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Shear Strength of Rock Joints and Its Estimation

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ABSTRACT

This study examined the shear behavior of rock joints and proposed an analytical model for estimating the shear strength at rock joints. Numerous experimental tests indicated that the shear behavior at rock joint depends on many factors including the distribution of initial joint asperity, asperity angle and strength, applied normal stress, and progressive degradation of asperity. Nevertheless, most pre-existing strength models do not consider these features on shearing at rock joints enough. In this study an improved shear strength model, which can consider the complex joint shearing features observed from many experimental tests, was developed and proposed. The improved model considers the joint features more reliably and overcomes the limitations of the pre-existing strength models. The proposed model was compared with some experimental test results along with some pre-existing strength models. The comparison clearly indicated that the improved model can estimate the shear strength at rock joints more reliably than the pre-existing models. From the study results it is expected that a better estimate of joint shear strength could be obtained in the future for various rock, joint, stress, and shearing conditions.

1. Introduction

The shear strength of rock joints plays the most important role when conducting stability analysis of slope, foundation, tunnel, and many other works in rock mass. Therefore, it is very important to understand the mechanism of the shear strength development at a rock joint and provide a reliable method to estimate the shear strength at joints. In general, the shear response at a rock joint shows very complex phenomena depending on rock properties, joint conditions, and in-situ stress conditions. When studying the shear strength of rock joints, the compressive strength of rock at the joint is one of most important factors to be considered. In addition, the geometrical condition of the joint affects the shear strength considerably and it should be taken into account carefully. Most important geometrical factors include the layout and inclination angle of joint asperities and their geometrical change during joint shearing. Though the geometrical characteristics of joint considerably influence the mechanical behavior of joint shear strength, they are very difficult to define in a simple way. In addition, the rock strength and in-situ stress conditions vary depending on the formation of rock, weathering, location, and so on.

For estimating the shear strength at rock joints many remarkable efforts have been made by numerous researchers (Patton, 1966; Goldstein et al., 1966; Ladanyi and Archambault, 1970; Jaeger, 1971; Barton, 1973; Schneider, 1976; Barton and Choubey, 1977; Barton and Bandis, 1982; Plesha, 1987; Johnston and Lam, 1989; Jing, 1990; Saeb, 1990; Maksimovic, 1996; Haberfield and Johnston, 1994; Kulatilake et al., 1995; Zhao, 1997a, 1997b; Grasselli, 2001; Grasselli and Egger, 2003; Belem et al., 2004; Oh, 2005; Jiang et al., 2006; Kulatilake et al., 2006; Kim and Lee, 2007; Ghazvinian et al., 2010; Lee et al., 2014; Zhang et al., 2016). Regardless of many significant contributions to the development of joint shear strength models, the importance of joint shear strength is too great and a better and more understanding is continuously required.

This study reviews pre-existing joint shear strength models, which are widely known, discusses the limitations and problems of the models, and provides an improved shear strength model. The improve strength model is compared with the results of experimental tests along with some pre-existing strength models.

2. Pre-Existing Shear Strength Models at Rock Joints

The shear strength models at rock joints which have usually

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nonplanar surface were developed based on the observed dilatant behavior of granular soils. A model of shear strength of granular soils, which considers the influence of dilatancy, was developed by Newland and Allely (1957) and Row et al. (1964) as follows:

$$\tau = \sigma_n \cdot \tan(i + \phi_u), \quad (1)$$

where τ = the maximum shear strength of a granular soil, σ_n = the applied effective normal stress, i = the average angle of deviation of particle displacements from the direction of the applied shear stress, ϕ_u = the angle of frictional sliding resistance between particles.

By assimilating the overall sliding surface to a plane containing inter-locked sawtooth irregularities, Patton (1966) developed the bilinear model for the joint shear strength even though he observed that failure envelopes with irregular surfaces are curved with the normal stress. He also described that "Failure envelopes for rocks would not reflect a simple change in the mode of failure but changes in the intensities of different modes of failure occurring simultaneously" (Patton, 1966, p. 513). In the bilinear model, the shear strength is assessed at low and high normal stresses differently (Fig. 1).

At a low normal stress, where the asperities are not sheared off, the shear strength is assessed as follows:

$$\tau = \sigma_n \cdot \tan(i + \phi_u). \quad (2)$$

At a high normal stresses, where the joint asperities are practically sheared off, the Mohr-Coulomb shear strength criterion is used as follows:

$$\tau = c_j + \sigma_n \cdot \tan(i + \phi_r), \quad (3)$$

where σ_n = the applied effective normal stress, ϕ_u = the angle of frictional sliding resistance, i = the inclination of teeth, c_j = the apparent joint cohesion, and ϕ_r = the residual friction angle.

Patton's bilinear model practically represents the shear strength of regularly spaced joint asperities and does not consider the gradual degradation of the joint asperities during shearing.

In general, when shearing takes place on an inclined joint asperity, there is a tendency to dilate toward the normal direction perpendicular to the direction of shear displacement and it decreases gradually with the increasing of normal stress. Ladanyi and Archambault (1970) developed a more general shear strength model which considers the change of dilatancy with the normal stress. The model was basically developed based on the stress dilatancy theory of sand and energy considerations (Rowe et al., 1964). By experimental tests, they examined the change of dilatancy and shearing-off area in terms of the ratio of applied normal stress, σ_n to the transition stress from brittle to ductile behavior, σ_T and considered the observations into their model as follows:

$$\tau = \frac{\sigma_n \cdot \left(1-\frac{\sigma_n}{\sigma_T}\right)^{1.5} \left[\left(1-\frac{\sigma_n}{\sigma_T}\right)^{4} \cdot \tan i + \tan\phi_u\right] + \left[1-\left(1-\frac{\sigma_n}{\sigma_T}\right)^{1.5}\right] \cdot (\sigma_n \cdot \tan\phi_o + s_o)}{1-\left(1-\frac{\sigma_n}{\sigma_T}\right)^{1.5} \cdot \left(1-\frac{\sigma_n}{\sigma_T}\right)^{4} \cdot \tan i \cdot \tan\phi_f}, \quad (4)$$

where σ_n = the applied effective normal stress, σ_T = the transition stress from brittle to ductile behavior, i = the inclination of teeth, ϕ_u = the angle of frictional sliding resistance, ϕ_o = the residual friction angle, ϕ_f = when sliding occurs along the irregularities of different orientations and for initially tightly interlocked rock surfaces it is similar to ϕ_u, S_o = the cohesive strength of the asperity.

They also considered the effect of a previous decrease in the degree of interlocking and provided the following strength model replacing σ_n by σ_n/η where η is called as the degree of interlocking and defined as shown in Fig. 2.

$$\tau = \frac{\sigma_n \cdot \left(1-\frac{\sigma_n}{\eta\sigma_T}\right)^{1.5} \cdot \left[\left(1-\frac{\sigma_n}{\eta\sigma_T}\right)^{4} \cdot \tan i + \tan\phi_u\right] + \left[1-\left(1-\frac{\sigma_n}{\eta\sigma_T}\right)^{1.5}\right] \cdot (\sigma_n \cdot \tan\phi_o + s_o \cdot \eta)}{1-\left(1-\frac{\sigma_n}{\eta\sigma_T}\right)^{1.5} \cdot \left(1-\frac{\sigma_n}{\eta\sigma_T}\right)^{4} \cdot \tan i \cdot \tan\phi_f}, \quad (5)$$

where Δx = the shear displacement, ΔL = the projected length of the ascending portion of the irregularities in shear direction.

In the Ladanyi and Archambault's model and other pre-existing models, the inclination of joint teeth is the average of the inclination angles of the first order asperities.

The model by Ladanyi and Archambault (1970) was modified by Saeb (1990) to provide theoretically more rigorous solution

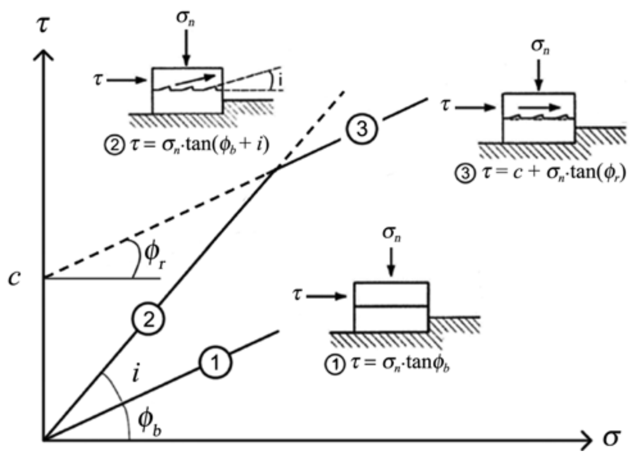

Fig. 1. Bilinear Failure Envelope for Multiple Inclined Surfaces (Patton, 1966)

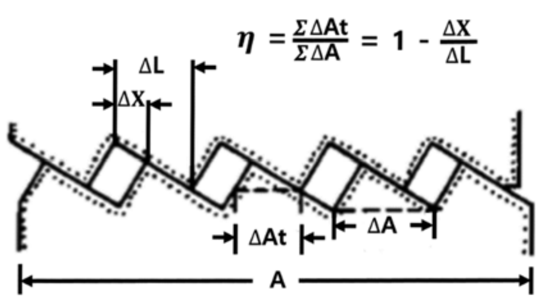

Fig. 2. Definition of the Degree of Interlocking, η

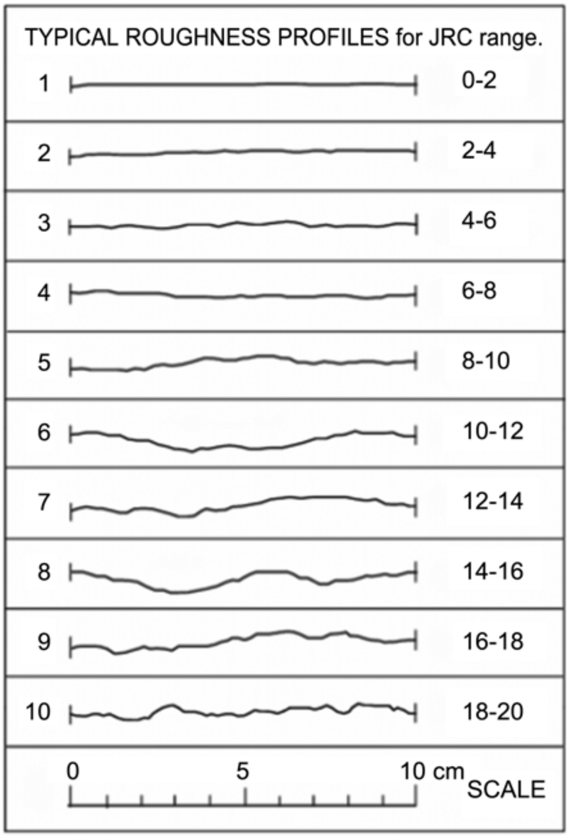

Fig. 3. Standard Profiles Used for Visual Estimation of JRC (Barton and Choubey, 1977)

and a simpler expression. The modified model is as follows:

$$\tau = \sigma_n \cdot \left(1 - \frac{\sigma_n}{\sigma_T}\right)^{1.5} \cdot \tan\left[\tan^{-1}\left(\left(1 - \frac{\sigma_n}{\sigma_T}\right)^4 \cdot \tan i\right) + \phi_u\right] + \left[1 - \left(1 - \frac{\sigma_n}{\sigma_T}\right)^{1.5}\right] \cdot (\sigma_n \cdot \tan\phi_o + s_o). \quad (6)$$

Barton (1973, 1976) developed a highly empirical shear strength model as shown below at relatively low normal stresses based on experimental tests and collected data analysis. The model considers the joint roughness coefficient (JRC), which represents the condition of joint surface and it varies from 0 for very smooth joint surface to 20 for very rough joint surface. The JRC values are shown in Fig. 3.

$$\tau = \sigma_n \cdot \tan\left[\mathrm{JRC} \cdot \log_{10}\left(\frac{\sigma_c}{\sigma_n}\right) + \phi_b\right], \quad (7)$$

where JRC = the joint roughness coefficient, σ_c = the uniaxial compressive strength at joint, ϕ_b = the basic friction angle.

Barton's original experiments were carried out at relatively low normal stresses which ranged between 0.001 and 0.3 in the value of σ_n/σ_c. In addition, the logarithmic term in Barton's model is logically not admissible because the shear strength becomes very high as the applied normal stress (σ_n) decreases to zero. Barton suggested that the maximum angle for the total friction should be 70º. Besides, at high normal stress conditions where most asperities are sheared off and the expected strength is not dependent on the joint surface conditions, the shear strength estimated by the model is not reasonable. Among others, the JRC value is difficult to quantify and varies by person. Nevertheless, many attempts (Tse and Cruden, 1979; Reeves, 1985; Lee et al., 2006) have been made to quantify the JRC value more accurately. However, the attempted methods themselves have some problems including they do not provide any information of the dependence of shearing direction in spite of that the joint shear strength is very influenced by the direction of shearing (Huang and Doong, 1990; Jing et al., 1992). In other words, to date there is no general agreement on the quantification of JRC.

Zhao (1997a, 1997b) introduced the joint-matching coefficient (JMC), modifying the Barton's model as follows:

$$\tau = \sigma_n \cdot \tan\left[\mathrm{JMC} \cdot \mathrm{JRC} \cdot \log_{10}\left(\frac{\sigma_c}{\sigma_n}\right) + \phi_b\right], \quad (8)$$

where JMC = the joint matching coefficient, JRC = the joint roughness coefficient, σ_c = the uniaxial compressive strength at joint, ϕ_b = the basic friction angle. The JMC is defined the approximate percentage area in contact between the upper and lower walls of the joint (Fig. 4) and has a value between 0 and 1. However, the model has the same limitations and problems as the Barton's model and the JMC value is also difficult to quantify.

Including the aforementioned shear strength models, many others have been proposed over the last several decades. However, it is still difficult to determine the shear strength at a joint due to complex joint condition and shearing process.

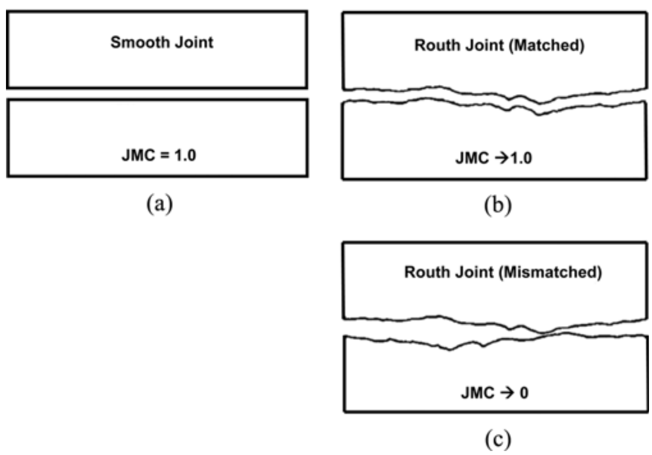

Fig. 4. Examples of Joint Matching Coefficient (JMC) (Zhao, 1997a): (a) Smooth Joint, (b) Rough Joint (Mached), (c) Rough Joint (Mismatched)

3. Development of a Shear Strength Model at Rock Joints

While the understanding of the shear strength at a rock joint has been improved by the many efforts, the limitations and problems of the pre-existing models should also be recognized. Most pre-existing shear strength models consider the entire joint surface to be involved simultaneously for joint sliding and shearing interlocking. In many cases, however, some portion of a joint surface shows little or no interaction of joint asperities from the beginning of shear so that only the residual shear strength could be mobilized at the portion. In addition, the pre-existing models considers the inclination of joint asperity, i as the average value of the inclination angles of the first-order joint asperities. However, the shearing on a joint surface starts from the steepest joint asperity and after some degradation of the asperity, it continuously proceeds to the next steepest asperity. Fig. 5 shows various joint asperities which can be encountered in the field.

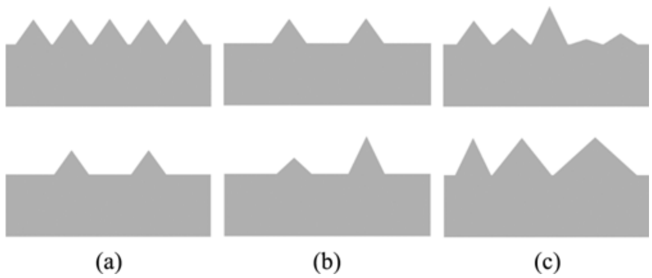

Fig. 5. Schematic View of Various Joint Asperities: (a) Different Joint Asperity Area, (b) Different Asperity Inclination Angle, (c) Different Asperity Configuration

In attempt to overcome the foregoing limitations and problems of the pre-existing strength models and to represent the joint behavior more reasonably, an improved shear strength model, which was to be described afterwards, was developed to provide a better estimate of joint shear strength in various rock, joint, stress, and shearing conditions. The developed model considered the joint shear features observed from various experimental tests by Patton (1966), Ladanyi and Archambault (1970), Barton (1973), Jing (1990), Grasselli (2001), and Asadi and Rasouli (2012). The observations from several experimental tests were very valuable for the development of new strength model and therefore they are described here briefly.

From the results of controlled experimental tests, Patton (1966) showed that the shear strength at very low normal stress is predominantly influenced by the inclination angle of the asperity but as the stress increases the cohesive effect of rock substance increases whereas the effect of the asperity inclination decreases showing a curved failure envelope which reflects changes in the mode of failure. Patton (1966) concluded that "Changes in the mode of failure are related to the physical properties of the irregularities along the failure surface" (p. 512). When the normal stress reached to the joint compressive strength the effect of asperity inclination disappears and the cohesive effect is fully mobilized. Comparing two different number of teeth (2 and 4) cases with keeping other respects identical, Patton observed that the shear strength of 4 teeth case after shearing off of the teeth was about twice as far above the residual strength as the strength of 2 teeth. From the comparison of another test set where the internal strength was only different, he observed that "Increasing the strength of the specimen teeth has an effect similar to that of increasing the number of teeth" (Patton, 1966, p. 512). Additionally, he pointed out that the internal friction angle, not the inclination of the joint asperity, does not change much throughout a wide range of normal stresses. Ladanyi and Archambault (1970) from their experimental tests observed that the area of shearing off of joint asperities is dependent on both joint compressive strength and applied normal stress. Their proposed model implied that the shearing off decreases the dilation of joint asperities but at the same time mobilizes the internal strength of the shearing substance of the asperities. Barton (1973, 1976) developed an empirical shear strength model at relatively low normal stresses based on experimental tests and collected data analysis. Even though Barton (1973) reported that the dilation angle along a joint is related to a logarithmic term of joint compressive strength and applied normal stress, revisiting of the test data indicated that the relation between the dilation angle and the ratio of the applied normal stress to the compressive strength of asperity (σ_n/σ_c) could be a polynomial type function rather than a logarithmic function. Jing (1990) also suggested a polynomial type function to model the degradation of dilatancy angle according to his experimental data. Grasselli (2001) from experimental works indicated several noticeable observations as follows. The geometry of joint had a significant effect on the stress distribution and shearing process across the joint surface and the damaged areas depended on the degree of stress and shear direction. In addition, the joint areas facing the shear direction with the steepest inclination controlled the joint damage behavior and the shape of damage zones depended on the characteristics of the joint asperities as well as the mechanical properties of the rock. The damage induced by shearing resulted in a more uniform geometry condition. Asadi and Rasouli (2012) investigated the effects of joint surface roughness on asperity contact degradation of synthetic and real rock samples. They observed that the significant damage occurs at the steepest asperities and the asperity degradation increases significantly by increasing the normal stress.

However, most pre-existing strength models do not fully consider these observed shearing features at rock joints, so it is worth developing a shear strength model that can more reasonably reflect complex shearing features.

In this study, an improved shear strength model was developed to overcome the aforementioned limitations and problems of existing strength models, to account for the anticipated coincidence of complex shearing processes, and to represent more rational joint behavior, and ultimately to provide a better estimate of joint shear strength in various rock, joint, stress, and shearing conditions. The model was developed by considering, analyzing, and comparing the joint shearing features and results observed from various experimental tests by Patton (1966), Ladanyi and Archambault (1970), Barton (1973), Jing (1990), Grasselli (2001), and Asadi and Rasouli (2012).

The following model is proposed as an improved model of Ladanyi and Archambault (1970) model.

$$\tau = \sigma_n \cdot f^{1.5} \cdot \tan(i_s \cdot f^2 + \phi_u) + (1 - f^{1.5}) \cdot (r \cdot c + \sigma_n \cdot \tan\phi_r), \quad (9)$$

where τ = the peak shear strength, σ_n = the applied effective normal stress, f = the joint degradation resistance factor, $1 - \frac{\sigma_n}{r \cdot \sigma_c}$, ($\sigma_c$ = the uniaxial compressive strength), r = the ratio of the projected area of the interlocking asperity (A_a) to total shear area (A), A_a/A, which considers the induced stress concentration, i_s = the steepest first order asperity inclination angle, ϕ_u = the angle of frictional sliding resistance, c = the cohesive strength of asperity, and ϕ_r = the residual friction angle.

The improved model considers that the shearing and degradation is started at the steepest asperity and once the angle of the degraded asperity is the same as the second steepest asperity the shearing is mobilized along both asperities together. This process is repeated until all the asperities are sheared off (Fig. 6). In addition, the model considers the fact that the shearing off decreases the tendency of dilation at a joint asperity but at the same time mobilizes the internal strength of the shearing substance of the joint asperity. Furthermore, the model also considers the stress concentration and redistribution (Fig. 7) due to the initial condition of asperity distribution and the pre-shear condition along a joint.

Grasselli (2001) indicated that the constant normal load (CNL) and constant normal stiffness (CNS) tests have the same behavior up to a peak shear stress and therefore, the proposed model in this study could be valid for both boundary conditions up to a peak stress. The scale of model test can affect the result of joint shear strength. Nevertheless, Ueng et al. (2010) from their controlled joint shear strength tests reported that there is little scale effect on the peak shear strength of the joint specimen whose surface geometry is enlarged or reduced proportionally in both length and height. In other words, the proposed model could be used for both model and prototype scales as far as the proportionality in both length and height is kept constant.

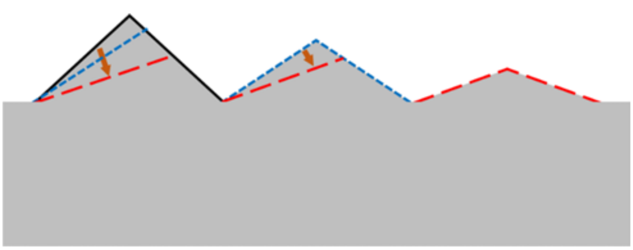

Fig. 6. Schematic View of Progressive Asperity Degradation

4. Comparison between the Proposed Model and Pre-Existing Models

The proposed shear strength model was compared with the results of experimental tests (Patton, 1966) along with two mostly used pre-existing models. The material properties for the model comparison were from the experimental test results (Table 1). The JRC value in the Barton's model was assumed to be 20 for the asperity angle.

Figures 8 and 9 show the comparison with Patton's experimental test results for varying the number of teeth and the teeth strength, respectively. All the teeth had a height of 5 mm and the inclination angle (i) of the teeth with respect to the direction of application of the shearing force was 45°. Plaster of Paris and Kaolinite were used for the test specimens. Fig. 8 shows the effect of teeth number keeping other respects identical. The steep initial portions of the failure envelopes except for Barton's model are approximately $\phi_u + i$ (teeth inclination angle) and the second gentle slope of the failure envelops are approximately ϕ_r. As indicated before the logarithmic term in Barton's model caused the shear strength very high at the low normal stress (σ_n). The doubling the teeth number increased the shear strength approximately twice as far above the residual strength as the failure envelope of the case of two teeth. Comparing the different strength models, the proposed model showed the effect of teeth numbers more reasonably. The model provided a very good agreement with the test results for both peak and residual strengths of the two different teeth number cases. The effect of teeth number can be considered in Ladanyi & Archambault model, but Barton's model cannot consider it.

Figure 9 shows the test results on two series of specimens with all the respects identical but the internal strength. The material properties for the model comparison were from the experimental test results (Table 1). The specimen with more plaster showed a higher internal strength and therefore a higher peak shear strength. For the stronger specimen, the change from the steeper portion of the failure envelope to the gentler portion occurred at a higher normal stress. The increase of material strength increased the shear strength of joint. The proposed

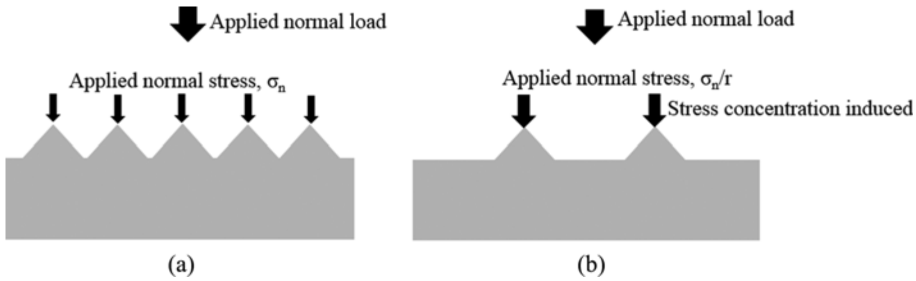

Fig. 7. Schematic View of Stress Concentration: (a) Uniform Stress Condition, (b) Stress Concentration Condition

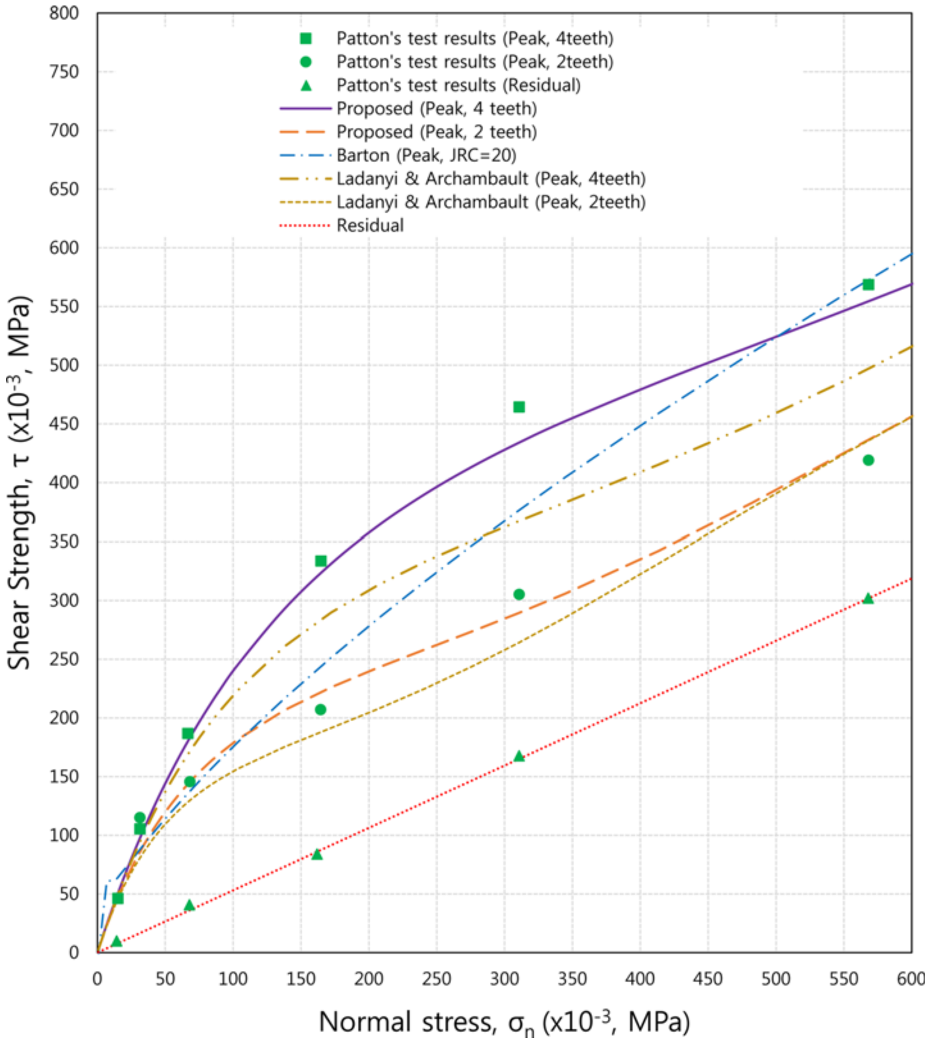

Fig. 8. Comparison with Patton's Experimental Test Results [varying the number of teeth, Kaolinite : Plaster (1:1)]

Table 1. Properties for Test Specimens

Cases	σ_c (MPa)	c (MPa)	σ_t (MPa)	ϕ_u (°)	ϕ_r (°)	i (°)
Kaolinite : Plaster (1:1)	4.14	0.9	0.48	28	28	45
Kaolinite : Plaster (1:2)	6.81	1.38	0.62	30	28	45

σ_c : Uniaxial compressive strength of asperity, c : Cohesive strength of asperity, σ_t : Tensile strength of asperity, ϕ_u : Angle of frictional sliding resistance, ϕ_r : Residual friction angle, i : Inclination angle of asperity

model showed a good agreement with the test results regardless of the peak and residual strengths.

The test results showed the specimen with a steeper asperity angle has a higher shear strength and the change from the steeper portion of the failure envelope to the gentler portion at a higher normal stress. In addition, the proposed model showed the stress dependent strength feature more reliably.

From the comparison with the experimental test results along with the pre-existing strength models, the proposed shear strength model could estimate the shear strength at a rock joint more reasonably by better considering the complex joint characteristics of asperity angle, distribution, and strength.

5. Conclusions

In this paper, the characteristics of shear strength at rock joints were described and the limitations of the pre-existing strength models were examined. A rock joint could have the asperities with various inclination angles and base lengths and a portion of the joint surface may be sheared off previously. In addition, a rock joint is sheared progressively from the steepest asperity and at the same time the degradation of the joint is induced. Therefore, to estimate the shear strength at a rock joint more accurately and reasonably a shear strength model should consider these complex characteristics of shearing process as much as it

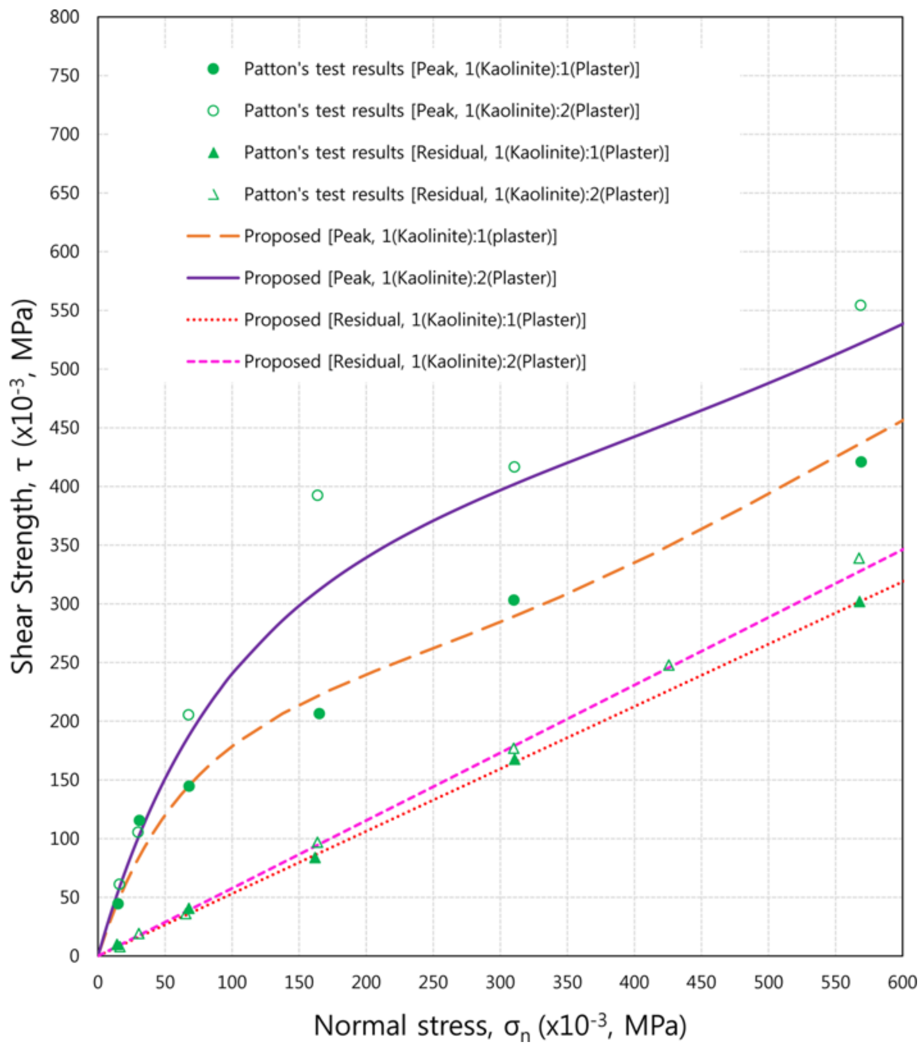

Fig. 9. Comparison with Patton's Experimental Test Results (varying the strength of teeth)

can. However, most pre-existing strength models do not consider these complex shearing features at rock joints enough.

An improved shear strength model, which can consider the complex joint shearing features observed from various experimental tests, was developed and proposed in attempt to overcome the limitations and problems of the pre-existing strength models, to take into account the anticipated simultaneous occurrence of the complex shearing processes, to represent the joint behavior in reality more reliably, and ultimately to provide a better estimate of joint shear strength in various rock, joint, stress, and shearing conditions. The proposed model was compared with some experimental test results along with pre-existing strength models. The comparison indicated that the improved model can estimate the shear strength at rock joints more accurately and reasonably than the pre-existing models.

Though more comparison with other test results could give a further assurance for the proposed model, it is expected that the proposed model, which can consider complex joint characteristics better, could play an important role in practice for estimating the shear strength at rock joints based on the limited study results.

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