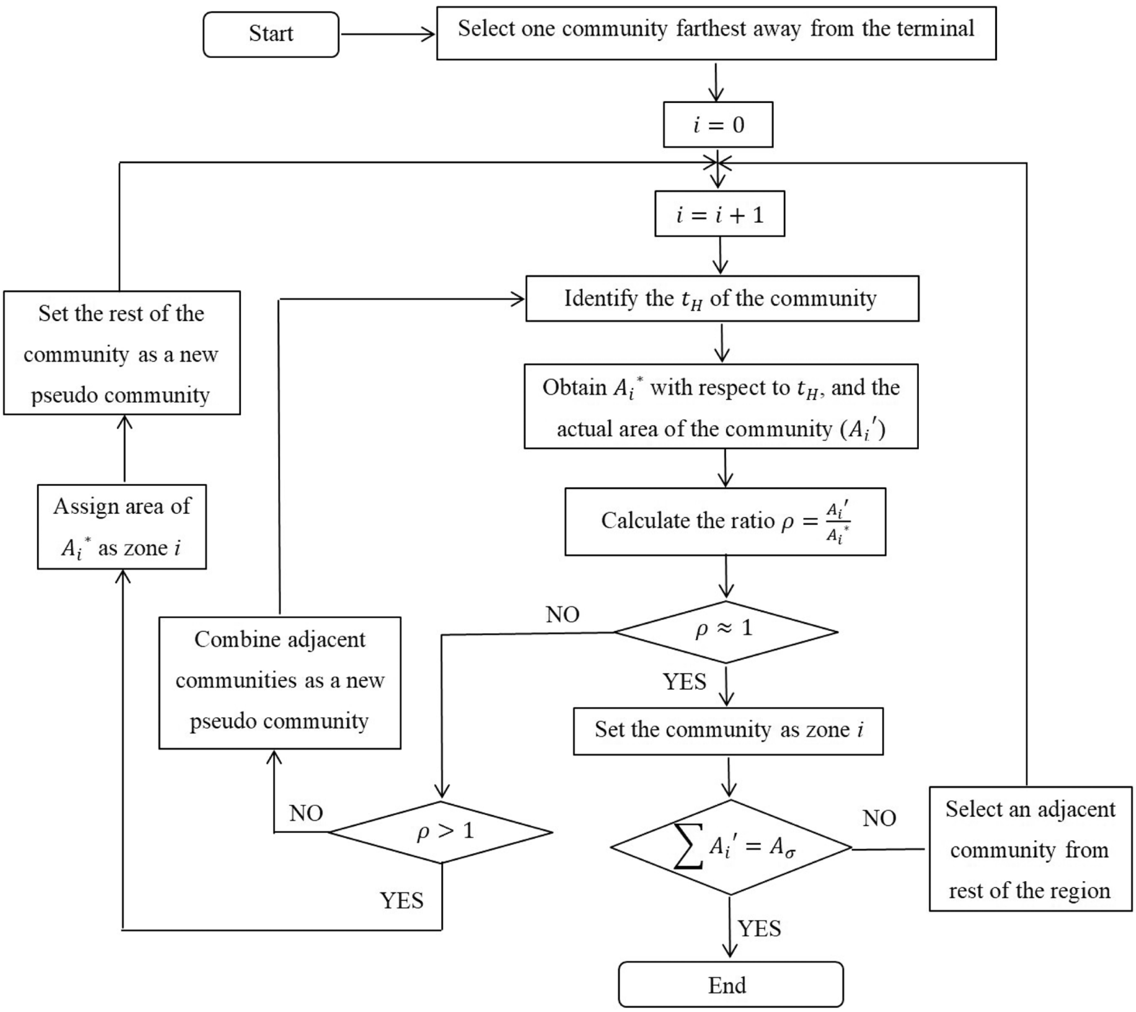



**Fig. 2.** Method for Zone Partition

ends. Otherwise, select an adjacent community near zone $i$ and go to Step 2 to form next zone and update $i$ as $i + 1$.

## 4. Case Study

The area of the study region shown in Fig. 3 is 86.9 km$^2$, consisting of 33 residential and 3 industrial communities (Names and layout of the communities shown in Appendix II). The green area is the Nose Hill Park which is a recreational area without residents. The proposed DRT terminal is located at the Calgary Tower near the town center where the DRT vehicles are dispatched.

### 4.1 Optimal Results

The baseline values of model parameters are summarized in Appendix I. The operating costs of vehicles with different capacities are estimated based on the data provided by Calgary Transit. The line-haul travel time is extracted from Google Maps. The demand densities of communities are estimated based on Calgary's public transit data. Expected passenger wait time at pick-up locations is suggested by Watkins et al. (2011). The values of in-vehicle and wait times are suggested by Hossain (2011).

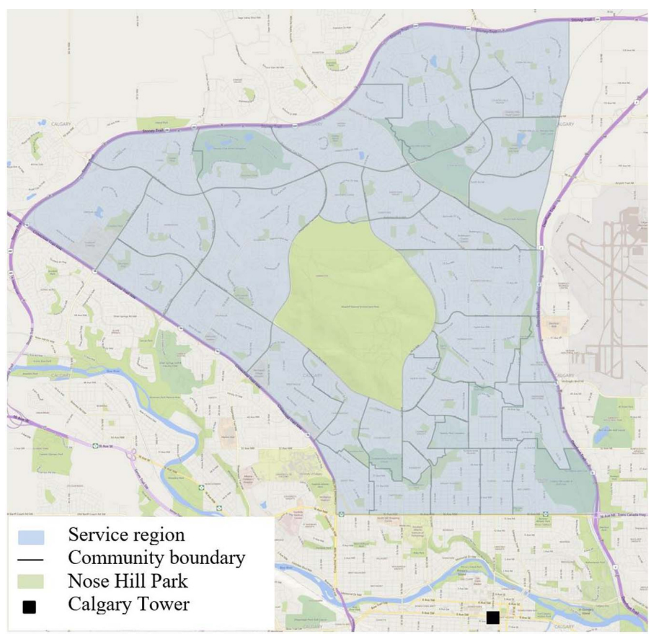
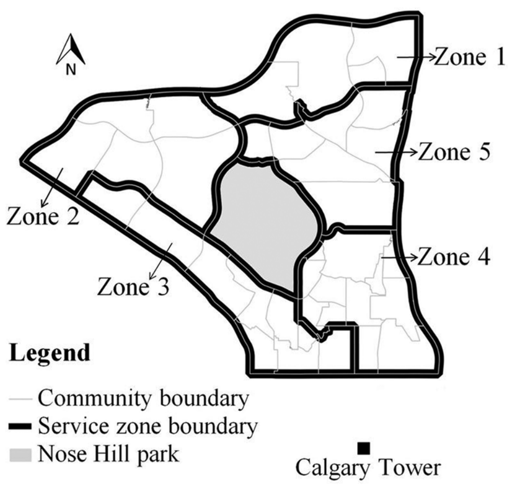



**Fig. 3.** Service Region and the Terminal (Calgary Tower)




| Zone ID | $q_i$ (pass/km²/hr) | $A_i'$ (km²) |
|---|---|---|
| 1 | 3.14 | 15.7 |
| 2 | 2.28 | 20.5 |
| 3 | 2.29 | 17.1 |
| 4 | 1.73 | 15.9 |
| 5 | 2.32 | 17.7 |
| Total | | 86.9 |

**Fig. 4.** Configuration of Partitioned Zones and Zone Attributes

**Table 1.** Optimal Results under PTS

| Zone ID | $A_i$ (km$^2$) | | $h_i$ (hr) | | | $C_i$ (\$/pass) | | | $C_{oi}$ (\$/pass) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $A_i^*$ | $D^\#$ | $h_i^*$ | $h_i'$ | $D$ | $C_i^*$ | $C_i'$ | $D$ | $C_{oi}^*$ | $C_{oi}'$ | $D$ |
| 1 | 14.7 | 6.8% | 0.44 | 0.44 | 0.0% | 22.7 | 22.8 | 0.4% | 13.5 | 13.2 | -2.2% |
| 2 | 18 | 13.9% | 0.49 | 0.47 | -4.1% | 23.5 | 23.6 | 0.4% | 13.9 | 13.5 | -2.9% |
| 3 | 15.7 | 9.6% | 0.47 | 0.47 | 0.0% | 21.3 | 21.4 | 0.5% | 13.2 | 12.9 | -2.3% |
| 4 | 16.4 | -3.0% | 0.5 | 0.54 | 8.0% | 20.3 | 20.4 | 0.5% | 13 | 12.7 | -2.3% |
| 5 | 15.9 | 11.3% | 0.47 | 0.47 | 0.0% | 21.6 | 21.7 | 0.5% | 13.3 | 13 | -2.3% |

$D^\#$: deviation of the adjusted value from the optimal value

**Table 2.** Optimal Results under RTS

| Zone ID | $A_i$ (km$^2$) | | $h_i$ (hr) | | | $C_i$ (\$/pass) | | | $C_{oi}$ (\$/pass) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $A_i^*$ | $D^\#$ | $h_i^*$ | $h_i'$ | $D$ | $C_i^*$ | $C_i'$ | $D$ | $C_{oi}^*$ | $C_{oi}'$ | $D$ |
| 1 | 16 | 1.9% | 0.49 | 0.44 | -2.2% | 21.6 | 21.6 | 0.0% | 12.9 | 12.6 | -2.3% |
| 2 | 19.7 | -3.9% | 0.56 | 0.47 | 2.0% | 22.3 | 22.4 | 0.4% | 13.4 | 12.9 | -3.7% |
| 3 | 17.1 | -0.6% | 0.55 | 0.47 | 0.0% | 20.3 | 20.3 | 0.0% | 12.7 | 12.4 | -2.4% |
| 4 | 17.9 | 12.6% | 0.55 | 0.54 | -8.9% | 19.5 | 19.5 | 0.0% | 12.6 | 12.3 | -2.4% |
| 5 | 17.4 | -1.7% | 0.56 | 0.47 | 0.0% | 20.6 | 20.6 | 0.0% | 12.8 | 12.5 | -2.3% |

$D^\#$: deviation of the adjusted value from the optimal value

After applying the method of zone partition with the suggested zone areas, the optimal zone partition with areas are shown in Fig. 4. The service area is partitioned differently into 5 zones with optimal results under PTS and RTS scenarios, illustrated in Tables 1 and 2, respectively. It was found that the optimal zone area and headway under RTS are larger than those under PTS because of less wait cost. Note that as the actual zone area, denoted as $A'$, is slightly adjusted from the optimal zone area $A^*$, subject to the street pattern and other geographic constraints, headway $h'$ is re-calculated based on $A'$.

It was also found that the solution space nearby the optimum is flat. The average costs under PST and RST shown in Fig. 5 vary with zone area and headway. The dark and flat areas suggest that it is quite flexible for the operator to justify zone area and service headway to accommodate the heterogeneity of each zone (i.e., street pattern and demand density), which will slightly increase the cost. Hence, a small deviation of the actual zone area from the optimal one in Tables 1 and 2 slightly impact the optimal results, such as headway and average cost.

## 4.2 Sensitivity Analysis

The purpose of sensitivity analysis is to explore the relationship among various system parameters on optimal solutions and minimized costs.

Figures 6 through 8 illustrate minimized average cost $C^*$, optimized headway $h^*$ and optimized zone area $A^*$ for varying demand density $q$ and vehicle capacity $c$ under PTS and RTS scenarios. For any given $c$, $h^*$ and $A^*$ decrease as $q$ increases. As

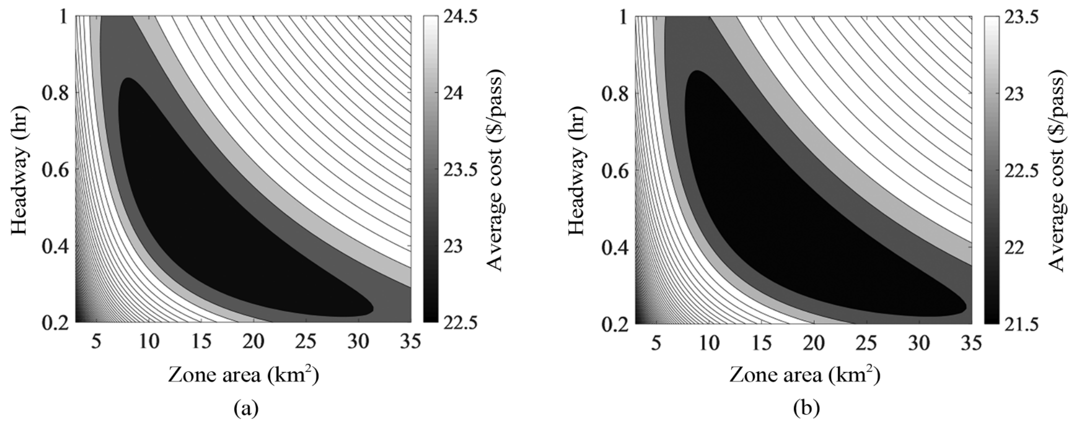



Fig. 5. Average Cost vs. Zone Area and Headway: (a) PTS, (b) RTS

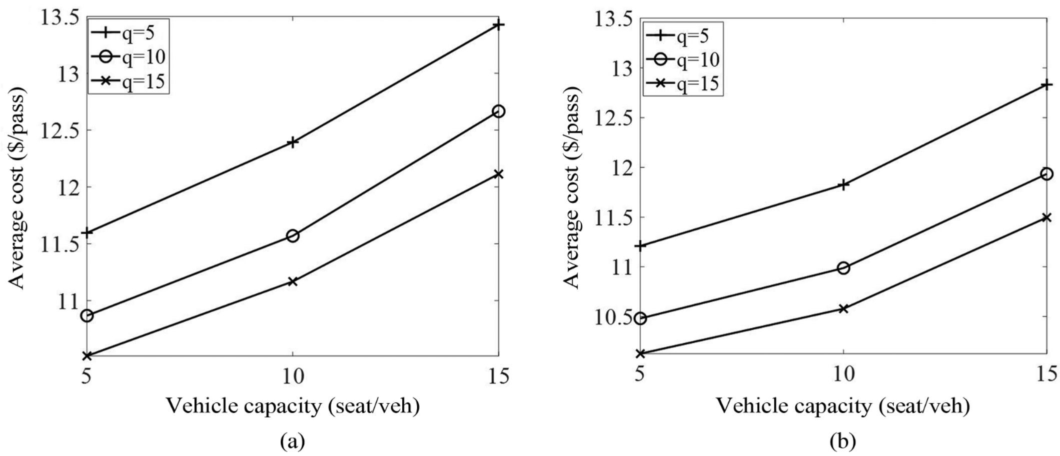



Fig. 6. Average Cost vs. Demand Density and Vehicle Capacity: (a) PTS, (b) RTS

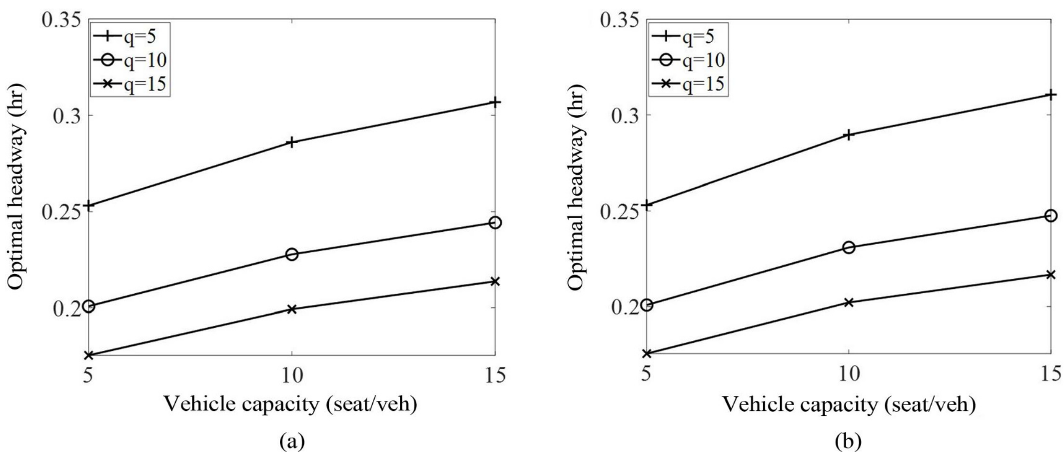



Fig. 7. Optimal Headway vs. Demand Density and Vehicle Capacity: (a) PTS, (b) RTS

$c$ increases, $h^*$, $A^*$, and $C^*$ increase. However, $A^*$ slightly change as $c > 10$ seats/veh. It is worth noting that under RTS, a lower average cost can be expected especially as demand is low.

As shown in Figs. 9 and 10, as q increases, $h^*$ and $A^*$ decrease. Similarly, $h^*$ and $A^*$ increase as $t_H$ increases. Since the increase of $t_H$ tends to increase user and operator's cost, serving more passengers per vehicle tends to reduce the average cost. However, as $t_H$ exceeds a critical value (e.g. 0.8 hrs in Fig. 10), $h^*$ and $A^*$ tend to remain constant at which vehicles are full loaded (see Table 3).

Land use, residence type, and service period will affect the fractions of passenger types. Figs. 11 and 12 suggest that as $\alpha$ (fraction of type 1 passengers) increases, $h^*$ decreases but $A^*$ increases for various $q$. Therefore, the average wait time of type 1 passengers at the destination may be reduced. However, $h^*$ and

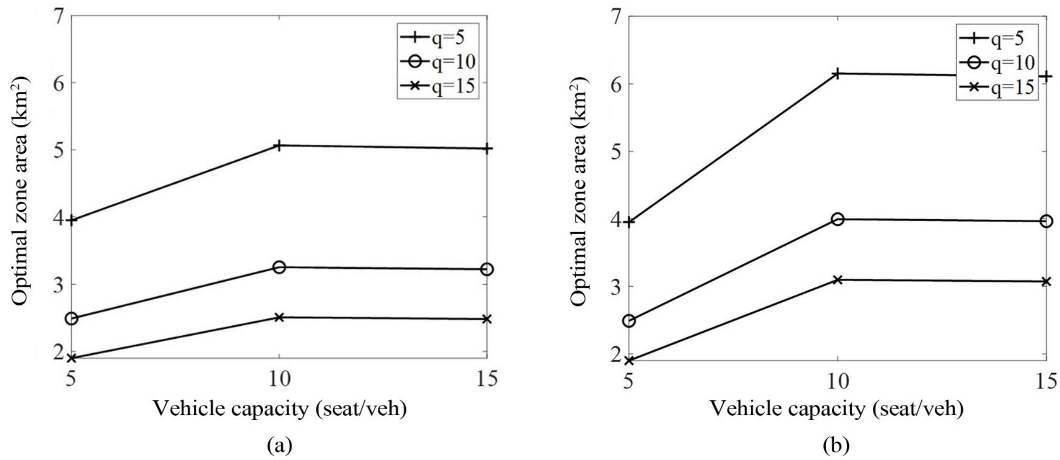



**Fig. 8.** Optimal Zone Area vs. Demand Density and Vehicle Capacity: (a) PTS, (b) RTS

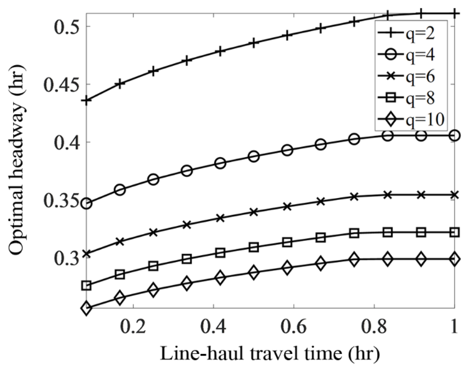



**Fig. 9.** Optimal Headway vs. Line-Haul Travel Time for Different Demand under PTS

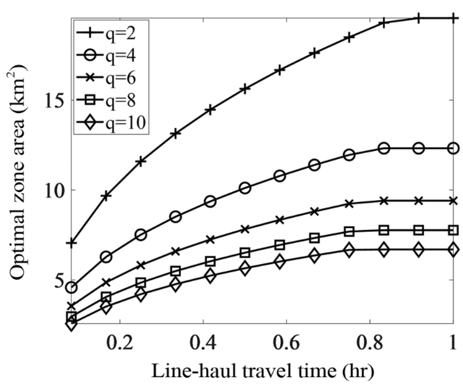



**Fig. 10.** Optimal Zone Area vs. Line-Haul Travel Time for Different Demand under PTS

**Table 3.** Vehicle Utilization vs. Line-Haul Travel Time and Demand Density under PTS

| $t_H$ (hr) | $q$ (pass/km$^2$/hr) | | | | |
|---|---|---|---|---|---|
| | 2 | 4 | 6 | 8 | 10 |
| 0.08 | 30.7% | 31.8% | 32.3% | 32.7% | 32.9% |
| 0.33 | 61.8% | 63.9% | 64.9% | 65.6% | 66.1% |
| 0.67 | 87.8% | 90.7% | 92.1% | 93.1% | 93.8% |
| 1 | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |

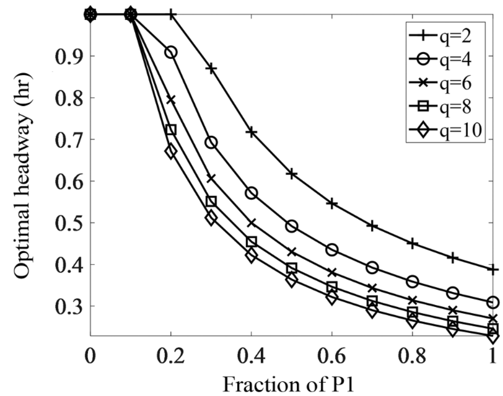



**Fig. 11.** Optimal Headway vs. Fraction of P1 for Different Demand under PTS

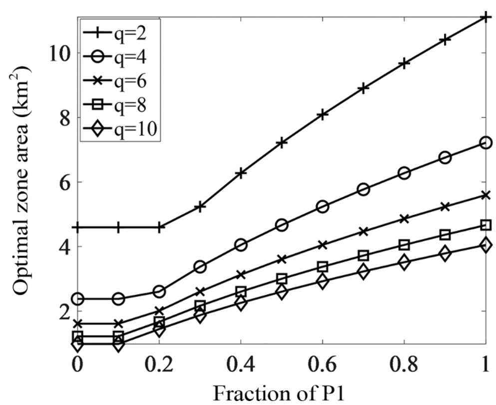



**Fig. 12.** Optimal Zone Area vs. Fraction of P1 for Different Demand under PTS

$A^*$ remain constant if $\alpha$ is less than 0.1 at which the policy headway constraint will hold ($h^* = h_{max}$). Vehicle utilization under various $\alpha$ and $q$ are also analyzed and the results are summarized in Table 4. It is found that as $q$ increases and $\alpha$ decreases, vehicle utilization increases.

Figure 13 shows that the increase of $\alpha$ results in a decrease in $h^*$ and an increase in $A^*$. The decrease of $h^*$ tends to reduce the

**Table 4.** Vehicle Utilization vs. Fraction of P1 for Different Demand under PTS

| $\alpha$ | $q$ (pass/km²/hr) | | | | |
|---|---|---|---|---|---|
| | 2 | 4 | 6 | 8 | 10 |
| 0.0 | 46.0% | 47.6% | 48.3% | 48.8% | 49.2% |
| 0.2 | 46.0% | 47.4% | 47.9% | 48.3% | 48.5% |
| 0.4 | 45.0% | 46.3% | 46.9% | 47.4% | 47.7% |
| 0.6 | 44.2% | 45.6% | 46.3% | 46.8% | 47.1% |
| 0.8 | 43.6% | 45.0% | 45.8% | 46.3% | 46.7% |
| 1 | 43.1% | 44.6% | 45.4% | 45.9% | 46.3% |

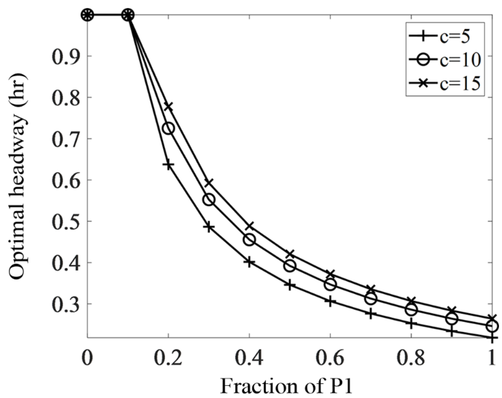



**Fig. 13.** Optimal Headway vs. Fraction of P1 for Different Vehicle Capacities under PTS

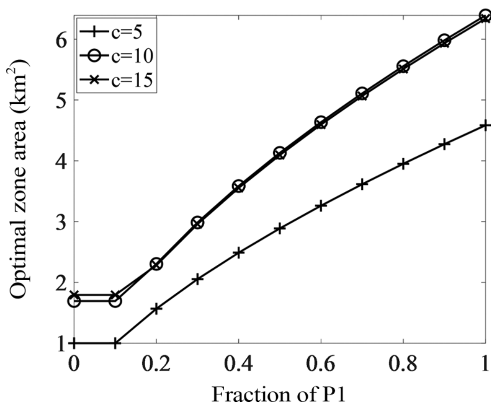



**Fig. 14.** Optimal Zone Area vs. Fraction of P1 for Different Vehicle Capacities under PTS

wait time of P1. Not that $h^*$ and $A^*$ remain constant when $\alpha < 0.1$ at which the policy headway constraint holds. When $c > 10$ seats/veh, $A^*$ is not sensitive to $c$. Vehicle utilization under various $\alpha$ and $c$ are analyzed and the results are summarized in Table 5. It is found that as $c$ and $\alpha$ decrease, vehicle utilization increases.

Figures 15 and 16 explore the relationship between demand density $q$ and average cost $C^*$ (as well as average operating cost $C_o^*$) for various vehicle capacity $c$. We found that $C^*$ and $C_o^*$ decreases as $q$ increases, and the decisions of selecting $c$ under the system and operator perspectives are different. From system's viewpoint, smaller vehicles (i.e., 5 seats/veh) would be cost-effective. Larger vehicles tend to pick-up more passengers,

**Table 5.** Vehicle Utilization vs. Fraction of P1 for Different Vehicle Capacities under PTS

| $\alpha$ | $c$ (seat/veh) | | |
|---|---|---|---|
| | 5 | 10 | 15 |
| 0.0 | 100.0% | 84.7% | 59.8% |
| 0.2 | 100.0% | 83.6% | 59.2% |
| 0.4 | 100.0% | 81.7% | 57.9% |
| 0.6 | 100.0% | 80.4% | 57.0% |
| 0.8 | 100.0% | 79.5% | 56.4% |
| 1 | 100.0% | 78.7% | 55.8% |

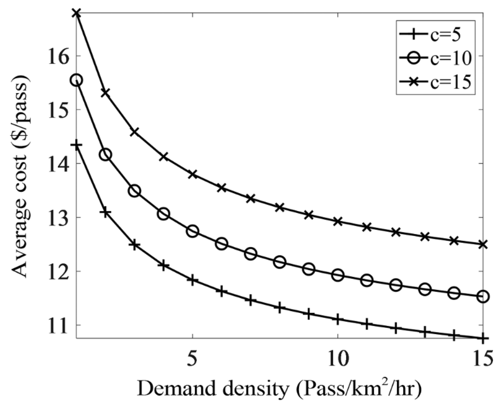



**Fig. 15.** Average Cost vs. Demand Density for Different Vehicle Capacities under PTS

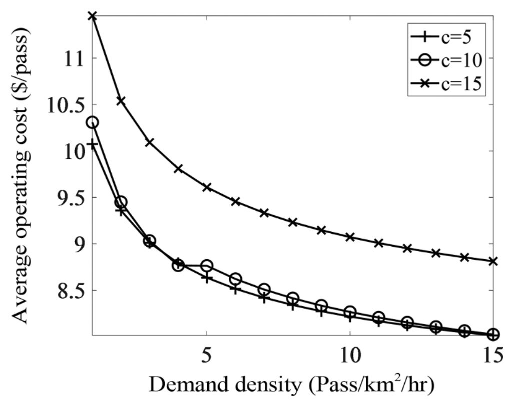



**Fig. 16.** Avg. Operating Cost vs. Demand Density for Different Vehicle Capacities under PTS

which will lead to the increase of passenger travel time (i.e., the sum of in-vehicle time and wait time). However, from operator's perspective, the difference between $C_o^*$ with smaller vehicles (5 and 10 seats/veh) is minor.

## 5. Conclusions

This paper proposes a mathematical model that is applied to jointly optimize zone areas and headways for demand responsive transit (DRT) which minimize the average cost, considering a heterogeneous environment under the advent of MaaS. Unlike previous models, the wait cost formulated here considers wait time at pick-up locations as well as early arrival penalties at destinations. Passengers notified with predetermined pick-up time or real-time vehicle arrival information via MaaS are analyzed and discussed separately.

A case study is conducted, in which a service region within the city of Calgary in Canada is applied to demonstrate the effectiveness of the model. With the optimized zone areas, the region is partitioned into five zones under both PTS and RTS scenarios with different optimal zone areas and headways as well as minimized average costs. Considering irregular community boundary, land use, demand density, and street network, the actual zone areas are slightly deviated from the optimized areas.

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Not Applicable

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## Appendix I

**Table 6.** Map of Service Region with Community Names

| Symbol | Definition | Baseline value | Unit |
|---|---|---|---|
| a | Fixed wait time per passenger | 0.017 | hr |
| $A_i$ | Area of zone *i* | - | km$^2$ |
| $A_\sigma$ | Area of the service region | 86.9 | km$^2$ |
| $A_i^*$ | Optimal area of zone *i* | - | km$^2$ |
| $A_i'$ | Actual area of zone *i* | - | km$^2$ |
| $b$ | Additional wait time per stop | 0.008 | hr/pass |
| $c_i$ | Capacity of vehicles applied in zone *i* | 20 | seat/veh |
| $C_i$ | Average cost per trip of zone *i* | - | \$/pass |
| $C_{oi}$ | Average operating cost of zone *i* | - | \$/pass |
| $C_{ri}$ | Average in-vehicle cost of zone *i* | - | \$/pass |
| $C_{wi}$ | Average wait cost of zone *i* | - | \$/pass |
| $C_{ti}^*$ | Minimum average cost per trip of zone *i* | - | \$/pass |
| $C_{oi}'$ | Adjusted average operating cost of zone *i* | - | \$/pass |
| $C_{ti}'$ | Adjusted average cost per trip of zone *i* | - | \$/pass |
| $D_i$ | Vehicle travel distance within zone *i* | - | km |
| D | Deviation of the adjusted value from optimal value | - | |
| EAP | Early arrival penalty per passenger | | \$/pass |
| $h_i$ | Headway of zone *i* | - | hr |
| $h_{max}$ | Policy headway | 1 | hr |
| $h_i^*$ | Optimal headway of zone *i* | - | |
| $h_i'$ | Adjusted headway of zone *i* | - | |
| $i$ | Index of zones | - | |
| $k$ | A constant related to road network pattern | 1.15 | |
| $n_i$ | Number of stops within one trip of zone *i* | - | pass/zone |
| $N$ | Number of zones | - | |
| $q_i$ | Demand density of zone *i* | 5 | pass/km$^2$/hr |
| $t_{Hi}$ | Round-trip line-haul travel time (zone *i* – terminal) | 0.17 | hr |
| $t_{Si}$ | Stop delay of zone *i* | - | hr |
| $t_{wi}$ | Average wait time per passenger of zone *i* at pick-up locations | - | hr/pass |
| $T_i$ | Round-trip vehicle travel time within zone *i* | - | hr |
| $V$ | Average vehicle running speed | 40 | km/hr |
| $\alpha_i$ | Fraction of type 1 passenger within zone *i* | 0.8 | |
| $\beta_i$ | Wait time under real-time transit information scenario | 0.125 | hr |
| $\gamma_o$ | Unit operating cost | 116 | \$/hr/veh |
| $\gamma_r$ | Value of passenger in-vehicle time | 5.13 | \$/pass/hr |
| $\gamma_w$ | Value of passenger wait time | 11.63 | \$/pass/hr |
| $\rho$ | Ratio of actual area over optimal area of a zone | - | |
| $\tau$ | Average stop delay | 0.05 | hr/stop |

## Appendix II

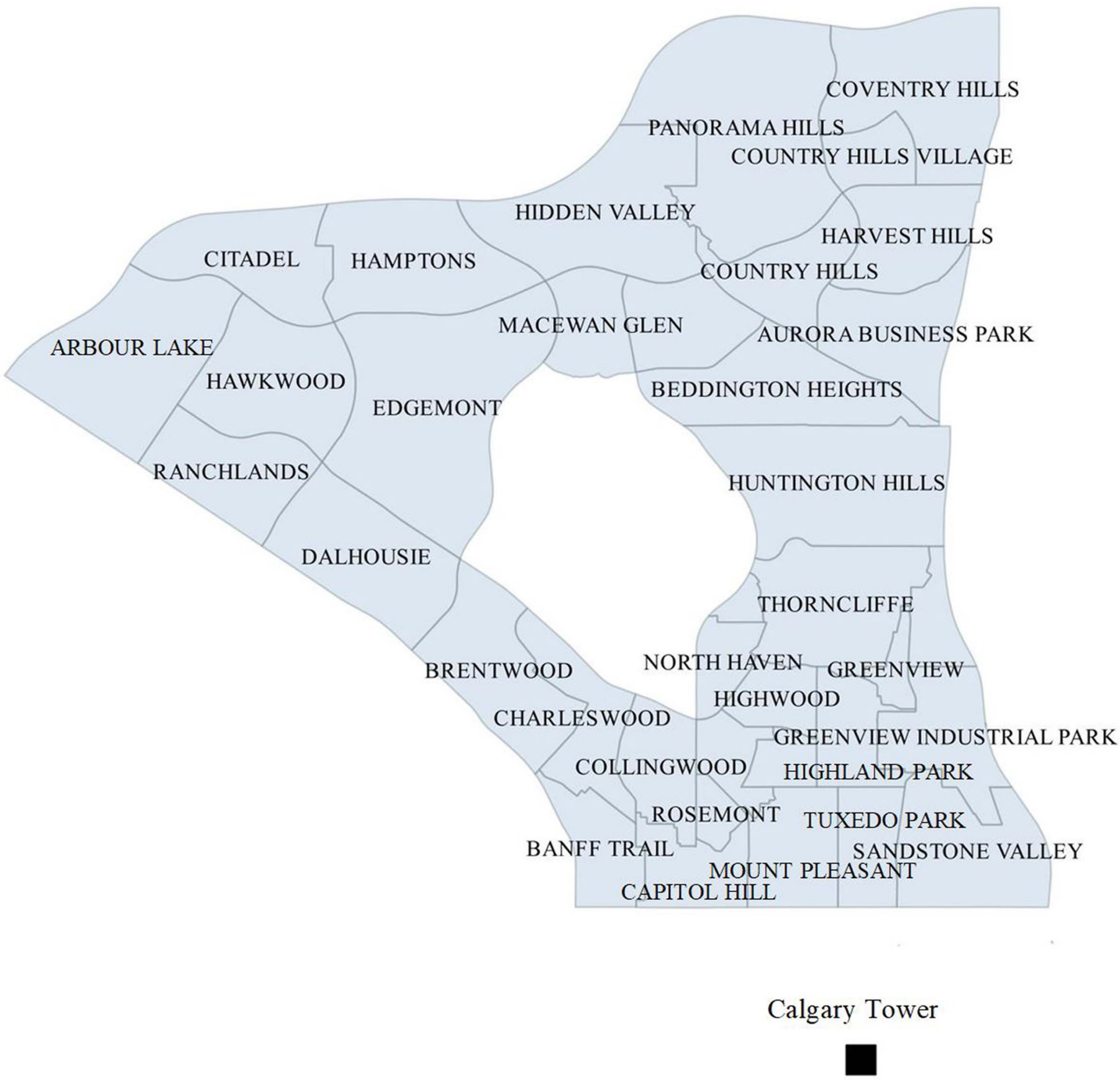


**Fig. 17.** Map of Service Region with Community Names