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Developing Short Term Drought Severity-Duration-Frequency Curves for Kerala Meteorological Subdivision, India Using Bivariate Copulas

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Abstract

Severity-duration-frequency (S-D-F) curve is a useful hydrological tool for drought mitigation. This paper presents a bivariate copula approach for developing SDF relationships for Kerala meteorological subdivision, India based on the three month standardized precipitation index (SPI-3). First the univariate models were developed considering Gamma and Weibull distributions to represent the probability of severity and duration of SPI-3 series of the subdivision prepared from the area weighted precipitation data. The joint association of these drought variables is modeled by three Archimedean copulas namely Clayton, Frank and Gumbel-Hougaard (GH). The results showed that the difference between the joint SDF relations with the type of copulas is marginal upto return period 100 (~) whereas the conditional dependency of drought variables showed distinct differences. The conditional SDF relationships between severity and return period showed larger differences for longer duration and the behaviour is identical in the relationships between duration and return period for conditional severity values. Further, to outline the spatial variability of short term drought, a detailed examination on SDF relations of different districts in Kerala is made by considering representative fine resolution gridded rainfall (0.25° × 0.25°) datasets. SDF relationships of different locations in Kerala showed a shorter duration drought of less severity magnitude is prominent in districts like Wayanad, Kottayam and Pathanamthitta. The highest value of equilibrium severity for longer drought duration for all return periods is noticed for Palakkad while lowest value is observed for the district of Wayanad, which are the two low rainfall regions of the subdivision.

Keywords: *drought, severity, duration, return period, copulas*

1. Introduction

Drought is one of the less attended natural calamities which represents a period of under-average rainfall at a region and leads to scarcity of water resource in different forms. Drought is one of the most complex and less understood among the different natural hazards which affects more people than any other hazard. Indian economy is highly dependent on agricultural production which in turn heavily relies on natural rainfall. Any anomaly in the rainfall could lead to droughts and severe impact on the economy of the country. Meteorological drought is quantified mainly based on the precipitation available at a location with respect to the normal precipitation (IFAS, 1998). The reduction in precipitation may lead to crop failure, create disturbance to habitats, lead to soil degradation and subsequently to desertification (Nicholson *et al.,* 1990; Pickup, 1998) and short term drought has severe impacts on the agricultural sector of any region. Therefore, monitoring, prediction and analysis of drought are important for agricultural planning and management. In the past, many studies centered on characterization and variability of droughts in different parts of India (Pai *et al.,* 2011; Reddy and Ganguli, 2012; Ganguli and Reddy, 2014; Thomas *et al*., 2015; Mallya *et al*., 2016). Understanding of the drought conditions over an area, its evaluation and forecasting is made possible by the researchers with the introduction of drought indices. Numerous drought indices have been proposed over the years which were formulated based on primary factors leading to drought such as precipitation, evapotranspiration, vegetation conditions etc. (Zargar *et al.,* 2011). However the drought index based on precipitation namely the Standardized Precipitation Index (SPI) proposed by McKee *et al.* (1993) is the most popular index owing to the flexibility of time scale, the requirement of fewer input variables, and the ease in estimation (Bazrafshan *et al.,* 2013). Moreover, Kerala being a monsoon dominated state, the drought characterization based on precipitation is an appropriate choice and of great significance in the management of its water

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resources. SPI of different time scales (3 months, 6 months, 12 months etc.) refers short-term or long-term droughts, which significantly influence different components of hydrologic cycle. For example, the medium to long term indices (SPI-6 and SPI-12) have implications on stream flow and ground water components respectively, while the seasonal drought index SPI-3 influences soil moisture. Water stress and crop failures may be more sensitive to the short term 3-month (Thomas *et al.,* 2015), which may be more crucial for monsoon dominated and agronomy dependent regions.

For the mitigation of drought an effective tool is essential for which the SPI based characterization of drought is helpful. Droughts are generally characterized by its severity, duration and frequency of occurrences. Severity of drought indicates the degree to which the rainfall is lower than a specified threshold, i.e., the deficit of rainfall (precipitation) accumulated over the duration of drought. Drought duration is the period when there is a shortage of precipitation preceded and followed by periods of no deficiency. The severity–duration–frequency (S-D-F) relationships depicted as a 3-D plot is helpful in estimating the potential risk of occurrences of droughts in future. Univariate analysis were used earlier for investigating the drought properties (Tallaksen *et al.,* 1997; Chung and Salas, 2000; Cancelliere and Salas, 2004) while some researchers used conventional practices such as extreme value analysis or theory of run for developing the SDF relations (Dalezios *et al.,* 2000; Saghafian *et al.,* 2003). But as the drought variables (such as severity and duration) are highly correlated, a more reliable alternative approach which can trace the joint dependency of drought variables is desirable to prepare SDF curves. Copulas are such mathematical functions which can model the joint dependency of candidate variables irrespective of the type of their probability distributions. Hence copulas have been widely used for frequency analysis of drought in many countries (Shiau *et al*., 2006, 2007; Shiau and Modaress, 2009; Mirabbassi *et al.,* 2011; Reddy and Ganguli, 2012; Yusof *et al*., 2013; Lee *et al*., 2013; Rauf and Zeephongsekul, 2014; Kwak *et al.,* 2015; Zhang *et al.,* 2015; Tosunoglu and Can, 2016). Owing to the simplicity, most of the studies used bivariate approach while a few studies followed trivariate or multivariate approaches (Chen *et al.,* 2013; Saghafian and Mehdikhani, 2014; Liu *et al.,* 2016) for drought characterization.

The climatology of the monsoon dominated state of Kerala is unique from many other part of the country and any imbalance in the monsoon may affect the hydrology and agriculture of the region. A detailed examination of the rainfall variability of Kerala was made by different researchers in the past (Krishnakumar *et al.,* 2009; Nair *et al*., 2015; Adarsh and Janga Reddy, 2015), while Thomas and Prasannakumar (2016) presented an extensive study on rainfall trend and drought characteristics of Kerala. Also the recent media reports show that for the past few years Kerala has been declared as drought prone by the state and/or central government, and in the present year (2017) there is 30% reduction in monsoon rainfall, which shows the urgent necessity of preparation of SDF curve for the state of Kerala as a management measure of its water resources. Moreover, an in-depth analysis on the similarity or differences of SDF curves for different locations in Kerala is essential for understating the spatial variability of drought characteristics of the subdivision. In view of above reasons, this paper presents Copula approach for derivation of drought SDF curves for Kerala based on 3-month SPI.

The next section presents materials and methods used in the study. Sect. 3 presents the data details, results of different analysis along with discussions etc. Sect. 4 presents the major conclusions drawn from the study.

2. Materials and Methods

This section presents the details of different methodologies used in this study.

2.1 Standardized Precipitation Index (SPI) and Drought Characteristics

SPI is a measure which has been used widely to identify droughts of different types such as agricultural and meteorological etc. The computation of SPI involves :

(i) preparation of aggregated precipitation series for specified accumulation time-scale (say 3 months, 6 months etc.);

(ii) fitting of Gamma distribution function upon this series. The Cumulative Distribution Function (CDF) is to be computed as $F_X(x) = q + (1-q)G_X(x)$ to account the zero values, as the two parameter Gamma distribution is not defined for zero precipitation values, where q is the probability of zero precipitation in historical records; $G_X(x)$ is the CDF of non zero precipitation records and $F_X(x)$ is the CDF of actual precipitation series

(iii) performing an equi-probability transformation between CDF of mixed distribution and standard normal distribution, to get the SPI for given time scale, i.e., $Z = \psi^{-1}(F_X(x))$, where $\psi^{-1}(.)$ is the inverse of the CDF. For the characterization of drought, SPI values (estimated in monthly scale) were used to develop the relationships between severity, duration and frequency of occurrences of drought.

Drought duration (D) is defined as the number of consecutive months where SPI remains below a specified threshold and the minimum duration of drought is 1 month. Drought severity (S) is the values of SPI accumulated within the duration of drought, which represents the magnitude of dry events. Generally severity of drought event i is represented as a positive entity in the form

$$S_i = -\sum_{t=1}^{D} SPI_{i,t}, \quad i = 1, 2, ..., N \tag{1}$$

where N is the number of observed data points. Frequency of drought represents the recurrence interval of a specific severity value occurs in the study region. The expected drought inter-arrival time $E(L)$ is an important property used in the copula based drought frequency analysis. The drought inter-arrival time is defined as the period between the beginning of a drought and the beginning of the next drought (Mirabbasi *et al*., 2011; Kwak *et al*., 2016) and the

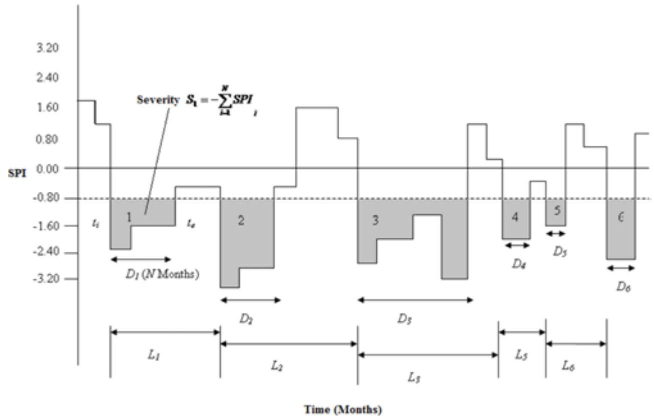

Fig. 1. Illustration of Drought Properties-severity (S), Duration (D) and Inter Arrival Time (L). Six Drought Events (1 to 6) are Shown by Shaded Regions in the Plot; t_i and t_e Show Initiation and Termination of the Drought Event 1, Its Severity is S_1 and Duration is N Months. Inter- Arrival Time between Successive Droughts are L_1, L_2, …, L_5 and the Expected Inter-arrival Time Can be Estimated as the Mean of These Values

Table 1. Classification of Drought Based on SPI (McKee *et al.*, 1993)

SPI value	Category
≥ 2	Extremely wet
1.5 to 1.99	Severely wet
1.00 to 1.49	Moderately wet
-0.99 to 0.99	Near normal
-1.00 to -1.49	Moderate drought
-1.5 to -1.99	Severe drought
≤ 2	Extreme drought

expected inter-arrival time $E(L)$ is the mean of inter arrival times of several drought events estimated over the study period.

The different drought characteristics and their estimation over a specific duration are presented in Fig. 1.

The S-D-F diagram of developed for a specific region can be used to determine when the region will experience severe drought and when the drought of specific duration may repeat in future. S-D-F curves are very much helpful for regional drought characterization and it serves as a useful tool for developing water management policies in drought affected areas (Ganguli and Reddy, 2014). Based on SPI values the drought can be classified and such a classification provided by McKee *et al*., (1993) is provided in Table 1.

2.2 Theory of Copulas

Copulas are joint distribution functions of standard uniform random variables. A bi-variate copula can be represented as:

$$C[0, 1]^2 \rightarrow [0, 1]$$

It should satisfy the following conditions:

(i) $C(1, u) = C(u, 1) = u$ and $C(u, 0) = C(0, u) = 0$;

(ii) $C(u_1,u_2) + C(v_1,v_2) - C(u_1,v_2) - C(v_1,u_2) \geq 0$, if $u_1 \geq v_1$, $u_2 \geq v_2$ and $u_1, u_2, v_1, v_2 \in [0, 1]$.

The Sklar's theorem (1958) is the foundation of the theoretical concept of copulas. A numerous families of copula have been proposed over the years, which include (i) Elliptical (normal and *t*); (ii) Archimedean (Clayton, Gumbel-Hougaard (GH), Frank, and Ali-Mikhail-Haq); (iii) Extreme Value (Gumbel, Husler-Reiss, Galambos, Tawn and *t*-EV); and (iv) other families (Plackett and Farlie-Gumbel-Morgenstern). Among which, Archimedean copula is more popular for hydrologic applications because of (i) easiness in its construction and implementation; (ii) availability of wide variety of formulations ; (iii) flexibility in application for both positively and negatively correlated variables; (iv) many attractive properties and uni-parameter property. In the present study, Frank, Clayton and Gumbel-Hougaard (GH) bivariate copulas are considered in the present study. A detailed theoretical background on copulas is presented by Nelsen (2006) and more details on statistical aspects of Archimedean family can be found in Genest and Rivest (1993).

2.3 Methodology of Developing SDF Curves

The overall framework of developing SDF curves using copulas is presented by many researchers (Kwak *et al*., 2012 a,b)

The different steps involved in the preparation of SPI-3 based SDFs using bivariate copulas are :

1. Compute SPI-3 at from the monthly rainfall data
2. Prepare severity (*S*) by setting a threshold SPI (say < -1.0) and duration (*D*) series
3. Fit appropriate probability distributions for severity and duration and their CDFs
4. Use Archimedean copula to obtain joint CDF of duration and severity by estimating the copula parameters
5. Determine the joint return periods of drought duration (*D*) and severity (*S*), which are defined for two cases : (i) T_{DS}, the return period for the case ($D \geq d$ and $S \geq s$) and (ii) $T'_{DS,}$ the return period for the case ($D \geq d$ or $S \geq s$).

The equations involved in their estimation are :

$$T_{DS} = \frac{E(L)}{P(D \geq d, S \geq s)} = \frac{E(L)}{1 - FD(d) - DFS(s) + C(FD(d), FS(s))} \quad (2)$$

$$T'_{DS} = \frac{E(L)}{P(D \geq d, or\ S \geq s)} = \frac{E(L)}{1 - C(FD(d), FS(s)} \quad (3)$$

In the above expressions, $E(L)$ is the expected inter arrival time of drought (Mirabbassi *et al.,* 2011).

The contour maps or surface plots can be used to demonstrate the joint return period for various drought, severity and duration. Another flexibility of copula based approach is its potential to provide conditional return periods, which have more practical appeal in preparing drought management plans and developing policies for the same (Mirabbassi *et al.,* 2011).

3. Results and Discussion

This section presents the particulars of estimation of drought variables and preparation of SDF relationships using copulas.

3.1 Data and Estimation of Drought Variables

Indian Institute of Tropical Meteorology (IITM), Pune (http://www.tropmet.res.in/) has defined 36 meteorological subdivisions in India. The state of Kerala (latitude 8°15' - 12°50' N and longitude 74°50' - 77°30'), popularly known as 'gateway of Indian monsoon' located in the south west of India is considered as one of the meteorological subdivision which receive an average annual rainfall of ~300 cm. The state is monsoon dominated and agro dependent, the soil moisture conditions are more sensitive to SPI-3, which in turn can accurately represent the crop failures and water stress. Therefore the 3-month SPI is considered for preparing SDF curves. The monthly rainfall values of Kerala for the period 1871-2014 collected from are used for the estimation of SPI-3. Here the benchmark SPI of -0.8 is fixed as the threshold and when the SPI value becomes less than this value it indicates a possible drought event. Here 90 such drought events are identified between 1871 and 2014. The statistical properties of drought variables are provided in Table 2.

The extracted severity values along with the corresponding duration (in months) are provided as scatter plot in Fig. 2(a). For the preparation of SDF curves, first, the CDFs of drought severity and duration (estimated from SPI-3 series) are developed. For selection of appropriate univariate distributions for modeling drought severity and duration, first four candidate distributions log normal, Gamma, exponential and Weibull are tested for fitting the respective series. Also, an empirical CDF (ECDF) is fitted for the aforementioned series using the popular Gringorten plotting position formula (Gringorten, 1963). The differences of the CDFs fitted by the candidate distribution from the ECDF are quantified in terms of Mean Square Error (MSE) statistics and the Kolmogorov-Smirnov (K-S) statistics. Then the appropriateness of the distribution is assessed by estimating Akaike Information Criteria (AIC) (Akaike, 1974) and maximum deviation (d_{max}) criteria. The results are provided in Table 3.

Table 2. Statistical Properties of Characteristics Short Term Drought in Kerala. Statistical Properties of Duration Except CV are in Months

Statistical Property	Drought variable	
	Severity	Duration
Mean	2.863	4
Standard Deviation	2.608	2.16
Coefficient of variation (CV) (in %)	91.08	54.01
Minimum	0.823	1
Maximum	12.379	7

Table 3. Comparison of Performance of Different Probability Distributions for Fitting Short-term Drought Severity and Duration Series of Kerala Subdivision. The Figures in Italics Indicate the Best Estimates

Parameter	Distribution	MSE	AIC	d_{max}
Severity				
	Log normal	0.0061	-454.93	0.1265
	Gamma	*0.0044*	*-484.42*	*0.0709*
	Weibull	0.0066	-447.83	0.1446
	Exponential	0.0071	-443.79	0.1613
Duration				
	Log normal	0.0028	-526.48	0.1555
	Gamma	0.0035	-505.26	0.1310
	Weibull	*0.0021*	*-549.56*	*0.1245*
	Exponential	0.0321	-307.45	0.1474

From Table 3, it is noted that for a sample size of N=90, the Gamma and Weibull gives the best estimate among the four candidate distributions with the least values for AIC (-484.92 and -549.56) and d_{max} (0.071 and 0.124) respectively for severity and duration series. It may be noted that at 5 % significance level the critical value for sample size of 90 is 0.141 and a value of d_{max} less than this threshold shows that the selected distributions are suitable for modeling severity and duration. The parameters of the gamma distribution are 1.7238 and 1.6612 (respectively the scale and shape) while the parameters of Weibull distribution are 4.52 and 2.1169. The plots of severity and duration series along with the respective CDFs are portrayed in Fig. 2.

3.2 Copulas for Developing SDF Curves

A probabilistic approach, copula is used to develop drought SDF curves which are functions of univariate distributions. The linear correlation between severity and duration for SPI-3 series is found to be 0.978, which clearly indicate a strong association

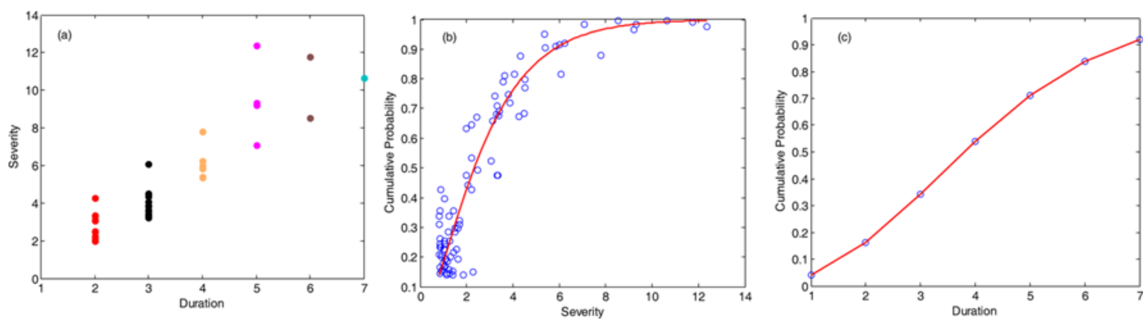

Fig. 2. (a) Scatter Plot between Severity and Duration, (b) the CDF of Severity Series of SPI-3 Along with Fitted Gamma Distribution, (c) the CDF of Duration Series

between the two series, which further indicate the necessity of following a bi-variate approach for modeling SDF relationships i.e., a joint dependency of severity and duration are to be considered in the modeling. The range of the dependence levels that a particular copula function can describe is an important factor influencing the appropriate selection of copulas, for estimation of which Kendall's τ can be used (Mirabbassi *et al.,* 2011). For example, the Gumbel-Hougaard (GH) copula is suitable only for positive dependence cases, while the Clayton and the Frank copulas can be used for cases of both positive and negative dependencies. The Ali-Mikhail-Haq copula is suitable for the range of Kendall's τ $-0.1807 < \tau < 0.3333$ (Nelsen, 2006). The Kendall's tau between the severity and duration series (for SPI-3) variables is found to be 1.002, which infer that Frank, Clayton and GH copulas in Archimedean family are suitable for the present dataset. Since the drought variables are modeled by different CDF's, copulas are used to connect the distributions fitted.

For estimation of copula parameters different methods such as method of moments (MoM), Exact Maximum Likelihood Method (EML), Inference From Margins (IFM), maximum pseudo-likelihood estimation method (MPL), Canonical Maximum Likelihood (CML) (Genest and Rivest, 1993; Joe 1997; Genest *et al.*, 1995; Ganguli and Reddy, 2012, 2014) etc can be used. In the MPL method, empirical distributions are used as marginal distribution of the dependent variables and no need to specify the type of distribution *apriori*, which may influence the copula parameter in particular if the number of observations is limited. In this method, first the marginal variables are transformed to uniformly distributed vectors using its empirical CDF. The rank based empirical CDF is computed as :

$$U_{i,d} = \frac{Ranked\ data\ of\ X_{i,d}}{N+1}, \forall i = 1,2,...,N \tag{4}$$

where $X_{i,d}$ refers the vectors of bivariate distribution (here $d = 1,2$ severity and duration)

The obtained CDF is substituted into bivariate copula density and on taking the logarithm in both sides of the expression provide the log-likelihood function in the form (Genest and Favre 2007):

$$\ln L_U(\theta) = \sum_{i=1}^{N} \ln\left[c_\theta\{U_{i,1}, U_{i,2}\}\right] = \sum_{i=1}^{N} \ln\left\{c_\theta\left(\frac{r_{1i}}{N+1}, \frac{r_{2i}}{N+1}\right)\right\}, \forall i = 1,2,...N \tag{5}$$

where r_{1i} and r_{2i} denote the ranks of the two dependent variables. The maximization of the pseudo log-likelihood function gives an estimate of copula parameter, which can be performed by numerical integration.

Then the joint CDF (JCDF) of the two drought variables are developed after estimating the expected inter arrival time as 0.9912 and copula parameter. The JCDFs derived by the three types of Copulas is presented in Fig. 3.

The copula parameters of are found to be 60.138, 22.12 and 9.23 respectively for Frank, Clayton and GH copula types. The surface plots of the joint return period-S-D relations by the three Copulas is provided in Fig. 4 and the contour plots of SDF curves prepared for specific return periods 2 years, 5 years, 10 years, 25 years, 50 years and 100 years are presented in Fig. 5.

Further to quantify the difference of estimates by the three

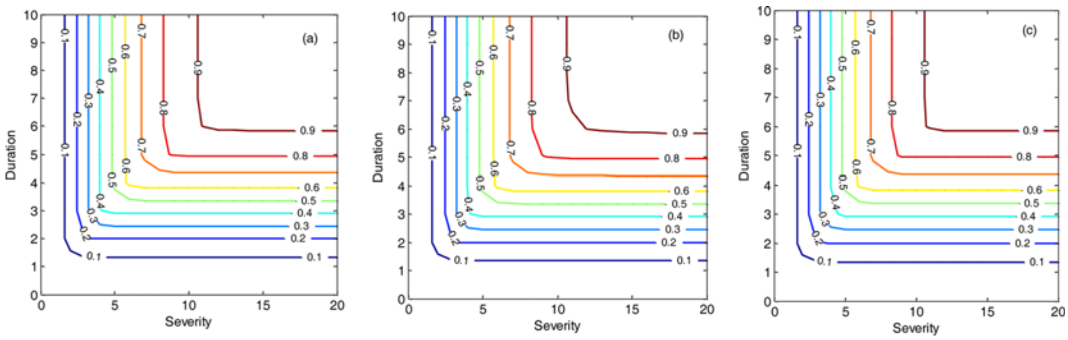

Fig. 3. Plots of Joint CDFs using the Three Copulas: (a) Frank, (b) Clayton, (c) GH

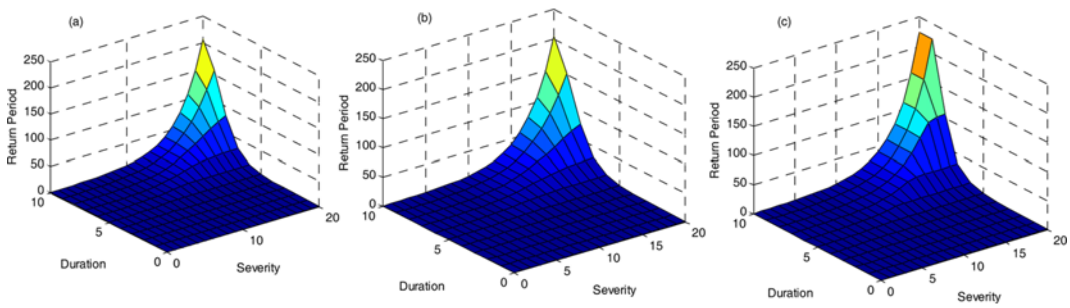

Fig. 4. Plots of Joint Return Period-S-D Relations of Drought in Kerala using Three Copulas: (a) Frank, (b) Clayton, (c) GH

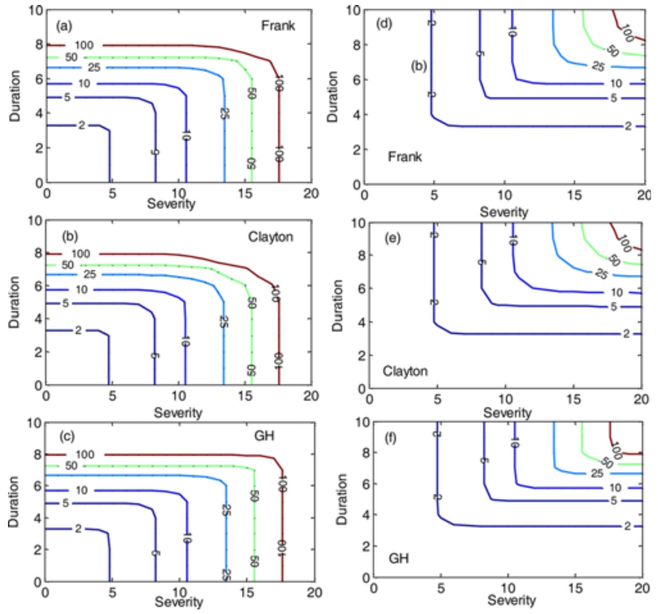

Fig. 5. The SDF Relations Considering Joint Return Periods (in years) of Drought in Kerala using Different Copulas : (a-c) T_{DS} ($D \geq d$ and $S \geq s$) (d-f) T'_{DS} ($D \geq d$ or $S \geq s$) in the form of Contour Plots

Table 4. Comparison of Performance of Different Copulas for the Characterization of Short Term Drought of Kerala Subdivision. The Figures in Italics Indicate the Best Estimates

Copula type	MSE	AIC	d_{max}
Frank	0.033	-151.506	0.141
Clayton	*0.020*	*-174.041*	*0.135*
GH	0.021	-171.845	0.137

copulas, an empirical CDF is fitted by using a modified form of Gringorten plotting position formula as follows

$$F_{S,D}(s,d) = P(S \leq s, D \leq d) = \frac{No\ \ of (s_j \leq s_i \ and\ d_j \leq d_i) - 0.44}{N + 0.12}, \quad \forall i,j = 1,2,....,90 \tag{6}$$

Further the AIC and K-S statistics are used to examine the best estimate copulas and the results are summarized in Table 4. From Table 4 it is noted that the Clayton copula provides the least AIC and d_{max} estimates with marginal differences from that of GH copula. But from joint return period-S-D relations plotted for the three types of copulas (Fig. 5) it is noted that only marginal difference exists in the severity values on considering the joint return period upto 100 years, and on the other hand, the SDF relationship show differences for the larger joint return periods (Fig. 4). Further, the equilibrium value of drought severity for longer durations (d) and a given return period can be estimated mathematically. Since $F_D(d)$ is approximately equal to unity for very large values of d, the equilibrium value of severity for given return period can be determined as (Shiau and Modaress, 2009):

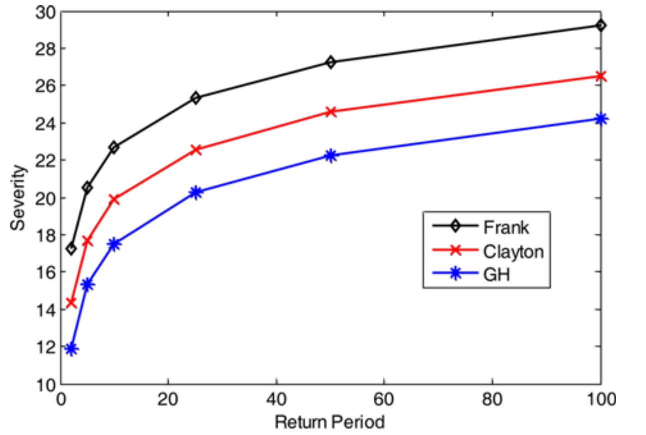

Fig. 6. The Plot of Constant Severity Versus Return Period (in years) for Longer Durations for Different Copulas

$$s = F_s^{-1}\left\{\left(1 - \frac{1}{E(L)T}\right)^{\frac{1}{1+\theta}}\right\} \tag{7}$$

The equilibrium severity value estimated using the three copulas, for different recurrence intervals are provided in Fig. 6. From Fig. 6, it is noticed that there exists significant differences between the constant severity value, based on the type of copulas and the difference increases with increase in return period. Here the Frank copula provides the largest estimate followed by Clayton and GH for different return periods.

Along with the joint return periods, the conditional return periods of drought severity and duration helps for risk evaluation and therefore they are also used by water resources managers as a hydraulic design criterion (Mirabbassi *et al.,* 2011). For example, if a water distribution conduit cannot provide enough water under a scenario for which severity of drought (S) exceeds a specified threshold for a given condition that duration D exceeds d months, the return period estimate for this scenario can be represented as $T_{S/D \geq d}$.

The conditional return period for duration of drought and its severity are generally defined for the two cases : (i) the recurrence interval of drought duration for the given condition that severity exceeds a specified threshold; and (ii) the recurrence interval of drought severity given condition that the duration exceeds a specified threshold. In copula based approach, the two cases of conditional return period of drought events can be represented as :

$$T_{D/S \geq s} = \frac{T_S}{P(D \geq d, S \geq s)} \tag{8}$$

$$T_{D/S \geq s} = \frac{E(L)}{[1 - F_S(s)][1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s)]} \tag{9}$$

$$T_{S/D \geq d} = \frac{T_D}{P(D \geq d, S \geq s)} \tag{10}$$

$$T_{S/D \geq d} = \frac{E(L)}{[1 - F_D(d)][1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s)]} \tag{11}$$

Where $T_{D/S \geq s}$ denotes the conditional return period for D

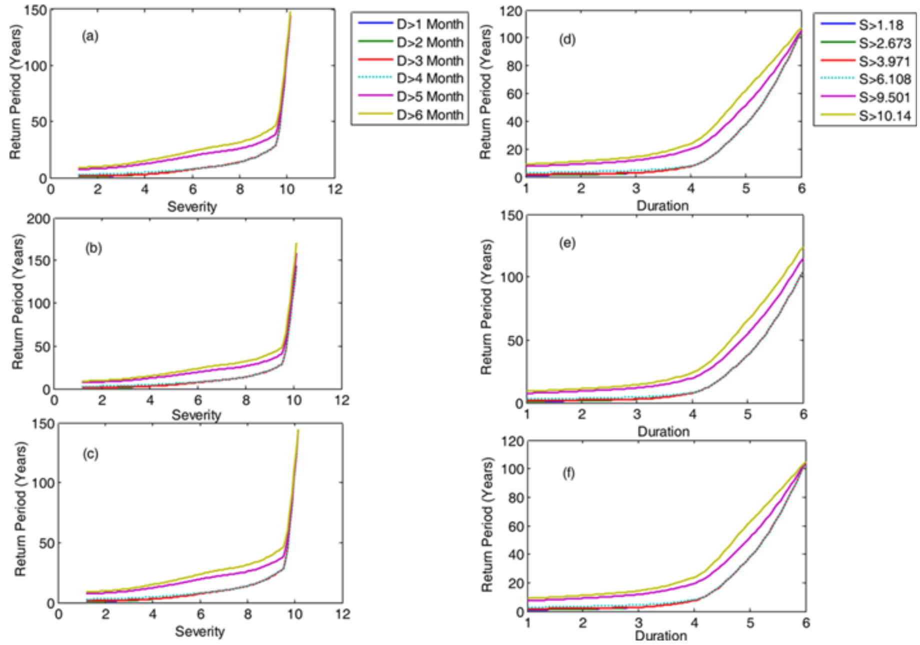

Fig. 7. (a)-(c) The Conditional Return Period of Drought Severity of Kerala Given that the Duration is Greater than a Certain value, *d*; (d-f) the Conditional Return Period of Drought Duration Given that the Severity is Greater Than a Certain Value, *s*, Mean Severity for Different Durations

Table 5. Latitudes and Longitudes of Representative Locations Over Kerala. LN is the Location Number

LN	Place	Latitude	Longitude	LN	Place	Latitude	Longitude
1	Thiruvananthapuram	8.5241°*N*	76.9366°E	8	Thrissur	10.5276°*N*	76.2144°E
2	Kollam	8.8932°*N*	76.6141°E	9	Palakkad	10.7867°*N*	76.6548°E
3	Pathanamthitta	9.2648°*N*	76.7870°E	10	Malappuram	11.0732°*N*	76.0740°E
4	Alappuzha	9.4981°*N*	76.3388°E	11	Kozhikode	11.2588°*N*	75.7804°E
5	Kottayam	9.5916°*N*	76.5222°E	12	Wayanad	11.6854°*N*	76.1320°E
6	Idukki	9.9189°*N*	77.1025°E	13	Kannur	11.8745°*N*	75.3704°E
7	Eranakulam	10.0718°*N*	76.5488°E	14	Kasarakod	12.4387°*N*	75.2012°E

given $S \geq s$; and $T_{S/D \geq d}$ represents the conditional return period for severity (*S*) for the condition $D \geq d$. The conditional return periods of drought duration and severity using different copulas are provided in Fig. 7.

On comparing the SDF curves prepared by the three copulas (Fig. 7(a-c)), the variation of conditional return period of drought with severity, significant difference is noted for the return period values for larger duration, whereas for smaller duration the difference is marginal. A similar pattern is followed for the variation of return period with duration estimated for specified severity condition (Fig. 7(d-f)).

3.3 Spatial Variability of SDF Relationships Over Kerala

In order to track the spatial variability of SDF relationships over Kerala, daily gridded data for 14 locations in Kerala for the period 1901-2013 (0.25° × 0.25° resolution) are collected from India Meteorological Department (IMD), Pune. The monthly rainfall values are then calculated from the daily gridded data for each location. The latitude and longitude of grid points and the map showing grids over Kerala are provided in Table 5 and Fig. 8 respectively. From Table 4, it is noticed that Clayton copula

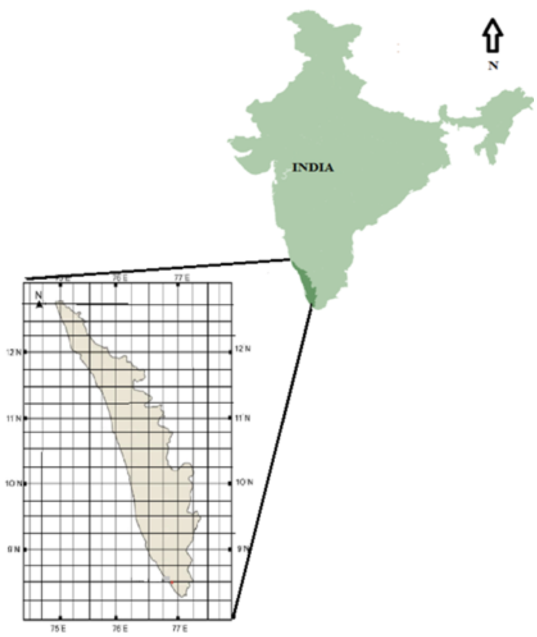

Fig. 8. Map of Kerala Subdivision Showing Grids of 0.25° × 0.25° Size Superposed

Table 6. Number of Drought Events, Expected Inter Arrival Time ($E(L)$) and Copula Parameter of Different Locations in Kerala Subdivision

Place	Number of drought events	$E(L)$	Copula parameter	Place	Number of drought events	$E(L)$	Copula parameter
Thiruvananthapuram	55	0.9926	20.07	Thrissur	70	0.9794	13.22
Kollam	54	0.9882	9.78	Palakkad	60	0.9919	20.17
Pathanamthitta	48	0.9897	5.58	Malappuram	71	0.9823	24.84
Alappuzha	61	0.9971	12.08	Kozhikode	79	0.9882	15.24
Kottayam	36	0.9609	6.87	Wayanad	34	0.9646	7.82
Idukki	55	0.9963	28.58	Kannur	70	0.9764	15.35
Eranakulam	70	0.9882	7.30	Kasarkode	75	0.9823	26.71

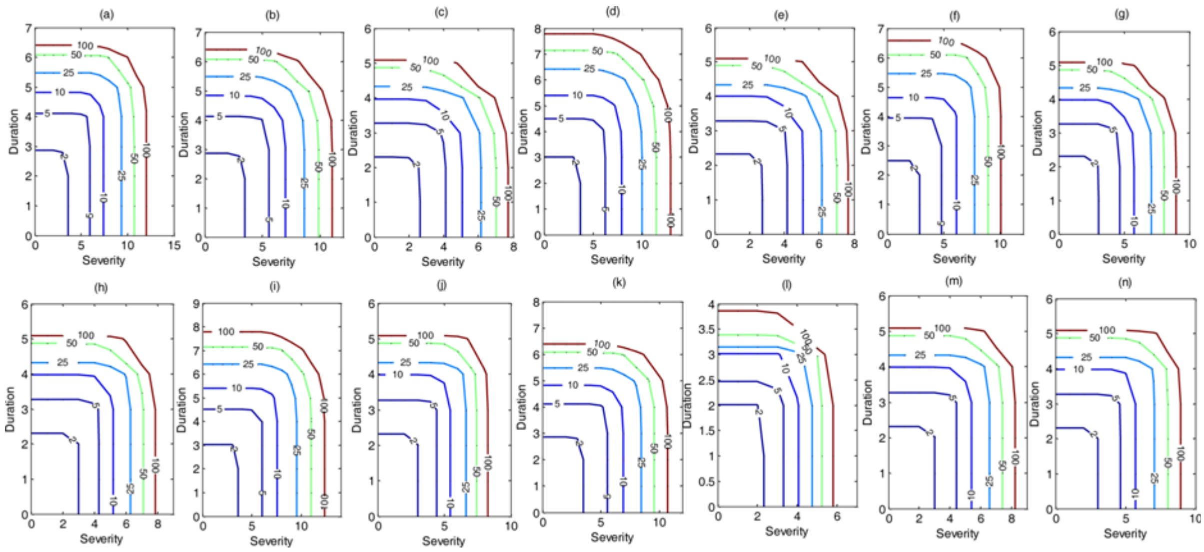

Fig. 9. The SDF Relations for Different Locations in Kerala Considering Joint Return Periods (in years) using Clayton Copula (for TDS case (D ≥ d and S ≥ s)): (a) Thiruvananthapuram, (b) Kollam, (c) Pathanamthitta, (d) Alappuzha, (e) Kottayam, (f) Idukki, (g) Eranakulam, (h) Thrissur, (i) Palakkad, (j) Malappuram, (k) Kozhikode, (l) Wayanad, (m) Kannur, (n) Kasarkode

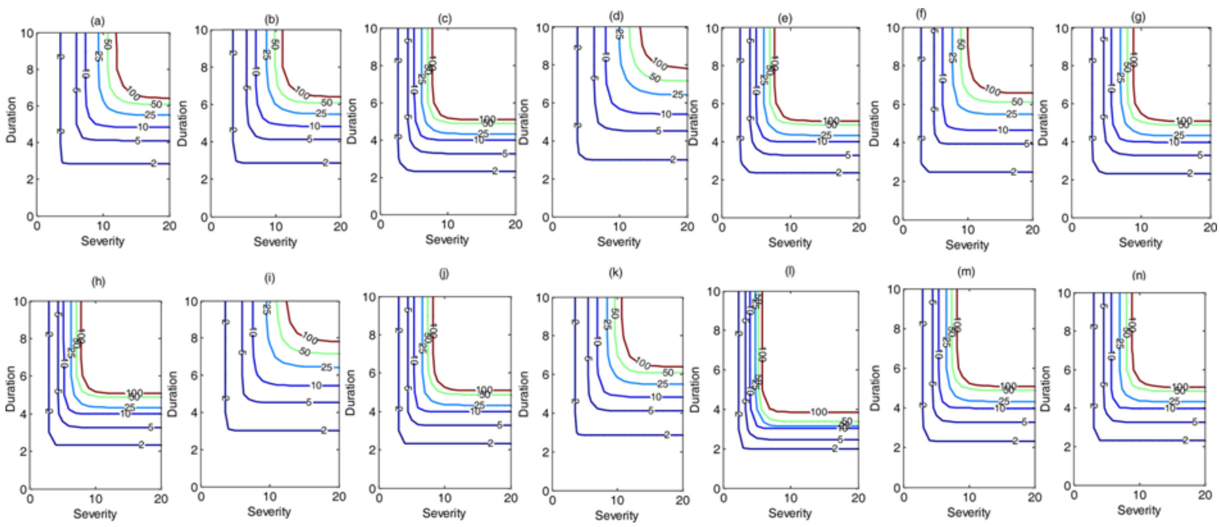

Fig. 10. The SDF Relations for Different Locations in Kerala Considering Joint Return Periods (in years) using Clayton Copula (for T'DS case (D ≥ d or S ≥ s)): (a) Thiruvananthapuram, (b) Kollam, (c) Pathanamthitta, (d) Alappuzha, (e) Kottayam, (f) Idukki, (g) Eranakulam, (h) Thrissur, (i) Palakkad, (j) Malappuram, (k) Kozhikode, (l) Wayanad, (m) Kannur, (n) Kasarkode

provides best estimate of drought characterization of Kerala and it is understood that it provides an intermediate estimate in providing the severity values for the limiting case of equilibrium severity for infinite duration (Fig. 6). Therefore Clayton copula is invoked for developing the SDF curves for different locations in Kerala. The mean inter arrival time, drought duration and parameter of Clayton

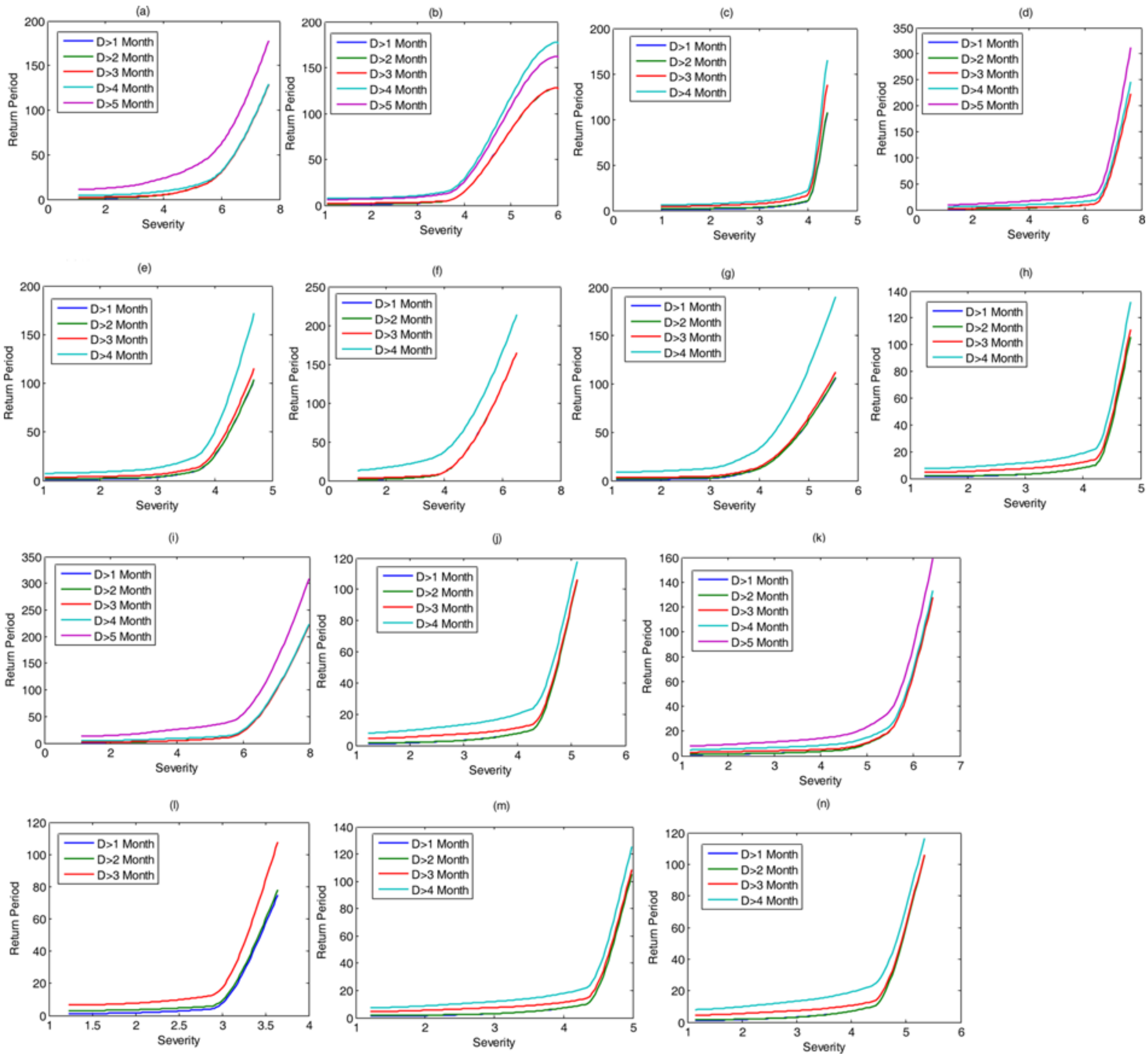

Fig. 11. The Conditional Return Period Estimates of Drought Severity of Different Locations in Kerala Given that the Duration is Greater Than a Certain Value, *d*: (a) Thiruvananthapuram, (b) Kollam, (c) Pathanamthitta, (d) Alappuzha, (e) Kottayam, (f) Idukki, (g) Eranakulam, (h) Thrissur, (i) Palakkad, (j) Malappuram, (k) Kozhikode, (l) Wayanad, (m) Kannur, (n) Kasarkode

copula for all the cases are summarized in Table 6. The obtained SDF curves for $T_{DS}(D \geq d$ and $S \geq s)$ are provided in Fig. 9 and that for $T'_{DS}(D \geq d$ or $S \geq s)$ are provided in Fig. 10.

From different contour plots (Fig. 9 and Fig. 10), a maximum of 6-7 month drought duration are observed at different locations. The value of severity is as large as 10 for larger drought duration such as 5-7 months. The value of severity is less for smaller drought duration and larger for longer duration, which again clearly indicates a strong relation between severity of short term drought of the subdivision and its duration. Further the conditional return period for severity of drought at different locations given that the duration exceeds a specified value, *d* are estimated and presented in Fig. 11. Similarly the conditional return period of drought duration given that the severity exceeds a specified value, *s*, mean severity for different durations is provided in Fig. 12. Moreover the constant severity values corresponding to different return period for longer duration for all locations is summarized in Table 7.

From Table 6 it is noticed that the expected inter arrival time is similar in different locations in Kerala. The number of droughts events is observed to be the least at Wayanad, followed by Kottayam, Pathanamthitta and Kollam. From Table 7, it is noticed that the maximum value of constant severity observed for all return periods for Palakkad while minimum value is observed for the district of Wayanad. The severity values of all almost all districts lies above 10 for 100 year return period expect for few locations such as Wayanad, Kottayam and Pathananmthitta. The average annual rainfall for different locations based on 1901-2013 period is provided in Fig. 13, which clearly indicate that the rainfall of the locations Palakkad, Thiruvananthapuram and Wayanad are the least. Based on the severity value and AAR of the regions, some broad inferences can be drawn. Highest

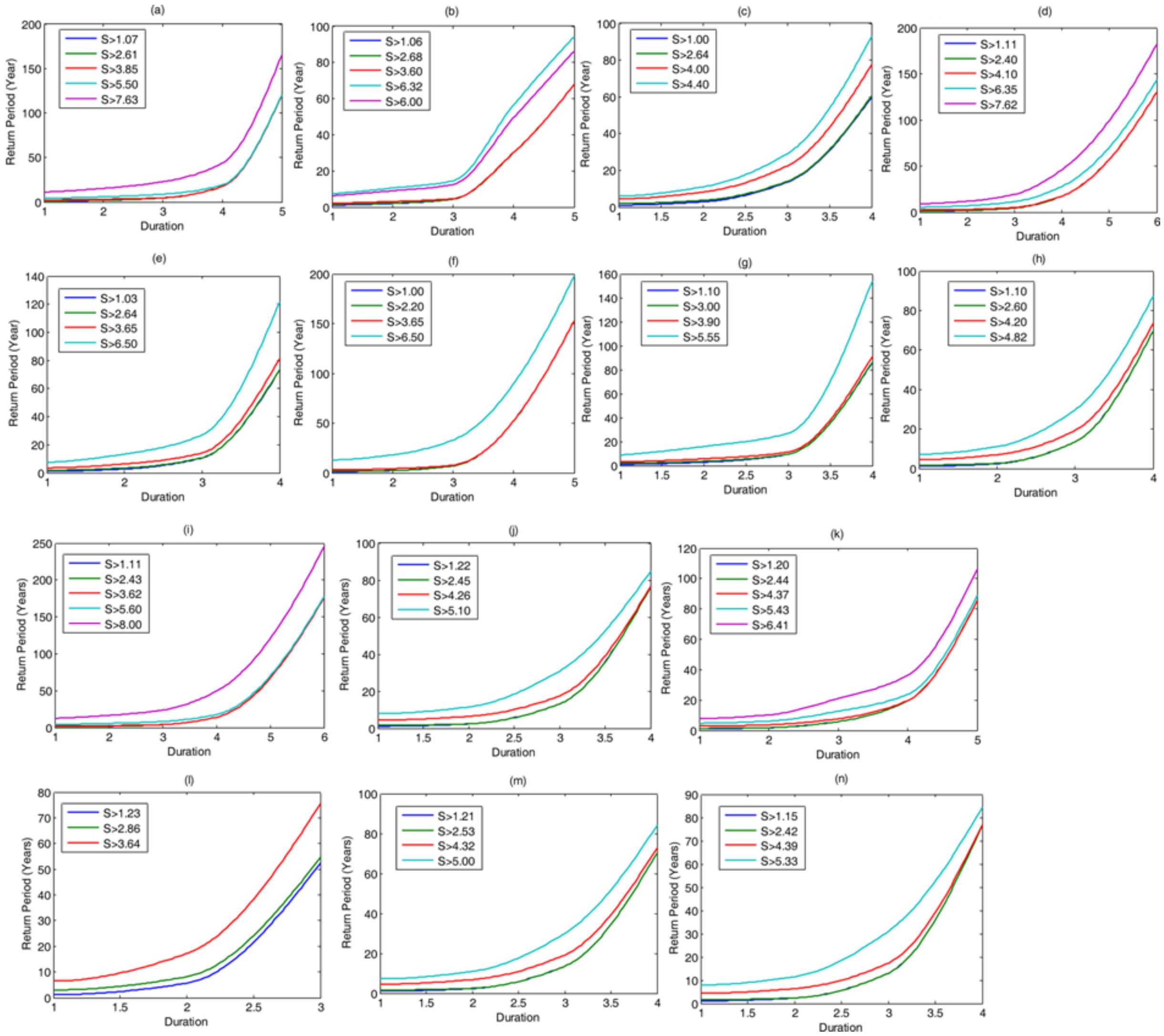

Fig. 12. The Conditional Return Period Estimates of Drought Duration of Different Locations in Kerala Given that the Severity is Greater than a Certain Value, *s*, Mean Severity for Different Durations: (a) Thiruvananthapuram, (b) Kollam, (c) Pathanamthitta, (d) Alappuzha, (e) Kottayam, (f) Idukki, (g) Eranakulam, (h) Thrissur, (i) Palakkad, (j) Malappuram, (k) Kozhikode, (l) Wayanad, (m) Kannur, (n) Kasarkode

Table 7. Constant Severity Values Corresponding to Different Return Period (RP) for Longer Duration for All Districts. The Maximum Values for Each Return Period are Highlighted with Bold Italics and Minimum Values are Highlighted in Italics

RP	Thiruvanthapuram	Kollam	Pathanamthitta	Alappuzha	Kottayam	Idukki	Eranakulam
2	9.797	7.848	5.084	9.409	5.225	8.697	6.120
5	11.965	9.825	6.419	11.798	6.542	10.544	7.676
10	13.361	11.088	7.264	13.338	7.370	11.740	8.664
25	15.091	12.644	8.295	15.245	8.380	13.226	9.876
50	16.353	13.775	9.040	16.636	9.108	14.313	10.752
100	17.592	14.882	9.765	18.000	9.818	15.382	11.608
RP	Thrissur	Palakkad	Malappuram	Kozhikode	Wayanad	Kannur	Kasarkode
2	6.149	***9.993***	7.165	4.787	4.272	6.602	7.704
5	7.395	***12.257***	8.496	6.797	5.160	7.944	9.190
10	8.181	***13.717***	9.345	8.073	5.712	8.795	10.141
25	9.144	***15.527***	10.389	9.628	6.380	9.838	11.315
50	9.839	***16.850***	11.148	10.746	6.859	10.594	12.170
100	10.517	***18.148***	11.889	11.831	7.324	11.332	13.006

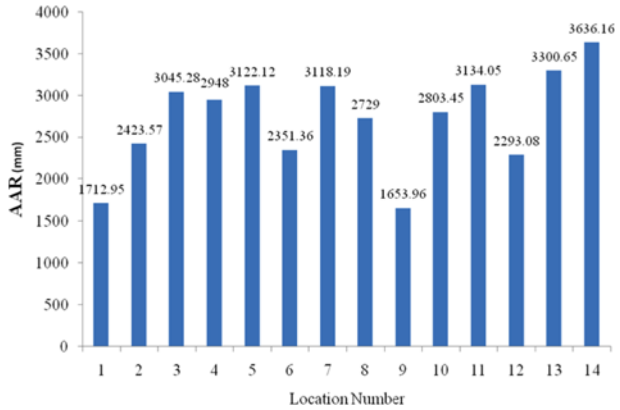

Fig. 13. Average Annual Rainfall (AAR) of Different Loactions in Kerala. Locations are Sequentially Numbered from Trivandrum to Kasarkode

severity is observed at place with less rainfall (for Palakkad) which is quite expected while lowest severity is noticed in Wayanad, which is again a low rainfall receiving region and recognized to be a region vulnerable to drought. This anomaly may be because of the fact that apart from the AAR, rainfall fluctuations may be playing a role of drought severity in such regions which are to be further investigated to corroborate the inferences. It is recommended that in such vulnerable regions the estimation of drought and the subsequent risk analysis is to be performed incorporating other meteorological factors such as standardized precipitation-evaptranspiration index (SPEI) or other multivariate drought indices.

In this study, a bivariate copula approach is followed while it can also be performed using multivariate copulas by including the characteristics such as drought inter–arrival time, peak severity, and minimum SPI values. In the present study the gridded rainfall data of representative grid point from different districts is used in developing SDF relationships. However the development of a mean SDF curve as a signature of drought based on different SDF curves devised for different grid points of the district can be performed as an extension of the present study. Also the non-stationary issues are not addressed in the present study and developing a non-stationary SDF curve (Sarhadi *et al*., 2016) is a problem in drought risk assessment which is gaining lot of scientific attention in recent days.

4. Conclusions

This study followed the Copula approach to account the joint dependency of the two drought variables- severity and duration for developing the drought SDF curves based on 3-month SPI for Kerala meteorological subdivision in India.

The important conclusions of the study are :

1. The short term joint SDF relationships for Kerala is independent of the type of Archimedean Copula function used, on considering the return period upto 100 years (~) while conditional SDF relationships differs with type of copula
2. The conditional SDF relationships between severity and return period showed larger differences for longer duration and the behaviour is identical in the relationships between duration and return period for conditional severity values
3. SDF relationships of different districts in Kerala showed that a shorter duration drought of less severity magnitude is prominent in districts like Wayanad, Kottayam and Pathanamthitta
4. The highest value of constant severity for longer duration is noticed for Palakkad while lowest value is observed for the district of Wayanad for all return periods
5. The copula based SDF relationships devised in this study are helpful for planning and management of water resources and for implementing appropriate mitigation strategies for drought in Kerala.

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