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TECHNICAL NOTE

Active Earth Pressure Against Rigid Retaining Wall Considering Effects of Wall-soil Friction and Inclinations

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Abstract

This paper reports a new method for calculating the active earth pressure acting on inclined rigid retaining wall with inclined backfill, considering wall-soil interface friction angle. Based on Mohr-Coulomb strength theory and Mohr stress circle, new formulae of the active earth pressure and the active rupture angle are derived. The effects of internal friction angle, backfill-surface inclination, wall-back inclination on the active earth pressure and the active rupture angle are investigated. In order to facilitate calculation, special solutions of presented formulae are discussed under various particular conditions. Finally, the calculated results from the presented formula and existing formulae are compared with existing small-scale test results. The comparison shows that the presented method satisfactorily predicts active earth pressure.

Keywords: *active earth pressure, active rupture angle, wall-soil friction angle, wall-back inclination, backfill-surface inclination*

1. Introduction

It is very important to evaluate the earth pressures with higher accuracy in the design of many geotechnical engineering structures, especially rigid retaining walls. Coulomb (1776) and Rankine (1857) earth pressure theories have been widely used to calculate earth pressures acting on rigid retaining walls by civil engineers. The Coulomb earth pressure theory is based on ultimate force equilibrium of the sliding non-cohesive backfill wedge behind rigid retaining wall, but the Rankine earth pressure theory is based on ultimate stress equilibrium at any point of semi-infinite non-cohesive soil mass.

The calculation methods of earth pressure proposed by Coulomb (1776) and Rankine (1857) are aimed at non-cohesive backfill. Because of considering the effects of wall-soil interface friction angle, backfill-surface inclination, wall-back inclination, Coulomb's method is revised and improved by some researchers (Motta, 1994; Wang, 2000; Shukla *et al*., 2009; Peng and Chen, 2013). However, they did not give the distribution of earth pressure along the wall. In recent decades, other researchers (Handy, 1985; Paik and Salgado, 2003; Shubhra and Patra, 2008; Li and Wang, 2014; Tu and Jia, 2014; Zhou *et al*., 2017; Zhou *et al.*, 2018) attempted to derive formulas for evaluating active earth pressure on translating rigid retaining walls, considering soil arching effects and based on Rankine failure angle. But the trajectory of minor principal stress adopted by them did not meet that the integration of a shear stress along horizontal direction in sliding backfill mass is zero behind translating rigid retaining wall.

Several researchers (Terzaghi, 1943; Chu, 1991; Mazindrani and Ganjali, 1997; Gnanapragasam, 2000) extended Rankine's method. Terzaghi (1943) and Rankine (1857) presented formulae for calculating earth pressure against vertical rigid retaining walls with slippery wall and horizontal backfill surface. Terzaghi (1943) presented graphical method for inclined backfill surface, but he did not present an analytical solution based on several Mohr's stress circles. Chu (1991) presented those formulae for inclined wall back and inclined backfill surface with non-cohesive backfill, but he did not consider effect of wall-back inclination on failure angle. Several researchers (Mazindrani and Ganjali, 1997; and Gnanapragasam, 2000) presented analytical solutions for inclined backfill surface with cohesive backfill, but they did not present a formula of rupture angle. However, they (Rankine, 1857; Terzaghi, 1943; Chu, 1991; Mazindrani and Ganjali, 1997; Gnanapragasam, 2000) did not consider the effect of wall-soil friction angle on static active earth pressure.

In this study, based on the concept of Rankine earth pressure theory, considering wall-soil friction angle, a new formula for calculating active earth pressure acting on the inclined rigid retaining walls with non-cohesive backfill is put forward. A new formula for calculating active rupture angle is also presented. To verify the new method, the predicted results by the presented formulae are compared with those from existing experimental studies and some existing theories. Furthermore, those solutions under various special cases are investigated.

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2. Theoretical Analysis

Figure 1 shows a rigid retaining wall of height H with inclined non-cohesive backfill at an angle β to the horizontal, inclined wall back at an angle η to the plumb line, with internal friction angle φ and unit weight g of backfill, and wall-soil friction angle δ. The non-cohesive backfill is assumed to be homogeneous and isotropic, and it is assumed to be in active limit equilibrium state owing to sufficient displacement and deformation of the wall under the active earth force.

Figure 1 shows the differential triangle element Δabc, located in wall back at the depth z from the surface of backfill. $\overline{ac}$ is parallel to the surface of backfill, $\overline{bc}$ is the sliding plane. The σ_w is the total stress on $\overline{ac}$, and the angle between σ_w and the normal of $\overline{ac}$ is β. σ_a is the active earth pressure on $\overline{ab}$, and the angle between σ_a and the normal of $\overline{ab}$ is δ. σ_R is the soil reaction stress on $\overline{bc}$, and the angle between σ_R and the normal of $\overline{bc}$ is φ.

Let $\overline{mm}$ parallel to the surface of backfill or $\overline{ab}$, then let $\overline{OC}$ be perpendicular to $\overline{mm}$ through point O of $\overline{mm}$. Let Mohr-Coulomb strength line $\overline{OS}$ at angle φ to $\overline{OC}$, and let plumb line $\overline{OG}$, which represents the vertical stress σ_w on $\overline{ac}$. Then let dashed circle, of which the point N of the line $\overline{OC}$ is center, intersect $\overline{OG}$ at the points G and B, and is tangential to $\overline{OS}$ at the point T. Let $\overline{GH}$ at angle η to $\overline{OG}$. Thereby, the dashed circle shown in Fig. 2 represents Mohr stress circle of non-cohesive soil without regard to the influence of wall-soil friction.

The point B on the dashed circle represents the total stress acting on the plumb plane. Let $\overline{NH}$ at the angle 2η to $\overline{NB}$, which intersects the dashed circle at the point H, then $\overline{OH}$ represents the active earth pressure σ_a, further μ represents the inclined angle of the active earth pressure, that is an angle between the active earth pressure and the normal of wall back. If μ is less than the wall-soil friction angle δ, the stress represented by $\overline{OH}$ can't make the backfill slide relative to the wall, and the backfill at wall back is in the active un-limit state, then $\overline{OH}$ doesn't represent the active earth pressure. If μ is greater than the wall-soil friction angle δ, when the inclined angle of earth pressure

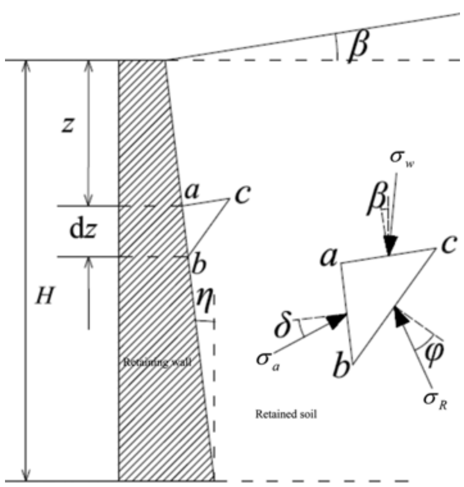

Fig.1. Force on Differential Triangle Element

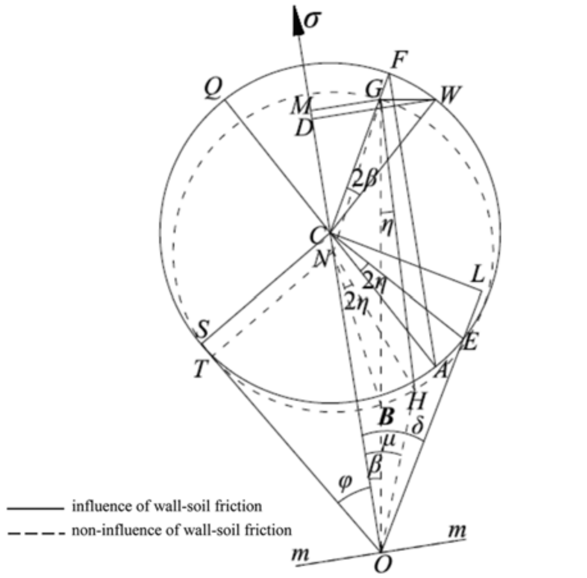

Fig. 2. Mohr-Coulomb Strength Line and Mohr Stress Circle for $\delta \geq \mu$

reaches δ, wall and backfill are separate, and the inclined angle would not further increase, thereby the inclined angle of the active earth pressure does not reach μ, and $\overline{OH}$ doesn't represent the active earth pressure. Thus, when μ isn't equal to δ, the dashed circle doesn't represent Mohr stress circle of backfill. When $\mu \neq \delta$, the direction of the total stress on $\overline{ac}$ isn't vertical, so the vertical stress is only a component stress of the total stress in the vertical direction.

Considering wall-soil friction angle and active limit equilibrium state of backfill, the Mohr stress circle is drawn as follows. Let $\overline{OE}$ at the angle δ to $\overline{OC}$, then let continuous circle, of which the point C of the line $\overline{OC}$ is center, intersect $\overline{OE}$ at the point E, and is tangential to $\overline{OS}$ at the point S. So $\overline{OE}$ represents the total stress acted on the wall back, that is the active earth pressure σ_a. Let $\overline{CA}$ at the angle 2η to $\overline{CE}$, and $\overline{CA}$ intersects the continuous circle at the point A, so $\overline{OA}$ represents the total stress acted on plumb place. And let $\overline{AF}$ parallel to $\overline{OC}$ and intersect the continuous circle at the point F, so $\overline{OF}$ represents the total stress acted on horizontal place. Then let $\overline{CW}$ at the angle 2β to $\overline{CF}$, and $\overline{CW}$ intersects the continuous circle at the point W, so $\overline{OW}$ represents the total stress acted on the inclined plane at the angle β to the horizontal, $\overline{OG}$ represents the vertical components stress of the total stress $\overline{OW}$, and $\overline{CW}$ represents the horizontal components stress. Well then, the continuous circle for $\mu < \delta$ shown in Fig. 2 represents Mohr stress circle of the non-cohesive backfill considering wall-soil friction.

Figure 2 shows the Mohr-Coulomb strength line and Mohr stress circle for $\delta \geq \mu$. Similarly, those for $\delta \leq \mu$ are obtained as shown in Fig. 3.

2.1 Active Earth Pressure

According to the law of cosines in ΔOCE, we can get the active earth pressure

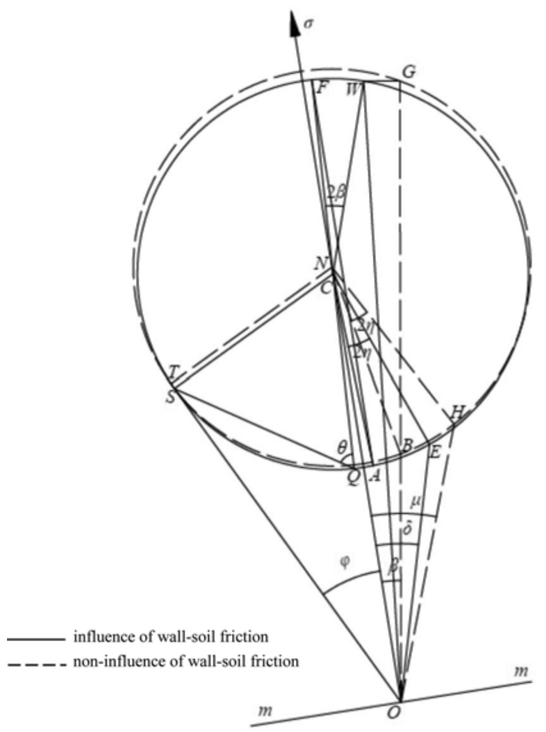

Fig. 3. Mohr-Coulomb Strength line and Mohr Stress Circle for $\delta \le \mu$

$$\sigma_a = \overline{OE} = \sqrt{\overline{CE}^2 + \overline{OC}^2 - 2\overline{OC}\cdot\overline{CE}\cdot\cos\angle OCE} \tag{1}$$

Similarly, we can get in ΔOCW

$$\overline{OW} = \sqrt{\overline{CW}^2 + \overline{OC}^2 + 2\overline{OC}\cdot\overline{CW}\cdot\cos\angle DCW} \tag{2}$$

Assumed that the radius of continuous circle in Fig. 2 is R, the following relations can be got

$$\overline{CA} = \overline{CS} = \overline{CE} = \overline{CF} = \overline{CW} = R \tag{3}$$

$\overline{OS}$ is tangential to the continuous circle, then $\overline{OS} \perp \overline{CS}$, and then we can get

$$\overline{OC} = \frac{\overline{CS}}{\sin\varphi} = \frac{R}{\sin\varphi} \tag{4}$$

Let $\overline{CL} \perp \overline{OE}$ through the point C, and $\overline{CL}$ intersect $\overline{CE}$ at the points L, then we can get

$$\angle CEL = \arcsin\frac{\overline{CL}}{\overline{CE}} = \arcsin\frac{\overline{OC}\cdot\sin\delta}{\overline{CE}} = \arcsin\frac{\sin\delta}{\sin\varphi} \tag{5}$$

According to the exterior angle theorem in ΔOCE, the following relation can be got

$$\angle OCE = \angle CEL - \delta = \arcsin\frac{\sin\delta}{\sin\varphi} - \delta \tag{6}$$

Because of $\overline{AF} // \overline{OC}$ and the isosceles triangle ΔCAF, we can get

$$\angle DCF = \angle OCA \tag{7}$$

From Fig. 2, we can get

$$\angle OCA = \angle OCE - 2\eta \tag{8}$$

and

$$\angle DCW = \angle DCF + 2\beta \tag{9}$$

Simultaneous Eqs. (6-9) can yield

$$\angle DCW = \arcsin\frac{\sin\delta}{\sin\varphi} - \delta - 2\eta + 2\beta \tag{10}$$

When $\mu \ne \delta$, $\overline{OW}$ represents the total stress σ_w on $\overline{ac}$, and $\overline{OG}$ represents the vertical components stress σ_{wv} of the total stress σ_w, and $\overline{GW}$ represents the horizontal components stress σ_{wh} of the total stress σ_w. Let $\overline{MG} \perp \overline{OC}$ and $\overline{DW} \perp \overline{OC}$, then $\overline{OD}$ represents the normal stress on $\overline{ac}$, and $\overline{DW}$ represents the tangential stress on $\overline{ac}$. $\overline{OM}$ and $\overline{MG}$ respectively represents the components stresses of the vertical stress σ_v at the vertical and the horizontal. Thus

$$\overline{GW} = \sqrt{\left(\overline{DW} - \overline{MG}\right)^2 + \overline{MD}^2} \tag{11}$$

So that

$$\tan\angle DWG = \frac{\overline{MD}}{\overline{DW} - \overline{MG}} = \frac{\overline{GW}\cdot\sin\beta}{\overline{GW}\cdot\cos\beta} = \tan\beta$$

Namely

$$\angle DWG = \beta \tag{12}$$

According to Eq. (12) and $\overline{DW} \perp \overline{OC}$, we can get $\overline{GW} \perp \overline{OG}$. So $\overline{GW}$ represents the horizontal components stress σ_{wh} of $\overline{OW}$, that is $\sigma_{wh} = \overline{GW}$, and we can get the total stress σ_w on $\overline{ac}$

$$\sigma_w = \overline{OW} = \frac{\overline{OG}}{\cos\angle GOW} \tag{13}$$

$\overline{OG}$ represents the vertical stress on $\overline{ac}$, and can be get as follows (Terzaghi, 1943; Chu, 1991; Mazindrani and Ganjali, 1997):

$$\overline{OG} = \gamma z\cdot\cos\beta \tag{14}$$

From Fig. 2, we can get

$$\angle COW = \angle GOW + \beta \tag{15}$$

According to the exterior angle theorem in ΔOCW, we can get

$$\angle DCW = \angle CWO + \angle COW = \arcsin\frac{\sin\angle COW}{\sin\varphi} + \angle COW \tag{16}$$

Substitution of Eqs. (10) and (15) into Eq. (16) yields

$$\arcsin\frac{\sin\left(\angle GOW + \beta\right)}{\sin\varphi} + \angle GOW + \beta = \arcsin\frac{\sin\delta}{\sin\varphi} - \delta - 2\eta + 2\beta \tag{17}$$

Solving Eq. (17) can yield

$$\angle GOW = \arctan\frac{\sin\varphi\sin\left(\arcsin\frac{\sin\delta}{\sin\varphi} - \delta - 2\eta + 2\beta\right)}{1+\sin\varphi\cos\left(\arcsin\frac{\sin\delta}{\sin\varphi} - \delta - 2\eta + 2\beta\right)} - \beta \tag{18}$$

Substituting Eqs. (14) and (18) into Eq. (13) and simplifying can yield

$$\overline{OW}=\frac{\sqrt{1+\sin^2\varphi+2\sin\varphi\cos\left(\arcsin\frac{\sin\delta}{\sin\varphi}-\delta-2\eta+2\beta\right)}}{\cos\beta+\sin\varphi\cos\left(\arcsin\frac{\sin\delta}{\sin\varphi}-\delta-2\eta+\beta\right)}\gamma z\cdot\cos\beta \tag{19}$$

Dividing Eq. (1) by Eq. (2) and substituting of Eqs. (3, 4, 6, 10 and 19) can yield

$$\begin{aligned}\sigma_a&=\overline{OW}\sqrt{\frac{1-2\sin\varphi\cos\angle OCE+\sin^2\varphi}{1+2\sin\varphi\cos\angle DCW+\sin^2\varphi}}\\&=\frac{\sqrt{1-2\sin\varphi\cos\left(\arcsin\frac{\sin\delta}{\sin\varphi}-\delta\right)+\sin^2\varphi}}{\cos\beta+\sin\varphi\cos\left(\arcsin\frac{\sin\delta}{\sin\varphi}-\delta-2\eta+\beta\right)}\gamma z\cdot\cos\beta\end{aligned} \tag{20}$$

Equation (20) is the formula of active earth pressure for $\delta\geq\mu$, according to Fig. 2. Similarly, according to Fig. 3, the formula of active earth pressure for $\delta\leq\mu$ can be derived and is the same as Eq. (20). Therefore, Eq. (20) is fit to $\delta\geq\mu$ and $\delta\leq\mu$.

Simplified Eq. (20), an analytical solution of the active earth pressure is presented as follows:

$$\sigma_a=\frac{\gamma z\left(\cos\delta-\sqrt{\cos^2\delta-\cos^2\varphi}\right)\cos\beta}{\cos\beta+\sqrt{\cos^2\delta-\cos^2\varphi}\cos(\delta+2\eta-\beta)+\sin\delta\sin(\delta+2\eta-\beta)} \tag{21}$$

Dividing Eq. (21) by γz can yield a new coefficient of active earth pressure as follows:

$$K_a=\frac{\sigma_a}{\gamma z}=\frac{\left(\cos\delta-\sqrt{\cos^2\delta-\cos^2\varphi}\right)\cos\beta}{\cos\beta+\sqrt{\cos^2\delta-\cos^2\varphi}\cos(\delta+2\eta-\beta)+\sin\delta\sin(\delta+2\eta-\beta)} \tag{22}$$

So an analytical solution of the normal active earth pressure on the wall back is presented as follows:

$$\begin{aligned}\sigma_{na}&=\sigma_a\cos\delta\\&=\frac{\gamma z\left(\cos\delta-\sqrt{\cos^2\delta-\cos^2\varphi}\right)\cos\beta\cos\delta}{\cos\beta+\sqrt{\cos^2\delta-\cos^2\varphi}\cos(\delta+2\eta-\beta)+\sin\delta\sin(\delta+2\eta-\beta)}\end{aligned} \tag{23}$$

2.2 Active Rupture Angle

Active rupture angle θ is an angle between the rupture surface and the horizontal under the active limit state. $\overline{OS}$ is Mohr-Coulomb strength line, and the continuous Mohr stress circle is tangential to $\overline{OS}$ at the point S, then $\overline{OS}$ represents a total stress acted on the rupture surface. So $\overline{OS}\perp\overline{CS}$. Let $\overline{CQ}$, which and $\overline{CA}$ is symmetrical on $\overline{OC}$, intersect the continuous circle at the point Q, Then the point Q represents a stress on the vertical line in the backfill. So the active rupture angle is obtained as follows:

$$\angle SQC=\theta \tag{24}$$

From the isosceles triangle ΔSCQ as shown in Fig. 2, we can get

$$\angle SQC=\frac{\pi-\angle SCQ}{2}=\frac{\pi-\angle SCO+\angle OCQ}{2} \tag{25}$$

$\overline{CQ}$ and $\overline{CA}$ be symmetrical on $\overline{OC}$, then we can get

$$\angle OCQ=\angle OCA \tag{26}$$

Simultaneous Eqs. (6) and (8) yields

$$\angle OCA=\arcsin\frac{\sin\delta}{\sin\varphi}-\delta-2\eta \tag{27}$$

Substituting Eq. (27) into Eq. (26) yields

$$\angle OCQ=\arcsin\frac{\sin\delta}{\sin\varphi}-\delta-2\eta \tag{28}$$

According to $\overline{OS}\perp\overline{CS}$ and the interior angle theorem in ΔOCS, we can get

$$\angle SCO=\frac{\pi}{2}-\varphi \tag{29}$$

Substituting Eqs. (24, 28) and (29) into Eq. (25), a formula of the active rupture angle is presented as follows

$$\theta=\frac{\pi}{4}+\frac{1}{2}\left(\varphi-\delta-2\eta+\arcsin\frac{\sin\delta}{\sin\varphi}\right) \tag{30}$$

2.3 Active Earth Force

By integrating Eq. (21) with respect to z, the active earth force can be yielded

$$\begin{aligned}E_a&=\int_0^{H\cos(\beta-\eta)/\cos\beta/\cos\eta}\sigma_a\mathrm{d}z\\&=\frac{1}{2}\gamma H^2\frac{\left(\cos\delta-\sqrt{\cos^2\delta-\cos^2\varphi}\right)\cos^2(\beta-\eta)\sec^2\eta\sec\beta}{\cos\beta+\sqrt{\cos^2\delta-\cos^2\varphi}\cos(\delta+2\eta-\beta)+\sin\delta\sin(\delta+2\eta-\beta)}\end{aligned} \tag{31}$$

3. Discussions

3.1 The Case of $\delta=\beta=\eta=0$

For $\delta=\beta=\eta=0$, the expression (21) of active earth pressure can be turned into

$$\sigma_a=\gamma z\tan^2\left(\frac{\pi}{4}-\frac{\varphi}{2}\right) \tag{32}$$

Substituting $\delta=\beta=\eta=0$ into Eq. (30) can yield the active rupture angle:

$$\theta=\frac{\pi}{4}+\frac{\varphi}{2} \tag{33}$$

Expressions (32 and 33) are the same as formulae of Rankine active earth pressure and Rankine (1857) active rupture angle. So Rankine active earth pressure is a special solution of this study.

3.2 The Case of $\delta = \beta = 0$

For $\delta = \beta = 0$, the active earth pressure by expression (21) can be got as follows

$$\sigma_a = \frac{1-\sin\varphi}{1+\sin\varphi\cos 2\eta}\gamma z \tag{34}$$

Substituting $\delta = \beta = 0$ into Eq. (30), the active rupture angle can be got as follows

$$\theta = \frac{\pi}{4}+\frac{\varphi}{2}-\eta \tag{35}$$

3.3 The Case of $\delta = \eta = 0$

For $\delta = \eta = 0$, the rupture angle by expression (30) is Rankine rupture angle, and the active earth pressure by expression (21) is Rankine earth pressure

3.4 The case of $\beta = \eta = 0$

For $\beta = \eta = 0$, the active earth pressure by expression (21) can be got as follows

$$\sigma_a = \frac{\gamma z\left(\cos\delta-\sqrt{\cos^2\delta-\cos^2\varphi}\right)}{1+\sqrt{\cos^2\delta-\cos^2\varphi}\cos\delta+\sin^2\delta} \tag{36}$$

Substituted of $\beta = \eta = 0$ into Eq. (31), the active rupture angle can be got as follows

$$\theta = \frac{\pi}{4}+\frac{1}{2}\left(\varphi-\delta+\arcsin\frac{\sin\delta}{\sin\varphi}\right) \tag{37}$$

3.5 The Case of $\delta = 0$

For $\delta = 0$, the active rupture angle by expression (30) is Eq. (35), and the active earth pressure by expression (21) can be got as follows

$$\sigma_a = \frac{\gamma z(1-\sin\varphi)\cos\beta}{\cos\beta+\sin\varphi\cos(2\eta-\beta)} \tag{38}$$

3.6 The Case of $\eta = 0$

For $\eta = 0$, the active rupture angle by expression (30) is Eq. (37), and the active earth pressure by expression (21) can be got as follows

$$\sigma_a = \frac{\gamma z\left(\cos\delta-\sqrt{\cos^2\delta-\cos^2\varphi}\right)\cos\beta}{\cos\beta+\sqrt{\cos^2\delta-\cos^2\varphi}\cos(\delta-\beta)+\sin\delta\sin(\delta-\beta)} \tag{39}$$

3.7 The Case of $\beta = 0$

For $\beta = 0$, the active rupture angle by expression (30) is Eq. (30), and the active earth pressure by expression (21) as follows

$$\sigma_a = \frac{\gamma z\left(\cos\delta-\sqrt{\cos^2\delta-\cos^2\varphi}\right)}{1+\sqrt{\cos^2\delta-\cos^2\varphi}\cos(\delta+2\eta)+\sin\delta\sin(\delta+2\eta)} \tag{40}$$

3.8 The Case of $\delta = \mu$

From the dashed circle in Fig. 2 or Fig. 3, we can get

$$\angle ONH = \arcsin\frac{\sin\beta}{\sin\varphi}-\beta+2\eta \tag{41}$$

and

$$\angle ONH = \arcsin\frac{\sin\mu}{\sin\varphi}-\mu \tag{42}$$

Substituting Eq. (41) into Eq. (42) can yield

$$\arcsin\frac{\sin\mu}{\sin\varphi}-\mu = \arcsin\frac{\sin\beta}{\sin\varphi}-\beta+2\eta \tag{43}$$

From Eq. (43), $\mu = \beta$ for $\eta = 0$, and $\mu > \beta$ for $\eta > 0$. Solving above equation can yield for $\eta>0$:

$$\mu = \arctan\frac{\sin\varphi\sin\left(\arcsin\frac{\sin\beta}{\sin\varphi}-\beta+2\eta\right)}{1-\sin\varphi\cos\left(\arcsin\frac{\sin\beta}{\sin\varphi}-\beta+2\eta\right)} \tag{44}$$

Substituting $\delta = \mu$ into Eq. (43) can yield

$$\arcsin\frac{\sin\delta}{\sin\varphi}-\delta = \arcsin\frac{\sin\beta}{\sin\varphi}-\beta+2\eta \tag{45}$$

Substituting Eq. (45) into Eqs. (30) can yield the active rupture angle for $\delta = \mu$

$$\theta = \frac{\pi}{4}+\frac{\varphi-\beta}{2}+\frac{1}{2}\arcsin\frac{\sin\beta}{\sin\varphi} \tag{46}$$

From Fig. 2 or Fig. 3 $\overline{OH}$ represent the active earth pressure. Substituting Eq. (45) into Eq. (21) can yield the active earth pressure for $\delta = \mu$

$$\sigma_a = \frac{\sqrt{\left[\cos(\beta-2\eta)-\sqrt{\cos^2\beta-\cos^2\varphi}\right]^2+\left[\sin\beta-\sin(\beta-2\eta)\right]^2}}{\cos\beta+\sqrt{\cos^2\beta-\cos^2\varphi}}\gamma z\cos\beta \tag{47}$$

Substituting $\eta = 0$ into Eq. (48) can yield the same expression of the active earth pressure with other methods by Mazindrani and Ganjali (1997) and Chu (1991).

3.9 The Case of $\delta \neq \mu$

From the dashed circle in Fig. 2 or Fig. 3, the active earth pressure which is represented by $\overline{OE}$ for $\delta > \mu$ is more than that for $\delta = \mu$, but can the active earth pressure for $\delta < \mu$ is less than that for $\delta = \mu$. To security, the active earth pressure for $\delta < \mu$ is suggested to be calculated using Eq. (47).

3.10 Comparisons

In order to verify the accuracy of the presented method by this

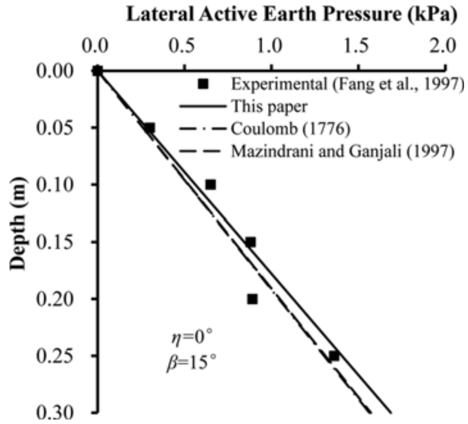

Fig. 4. Comparison between Calculated And Measured Distributions of Lateral Active Earth Pressure

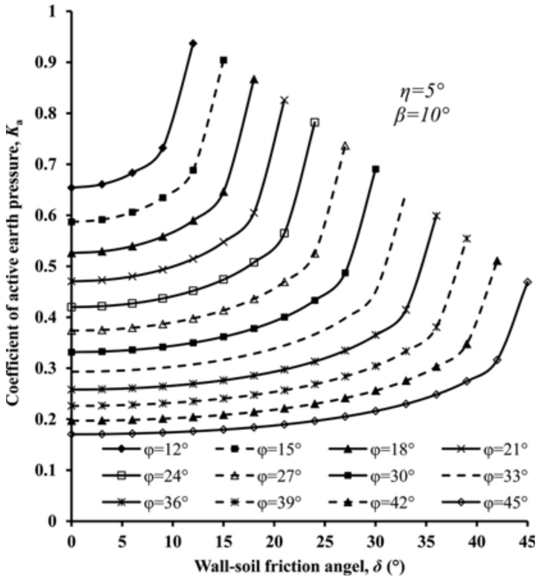

Fig. 5. Variation of Coefficient of Active Earth Pressure with δ and φ

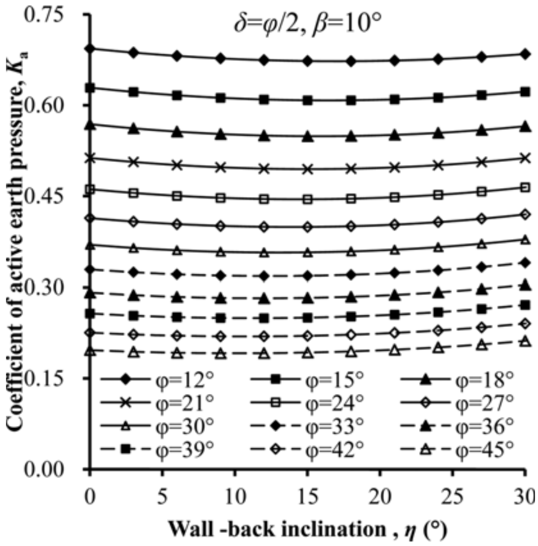

Fig. 6 .Variation of Coefficient of Active Earth Pressure with η

study, the predications of the active earth pressure by the new and existing methods are compared with existing test results by Fang *et al.* (1997). Fang *et al.* (1997) measured the horizontal earth pressure acting against a vertical rigid retaining wall at five different depths (0.05, 0.10, 0.15, 0.20 and 0.25 m) from backfill surface, and this retaining wall horizontally moved away from a mass of dry sand with an inclined surface. In the existing small-scale experiment by Fang *et al.* (1997), the internal friction angle φ was 30.9°, the backfill inclination β was 15°, the wall-back inclination η was 0, the backfill unit weight g was 15.5 kN/m^3, the wall-soil friction angle δ was 19.2°, and the wall height was 0.3 m.

Figure 4 shows the distributions of the normal active earth pressure that are calculated by using Eq. (23) and measured by Fang *et al*. (1997). The Fig. 4 also shows the distributions that are obtained by using the analyses of Coulomb (1776) and Mazindrani and Ganjali (1997) or Chu (1991). In Fig. 4, the predicted values by this study and existing methods (Coulomb 1776; Mazindrani and Ganjali, 1997) are mostly less than the measured values, and the calculated values by Mazindrani and Ganjali (1997) are slightly larger 0.58% than those by Coulomb (1776). And the average predicted value by this study is larger 1.4% than the average measured value, but the average predicted value by Coulomb (1776) and Mazindrani and Ganjali (1997) are smaller 7.84% and 7.16% than the average measured value respectively. So, the predicted values by this study are more approximate to the measured values than those by Coulomb and Mazindrani & Ganjali.

4. Parametric Studies

The active earth pressure σ_a calculated by presented formula is the product of K_a and γz. K_a calculated by presented formula is independent of γ and z, but is dependent on φ, δ, β and η. Therefore, the variations of σ_a with φ, δ, β and η are the same as those of K_a as shown in Figs. 5-7. And the variations of active rupture angle calculated by presented formula are shown in Figs. 8 and 9.

4.1 Internal Friction Angle and Wall-soil Friction Angle

Figure 5 shows the variation of coefficient of active earth pressure with φ and δ for η = 5° and β = 10°. From Fig. 5, it is found that: the coefficient of the active earth pressure calculated by Eq. (22) increases as δ increases, decreases as φ increases, and its increase rate increases as δ increases.

4.2 Wall-back Inclination

Figure 6 shows the variation of coefficient of active earth pressure with η for different φ when $\delta = \varphi/2$ and β = 10°. From Fig. 6, it is found that: coefficient of active earth pressure first decreases and then increases as η increases for $\delta = \varphi/2$ and β = 10°, and it decreases as φ increases; η yielding the minimal value

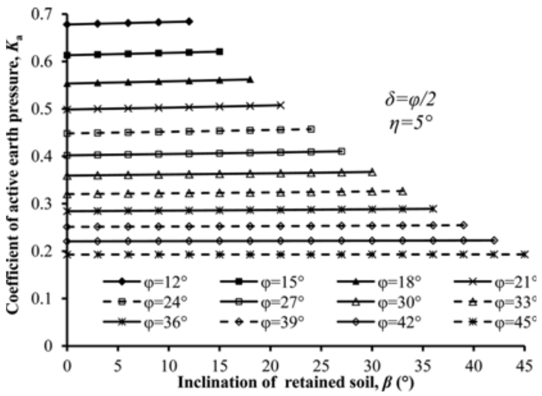

Fig. 7. Variation of Coefficient of Active Earth Pressure with β

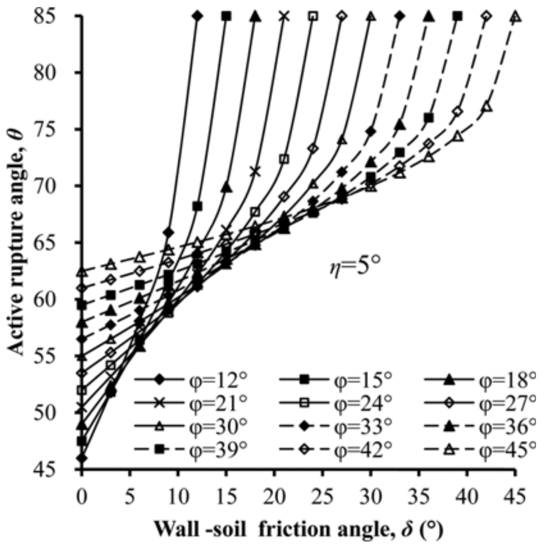

Fig. 8. Variation of Active Rupture Angle with Wall-soil Friction Angle

of K_a increases from 18° to 9° as φ increases from 12° to 45°.

4.3 Backfill-surface Inclination

Figure 7 shows the variation of coefficient of active earth pressure with β for different φ when $\delta = \varphi/2$ and $\eta = 5°$. From Fig. 7, it is found that coefficient of active earth pressure slowly increases as β increases for $\delta = \varphi/2$.

4.4 Active Rupture Angle

Figure 8 and Fig. 9 show the variation of the active rupture angle with the wall-soil friction angle, backfill internal friction angle and the δ/φ ratio. From Fig. 8, Fig. 9 and Eq. (30), it is found that: the active rupture angle respectively non-linearly increases as δ and φ decrease as well as δ/φ, and linearly decreases as η increases, but is independent of β.

5. Conclusions

In this study, the concept of Rankine earth pressure theory is adopted, and the vertical stress on upper surface of differential triangle element is thought to be not total stress but component

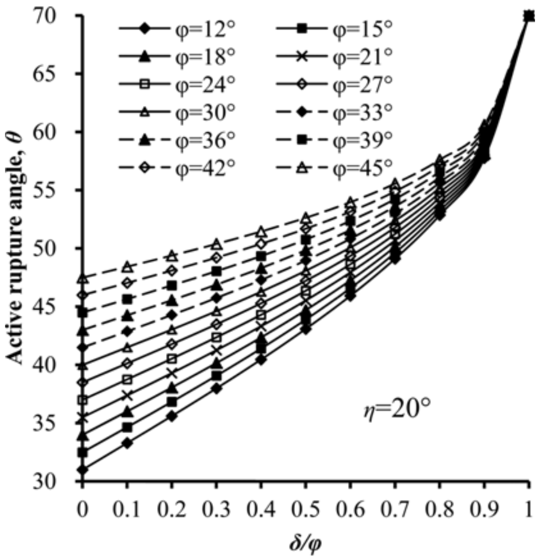

Fig. 9. Variation of Active Rupture angle with δ/φ Ratio

stress, due to wall-soil friction. According to Mohr stress circle and Mohr-Coulomb strength theory, the new formula for calculating active earth pressure against inclined rigid retaining walls with inclined non-cohesive backfill is proposed. It shows that the active earth pressure depends on the internal friction angle and unit weight of backfill, wall-soil friction angle, wall-back inclination and backfill. The new formulae of the active rupture angle and the coefficient of active earth pressure are also derived.

The effects of parameters on the presented formulae are investigated, and special solutions of presented formulae of the active earth pressure and active rupture angle are discussed under various special conditions. The active earth pressure or its coefficient using proposed formulas in this study respectively increases as δ increases, decreases as φ increases, and first decreases and then increases as η increases for $\delta=\varphi/2$ and $\beta=10°$, slowly increases as β increases for $\delta=\varphi/2$

To validate this analysis, the predicted values by the presented formula and existing methods are compared with existing experimental results, the comparison shows that the presented formula satisfactorily predicts active earth pressure.

Notations

dz = Thickness of differential triangle element
E_a = Active earth force on wall back
H = Height of the rigid retaining wall
K_a = New coefficient of active earth pressure
z = Depth from surface of backfill
β = Angle of surface of backfill to the horizontal
δ = Wall-soil friction angle
γ = Unit weight of the backfill
η = Angle of wall back to plumb line
φ = Internal friction angle of backfill
σ_a = Active earth pressure on wall back

σ_{na} = Normal stresses of the active earth pressure
σ_R = Reaction stress on sliding surface
σ_w = stress on upper surface of differential triangle element
θ = Angle of active sliding surface to the horizontal

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