# Recurrent Congestion Impact based on Spatiotemporally Historic Congested Information - Case Study: Separating Collision-Induced Congestion

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#### Abstract

Transportation jurisdictions should monitor mobility and reliability of roadway systems in order to adequately invest capacity expansion and deployment of ITS technologies to alleviate congestion effectively and efficiently. In recent years, several link-based bottleneck identification schemes have estimated bottleneck impact factors on freeways based on the characteristics of congestion. However, those have used congestion data with no attention to distinguishing recurrent level of at the same "bottleneck" location. Most existing studies that distinguish between recurrent and non-recurrent congestion have focused on separating non-recurrent congestion from recurrent congestion only for intensity of congestion using parameters for the speed distribution in a time of day in a segment or point. As such, this study introduced a data-driven procedure for quantifying spatiotemporal "recurrent" congestion impact. In addition, this study used spatiotemporally historic congestion information and generated stochastic spatiotemporal congestion distributions in terms of congestion types. Using the relationship between the distributions of recurrent and non-recurrent congestion occurring at bottlenecks, the bottleneck impacts were estimated by capturing spatial and temporal impact of recurrent bottleneck from that of non-recurrent congestion occurring at recurrent bottleneck. The proposed approach represents a significant improvement in the understanding and monitoring of mobility on freeways. This can be directly applied to evaluate and rank bottlenecks.

Keywords: recurrent congestion impact, historical congested impact area, data-driven approach, recurrent bottleneck identification, probe vehicle data

# 1. Introduction

Traffic congestion is a major obstacle to continued economic growth, deteriorating mobility, travel time reliability, and the environment. Traffic congestion is generally classified into one of two types: recurrent and non-recurrent congestion. Recurrent congestion is well-known as the condition in which demand exceeds capacity at a facility over a specified time period. Nonrecurrent congestion references any delays caused by an unexpected event (Hallenbeck et al., 2003; Skabardonis et al., 2003; Dowling et al., 2004; Chung, 2013; Chung, 2017). Of these, non-recurrent congestion can also be classified by the location where it occurs in relationship to recurrent bottlenecks: 1) at recurrent bottleneck but bottleneck is inactive before the nonrecurrent congestion occurrence, 2) in recurrent bottleneck impact area, and 3) outside of recurrent bottleneck impact area. The two first cases result in extra congestion from recurrent congestion and thereby deteriorate mobility and reliability of roadway systems considerably (Khattak et al., 2012; Zhang et al., 2012). Accordingly, it is imperative that those two congestion cases should be identified and quantified separately when it comes to identifying and monitoring bottlenecks and their associated impacts more accurately.

Although the literature is replete with attempts to identify bottlenecks (Chen *et al.*, 2004; Jiang, 2010; Wieczorek, 2010), a few link-based approaches for identifying bottlenecks included quantifying bottlenecks' impact based on queue length, duration, and frequency (FDOT, 2011; RITIS, 2016). Another study by Liu and Fei (2010) proposed a fuzzy-logic-based approach that can diagnose the severity of bottleneck based on travel delay and frequency. Those approaches made no attempt to distinguishing those cases occurring in recurrent bottleneck impact areas; therefore, the research may have overestimated recurrent bottleneck impacts.

Many studies attempting to distinguish between recurrent and non-recurrent congestion have been conducted (Hallenbeck *et al.*, 2003; Skabardonis *et al.*, 2003; Dowling *et al.*, 2004; Chung 2013). They used parameters for the speed distribution in a time of day at a segment to distinguish between recurrent and nonrecurrent congestion. In other words, non-recurrent congestion was noted when a specified speed falls below a representative speed threshold (Chung, 2011) regardless of cause of congestion. Despite the fact that the extra congestion occurring in recurrent bottleneck impact area extends spatiotemporally, these only considered the severity of congestion. This calls for separating extra congestion from recurrent congestion spatiotemporally to quantify bottleneck impact. Furthermore, recurrent bottleneck

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impacts may have a site-specific spatiotemporal shockwave phenomenon (May, 1990). It indicates that associated links with time need to be identified in quantifying bottleneck impacts.

With these considerations in mind, the objective of this study is to develop a dynamic data-driven approach for quantifying recurrent congestion impacts. The proposed method is based on spatiotemporally historic congestion information. In this study, congestion impacts are classified by contributing event: 1) active bottleneck, 2) crash occurring at a recurrent bottleneck but bottleneck is inactive before the crash, and 3) crash occurring within an active bottleneck period. To quantify recurrent congestion impact, this study develops an easily implementable approach to capture the spatiotemporal recurrent bottleneck impact areas. In addition, a new procedure is proposed to define the stochastic variation of spatiotemporal congestion impacts. Finally, this study introduces a formulation to calculate recurrent congestion impacts by each recurrent bottleneck impact area.

This paper is organized as follows. Following the introductory section with review of relevant works we present the overall procedure for this study. The procedure is followed by a detailed description of each component in the procedure and a case study of applying it to freeway across North Carolina. In the next section, several possible applications we discussed. Finally, the authors conclude with the key findings and recommendations for future work.

# 2. Literature Review

Relevant studies on congestion have been mainly focused on

theoretical estimation of impacted area caused by non-recurrent congestion. Identifying non-recurrent congestion and its impacted area is to prevent secondary incidents by providing appropriate real-time incident management. This calls for developing sophisticated methodologies to estimate spatiotemporal impacts of non-recurrent congestion. On the other hands, a few studies developing practical approaches for quantifying or ranking bottleneck impacts have been conducted for a decade. Quantification and ranking bottlenecks' impact aims for removing recurrent congestion by making an appropriate investment in current facilities rather than to provide real-time traffic management to relieve congestion.

# 2.1 Non-recurrent Congestion

Both static and dynamic approaches have been used for classification and estimation of spatiotemporal impacted area caused by non-recurrent congestion (Karlaftis *et al.*, 1999; Moore *et al.*, 2004; Zhan *et al.*, 2009; Chou and Miller-Hooks, 2010; Vlahogianni *et al.*, 2010) as shown in Table 1. Various statistical methods such as regression model (Chou and Miller-Hooks, 2010), Bayesian network model (Vlahogianni *et al.*, 2010), and frailty model (Chung and Recker, 2015) have been applied to estimate spatiotemporal congestion impacted area. Recent studies conducted on estimating non-recurrent congestion impacts have used integer programming models (Chung and Recker, 2012; Chung, 2013; Wang *et al.*, 2018). Chung and Recker (2012) and Chung (2013) classified the non-recurrent congestion impact area by identifying the end points where the speed drop due to incident is recovered. However, these models developed may

		-	-	•		
Tasks on congested impact area	Study	Event type considered	Methods	Variables or factors		
	(Karlaftis et al., 1999)	Non-recurrent (only crash)	Fixed	15 min. and 1 mile		
	(Moore <i>et al.</i> , 2004)	Non-recurrent	Fixed	2 hours and 2 miles		
Classification	(Zhan et al., 2009)	Non-recurrent	Dynamic	Maximum queue and dissipation time of the potential lane-blockage primary incident		
	(Chung and Recker, 2012)	(only crash)	Dynamic	Binary speed contour plot on a representative		
	(Chung, 2013)		Dynamic	speed contour map when a crash occurs		
	(Chou and Miller-Hooks, 2010)	Non-recurrent	Regression model	Corner points of incident boundary		
	(Vlahogianni et al., 2010)	Non-recurrent	Bayesian network model	Queue, duration, and density		
	(Chung and Recker, 2015)	(only crash)	Frailty model	Queue, duration, and defisity		
Estimation	(Chen et al., 2016)	Non-recurrent	K-Nearest Neighbor	Delay, VHT		
	(Yang et al., 2017)	Non-recurrent (only crash)	Fuzzy c-means	Characteristics of vehicle trajectory (speed, loca- tion, and angle)		
	(Wang et al., 2018)	Non-recurrent	Integer programming model	Speed, time, distance, shockwave propagation		
	(Emam and Al-Deek, 2006)		Data-driven	Travel time reliability		
	(Liu and Fei, 2010)		Fuzzy logic approach	Travel delay and frequency		
Quantification and ranking	(Zhao et al., 2013)	All congestion types	Data-driven	Travel time reliability (unreliable, reliably slow and reliably fast)		
and runking	(Lund et al., 2016)	0,003	Data-driven impact factor	Occurrence, element, blob		
	(RITIS, 2016)		Data-driven impact factor	Average duration, average maximum queue length of queue, and number of occurrences		

Table 1. Comparison of Relevant Works for Congestion Impacted Area

produce the results where the spatiotemporal impacted area is separated into two clusters. In addition, both Chen *et al.* (2016) and Yang *et al.* (2017) studied estimation of the spatiotemporal impacted area based on K-Nearest Neighbor and Fuzzy c-means, respectively. However, these results may violate that the incident shockwave must propagate uninterruptively in a spatiotemporal impacted cells. Recently, Wang *et al.* (2018) suggested an integer modelling method to address the above limits regarding shockwave propagation. This state-of-the-art method facilitates more reliable estimation of non-recurrent congestion by reducing computing time significantly compared to traditional methods. This still calls for developing more simplified as well as reliable method for practical application of the real-time incident management in a real.

#### 2.2 Recurrent Congestion

Identification of recurrent congestion stems from distinguishing congestion and bottleneck. Many relevant works for congestion identification have been conducted for decades with three major concepts: breakdown, bottleneck, and congestion. Those terms have been used interchangeably. Breakdown is usually defined as a phenomenon of transition from free flow traffic condition to congested states (Wang *et al.*, 2010; Jia *et al.*, 2010; Zhang and Levinson, 2004). Bottleneck is usually defined as a physical location where speed drops due to the lack of capacity (Neudorff *et al.*, 2011). However, previous studies defined bottleneck with a significant speed difference from the free-flow traffic state (Chen *et al.*, 2004; Warita *et al.*, 2006; FDOT, 2011). For instance, Warita *et al.* (2006) used 85% of free-flow speed as the cut-off threshold and Florida (2011) used 75%.

A consensus on the definition of non-recurrent congestion has been formed among the studies, whereas various definitions exists for recurrent congestion (Hallenbeck *et al.*, 2003; Skabardonis *et al.*, 2003; Dowling *et al.*, 2004; Chung, 2013). Recurrent congestion was defined only for the purpose of data-driven separation of non-recurrent congestion using the speed distribution in the above studies. Recently, a study developed by Song *et al.* (2018) proposes a data-driven methodology for identifying spatiotemporal recurrent bottlenecks. In this study, recurrent bottlenecks are defined as congestion repeatedly occurring at a specified location in a same time span during a study period unlike an unexpected congestion. However, this methodology only identifies recurrent bottleneck location and its frequency and time span.

As mentioned earlier, studies on quantifying and ranking bottlenecks have only focused on practical applications for better decision making by jurisdictions. Travel time reliability was the most frequently used index as suggested in Table 1. Recently, RITIS (2016) provides bottleneck impact factor by a function of average duration, average maximum queue length of queue, and number of occurrences with the simple definition of bottleneck when speed drops 65% to free-flow speed in a link. That is, all focused on quantification of impacts regardless of congestion type to rank bottleneck points. Also, none of these approaches consider the concept of recurrent bottlenecks. Therefore, as mentioned above, impacts of non-recurrent congestion occurring in recurrent congestion area should be separated from mixed congestion impacts. It is essential that an approach is developed focusing on quantification of impacts occurring solely by bottleneck activations.

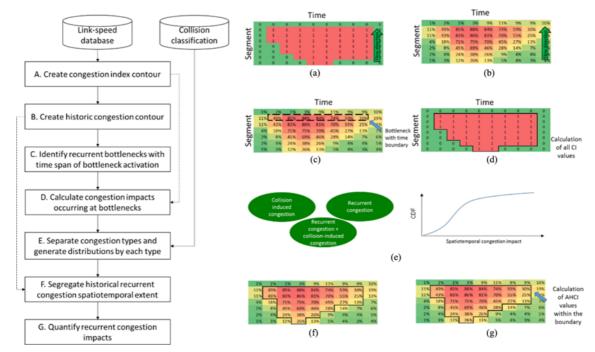


Fig. 1. Overall Historic Congestion Impact Quantification Framework: (a) CI Contour, (b) AHCI Contour, (c) Bottleneck Identification, (d) Congestion Impacts at Bottlenecks, (e) Separate Congestion Types and Generate Distributions by Each Type, (f) Segreate Historical Recurrent Congestion Spatiotemporal Extent, (g) Quantify Recurrent Congestion Impacts

# 3. Methodology

# 3.1 Overall Framework

The overall framework for this study is outlined in Fig. 1: the left figure describes the study processes and outlines the main components, while the right figures depict simple schematic examples of the main tasks achieved by each component (A through G). As a first step, study location and time period should be decided. In this study, link-based speed data is used to create a daily congestion index (CI) contour map (A). An average historic congestion index (AHCI) contour map (B) is created by summing the CIs created for a specified study period. These terms were developed by Song *et al.* (2018). As the AHCI contour map becomes available, it is used to identify recurrent bottlenecks with time span of bottleneck (C). In the next step, spatiotemporal congestion impacts occurring at recurrent bottlenecks identified are calculated (D). This information is stored into a knowledge based and supports the remaining applications of the framework.

The key objective of this study is to isolate spatiotemporal recurrent congestion from non-recurrent congestion. As stated above, in a recurrent congestion area, non-recurrent congestion can occur either at 1) a bottleneck location before the bottleneck is active or 2) within the impact time of an active recurrent bottleneck. Those two types are classified in this research. This study uses crash data classified under different operation conditions to separate non-recurrent (or collision-induced) congestion and recurrent congestion.

This study performs separation of the impacts by congestion type (E). A spatiotemporal impact distribution is then generated for each type of congestion impact and is compared to the distributions of the recurrent congestion impacts in the specified study period. The distribution of the impacts of collision-induced congestion occurring at a bottleneck before bottleneck is active can be applied to segregate historical recurrent congestion extent in step (F). Eventually, quantification of recurrent congestion area is performed (G).

# 3.2 Spatiotemporal Congestion State and Historic Congestion Contours

Previous studies regarding congestion identification processes have been mainly conducted using a speed-based definition (Elefteriadou *et al.*, 1995; Chen *et al.*, 2004; Bertini and Leal, 2005; Jia *et al.*, 2010; Liu and Fei, 2010). The relevant studies using speed-based definitions were based on either a prespecified speed threshold or a precipitous speed drop. Each used a representative cut-off threshold to identify congestion. In this study, different cut-off thresholds are decided using the ratio of speed at capacity to free flow speed in the TRB (2010). Therefore, the cut-off thresholds change with the free flow speed (FFS). A spatiotemporal congestion index (CI) contour is drawn based on the following definition:

$$CI(i,t,m) = \begin{cases} 1 & \text{if } C(i,t,m) < \alpha \\ 0 & \text{otherwise} \end{cases}$$
(1)

## Where,

C(i, t, m) = reported speed/free flow speed at a spatiotemporal cell (*i*, *t*), and

CI(i, t, m) = Congestion Index on segment *i* in *t*.

$$i =$$
 segment id,

- m = a day in the study period,
- t= specified time interval id in a day (e.g., 8:00-8:15, 15 min),
- $\alpha$  = cut-off threshold (free flow speed: 55mph 0.9; 60mph - 0.85; 65mph - 0.8; 70mph - 0.78; 75mph - 0.75)

The major component for this study is to identify the characteristics of recurrence in congestion. This study sums of all CIs generated for M days in the study period and thereby an AHCI contour is generated. The AHCI, AHCI(i, t), is a value at a spatiotemporal cell (i, t) in the contour and is also defined as the fraction of days in the reporting period T (usually more than one year) where a segment i was congested at time t, based on the CI contours.

$$AHCI(i,t) = \frac{\sum_{m=1}^{M} CI(i,t,m)}{M}$$
(2)

Where,

AHCI(i,t) = Average Historic Congestion Index for segment *i* at time *t* 

M= the number of days in the study period (e.g., 250 weekdays in a year)

#### 3.3 Recurrent Bottleneck Identification

This study needs to identify recurrent bottlenecks and their time span of activation. A data-driven link-based approach by Song et al. (2018) was used for identifying spatiotemporal recurrent bottlenecks. The study identified recurrent bottlenecks based on the AHCI. A recurrent bottleneck is defined as "a segment with an AHCI(*i*, *t*) which exceeds  $\beta$  and a significant spatial difference, y, between the AHCI(i, t) and adjunct downstream AHCI(i-1,t)". The variable of  $\beta$  for this study indicates that congestion is more likely to occur than not occur on segment *i* at time *t*. Recurrent congestion may occur within a static historical time span of bottleneck activation. For instance, Exit 295 on I-40 Westbound shown in Fig. 2 was identified as a recurrent bottleneck which was activated from 16:30 to 18:00 in 2014.  $\delta$  is used as a threshold to separate the time span of bottleneck activation. In this study, the value of  $\delta$  is 0.2, which indicates congestion occurs on average at least one day a week on segment *i* and time *t*.

#### 3.4 Spatiotemporal Congestion Impact

To achieve the objective of this study, it is essential to develop an approach for quantifying the spatiotemporal congested impact area. This study introduces a new concept called "Congestion Spatiotemporal Impact Index (CSII)", *CSII*<sub>c</sub>, which takes into account the segment length, and the duration of congestion by an event or active bottleneck. Fig. 3 shows a schematic example of

$\beta \geq 0$	$\beta \ge 0.5$ $\gamma \ge 2 \text{ (ex. 0.84 } \ge 0.09 \text{ x } 2 \text{ at } 17:15) \longleftarrow \delta \ge 0.2$									
Link	Length	16.15	Time period 16:15 16:30 16:45 17:00 17:15 17:30 17:45 18:00 18:							10.15
(miles)	(miles)	16:15	16:30	16:45	17:00	17:15	17:30	17:45	18:00	18:15
1	0.5	0.01	0,02	0,01	0,03	0.09	0.11	0.09	0.09	0.10
2	0.7	0.1 <u>1</u>	<b>.</b> 0.49	<b>0</b> .85	<b>v</b> 0.88	<b>0.84</b>	<b>0</b> .74	<b>√</b> 0.59	<b>√</b> 0.30	0,20
3	1.2	0.11	0.43	0.83	0.86	0.81	0.70	0.55	0.25	U E
4	0.3	0.04	0.18	0.71	0.75	0.70	0.45	0.27	0.13	direction
5	0.8	0.02	0.08	0.45	0.69	0.46	0.28	0.14	0.07	
6	1.1	0.02	0.04	0.24	0.38	0.26	0.09	0.04	0.04	6 Flow
7	1	0.01	0.03	0.12	0.26	0.13	0.05	0.04	0.03	0.04

Fig. 2. A Schematic Example of Recurrent Bottleneck Identification Process in an AHCI Contour

Li (miles)		Congesti	ion occu	irrence	T	ime perio	od					$L_i \cdot \sum_{t=1}^{T(i)} CI(i,t) \cdot \frac{R}{60}, \forall CI(t) = 1$
0.5	0	0,-	0	0	0	0	0	0	0	0	0	0.00
0.7	0	+1	1	1	1	1	1	1	1	0	0	1.40
1.2	0	1	1	1	1	1	1	1	1	0	$\triangle$	2.40
0.3	0	1	1	1	1	1	1	1	1	0	5	0.60
0.8	0	0	1	1	1	1	1	1	1	0	direction	1.40
1.1	0	0	1	1	1	1	1	1	1	0		1.93
1	0	0	0	1	1	1	1	1	1	0	Flow	1.50
0.9	0	0	0	0	1	1	0	0	0	0	Ļ	0.45
									С	SII(mile	es∙hours)	9.68

Fig. 3. Estimation of Congestion Spatiotemporal Impact Index (CSII)

estimation of the  $CSII_c$  with a CI contour. The proposed  $CSII_c$  is as follows:

$$CSII_{c}(miles \cdot hours) = \sum_{i=1}^{T(i)} \sum_{i=1}^{I} L_{i} \cdot CI(i,t) \cdot \frac{R}{60}, \forall CI(i,t) = 1$$
(3)

Where,

- c = congestion caused by an event, active bottleneck or both,
- I = total number of consecutive segments in the area of the impact,
- Li =length of segment (miles), and
- R =time resolution (i.e., 15 min. = 15).
- T(i) = congestion duration expressed in terms of time intervals on segment i

# 3.5 Separation of Congestion Type

As stated above there are two types of collision-induced congestion occurring in the recurrent congestion area shown in Fig. 4. A collision classification procedure developed by Song *et al.* (2015) is applied for this study to separate the impact of such collision-induced congestion types. In the study, collisions were classified by three different types of congestion when a crash occurs: 1) collision not in a congested area, 2) collision in a non-recurrent congestion area, and 3) collision in a recurrent congestion area recurrent congestion area – was used for this study.

There is an additional process to distinguish collisions occurring at a recurrent bottleneck location with time span of bottleneck activation but bottleneck is inactive before the collision and within an active recurrent bottleneck. This is processed based on the approach for identifying recurrent bottlenecks introduced above and comparing the bottleneck locations to the crash locations. As a final process in this selection, this study classifies recurrent congestion as congestion occurring within a time span of bottleneck activation with no evidence of any incident shown in Fig. 4(a).

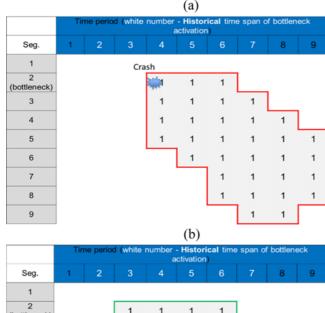
Congestion contributions are labeled by the types shown in Table 2 in each day of occurrence according to the following conditions occurring at recurrent bottlenecks. Type 0 is congestion due to an active bottleneck with no evidence of identified collision (referred to as recurrent congestion). Type 1 is defined as congestion due to a crash, but the bottleneck is inactive before the crash (referred as collision-induced congestion occurring at a recurrent bottleneck). Finally, Type 2 is congestion due to a crash within an active bottleneck (referred as recurrent congestion plus collision-induced congestion occurring at a recurrent bottleneck).

In order to distinguish the spatiotemporal impacts of recurrent and non-recurrent congestion,  $CSII_c$  is estimated and captured for congestion type by each recurrent bottleneck activation and collision occurrence during a specified study period. The authors define  $CSII_{c,s}$ , where *s* is: 0 = Type 0, 1 = Type 1, and 2 = Type 2. Distributions of  $CSII_{c,s}$  are then generated from the data.

#### 3.6 Recurrent Congestion Area

The key for quantifying recurrent congestion impacts is to

	Tin	ne perio	d (white		- Histor		e span o	fbottlen	eck
Seg.	1	2	3	4	5	6	7	8	9
1									
2 (bottleneck)		1	1	1	1	1	1		
3		1	1	1	1	1	1		
4			1	1	1	1			
5			1	1	1	1			
6			1	1	1				
7				1					
8					-				
9									



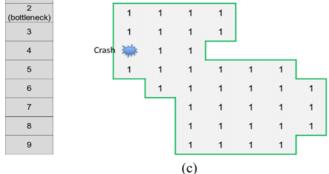


Fig. 4. Types of Congestion Occurring within a Recurrent Congestion Area: (a) Type 0 – Recurrent Congestion (bottleneck activation), (b) Type 1 – Congestion Due to a Crash But Bottleneck is Inactive before the Crash, (c) Type 2 – Congestion due to a Crash within an Active Bottleneck (recurrent congestion plus extra non-recurrent congestion)

identify the historical spatiotemporal recurrent congestion extent with a robust methodology. This starts by segregating recurrent congestion from the impact due to Type 1. In this study, identifying historical recurrent congestion spatiotemporal extent is based on AHCI contours. As mentioned earlier, each value of *AHCI* (i, t) in the AHCI contour is the probability of congestion at segment i

Table 2. Congestion Type Occurring in Recurrent Bottleneck Impact Area

Congestion type	Bottleneck activation	Crash occurrence
Type 0	Yes	No evidence
Type 1	No evidence	Yes
Type 2	Yes	Yes

in time t during a study period. In the contour, cell (i, t) which is further from the bottleneck activation, either in time and/or in space, has a lower probability of frequent congestion. This study proposes an approach for identifying the maximum recurrent congestion extent using the characteristics with the distribution of  $CSII_{c,0}$  and  $CSII_{c,1}$ . In the contour, the maximum value of AHCI(*i*,*t*) at a recurrent bottleneck, *b*, is overall congestion frequency,  $f_b$ , which in b of M days. The total congestion frequency is the summation of recurrent congestion frequency,  $f_r$ , and non-recurrent congestion frequency,  $f_{m}$ . This study assumes that  $CSII_{c0}$  (recurrent congestion) with values that exceed the minimum  $CSH_{c,l}$  values (collision-induced congestion) are considered to be exclusively incident-induced congestion. This is followed by three hypotheses. First is possible that the impact due to Type 1 may include Type 0, if a bottleneck is activated in the time span of a recurrent bottleneck activation. Second, traffic passing a recurrent bottleneck is usually high even though the bottleneck maybe inactive. Finally, AHCI contours include all no-recorded non-recurrent congestion but is assumed to be recurrent congestion.

The cut-off threshold,  $\lambda$ , of *AHCI*(*i*, *t*) needed to segregate a recurrent bottleneck impact can be derived from these assumptions alternatively it can be set at 0.2 (which is the conservative value of the probability that a congestion event occurs at least once a week for recurrent congestion). This leads to the following of models:

$$max \ AHCI(i,t) \times M = f_b, \forall i, t \in B$$
(4)

$$f_b = f_r + f_{nr} \tag{5}$$

$$\frac{f_{nr}}{f_b} = [1 - F(minCSII_{c,1})] \tag{6}$$

$$\therefore f_{nr} = f_b [1 - F(minCSII_{c,1})] \tag{7}$$

$$=\frac{J_{mr}}{M}$$
(8)

Where,

λ

B = spatiotemporal boundary of bottleneck activation, and

 $F(minCSII_{c,l})$  = the minimum cumulative density function of  $CSII_{c,l}$ 

#### 3.7 Quantification of Recurrent Congestion Impacts

To quantify recurrent congestion impacts, this study proposes a new concept called the "Recurrent Bottleneck Spatiotemporal Impact Index (RBSII)", which takes into account the segment length, the duration of the congestion, and the frequency of occurrences (AHCI's) by a recurrent bottleneck area. The proposed **RBSII** is as follows:

$$RBSII(miles \cdot hours \ per \ activation) = \sum_{i=1}^{T(i)} \sum_{i=1}^{I} L_i \cdot AHCI(i,t) \cdot \frac{R}{60} \cdot \forall AHCI(i,t) \ge \lambda$$
(9)

Where,

i = segment id,

- I = total number of consecutive segments in the area of the impact,
- $L_i$  = length of segments (miles),
- t = time interval (e.g., 15 min),
- T(i) = congestion duration expressed in terms of time intervals on segment *i*, and
  - R =time resolution (e.g., 15 min. = 15).

# 4. Case Study

#### 4.1 Data Description

In this study, processed speed data were downloaded from the RITIS.org database, which is generated from GPS-enabled vehicle probes using INRIX technology. This case study selected statewide interstates across North Carolina. The extension of the interstates is 2253 miles on 2287 Traffic Message Channel (TMC) segments, with an average TMC length of 1.11 miles. The posted speed limit of the interstates varies from 55 mph to 70 mph. The data covered the period from January 1, 2011, to December 31, 2014, aggregated at 15-min intervals.

In addition to speed data, crash location and time are used for separating collision-induced congestion impact from recurrent congestion. Crash data stored in the Traffic Engineering Accident Analysis System (TEAAS) maintained by North Carolina Department of Transportation was used for this study.

#### 4.2 Study Site Identification

As a basis for selecting a recurrent bottleneck as a study site, the first step is to identify recurrent bottlenecks where crashes occurred. However, a crash occurring within the spatiotemporal boundary of bottleneck activation is an extremely rare event. In fact, crash data collected by police officers/witnesses can lead to subjective and erroneous understanding regarding crash occurrences. Despite the large data set, there were recurrent bottlenecks where no crashes were observed. In addition, several recurrent congestion problems were not singular bottleneck occurrences, but were the result of several compounding bottlenecks within a recurrent bottleneck impact area. Given these constraints, a systematic process was developed for study site selection. The site selection criteria were:

- 1. At least one crash occurred within the spatiotemporal boundary of bottleneck activation;
- 2. At least three miles from the location (the TMC with AHCI value greater than 20%) of the impact area of the nearest downstream bottleneck as measured by the AHCI contour; and,
- 3. Presence of bottleneck activations at the recurrent bottleneck selected on at least 50% of all weekdays studied.

In order to conduct this process with above criteria, recurrent bottlenecks with time span of bottleneck activation should be identified first. Therefore, daily CI contours were generated for all interstates and thereby an AHCI contour per year was created to identify recurrent bottlenecks. In all cases, weekday data were used. The recurrent bottlenecks identified based on the approach of Step (C), shown Fig. 1, were 67, 69, 86, and 95 for 2011, 2012, 2013, and 2014, respectively.

With the crash data sorted by Type 1 and Type 2, six bottlenecks on interstates for the study period were finally selected as the study sample according to the three criteria above. The basic information for the recurrent bottleneck site is summarized in Table 3.

As mentioned earlier, it is possible that the crash data contains errors regarding location and time of the crash. This study identified and used crashes reported which can match Type 1 congestion starting point not only in time interval *t* on segment *i* ((2,3) as shown in Fig. 5), but also in adjacent times (t - 1 or t + 1)and/or segments (i - 1 or i + 1) (from (1,2) to (3,4) in Fig. 5).

				···· <b>,</b> ···			
Site		1	2	3	4	5	6
Road	1	I-540	I-40	I-77	I-77	I-77	I-40
Directi	on	EB	WB	EB	SB	WB	WB
Count	ty	WAKE	WAKE	MECKL.	MECKL.	MECKL.	DURHAM
TMC c	ode	125 + 05079	1125 + 04965	125n04792	125 - 04787	125 - 04783	125 + 04868
Length (1	mile)	1.52	0.39	0.63	0.48	0.57	0.64
Year		2012	2014	2012	2012	2012	2012
Time span of bottle	neck activation	17:00 - 18:15	7:15 - 8:45	6:45 - 9:15	7:45 - 9:00	7:30 - 8:45	17:30 - 18:15
Reported frequence activation		180	161	127	147	173	130
Distance to do bottlenect		NA	8.07	8.32	4.5	NA	NA
Reported number	Type 1	2	4	2	5	2	5
of crashes	Type 2	3	7	23	13	14	9

Table 3. Bottleneck Characteristics for Study Sites Selected in NC

Note: NA = no adjacent downstream bottleneck within 10 miles

Seg.			Ti	me perio	bd		
	1	2	3	4	5	6	7
1		(1,2)	(1,3)	(1,4)			
2 (bottleneck)		(2,2)	(2,3)	(2,4)			
3		(3,3)	(3,3)	(3,4)			
4							
5							

Fig. 5. Crash Location and Time Corresponding to Congestion

Table 4. Number of Type 1 Crashes Observed under Time and Location Cell

Crash location	Same time and location	Same time but next location	Previous or after time but same location	Both next time and segment cell
Number of Type 1 crashes	2	7	7	20

Table 4 shows the frequency of crashes reported in the nearby time and space range to congestion occurrence. Only two crashes were matched to congestion starting point. This study used crashes reported and matched to congestion within one spatiotemporal cell. A total of 20 crashes reported were used as seen in Table 4.

## 4.3 Probability Distribution of Congestion Spatiotemporal Impact Index

Next, the  $CSII_{c,s}$  of each congestion, c, occurring for each study site was calculated. Table 5 shows the descriptive statistics of the  $CSII_{c,s}$ , which include all congestion cases for Type 0, 1 and 2. Maximum values of the  $CSII_{c,s}$  for the study sites were the value for Type 2. The average  $CSII_{c,s}$  varied from 1.5 to 3 with the exception of study site 3. The value of  $CSII_{c,s}$  for site 3 was higher

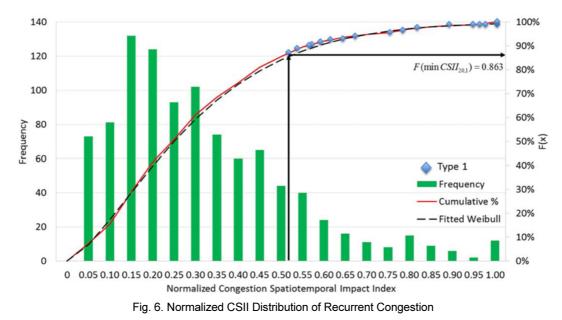
Table 5. Descriptive Statistics of Congestion Spatiotemporal Impact Index (CSII)

Site	1	2	3	4	5	6
Average CSII (miles·hours)	2.96	1.88	10.52	1.51	1.74	1.66
Stand deviation of CSII (miles hours)	1.42	0.99	5.97	1.48	1.19	1.45
Min. CSII (miles·hours)	0.23	0.19	0.32	0.12	0.14	0.06
Max. CSII (miles·hours)	6.79	7.3	26.54	6.62	7.28	7.31

Tested	Type 0	Type 2	Type 0	Type 2		
distribution	K-S st	tatistic	A-D statistic			
Weibull	0.028	0.097	1.26	0.623		
Gamma	0.053	0.118	7.52	1.321		
Normal	0.084	0.137	12.091	1.880		

than others which indicates that it is the recurrent bottleneck where has the biggest impact area among the study sites.

As mentioned earlier, it is hard to capture the historical recurrent congestion extent using Type 1 and Type 2 with significant samples of  $CSII_{c,s}$  due to the nature of crash occurrence. In addition, it is likely that some crashes are not reported in the crash data resulting in the statistics of  $CSII_{c,0}$  to include several Type 1 congestion occurrences. Therefore, this study developed normalized distributions of  $CSII_{c,s}$  across all study sites in terms of Type 0, 1, and 2. The Kolmogorov-Statistical (K-S) test and Anderson-Darling (A-D) tests, which are commonly used to test goodness of fit, were conducted to determine the probability distributions that reflect the stochastic characteristics of normalized congestion severity index (Jia *et al.*, 2010). The Weibull, gamma, and normal distributions were calculated. Two normalized distributions were generated except for Type 1 because of its limited sample



size, as mentioned above. Table 6 shows that the Weibull distribution yields the lowest K-S and A-D statistic values in terms of both congestion types.

Figure 6 illustrates the normalized  $CSII_{c,0}$  with CSIIc, 1 values for all study sites. The 20 values of  $CSII_{c,1}$  were superimposed onto the distribution of  $CSII_{c,0}$ . This figure gives the minimum percentile  $CSII_{c,1}$ ,  $F(\min CSII_{c,1})$ , compared to the percentile  $CSII_{c,0}$  derived from the distribution. It shows that  $F(CSII_{c,1})$ values were greater than 0.9 and  $F(\min CSII_{c,1})$  was 0.863. This can be simply converted to  $\lambda$  which is derived from Eqs. (7) and (8) for each study site in the AHCI contour to ascertain the thresholds for recurrent congestion.

# 4.4 Capturing and Quantifying Recurrent Congestion Impact

Given that the normalized CSII distributions present a representative value of  $F(\min CSII_{c,l})$  among all study sites, a recurrent congestion spatiotemporal extent is ascertained using  $\lambda$  for each bottleneck. Fig. 7(a) and 7(b) illustrate the recurrent congestion impact identified for study sites 1 and 2. The blooded value in the AHCI contour was the maximum value of *AHCI*(*i*,*t*), 0.89 and 0.86 for sites 1 and 2, respectively. Using these values, the value of  $\lambda$  for a recurrent bottleneck was estimated as 0.122

Site	1	2	3	4	5	6		
Maximum value of <i>AHCI</i> ( <i>i</i> , <i>t</i> ) (%)	89	86	92	73	82	56		
λ (%)	12.2	11.7	12.5	10.0	11.2	7.6		
<i>RBSII</i> (miles-hours per activation)	2.94	2.78	12.92	1.39	1.47	1.21		

Table 7. Quantification Result of Recurrent Congestion Impact

for site 1 and 0.117 for site 2.

In step (H) in Fig. 1, those areas were then calculated to quantify each impact. The quantification results of recurrent congestion impact for the six sites are summarized in Table 7. This informs the site 3 was the worst bottleneck, while the RBSII values of others were comparatively similar.

# 5. Applications

The proposed RBSII can be directly applied for ranking recurrent freeway bottlenecks. This facilitates monthly and/or annual analysis on a large scale network (i.e., nation or statewide). In addition, it allows for identification of degraded or improved recurrent bottleneck impact. For instance, Fig. 8 depicts the degraded recurrent bottleneck impact for a typical weekday

Link	Length (miles)	4:30 PM	4:45 PM	5:00 PM	5:15 PM	5:30 PM	5:45 PM	6:00 PM	6:15 PM	6:30 PM	6:45 PM
1	0.7	0.00	0.02	0.02	0.05	0.06	0.07	0.07	0.05	0.03	0.03
2	1.3	0.01	0.07	0.30	0.62	0.87	0.82	0.66	0.33	0.13	0.07
3	0.9	0.01	0.07	0.33	0.76	0.89	0.88	0.73	0.32	0.14	0.09
4	0.5	0.01	0.03	0.12	0.51	0.75	0.69	0.42	0.14	0.07	0.06
5	0.5	0.00	0.01	0.04	0.23	0.50	0.46	0.20	0.08	0.04	0.03
6	0.5	0.00	0.00	0.02	0.10	0.25	0.21	0.11	0.05	0.03	0.02
7	1.6	0.00	0.00	0.00	0.03	0.11	0.11	0.07	0.04	0.02	0.01
8	0.2	0.00	0.00	0.00	0.01	0.05	0.06	0.02	0.01	0.00	0.00

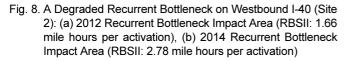
(a)

Link	Length (miles)	7:00 AM	7:15 AM	7:30 AM	7:45 AM	8:00 AM	8:15 AM	8:30 AM	8:45 AM	9:00 AM	9:15 AM
1	0.7	0.01	0.01	0.02	0.05	0.08	0.05	0.07	0.09	0.07	0.05
2	1.3	0.10	0.32	0.82	0.86	0.81	0.70	0.56	0.34	0.14	0.07
3	0.7	0.09	0.31	0.82	0.86	0.80	0.67	0.52	0.32	0.11	0.05
4	0.4	0.04	0.13	0.66	0.74	0.63	0.42	0.23	0.11	0.06	0.03
5	0.6	0.02	0.06	0.40	0.57	0.43	0.24	0.12	0.06	0.03	0.01
6	0.2	0.01	0.03	0.19	0.33	0.19	0.05	0.04	0.02	0.02	0.01
7	0.6	0.01	0.02	0.08	0.22	0.11	0.04	0.04	0.02	0.02	0.01
8	0.4	0.01	0.01	0.05	0.11	0.08	0.05	0.03	0.02	0.01	0.00
9	1.4	0.01	0.00	0.02	0.04	0.04	0.04	0.02	0.01	0.01	0.00

(b)

Fig. 7. Recurrent Congestion Spatiotemporal Extent: (a) Site 1:  $\lambda$  = 0.122, (b) Site 2:  $\lambda$  = 0.117

Louath (miles)	Time resolution (15 minutes)											
Length (miles)	7:15 AN	1 7:30 A	M 7:45	AM 8:0	00 AM	8:15 AM	8:30 AM	8:45 AM				
0.7	0.00	0.03	0.0	19 0	.09	0.08	0.08	0.07				
1.3	0.03	0.38	0.6	7 0	.66	0.43	0.25	0.10				
0.7	0.03	0.33	0.5	8 0	.59	0.31	0.14	0.08				
0.4	0.03	0.13	0.3	2 0	.18	0.09	0.05	0.04				
0.6	0.02	0.03	0.1	.0 0	.05	0.05	0.03	0.03				
0.2	0.02	0.02	0.0	2 0	.03	0.02	0.02	0.01				
0.6	0.01	0.02	0.0	2 0	.02 0.01		0.00	0.00				
(a)												
Length (miles)	Time resolution (15 minutes)											
Lengun (miles)	7:15 AM	7:30 AM	7:45 AM	8:00 AM	8:15 A	M 8:30 AN	1 8:45 AM	9:00 AM				
0.7	0.01	0.02	0.05	0.08	0.05	0.07	0.09	0.07				
1.3	0.32	0.82	0.86	0.81	0.70	0.56	0.34	0.14				
0.7	0.31	0.82	0.86	0.80	0.67	0.52	0.32	0.11				
0.4	0.13	0.66	0.74	0.63	0.42	0.23	0.11	0.06				
0.6	0.06	0.40	0.57	0.43	0.24	0.12	0.06	0.03				
0.2	0.03	0.19	0.33	0.19	0.05	0.04	0.02	0.02				
0.6	0.02	0.08	0.22	0.11	0.04	0.04	0.02	0.02				
0.4	0.01	0.05	0.11	0.08	0.05	0.03	0.02	0.01				
(b)												



using the RBSII. Part (a) of Fig. 8 shows the AHCI contour for site 2 in 2012; while the AHCI contour from 2014 is shown in (b). The RBSII in 2014 is greater than in 2012 by approximately 1.2 mile-hours per activation which indicates that traffic factors degraded the recurrent bottleneck impact. In addition to the RBSII comparison, Fig. 8 also shows the temporal increase in recurrent congestion duration (45 minutes increase) and AHCI values in 2014. The RBSII can be also applied to calculate the maximum queue length and the duration of congestion for each recurrent bottleneck. For example, the maximum length of queue in the AHCI of 2012 was 2.4 miles at the recurrent bottleneck, while it was 3.8 miles in 2014.

# 6. Conclusions

This paper introduced a data-driven approach for quantifying recurrent congestion impacts based on historically spatiotemporally congested information. The methodology presented here quantifies historical recurrent congestion impact. Unlike previous studies that quantified bottleneck impact with average maximum queue length and duration, the proposed method uses the segment length, duration of the congestion, and frequency of occurrences by each spatiotemporal cell that reflects historical congested impacts well identified by a recurrent congestion definition. This definition based on an average congestion history using probereported speeds. This study provided a stochastic normalized spatiotemporal congested impact distribution for distinguishing recurrent and collision-induced congestion impacts. The proposed methodology can support both road infrastructure decision makers and congestion managers in their efforts to implement mobility and reliability treatments that are precisely targeted and effective by providing critical information about which bottlenecks result

in the worst mobility and reliability.

Future research is required to enhance quantification of bottleneck impacts. Congested speed for each congestion event should be considered as a variable for intensity in quantifying bottleneck impact. This was not considered for this study because this study was focused on developing an easily implementable methodology that quantifies spatiotemporal congested impacts quickly. However, considering such intensity variable can lead to more accurate quantification of congested impacts in the world. This study was based on the link-based speed data that provides uniform traffic performance information spatially. Thus, this may cause some errors in quantifying congested impacts. In addition, there is a need to scientifically select an appropriate time resolution. Although this study employed normalized time solution, an appropriate aggregated time resolution needs to be found to reduce errors temporally. Finally, other causes of nonrecurrent congestion, such as adverse weather, will be considered to improve the proposed method.

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