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System Identification of Structures with Severe Closely Spaced Modes Using Parametric Estimation Algorithms Based on Complex Mode Indicator Function with Singular Value Decomposition

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ABSTRACT

The modal-interference phenomenon usually makes difficulty of parametric estimation, especially for some structural systems with severe modal interference caused from close or even repeated modes. The existence of severe modal interference will degrade the effectiveness of system identification, and may lead to the problem of insufficient model order due to the existence of repeated modes. Multiple input/multiple output modal estimation is therefore usually conducted effectively to meet the sufficient number of measurement channels. In this paper, the Complex Mode Indicator Function (CMIF) is introduced to estimate the number of significant modes of a structure with severe modal interference, and then the singular value decomposition (SVD) is employed to parametric estimation of the major modes of a structural system without additionally evaluating enhanced frequency response function. Numerical simulations and experimental validation of a practical rectangular steel plate confirm the effectiveness of the presented method for parametric estimation of systems with severe closely spaced modes under noisy conditions.

1. Introduction

Engineering structures often exhibits dynamic behaviors under excitation forces. Dynamic response of a structure may be determined by either theoretical or experimental analysis. Through the experimental analysis of structural vibration, it can help us understand many phenomena encountered in reality, and then probably carry out a better design or control over the structures. Modal testing is one of the most important areas in experimental analysis, and is usually the essential procedure of an effective modal estimation of a structure from measured data through system identification methods.

In many past modal estimation techniques, effective frequency-domain methods often rely on accurate estimation of frequency response functions on the basis of measured excitation and response data. The frequency response function of a system can be obtained from the Fourier transform of measured response and excitation data (Hougen and Wash, 1961). Because the corresponding experimental apparatuses and techniques are more available to implement effective modal estimations, the single input modal-estimation method had extensively applied to modal-parameter identification in 1970. However, some problems still exist in practical work when using the single input modal estimation. If excitation input locally concentrates on one point, it may induce the nonlinear characteristic of a system. When the excitation input acts on the node of a structure, some structural modes could not be excited and may be omitted. The identification results of closely spaced modes may not be accurate, and the modes with repeated eigenvalues can even not be identified (Tan et al., 2008). If the excitation points with different direction are available, the multi-input modal-identification method can be employed to effectively avoid the difficulties when using single-input modal-identification methods (Shye et al., 1987). The occurrence of nonlinear characteristic of system can then be reduced due to the relatively uniform distribution of energy from excitation input. In addition, the occurrence of omitted modes may be also reduced by using multi-input modal-identification methods, and closely spaced (Le and Caracoglia, 2015) or repeated modal frequencies (Qu et al., 2018) of structures can also be well estimated (Hwang and Kim, 2017). The frequency decomposition

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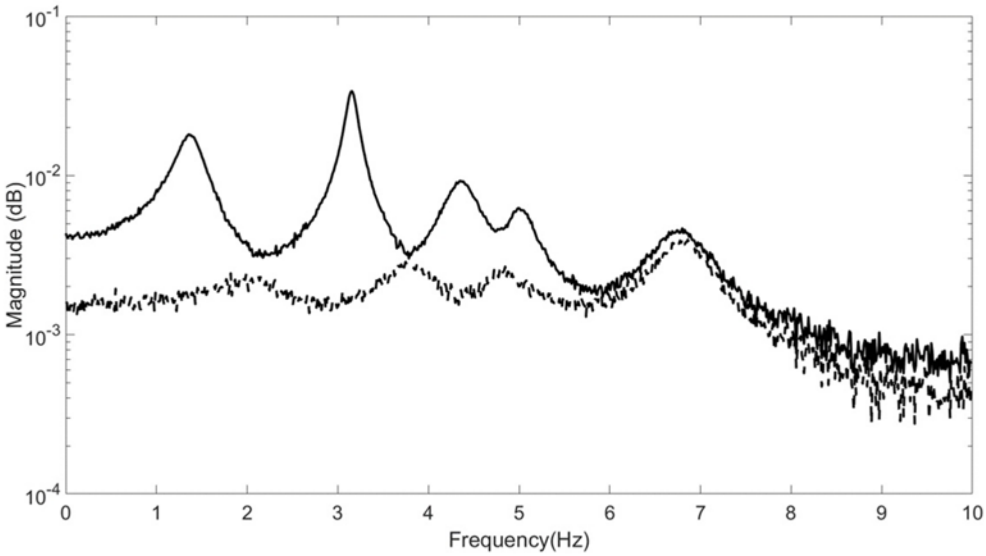

Fig. 1. Estimation of the Number of Significant Modes of a System with Severe Closely Spaced Modes through Modified CMIF Method

technique (FDD) is developed to extend the Complex Mode Indicator Function (CMIF) (Wei et al., 1987) for operational modal analysis (OMA) (Brincker et al., 2000), and employs the power spectral density (PSD) matrix instead of frequency response functions in conventional modal estimation. The separation procedure is implemented through the singular value decomposition (SVD) technique, and the number of important modes with major contribution in the structural system can then be estimated from the number of the largest singular values through SVD analysis.

When executing parametric identification of structures, the modal interference may conduct poor estimation results, even induce the problem of model identifiability. The phenomenon of modal interference usually makes difficulty of parametric estimation, especially for some structural systems having close (Kordkheili et al., 2018) or even repeated modes, large damping non-proportionality (Rayleigh, 1897), and high damping ratios. The existence of severe closely spaced modes (Kim et al., 1997) can cause complex vibration behavior of structures to make the difficulty of stress prediction of turbine bladed-disk assemblies (Lin and Lim, 1997). In this paper, on the basis of the theory of frequency response function in modal analysis, we use multiple references to implement the system identification of structures with severe closely spaced modes. The CMIF algorithm is employed to number estimation of significant modes of a system with severe modal interference (Allemang and Brown, 2006), as shown in Fig. 1, in which at least one pair of severe closely spaced modes exist around 7 Hz. The major modes of a structural system are then identified through the SVD without additionally evaluating enhanced frequency response function in conventional CMIF algorithm. The identification procedure of CMIF algorithm can therefore be simplified to improve the efficiency of modal estimation. Numerical simulations and experimental validation of an actual rectangular steel plate confirm the effectiveness of the proposed method for parametric identification of systems with severe modal interference under noisy conditions.

2. Modal Identification of a System with Severe Modal Interference

2.1 Modal Interference

Based on the theory of modal analysis, either proportional or nonproportional damping systems, the multiple degree of freedom (MDOF) system can be decoupled into many single degree of freedom (SDOF) systems by using the orthogonality of mode shapes with respect to the stiffness matrix or to the mass matrix, and the frequency response function (FRF) matrix of MDOF system can then be composed of many FRF's of SDOF systems through the modal superposition method. When the vibration energy of each structural mode in frequency domain may overlap with other modes within certain range, the phenomenon is so called modal interference (Kordkheili et al., 2012). The severe modal interference existing in these systems will degrade the effectiveness of modal identification, and may lead to the problem of insufficient model order due to the existence of repeated eigenvalues.

2.2 Modal Analysis of a System with Repeated Modes

According to the theory of modal analysis, the MDOF FRF matrix can be decoupled into many SDOF FRF's by using the orthogonality of the mode shapes with respect to the stiffness matrix or to the mass matrix. In the following, for a structural system has modes with repeated eigenvalues, the corresponding modal-shape vector of a structural mode will be investigated. Consider a N-DOF structural system with proportional damping, the corresponding modal-shape vectors can be determined by solving the eigenvalue problem of a system:

$$[\mathbf{K}-\omega_{nr}^2\mathbf{M}]\boldsymbol{\phi}_r=0 \tag{1}$$

Let ω_{ns}^2 be k repeated eigenvalues, i.e., $(\omega_{ns}^2)_1=(\omega_{ns}^2)_2=\cdots(\omega_{ns}^2)_k=\omega_{ns}^2$, and substitute ω_{ns}^2 into ω_{nr}^2, the Eq. (1) can be rewritten as:

$$[\mathbf{K}-\omega_{ns}^2\mathbf{M}]\boldsymbol{\phi}_s=0 \tag{2}$$

The relationship between the rank of matrix $[\mathbf{K}-\omega_{ns}^2\mathbf{M}]$ and the number of repeated eigenvalues can be expressed as follows:

$$\operatorname{rank}[\mathbf{K}-\omega_{ns}^2\mathbf{M}]\geq N-k \tag{3}$$

When the rank of $[\mathbf{K}-\omega_{ns}^2\mathbf{M}]$ is $N-k$, the $N-k$ independent equations but with N unknowns can be decoupled from Eq. (2) as follows:

$$\begin{aligned}&(\mathrm{K}_{1,1}-\omega_{ns}^2\mathrm{M}_{1,1})\phi_{s,1}+\cdots+(\mathrm{K}_{1,N\text{-}k}-\omega_{ns}^2\mathrm{M}_{1,N\text{-}k})\phi_{s,N\text{-}k}\\&\quad=-(\mathrm{K}_{1,N\text{-}k+1}-\omega_{ns}^2\mathrm{M}_{1,N\text{-}k+1})\phi_{s,N\text{-}k+1}-\cdots-(\mathrm{K}_{1,N}-\omega_{ns}^2\mathrm{M}_{1,N})\phi_{s,N}\\&\quad\vdots\\&(\mathrm{K}_{N\text{-}k,1}-\omega_{ns}^2\mathrm{M}_{N\text{-}k,1})\phi_{s,1}+\cdots+(\mathrm{K}_{N\text{-}k,N\text{-}k}-\omega_{ns}^2\mathrm{M}_{N\text{-}k,N\text{-}k})\phi_{s,N\text{-}k}\\&\quad=-(\mathrm{K}_{N\text{-}k,N\text{-}k+1}-\omega_{ns}^2\mathrm{M}_{N\text{-}k,N\text{-}k+1})\phi_{s,N\text{-}k+1}-\cdots-(\mathrm{K}_{N,N}-\omega_{ns}^2\mathrm{M}_{N,N})\phi_{s,N}\end{aligned} \tag{4}$$

Introduce the following k independent vectors into the Eq. (4), the following equation can be derived as:

$$\begin{bmatrix}(\phi_{s,N\text{-}r+1})_1\\(\phi_{s,N\text{-}r+2})_1\\\vdots\\(\phi_{s,N})_1\end{bmatrix}=\begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix},\quad\begin{bmatrix}(\phi_{s,N\text{-}r+1})_2\\(\phi_{s,N\text{-}r+2})_2\\\vdots\\(\phi_{s,N})_2\end{bmatrix}=\begin{bmatrix}0\\1\\\vdots\\0\end{bmatrix},\quad\cdots,\quad\begin{bmatrix}(\phi_{s,N\text{-}r+1})_k\\(\phi_{s,N\text{-}r+2})_k\\\vdots\\(\phi_{s,N})_k\end{bmatrix}=\begin{bmatrix}0\\0\\\vdots\\1\end{bmatrix} \tag{5}$$

the $N-k$ unknowns in each independent vector, $(\phi_{s,i})_j$ $(i=1\sim N-k,\ j=1\sim k)$, can then be obtained, and the corresponding eigenvectors of k repeated eigenvalues can then be expressed as:

$$\begin{aligned}(\boldsymbol{\phi}_s)_1&=\left[(\phi_{s,1})_1\quad(\phi_{s,2})_1\quad\cdots\quad(\phi_{s,N-r})_1\quad 1\quad 0\quad\cdots\quad 0\right]^{\mathrm{T}}\\(\boldsymbol{\phi}_s)_2&=\left[(\phi_{s,1})_2\quad(\phi_{s,2})_2\quad\cdots\quad(\phi_{s,N-r})_2\quad 0\quad 1\quad\cdots\quad 0\right]^{\mathrm{T}}\\&\vdots\\(\boldsymbol{\phi}_s)_k&=\left[(\phi_{s,1})_k\quad(\phi_{s,2})_k\quad\cdots\quad(\phi_{s,N-r})_k\quad 0\quad 0\quad\cdots\quad 1\right]^{\mathrm{T}}\end{aligned} \tag{6}$$

In the preceding, the corresponding eigenvectors of repeated eigenvalues can be obtained through the aforementioned method; these eigenvectors, however, cannot be met the condition that mode-shape vectors are orthogonal each other. Based on the mathematical definition, the linear combinations of these eigenvectors of repeated eigenvalues are also eigenvectors of the repeated eigenvalues, and mode-shape vector can further be obtained through the orthogonality properties with each other. If $(\bar{\boldsymbol{\phi}}_s)_1(\bar{\boldsymbol{\phi}}_s)_2\cdots(\bar{\boldsymbol{\phi}}_s)_k$ are mode-shape vectors of repeated modes meeting the condition that mode-shape vectors are orthogonal each other, one can assume:

$$(\bar{\boldsymbol{\phi}}_s)_1=(\boldsymbol{\phi}_s)_1 \tag{7}$$

$$(\bar{\boldsymbol{\phi}}_s)_2=c_1(\bar{\boldsymbol{\phi}}_s)_1+(\boldsymbol{\phi}_s)_2 \tag{8}$$

Equations (7) and (8) can be derived through the orthogonality of the mode shapes with respect to the mass matrix as follows:

$$c_1=-\frac{(\bar{\boldsymbol{\phi}}_s)_1^{\mathrm{T}}\mathbf{M}(\boldsymbol{\phi}_s)_2}{(\bar{\boldsymbol{\phi}}_s)_1^{\mathrm{T}}\mathbf{M}(\bar{\boldsymbol{\phi}}_s)_1} \tag{9}$$

$(\bar{\boldsymbol{\phi}}_s)_2$ can be obtained from Eq. (9), and then we can further assume:

$$(\bar{\boldsymbol{\phi}}_s)_3=c_2(\bar{\boldsymbol{\phi}}_s)_1+c_3(\bar{\boldsymbol{\phi}}_s)_2+(\boldsymbol{\phi}_s)_3 \tag{10}$$

Again, by using the orthogonality of the mode shapes with respect to the mass matrix, both Eqs. (7), (8) and (10) can be derived as follows:

$$\begin{aligned}c_2&=-\frac{(\bar{\boldsymbol{\phi}}_s)_1^{\mathrm{T}}\mathbf{M}(\boldsymbol{\phi}_s)_3}{(\bar{\boldsymbol{\phi}}_s)_1^{\mathrm{T}}\mathbf{M}(\bar{\boldsymbol{\phi}}_s)_1}\\c_3&=-\frac{(\bar{\boldsymbol{\phi}}_s)_2^{\mathrm{T}}\mathbf{M}(\boldsymbol{\phi}_s)_3}{(\bar{\boldsymbol{\phi}}_s)_2^{\mathrm{T}}\mathbf{M}(\bar{\boldsymbol{\phi}}_s)_2}\end{aligned} \tag{11}$$

$(\bar{\boldsymbol{\phi}}_s)_3$ can be obtained from Eq. (11), and all modal-shape vectors with repeated eigenvalues can be obtained by taking the aforementioned iteration. It should be mentioned that the results will be different as varying the assumption of $(\bar{\boldsymbol{\phi}}_s)_k$, and $(\bar{\boldsymbol{\phi}}_s)_k$ is therefore not unique but we can still obtain the mode-shape vectors with orthogonal each other. When the rank of $[\mathbf{K}-\omega_{ns}^2\mathbf{M}]$ is greater than $N-k$, this system is so-called defective system, where there exist the problems of defective eigenvectors. In dynamic problems of structures with non-proportional damping, the number of independent complex eigenvectors is less than that of repeated eigenvalues, the corresponding matrices can be defective. The insufficient mode-shape vectors can be obtained by using generalized eigenvectors, and the equation of motion of the aforementioned system can be rewritten as:

$$\dot{\mathbf{y}}(t)+\mathbf{G}\mathbf{y}(t)=\mathbf{P}(t) \tag{12}$$

where $\mathbf{G}=\begin{bmatrix}-\mathbf{I}&\mathbf{0}\\\mathbf{M}^{-1}\mathbf{C}&\mathbf{M}^{-1}\mathbf{K}\end{bmatrix}$, and $\mathbf{P}(t)=\begin{bmatrix}\mathbf{0}\\\mathbf{M}^{-1}\mathbf{f}(\mathrm{t})\end{bmatrix}$. Assume $\mathbf{T}$ is the mode-shape matrix of a system (including the generalized eigenvectors), and there exists the following equation for any real-coefficient matrix $\mathbf{G}$:

$$\mathbf{T}^{-1}\mathbf{G}\mathbf{T}=\mathbf{J} \tag{13}$$

In Eq. (13), the matrix $\mathbf{J}$ is Jordan normal form, which can be written as:

$$\mathbf{J}=\begin{bmatrix}\lambda_1&a_{12}&&\\&\lambda_2&\ddots&\\&&\ddots&a_{2N\text{-}1,2N}\\&&&\lambda_{2N}\end{bmatrix} \tag{14}$$

where a_{ij} is 0 or 1, which is determined by repeated eigenvalues and the corresponding independent eigenvectors. Applying the coordinate transformation to Eq. (13) yields:

$$\begin{Bmatrix} \dot{q} \\ \dot{q}^* \end{Bmatrix} + \mathbf{J}\begin{Bmatrix} q \\ q^* \end{Bmatrix} = \mathbf{T}^{-1}\mathbf{P}(t) \tag{15}$$

In the above, **J** is not a diagonal matrix when a system is defective, and this MDOF system can not be therefore completely decoupled into many SDOF systems by using the generalized eigenvectors. However, a system can not be determined to be defective from practical response data, and there exists a defective system only for some special conditions. In this paper, for most non-defective systems, we will decouple a MDOF system by mode-superposition method into many SDOF systems, and modal-parameter identification can then be performed.

2.3 Frequency-Domain Modal Identification of a System with Repeated Modes

Based on the theory of modal analysis, for a system having modes with repeated eigenvalues, the formulation of frequency response function can be employed to investigate the difference of a system having repeated modes, and the problem of identifiability induced when performing modal-parameter identification of a system with repeated modes. The FRF of a system can be expressed by the mode superposition as the linear combination of FRF's corresponding to each single mode in the following:

$$\mathrm{H}_{pq}(\omega) = \sum_{r=1}^{2N} \frac{Q_r \psi_{pr} \psi_{qr}}{j\omega - \lambda_r} \tag{16}$$

When there exist k repeated eigenvalues λ_s's of a system, according to mode-superposition method, the corresponding FRF of a system can be expressed as follows:

$$\begin{aligned} H_{pq}(\omega) = {} & \frac{Q_1\psi_{p1}\psi_{q1}}{j\omega-\lambda_1} + \frac{Q_2\psi_{p2}\psi_{q2}}{j\omega-\lambda_2} + \cdots + \frac{(Q_s)_1(\psi_{ps})_1(\psi_{qs})_1}{j\omega-(\lambda_s)_1} + \cdots \\ & + \frac{(Q_s)_k(\psi_{ps})_k(\psi_{qs})_k}{j\omega-(\lambda_s)_k} + \cdots \end{aligned} \tag{17}$$

where $(\lambda_s)_i$ denoted the i$^{\text{th}}$ repeated eigenvalue of λ_s. Due to $(\lambda_s)_1 = (\lambda_s)_2 = \cdots = (\lambda_s)_k = \lambda_s$, Eq. (17) can be rewritten as:

$$H_{pq}(\omega) = \frac{Q_1\psi_{p1}\psi_{q1}}{j\omega-\lambda_1} + \frac{Q_2\psi_{p2}\psi_{q2}}{j\omega-\lambda_2} + \cdots + \frac{\sum_{i=1}^{k}(Q_s)_i(\psi_{ps})_i(\psi_{qs})_i}{j\omega-\lambda_s} + \cdots \tag{18}$$

In Eq. (18), due to the fact that the repeated modes of a system have the same eigenvalues, the term $\dfrac{\sum_{i=1}^{k}(Q_s)_i(\psi_{ps})_i(\psi_{qs})_i}{j\omega-\lambda_s}$ of $H_{pq}(\omega)$ is expressed as the FRF of a single mode by using the mode superposition. When performing the modal-parameter identification, to evaluate the mode-shape vector of each mode, a column of FRF data is employed and can be expressed as follows:

$$\begin{aligned} \{\mathrm{H}(\omega)\}_q &= \sum_{r=1}^{2N} \frac{Q_r\{\boldsymbol{\psi}_r\}\psi_{qr}}{j\omega-\lambda_r} \\ &= \frac{Q_1\{\boldsymbol{\psi}_1\}\psi_{q1}}{j\omega-\lambda_1} + \frac{Q_2\{\boldsymbol{\psi}_2\}\psi_{q2}}{j\omega-\lambda_2} + \cdots + \frac{\sum_{i=1}^{k}(Q_s)_i\{(\boldsymbol{\psi}_s)_i\}(\psi_{qs})_i}{j\omega-\lambda_s} + \cdots \end{aligned} \tag{19}$$

In Eq. (19), when using a single column FRF data, a column of FRF matrix can be expressed as a linear combination of mode-shape vector of each mode. The summation of residuals corresponding to the repeated modes indicates the information of a single mode only, so no repeated modes can be determined. The residual vector of a single mode, in general, is proportional to the corresponding mode shape in the different column of FRF matrix. However, the different linear combination of the corresponding mode shape of repeated mode is not proportional to each other, the repeated modes can therefore be determined, and further extracted the other repeated mode shapes.

The preceding result indicates that it is necessary to use multiple references to estimate structural modes with repeated eigenvalues according to the formulation of frequency response function based on the theory of modal analysis. Because of insufficient model order due to the existence of structural modes with repeated eigenvalues, it may degrade the effectiveness of modal identification for a structure with severe closely spaced modes. Multiple input/multiple output (MIMO) modal estimation can therefore usually be lead effectively to reach the sufficient number of measurement channels.

3. Complex Mode Indicator Function

The CMIF method is an effective MIMO modal estimation technique, and has developed on the basis of SVD of FRF matrix of a system (Allemang and Brown, 2006). The CMIF is composed of singular values evaluated through SVD of MIMO FRF matrix at each spectral curve. According to the theory of modal analysis, the matrix of MIMO FRF of N-DOF system can be expressed as follows:

$$\mathbf{H}(\omega)_{N_o\times N_i} = \bar{\mathbf{\Phi}}_{N_o\times 2N} \begin{bmatrix} \ddots & & \\ & \dfrac{Q_r}{j\omega-\lambda_r} & \\ & & \ddots \end{bmatrix}_{2N\times 2N} \mathbf{L}^T_{N_i\times 2N} \tag{20}$$

where $\bar{\mathbf{\Phi}}$ and **L** are, respectively, the mode shape and modal participation factors matrices, whose column vectors are a mode shape and modal participation factor, respectively, of the r$^{\text{th}}$ mode. In Eq. (20), when a mode shape and modal participation factor are both expressed as unit vectors, the magnitude of FRF can then be expressed as the combination of diagonal term $\left|\dfrac{Q_r}{j\omega-\lambda_r}\right|$. By taking advantage of the SVD technique, $\mathbf{H}(\omega)_{N_o\times N_i}$

can be expressed as follows:

$$\mathbf{H}(\omega)_{N_o\times N_i}=\mathbf{U}(\omega)_{N_o\times N_i}\Sigma(\omega)_{N_i\times N_i}\mathbf{V}(\omega)^H_{N_i\times N_i} \quad (21)$$

In Eq. (21), the principal components of FRF matrix $\mathbf{H}(\omega)_{N_o\times N_i}$ can be obtained. The right and left singular matrices, $\mathbf{V}(\omega)_{N_i\times N_i}$ and $\mathbf{U}(\omega)_{N_o\times N_i}$, obtained from SVD analysis formed respectively orthogonal basis composed of unit vectors, so the dimension of FRF matrix in Eq. (21) can be determined by number of singular value, which is equivalent to $\left|\frac{Q_r}{j\omega-\lambda_r}\right|$ of the corresponding mode. Note that the singular values are the principal components of $\mathbf{H}(\omega)_{N_o\times N_i}$, and $\left|\frac{Q_r}{j\omega-\lambda_r}\right|$'s are the local maximum in FRF corresponding to the excited modes of a structure. Through the SVD of MIMO FRF corresponding to each frequency, the CMIF can be obtained from singular values. The corresponding frequencies of the peaks of CMIF curve are the natural frequencies of a system. Then, the number of the structural modes can be determined, and the mode shapes and modal participation factors can then be approximately estimated from the corresponding singular vectors. For a MDOF system, assume the mass matrix is an identity matrix, the corresponding mode shapes can be treated a unit vector. Then, all mode shapes in the form of unit vectors can then be formed a unity matrix. Therefore, in Eq. (21), by assuming the mass matrix of a system is an identity matrix, the singular vectors are also satisfied with the orthogonality conditions of mode-shape vectors with respect to mass and stiffness matrix, respectively.

In the aforementioned above, when the frequency of FRF reaches to the natural frequency ω_s of the sth mode, the local maximum exists in the FRF that mainly contributed from the sth mode. The MIMO FRF matrix can be expressed as:

$$\mathbf{H}(\omega_s)=\sum_{r=1}^{2N}\{\boldsymbol{\phi}_r\}\frac{Q_r}{j\omega_s-\lambda_r}\{\boldsymbol{l}_r\}^{\mathrm{T}}\approx\{\boldsymbol{\phi}_s\}\frac{Q_s}{j\omega_s-\lambda_s}\{\boldsymbol{l}_s\}^{\mathrm{T}} \quad (22)$$

where $\mathbf{H}(\omega_s)$ is the local maximum of the MIMO FRF, and thus the corresponding peak exists in the CMIF curve. The FRF matrix mainly contributed by the singular vectors corresponding to singular values can be expresses as:

$$\mathbf{H}(\omega_s)=\sum_{i}^{N_i}\{u_i(\omega_s)\}\sigma_i(\omega_s)\{v_i(\omega_s)\}^{\mathrm{H}}\approx\{u_1(\omega_s)\}\sigma_1(\omega_s)\{v_1(\omega_s)\}^{\mathrm{H}} \quad (23)$$

Comparing Eqs. (22) with (23), through the SVD of multi-input FRF matrix corresponding to the frequencies of peaks in $\mathbf{H}(\omega_s)$, we can obtain the left and right singular vectors approximated to the mode shapes and modal participation factors, respectively, and each singular value $\sigma(\omega)$ is equivalent to $\left|\frac{Q_r}{j\omega-\lambda_r}\right|$. When a structural system has close, even repeated modes, the number N_r of modes with main contribution is less than the number of input channel N_i, then N_r larger singular values obviously exist in the N_i singular values obtained from Eq. (21). Thus, we can observe the number of modes with main contribution in a system from different CMIF curves. The peaks corresponding to the same modal frequency appear in the different curves indicates that the modal frequencies correspond to the repeated modes exist in a system.

As mentioned above, we can estimate the number of the modes to be identified and determine the natural frequencies of excited modes corresponding to the peaks in the curve of CMIF. However, the accuracy of modal identification is influenced by the frequency resolution. To improve the accuracy of identification results of the modal frequencies and damping ratios, we use the aforementioned modal participation factors from SVD of MIMO FRF matrix of peaks in CMIF curve as weighting functions in conjunction with the orthogonality of mode shape vectors to obtain the enhanced frequency response function of each mode, and reduce the interference among the other modes of a system. CMIF method is implemented by using the data from Enhanced FRFs, and the Enhanced FRF of the sth mode is defined as follows:

$$\begin{aligned}\overline{\mathrm{H}}_s(\omega)&=\{u(\omega_s)\}^{\mathrm{H}}\mathbf{H}(\omega)\{v(\omega_s)\}\\&=\sum_{r=1}^{2N}\{u(\omega_s)\}^{\mathrm{H}}\{\phi_r\}\frac{Q_r}{j\omega-\lambda_r}\{l_r\}^{\mathrm{T}}\{v(\omega_s)\}\end{aligned} \quad (24)$$

Introduce the orthogonality condition of mode-shape vectors,

$$\begin{cases}\{u(\omega_s)\}^{\mathrm{H}}\{\phi_r\}\ll\{u(\omega_s)\}^{\mathrm{H}}\{\phi_s\}\\\{l_r\}^{\mathrm{T}}\{v(\omega_s)\}\ll\{l_s\}^{\mathrm{T}}\{v(\omega_s)\}\end{cases},\ \mathrm{s}\neq\mathrm{r} \quad (25)$$

where $\{u(\omega_s)\}$ and $\{v(\omega_s)\}$ are, respectively, the mode shape and the complex conjugate of modal participation factor corresponding to the sth mode through the SVD method. Eq. (24) can be rewritten by the aforementioned orthogonality condition of mode-shape vectors as follows:

$$\overline{H}_s(\omega)\approx\{u(\omega_s)\}^H\{\phi_s\}\frac{Q_s}{j\omega-\lambda_s}\{l_s\}^{\mathrm{T}}\{v(\omega_s)\}=\frac{A_s}{j\omega-\lambda_s} \quad (26)$$

Equation (26) is the enhanced FRF of the sth mode. $\overline{H}_s(\omega)$ can be employed to reduce the interference from the other modes n the frequency content where this mode exists, and approximated as the FRF of a single mode. An effective modal estimation can be carried out through the CMIF with the single-mode identification method. In the following, through the least-square method, Eq. (26) can be evaluated as:

$$\lambda_s\overline{H}_s(\omega)+A_s=(j\omega)\cdot\overline{H}_s(\omega) \quad (27)$$

Through the insertion of the data obtained from the neighborhood of the peak in $\overline{H}_s(\omega)$ into Eq. (27), the following matrix-form equation can then be derived

$$\mathbf{X\theta}=\mathbf{y} \quad (28)$$

where, $\mathbf{X}=\begin{bmatrix}\overline{\mathrm{H}}_s(\omega_1) & 1\\ \overline{\mathrm{H}}_s(\omega_2) & 1\\ \vdots & \vdots\end{bmatrix}$, $\boldsymbol{\theta}=\begin{bmatrix}\lambda_s\\ A_s\end{bmatrix}$, and $\mathbf{y}=\begin{bmatrix}(j\omega_1)\cdot\overline{\mathrm{H}}_s(\omega_1)\\ (j\omega_2)\cdot\overline{\mathrm{H}}_s(\omega_2)\\ \vdots\end{bmatrix}$.

By using the least-square method, the estimations θ can then be determined as follows:

$$\boldsymbol{\theta}=\left(\mathbf{X}^{\mathrm{T}}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y} \tag{29}$$

Thus, once the estimations θ is determined by using least-squares analysis from $\overline{H}_s(\omega)$, the modal parameters of structures can be further estimated by solving the eigenvalue problem associated with the estimations θ.

In the aforementioned theory, to avoid degrade the accuracy of identification results led by the large error from the data value involved in the peaks of response, an expansion matrix can be constructed by using the frequency response function matrix corresponding to frequency content close to the structural modes of interest. The SVD of the expansion matrix can then performed as follows:

$$\left[[\mathbf{H}(\omega_k-n\Delta\omega)]_{N_o\times N_i}\cdots[\mathbf{H}(\omega_k)]_{N_o\times N_i}\cdots[\mathbf{H}(\omega_k+n\Delta\omega)]_{N_o\times N_i}\right]_{N_o\times(2n+1)N_i}$$
$$=[\mathbf{U}(\omega_k)][\Sigma(\omega_k)][\mathbf{V}(\omega_k)]^{\mathrm{H}} \tag{30}$$

It can be observed that, in Eq. (30), the bigger the dimension of the expansion matrix composed of the overlapping adjacent frequency response function is, the more the singular values through the SVD of the expansion matrix are. Because the first N_i singular values and the corresponding singular vectors are main contribution to the expansion matrix, the number of identified structural modes and the modal frequencies corresponding to the first N_i singular values of expansion matrix can be determined. The corresponding mode-shape vectors (left singular vectors) can then be estimated by data averaging of the neighborhood of the modal frequency of a system in FRF, and the effectiveness of identification can thus be improved even the larger error existing in the FRF data of single frequency.

4. Modal Identification Using Singular Value Decomposition Only

From the aforementioned theory in the previous section, the complex mode indicator function is constructed by singular values through the SVD of MIMO FRFs, and is equivalent to $\left|\dfrac{Q_r}{j\omega-\lambda_r}\right|$ denoted as $\sigma(\omega)$ of the corresponding mode, which indicates that relationship between singular values and modal parameters. Thus, we can further perform modal identification by directly using the SVD analysis without evaluating EFRF in conventional indicator function. Note that in $\sigma(\omega)$,

$$\lambda_r=\alpha_r+j\beta_r \tag{31}$$

One can therefore derive:

$$(\alpha_r^2+\beta_r^2)\sigma^2(\omega)-2\beta_r\omega\sigma^2(\omega)-Q_r^2=-\omega^2\sigma^2(\omega) \tag{32}$$

Through the insertion of the data obtained from the neighborhood of the peak in $\sigma(\omega)$ into Eq. (32), the matrix-form terms $\mathbf{X}$, θ, and **y** in Eq. (28) can then be derived as:

$$\begin{bmatrix}\sigma^2(\omega_1) & -2\omega\sigma^2(\omega_1) & -1\\ \sigma^2(\omega_2) & -2\omega\sigma^2(\omega_2) & -1\\ \vdots & \vdots & \vdots\end{bmatrix},\ \begin{bmatrix}\alpha_r^2+\beta_r^2\\ \beta_r\\ Q_r^2\end{bmatrix},\ \text{and}\ \begin{bmatrix}-\omega_1^2\sigma^2(\omega_1)\\ -\omega_2^2\sigma^2(\omega_2)\\ \vdots\end{bmatrix},$$

respectively. It follows from the same procedure as used in Eq. (29), the estimations θ can then be determined through the least-square method. α_r and β_r can then be obtained from θ, and the modal parameters of a structural system can further determined by the eigenvalues of a system obtained from α_r and β_r.

5. Numerical Simulations and Experimental Verification

Most engineers depend on experience and knowledge to produce approaches to the problems they encountered. One can take advantage of the numerical simulations and experimental verification to confirm the effectiveness of the procedure of approaches. However, the exact modal information of a realistic structure is not usually available in advance when performing the experimental verification of large-scale structure, the validity of the proposed algorithm need to be confirmed by first using the numerical simulations. In the preceding, both complex mode indicator function and singular value decomposition have been employed

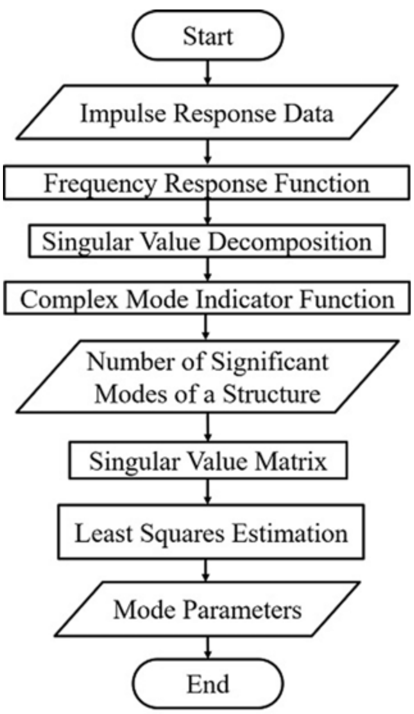

Fig. 2. A Flowchart of the Modified CMIF Method Indicates the Procedure of an Effective Modal Estimation of a Structure

to be combined to perform the modal identification. A flowchart of the proposed method, as shown in Fig. 2, indicates the procedure of an effective modal estimation of a structure. To confirm the validity of presented method, we consider a linear 3-DOF system having a pair of repeat modes. The mass matrix **M**, stiffness matrix **K**, and damping matrix **D** of the structural system with proportional damping are given as follows:

$$\mathbf{M}=\begin{bmatrix}4&0&0\\0&2&0\\0&0&4\end{bmatrix}\ N\cdot\sec^2/m \qquad \mathbf{K}=100\times\begin{bmatrix}7&-1&-1\\-1&3&-1\\-1&-1&7\end{bmatrix}\ N/m$$

$$\mathbf{D}=0.02\mathbf{M}+0.004\mathbf{K}\ \ N\cdot\sec/m\ .$$

The simulated impulse function employs as the excitation

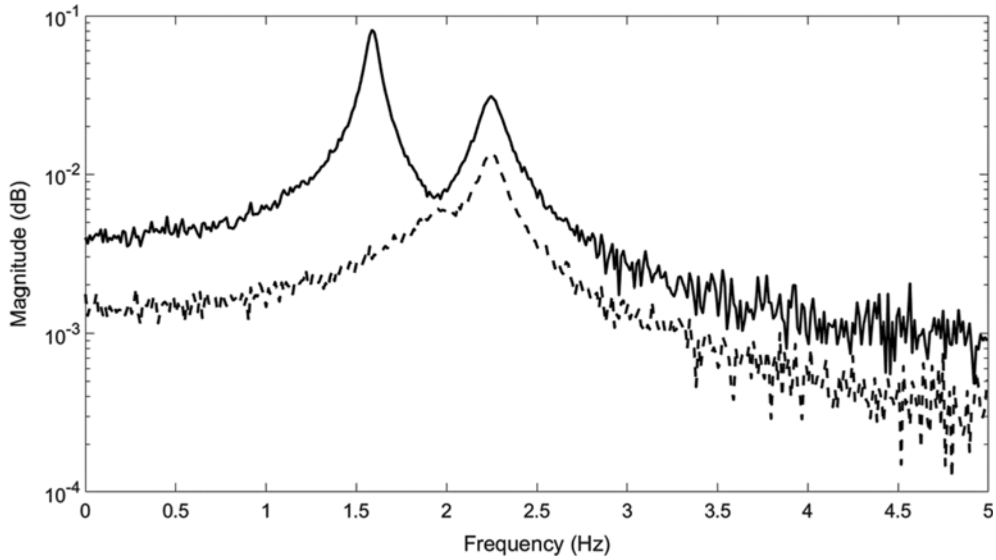

Fig. 3. A CMIF Curve of 3-DOF Structures with Repeated Modes (contaminated with SNR = 20% noise)

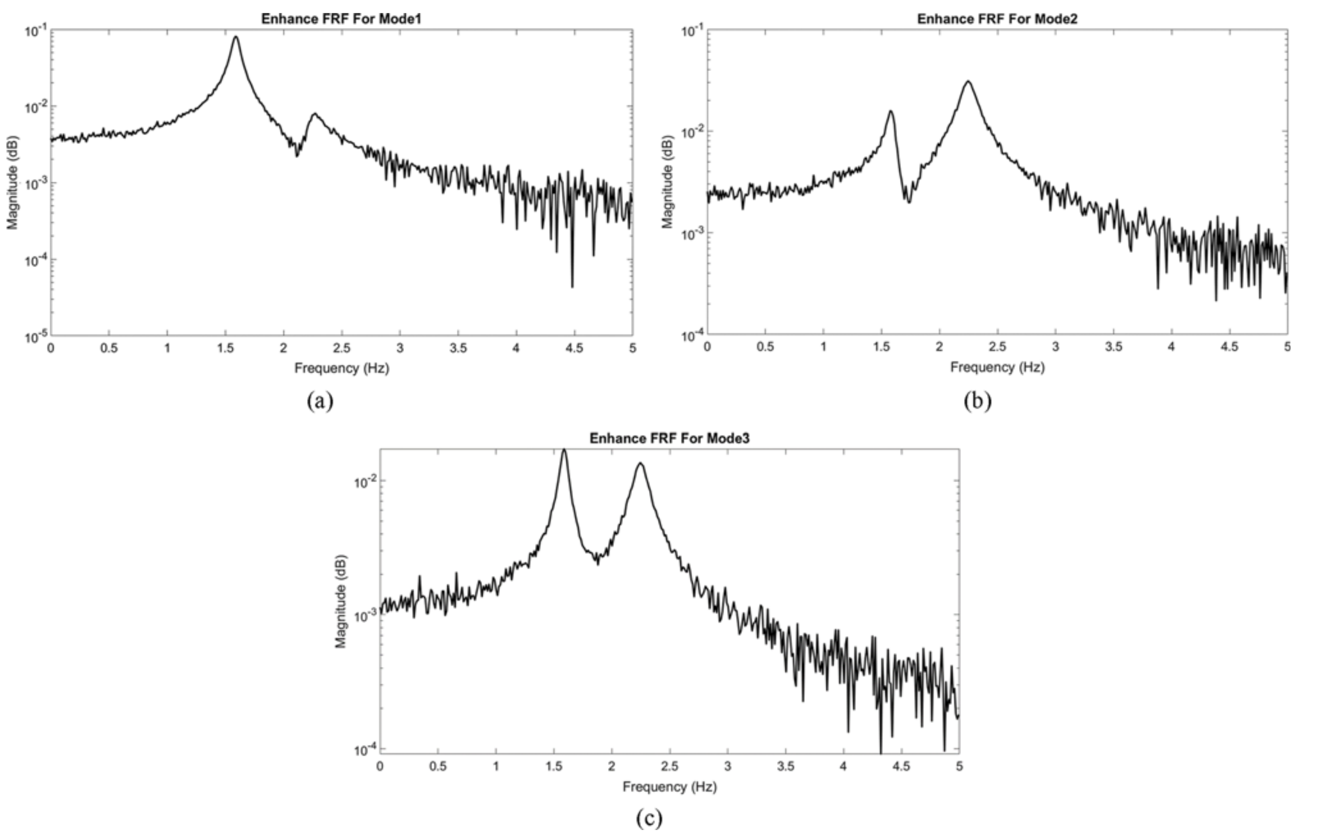

Fig. 4. An Enhanced FRF Plot of Each Mode of 3-DOF Structures with Repeated Modes (contaminated with SNR=20% noise): (a) 1st Mode, (b) 2nd Mode, (3) 3rd Mode

acting on each mass point of the system. In this case, we choose the sampling interval Δt and period T as 0.01 sec and 81.92 sec, respectively. The system is assumed initially at rest, the displacement responses of the system can be obtained using Newmark's method (Newmark, 1959), and the corresponding frequency response function of each degree of freedom of the system can be obtained through FFT. Consider the noise effect in reality, we execute the numerical demonstration from the simulated impulse response data contaminated with 20% noise, and average 10 samples through repeated procedure. Through Shannon's sampling theorem, the cut-off frequency f_c can be evaluated as 50 Hz, and the corresponding frequency resolution $\Delta f = 0.0122$ Hz.

In this example, the 3 × 3 frequency response function matrix corresponds to the linear 3-DOF system. To simulate the case of an effective modal estimation of a system with repeated modes using system model with insufficient order due to the repeated eigenvalues, we perform an effective modal identification from multi-input multi-output FRFs data consisting of the two columns data of FRFs only. We perform an effective modal estimation by using the CMIF method and the SVD analysis, respectively. Under the consideration of the simulated impulse response data contaminated with 20% noise, in the CMIF curve, as shown in Fig. 3, we can obviously see the peak corresponding to 2.5 Hz around of different curves, and estimate the number of identified modes, even existing even when there are two repeated modes in this case. We can thus estimate that there are two modes with the same modal frequency. The plot of Enhanced FRF corresponding to three modes of the structural system is shown in Fig. 4. The identification results through CMIF method and the SVD analysis are shown in Tables 1 and 2, respectively. The identification results of natural frequencies and damping ratios through CMIF method are acceptable. Each estimated mode is less affected by noise and has less variation at the peak of the curve, thus the identification results of natural frequencies and damping ratios through SVD analysis are good. It should be mentioned that the identification of data using SVD analysis can not only omit the step of calculating the enhanced FRF, but also is less computation and more efficient than the CMIF method.

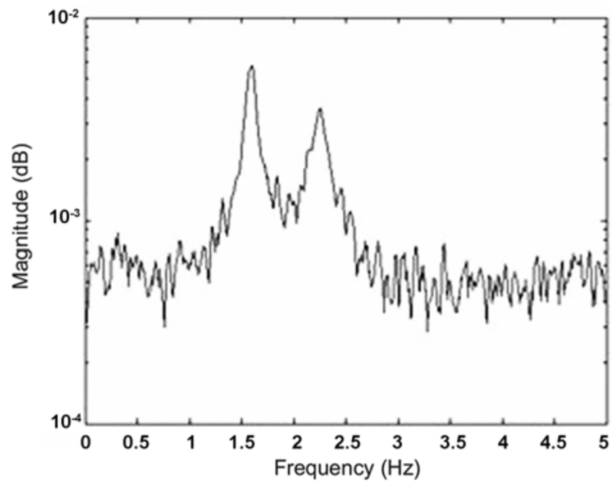

Fig. 5. A CMIF Curve of 3-DOF Structures with Repeated Modes (from overlapped data contmiated with SNR = 50% noise)

Furthermore, to perform modal estimation from the one-column data of frequency response function effectively, we consider the method of expansion matrix composed of the overlapping adjacent frequency response function. The SVD technique is employed to estimate modal parameter from the FRF data of left and right one frequency adjacent interval. Fig. 5 is a plot of the complex modal indication function, in which no repeated modes of the 3-DOF system are able to observe clearly. The results indicate that no modes with repeated eigenvalues can be identified from the data of frequency response function with single-input only. Fig. 6 shows the amplitude of the enhanced FRF of each mode. Tables 3 and 4 shows the identification results from non-overlapped and overlapped data contaminated with 50% noise. When the overlapped data is available to an effective modal estimation, the identification results of mode shape are more

Table 1. Estimation Results of Modal Parameters of 3-DOF Structures with Repeated Modes through the Conventional CMIF Method (contaminated with SNR = 20% noise)

Mode	Natural frequency (Hz)			Damping ratio (%)			MAC
	Exact	CMIF	Error (%)	Exact	CMIF	Error (%)	
1	1.59	1.59	0.17	2.10	2.12	0.94	1.00
2	2.25	2.25	0.19	2.90	2.91	0.51	0.92
3	2.25	2.25	0.22	2.90	3.04	4.80	0.90

Table 2. Estimation Results of Modal Parameters of 3-DOF Structures with Repeated Modes through the Modified CMIF Method (contaminated with SNR = 20% noise)

Mode	Natural frequency (Hz)			Damping ratio (%)			MAC
	Exact	MCMIF	Error (%)	Exact	MCMIF	Error (%)	
1	1.59	1.59	0.14	2.10	2.10	0.14	1.00
2	2.25	2.25	0.19	2.90	2.85	1.74	0.92
3	2.25	2.25	0.22	2.90	2.86	1.42	0.90

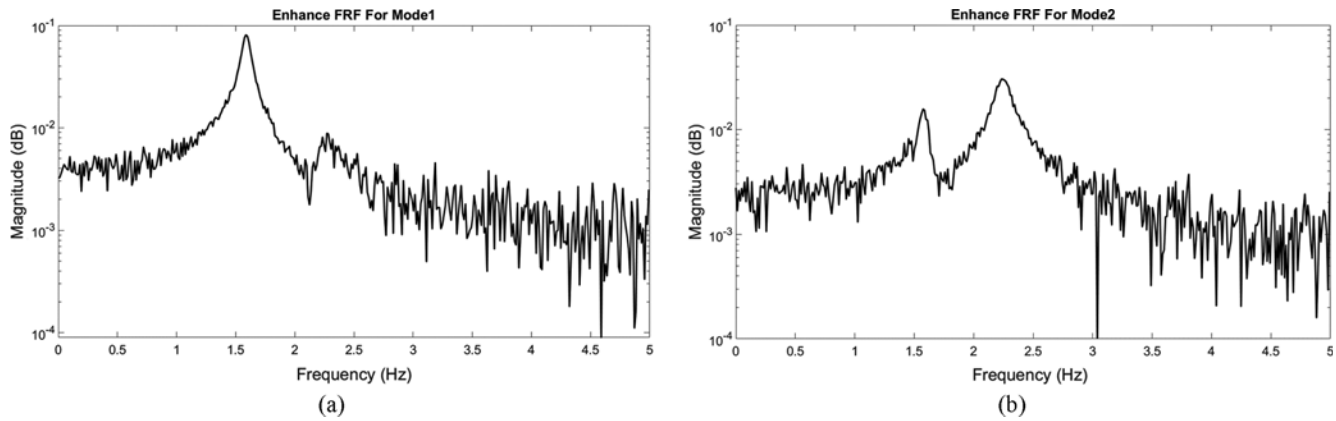

Fig. 6. An Enhanced FRF Plot of Each Mode of 3-DOF Structures with Repeated Modes (from overlapped data contaminated with SNR = 50% noise): (a) 1st Mode, (b) 2nd Mode (3rd mode is the repeated mode as the same as 2nd mode)

Table 3. Estimation Results of Modal Parameters of 3-DOF Structures with Repeated Modes from Simulated SIMO Data (from non-overlapped data, contaminated with SNR = 50% noise) through the Modified CMIF Method

Mode	Natural frequency (Hz)			Damping ratio (%)			MAC
	Exact	MCMIF	Error (%)	Exact	MCMIF	Error (%)	
1	1.59	1.60	0.23	2.10	2.24	6.45	0.99
2	2.25	2.25	0.26	2.90	2.82	2.83	0.98

Table 4. Estimation Results of Modal Parameters of 3-DOF Structures with Repeated Modes from Simulated SIMO Data (from overlapped data, contaminated with SNR = 50% noise) through the Modified CMIF Method

Mode	Natural frequency (Hz)			Damping ratio (%)			MAC
	Exact	MCMIF	Error (%)	Exact	MCMIF	Error (%)	
1	1.59	1.60	0.23	2.10	2.25	6.98	0.99
2	2.25	2.25	0.25	2.90	2.89	0.35	0.99

accurate. For the estimation of damping ratios from overlapped data, the identification of damping ratio of the second mode is more accurately, but the error of the identified damping ratio of the first mode is somewhat large. Note that the longer record is preferred for measurement data contaminated with noise, while the shorter record can be extracted the cleaner measurement data. The data length of measurement records depends on the vibration frequencies of the important or major modes of a system under consideration. A structure having the relatively low frequency requires relatively long measurement record to capture the sufficient number of cycles than that having the relatively high frequency. In addition, in our experience obtained from numerical simulation, we found that when using the expansion matrix composed of the frequency response function matrix of adjacent frequencies for identification, it can be overcome the problem that the accuracy of the identification result may be affected when the FRF data of a single frequency has a large error. However, it is also possible that the error caused by the data of the adjacent frequency is greater than the extent of improvement, and then identification results are less accurate.

To assess the effectiveness of the proposed modal estimation technique through experimental results from relatively realistic structures, we considered to supplement an experimental validation using an actual plate example. Modal testing is a form of vibration testing to estimate modal parameters of a structure. The procedure of modal testing contains acquisition and analysis phases. The common application of modal testing has been extensively to identification of dynamic characteristic and detection of structural damage. Depending on the source of excitation, the use of modal testing can be divided into experimental modal testing (EMA) or impact hammer testing and OMA or ambient vibration testing. A condition of impact hammer testing, including data acquisition system of Brüel & Kjær RT Pro Photon 7.0 with four input channels and one output channel used to transfer measurement signals data into the portable computer, and the corresponding cables with BNC connector, as shown in Fig. 7, is conducted a plate suspended by simple strings under a free-free boundary condition. The data acquisition system of Brüel & Kjær RT Pro Photon 7.0 is employed to extract, respectively, the excitation data induced from a PCB impulse hammer 086C01 with 444 N pk measurement range and 11.2 mV/N sensitivity impacting on the structure as well as the response data obtained from PCB piezoelectric accelerometers 352B10 with 10 kHz frequency range and 10.3 mV/g sensitivity.

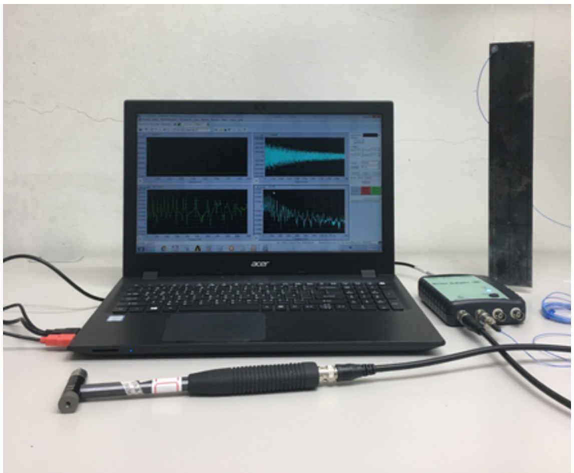

Fig. 7. Experiment Set Up of Modal Testing for a Rectangular SS400 Steel Plate

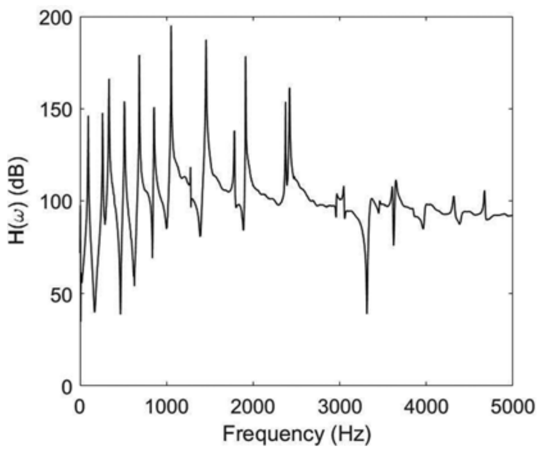

Fig. 8. The Amplitude Frequency Response Function $H_{15}(\omega)$ of the System Having at Least Three Pairs of Severe Closely Spaced Modes around 2,400 Hz, 3,000 Hz, and 3,500 Hz

The experimental testing consisting of a rectangular SS400 steel plate with the dimensions of 335 mm × 57 mm is employed to study in EMA. The sizes of thickness and sides of the plate are, respectively, about 2 mm and slightly different from 0.03 mm to 0.11 mm. Since the rectangular plate is a structure with axisymmetric model, the plate has closely spaced (even probably repeated) modes. To validate the identified modal parameters, initial finite element (FE) analysis of the testing steel plate is implemented. We estimate natural parameters of the rectangular steel plate with a simplified finite element model developed using ANSYS commercial software. The estimation results of natural frequencies show that the structure has at least three pairs of closely spaced (possibly even repeated) patterns of 2,400 Hz, 3,000 Hz, and 3,500 Hz, respectively, as shown in Fig. 8.

An impulse excitation impacts on the fifteen points marked on the plate, as shown in Fig. 9, and then employed to act the structure with a steel tip to carry out modal testing. During the impact hammer testing, a frequency span is chosen as 12,800 Hz, and making one measurement data block equals to 1.0 s, and the

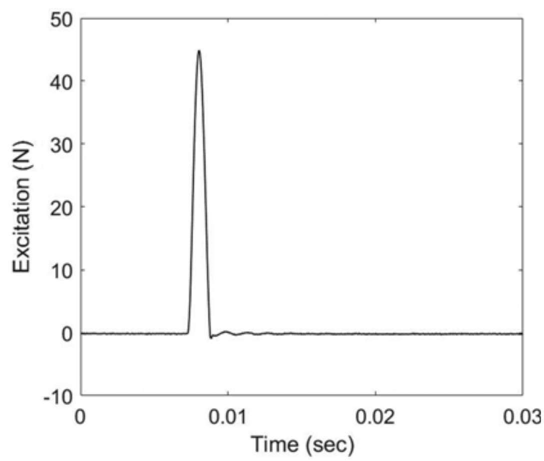

Fig. 9. Typical Impulse Excitation from an Impulse Hammer Acted through the Locations of the Plate

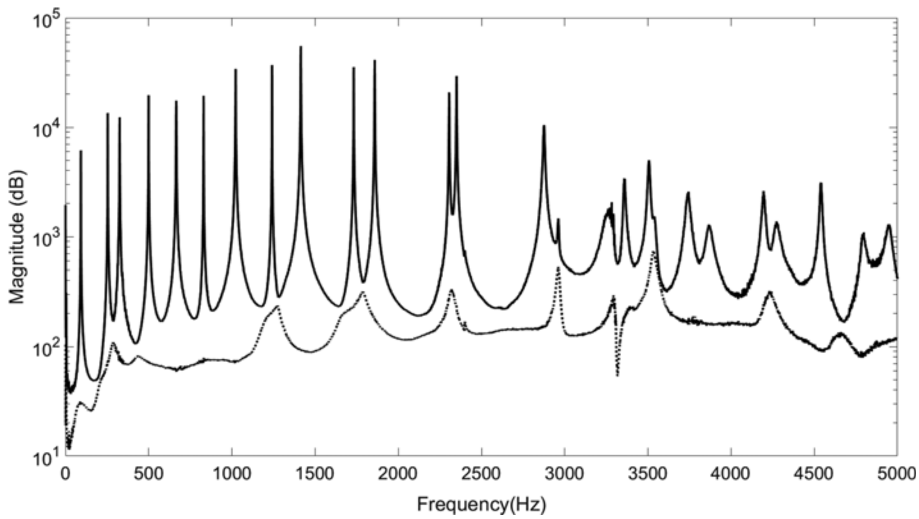

Fig. 10. Typical CMIF Curve of the Actual Plate with Three Pairs of Severe Closely Spaced Modes around 2,400 Hz, 3,000 Hz, and 3,500 Hz

Table 5. Estimation Results of Modal Parameters of a Practical Plate Compared through ME'Scope Commercial Software with the Modified CMIF Method

Mode	Natural frequency (Hz)			Damping ratio (%)			MAC
	EMA	MCMIF	Error (%)	EMA	MCMIF	Error (%)	
1	94.80	95.03	0.24	0.05	0.04	20.00	0.80
2	261.00	261.21	0.08	0.08	0.10	25.00	0.67
3	335.00	335.19	0.06	0.12	0.11	8.33	0.69
4	514.00	514.57	0.11	0.06	0.08	33.33	0.76
5	685.00	685.23	0.03	0.08	0.10	25.00	0.84
6	854.00	854.28	0.03	0.03	0.04	33.33	0.44
7	1050.00	1054.04	0.38	0.06	0.08	33.33	0.87
8	1280.00	1277.44	0.20	0.02	0.02	12.50	0.63
9	1450.00	1452.00	0.14	0.04	0.06	37.50	0.70
10	1780.00	1786.09	0.34	0.03	0.02	33.33	0.59
11	1910.00	1920.25	0.54	0.03	0.04	43.33	0.58
12	2370.00	2381.39	0.48	0.04	0.06	37.50	0.81
13	2420.00	2425.82	0.24	0.04	0.03	25.00	0.90
14	2960.00	2960.32	0.01	0.15	0.14	6.67	0.80
15	3050.00	2999.24	1.66	0.13	0.09	30.77	0.62
16	3450.00	3456.32	0.18	0.22	0.13	40.91	0.86
17	3610.00	3607.80	0.06	0.18	0.11	38.89	0.84
18	3850.00	3849.11	0.02	0.33	0.27	18.18	0.30
19	4320.00	4319.42	0.01	0.22	0.29	31.82	0.75
20	4390.00	4403.03	0.30	0.34	0.48	41.18	0.52

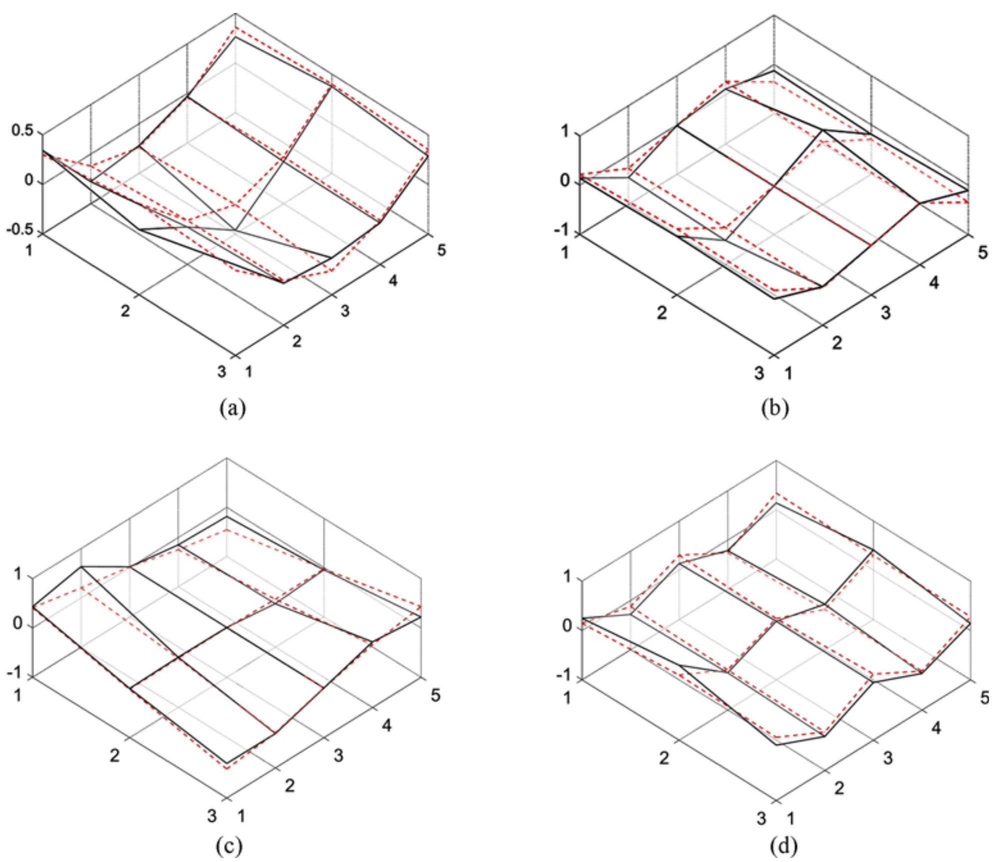

Fig. 11. Estimation Results of the Mode Shapes of a Realistic Plate Subjected to an Impulse Input through ME'Scope Commercial Software (dotted line) with the Modified CMIF Method (solid line): (a) 1st Mode, (b) 2nd Mode, (c) 3rd Mode, (d) 4th Mode

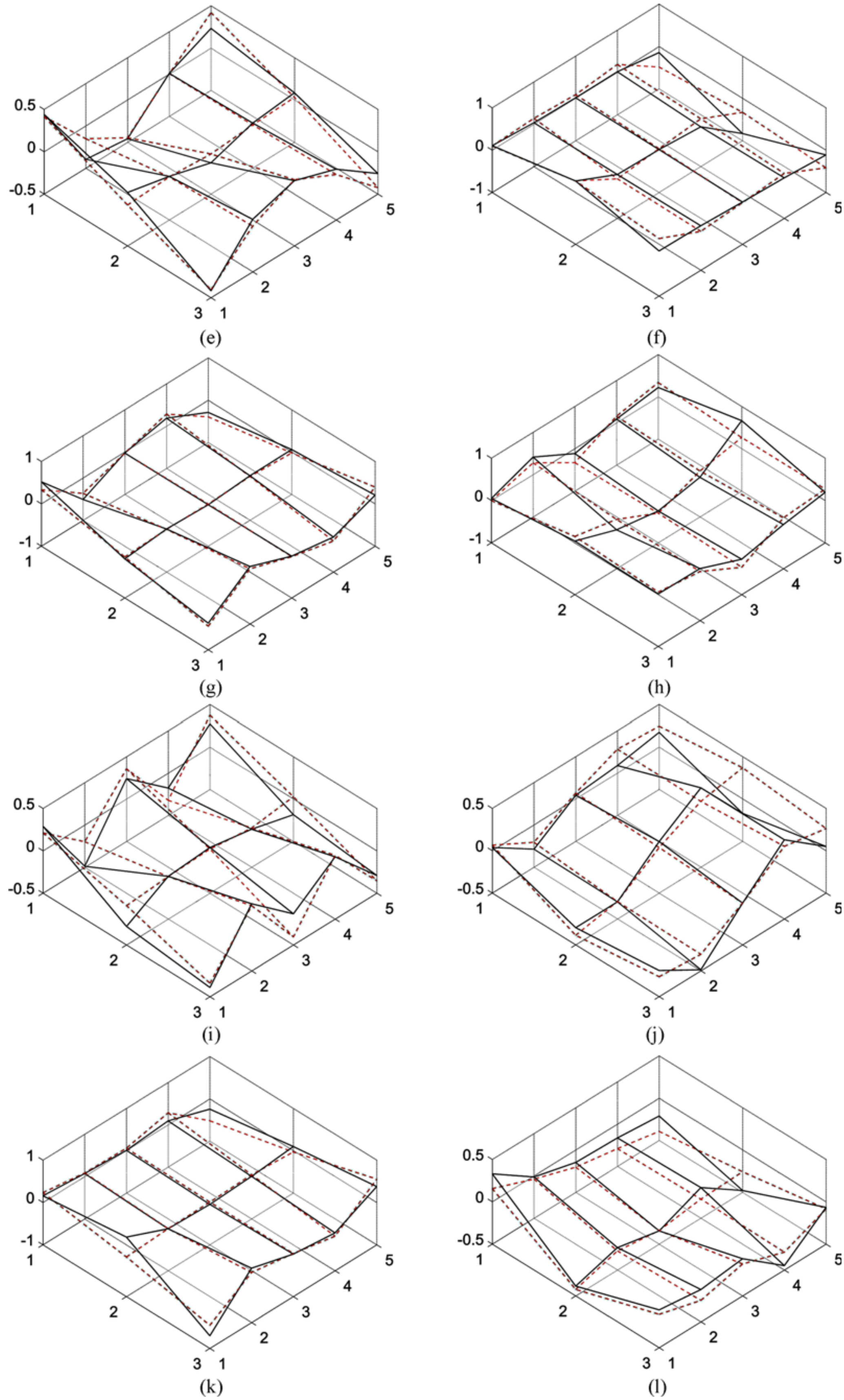

Fig. 11. (continued): (e) 5th Mode, (f) 6th Mode, (g) 7th Mode, (h) 8th Mode, (i) 9th Mode, (j) 10th Mode, (k) 11th Mode, (l) 12th Mode

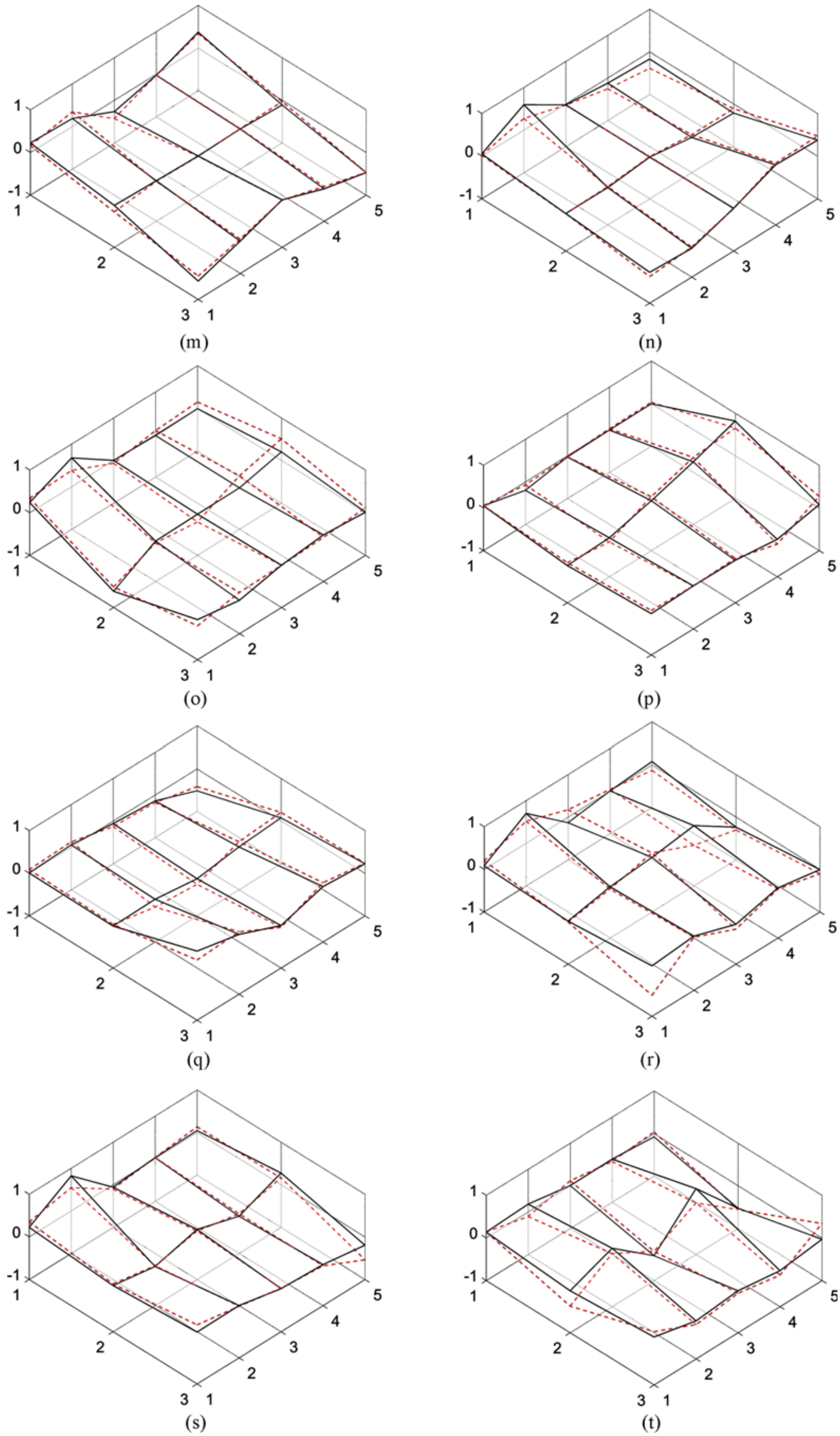

Fig. 11. (continued): (m) 13th Mode, (n) 14th Mode, (o) 15th Mode, (p) 16th Mode, (q) 17th Mode, (r) 18th Mode, (s) 19th Mode, (t) 20th Mode

20 data samples are then extracted from 225 measured acceleration responses. The number of significant modes of the rectangular SS400 steel plate can be estimated from the 20 extracted data samples through the modified CMIF method, as shown in Fig. 10. Then, without additionally evaluating enhanced frequency response functions, the corresponding modal parameters of rectangular SS400 steel plate can then be identified directly through SVD, as listed in Table 5. Note that the modal frequencies and damping ratios listed in Table 5, as well as the mode shapes shown in Fig. 11, obtained from the measured impulse response of the practical steel plate structure by using half power (bandwidth) method and curve-fitting technique of ME'Scope commercial software are actually used as the "exact" reference. The identification results of the natural frequencies of the corresponding severe closely spaced modes of a rectangular steel plate are reasonable through the modified CMIF method. However, the error of identification of damping ratios is somewhat higher but acceptable because the sensitivity of structural response to damping ratios is less than to natural frequencies. A good agreement is observed from the estimation results of the mode shapes of a practical steel plate subjected to an impulse input through ME'Scope commercial software and the modified CMIF method, as shown in Fig. 11. The verification results of Modal Assurance Criterion (MAC) signifying the consistency between the mode shapes identified through the modified CMIF method and EMA are also listed in Table 5, which indicates that the mode shapes of the corresponding severe closely spaced modes of a rectangular steel plate are consistent with those estimated by impact hammer testing.

6. Conclusions

In this paper, based on the theory of modal analysis, we perform the parametric identification of a system with severe modal interference by the multi-input frequency response function. Except for employing CMIF method to estimate the number of significant modes of a system with severe modal interference, we also use SVD algorithm to identify major modes of a structural system without additionally evaluating enhanced frequency response function. Through numerical simulation and the experimental validation of a rectangular SS400 steel plate, some conclusions of this study are presented as follows:

1. A CMIF curve can be employed to effectively estimate the number of modes to be identified, especially for the case of a system with close modes (even repeated modes), and further can be merge with the enhanced frequency response function as MIMO identification method to perform an effective modal estimation. Through the enhanced frequency response function, the response of the corresponding mode to be identified can be increased, and the interference induced from other modes and noise distortion can be reduced. Therefore, the robustness of CMIF method is better than SVD method.
2. In the procedure of CMIF method, we use expansion matrix consisting of frequency response function matrix associated with the proximity frequencies to implement system identification. It will be overcome that the data error from the single modes will be reduced, but the identification results may not be good because the errors induced from the proximity frequencies are probability larger than improvement extent of the aforementioned procedure.
3. The singular value data obtained from SVD method can be employed to directly implement system identification and effectively estimate modal parameters of structures under moderately noisy conditions because the corresponding peaks do not vary obvious in CMIF curve. Furthermore, it is done by SVD method without the calculation of enhanced frequency response function, so the computation will be reduced.
4. For a system with severe closely spaced modes, a column of FRF matrix can be expressed as a linear combination of mode-shape vector of each mode when using a single column FRF data, and the repeated modes cannot be determined. The different linear combination of the corresponding mode-shape vector of the repeated mode is not, however, proportional to each other, the repeated modes can thus be determined, and further extract the other repeated mode shapes.
5. According to the modal-estimation results from numerical simulation and the experimental testing of a rectangular steel plate, even if some modes may be very close (even probably repeated) in natural frequency, but with different (or similar) mode shapes, the proposed method can be effectively applicable to modal estimation.

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