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Traffic Flow Assignment Model with Modified Impedance Function

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Abstract

In this paper, we study a mixed-mode traffic network, in which Battery Electric Vehicles (BEVs) and Gasoline Vehicles (GVs) can be driven freely. We propose a new piecewise impedance function and, based on this new impedance function, we present a convex but nonsmooth optimization model for the traffic network. In order to deal with the nonsmoothness, we introduce a binary variable. Furthermore, we report some numerical examples to show the efficiency of the model. %is effective and robust. In particular, our numerical results illustrate that, if the distance limit of BEVs is long enough and the unit operating cost of BEVs is lower than GVs, more travelers prefer to choosing BEVs.

Keywords: *impedance function, BEVs, GVs, mixed-mode traffic network*

1. Introduction

During past several decades, due to increasing economic activities, the vehicles demand has had sustainable growth. How to assign traffic network flow effectively has been a hot problem. Recently, with the development of electric vehicle technologies, battery electric vehicles (BEVs) have been extensively utilized. It is a trend that BEVs will become significant vehicles in the near future. In this paper, we consider a mixed traffic network in which the users can choose BEVs and gasoline vehicles (GVs) freely.

We first focus on the so-called impedance functions, which reveal the relationships between travel costs and traffic conditions on the road and hence play a vital role in transportation network modeling. How to define an impedance function is one of the key problems in the traffic flow assignment problems.

A number of researchers have attempted to dispose the traffic assignment problems with different impedance functions. The first impedance function was presented to cope with the traffic assignment problems by Greenshields *et al.* (1935) and, since then, the traffic flow assignment problems have become a basic and hot topic. Underwood (1961) proposed a new impedance function applied to the case of low density situations and Drake *et al.* (1967) introduced a speed-density impedance function based on traffic statistical data. May and Keller (1967) presented another two-stage speed-density impedance function. Subsequently, Ben-Akiva (1996) stated a three-stage speed-density impedance function with two parameters. In particular, Yang and Qian (1994) constructed a link impedance function according to different kinds of urban roads. Del Castillo and Benitez (1995) put forward a family of curves with respect to balance function. Pei and Gai (2003) proposed a link impedance function by considering road pricing impact. Fu *et al.* (2003) gave a modified impedance function through analyzing relationship among average speeds of vehicles. Wang *et al.* (2004) constructed a link impedance function based on generalized speed considering factors of time, cost, traffic flow, toll stations and urban nodes comprehensively. Meng and Li (2005) analyzed characteristics of highway and trunk road of big cities and got some corresponding impedance functions. Wang *et al.* (2009) adopted two methods to obtain an improved impedance function. More recently, Song (2014) presented a new impedance function based on traffic wave theory. Li (2015) suggested a new impedance function based on subjective wishes or preference. Ryu *et al.* (2016) proposed two types of transit impedance functions for traffic assignment problems. Zhang and Yang (2016) suggested a new impedance function to solve a freight transportation problem.

In this paper, we propose a piecewise time-flow impedance function, which may be more reasonable in practice. See Section 2 for our motivation and its properties. Then, in Section 3, we present some optimization models for the considered mixed-mode traffic network and make a comprehensive analysis on its optimality conditions. In Section 4, we use two numerical examples to verify the efficiency and rationality of the established models.

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2. Modified Impedance Function

Among the existing time-flow impedance functions, the most popular one is the Bureau of Public Roads (BPR) function defined by

$$t_a(x_a) = t_a^0\left(1+\alpha\left(\frac{x_a}{C_a}\right)^{\beta}\right) \tag{1}$$

where x_a denotes traffic flow on link a, t_a^0 is the free-flow travel time on link a, $\alpha > 0$ and $\beta > 0$ are given parameters, and C_a represents the maximum capacity on link a within which vehicles can travel freely. In 1964, the Bureau of Public Roads conducted a traffic survey on a large number of road sections and presented the above impedance function through regression analysis. It is usual to set $\alpha = 0.15$ and $\beta = 4$ in the literature. Many researchers have dedicated to improve the accuracy of the BPR function; see, e.g., Yang and Qian (1994), Pei and Gai (2003), Fu *et al.* (2003), Wang *et al.* (2004), Meng and Li (2005), Wang *et al.* (2009), Song (2014), Li (2015), Ryu (2016), Zhang and Yang (2016).

The BPR function is presented through theoretical analysis based on free flow, without considering various congested conditions. From (1), we can observe the following properties of the BPR function:

- When $x_a \le C_a$, t_a has no distinct change.
- When $x_a > C_a$, t_a increases acutely, especially in the case of $x_a > 2C_a$.
- When x_a reaches the level that can result in traffic jam, t_a still increases and x_a is still changeable.

These properties, which are shown in Fig. 1, may not be consistent with actual states on roads. Actually, we think that the reality may satisfy the following properties:

(1) When the number of vehicles is smaller than or equal to the maximum value to ensure vehicles to travel freely, travel time is steady in general.

(2) When the number of vehicles is more than the threshold mentioned above, travel time changes gradually and increases sharply when vehicles are close to an amount resulting in traffic crowding.

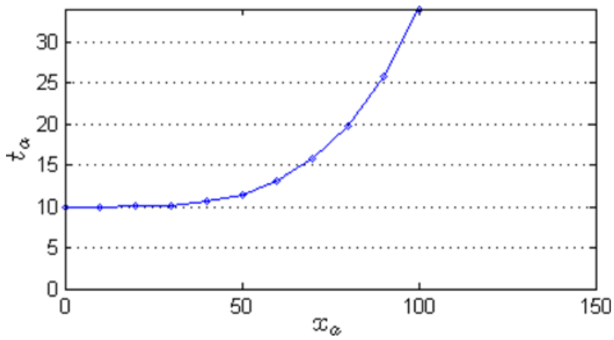


Fig. 1. BPR Function

(3) When jams occur, few vehicles can travel into the network.

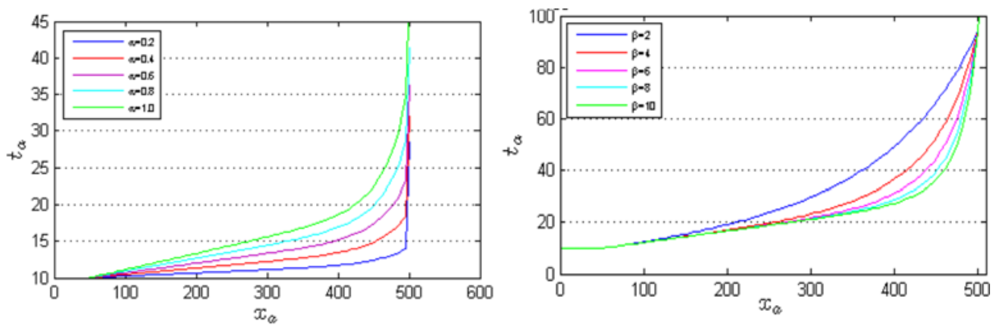
Based on these observations, a more rational impedance function is necessary.

For simplicity, we do not consider uncertain factors (weather, traffic accidents, etc.) here. In the light of the above three properties, we define a modification of the BPR function as follows:

$$t_a(x_a)\begin{cases} t_a^0, & 0 \le x_a \le C_a, \\ t_a^0\left(\dfrac{x_a - C_a}{Z_a - C_a} + e^{\alpha\left(\frac{x_a - C_a}{Z_a - C_a}\right)^{\beta}}\right), & C_a < x_a < Z_a, \\ t_a^0\left(\dfrac{x - Z_a + \varepsilon}{\varepsilon} + e^{\alpha\left(\frac{x_a - Z_a + \varepsilon}{\varepsilon}\right)^{\beta}}\right), & Z_a \le x_a \le Z_a + \varepsilon. \end{cases} \tag{2}$$

Here, x_a denotes traffic flow on link a, t_a^0 denotes free-flow travel time on link a, $\alpha > 0$ and $\beta > 0$ denote function parameters, C_a denotes the maximum capacity within which vehicles can travel freely, Z_a represents a threshold value over which there will be a traffic jam, and $\varepsilon > 0$ is a small constant. This new impedance function is depicted in Fig. 2.

Figure 2(a)-(b) correspond to the cases of $\beta = 10$ and $\alpha = 1$



(a) (b)

Fig. 2. New Impedance Function: (a) Different Values of α, (b) Different Values of β

respectively. It is obvious to see that the new piecewise impedance function (2) satisfies the realistic properties mentioned above. In addition, Fig. 2 demonstrates the effect of the parameters α and β: As α increases, the growth ratio of travel time rises synchronously. On the other hand, as β increases, the growth ratio of travel time decreases at first and then increases acutely.

The following lemmas are useful in proving the subsequent theorem.

Lemma 2.1 (Berkovitz, 2003, Theorem 2.2, Chapter III) *Let C be a convex set in* $\mathbb{R}$ *and, at each point* $x_0 \in C$, *f have a subgradient* ξ *at* x_0 *to satisfy the condition*

$$f(y)-f(x_0) \ge \xi(y-x_0), \ \forall y \in C \tag{3}$$

Then f is convex over C.

Lemma 2.2 *Let f, g be convex and smoothing over* $[a, b]$ *and* $[b, c]$ *respectively,* $f(b) = g(b)$, *and* $0 \le f'(x_1) \le g'(x_2)$ *for any* $x_1 \in [a,b]$ *and* $x_2 \in [b,c]$. Then the function

$$H(x)=\begin{cases} f(x), & x\in[a,b],\\ g(x), & x\in[b,c],\end{cases}$$

is convex over $[a, c]$.

Proof. By Lemma 2.1, it is sufficient to show that, for each $x_0 \in [a,c]$, H has a subgradient ξ at x_0 satisfying

$$H(y)-H(x_0) \ge \xi(y-x_0), \ \forall y \in [a,c] \tag{4}$$

Case 1: $x_0 \in [a,b)$. If $y \in [a,b]$, we have $H(y)-H(x_0)=f(y)-f(x_0)$.

Since f is differentiable and convex over $[a, b]$, we have $f(y)-f(x_0) \ge f'(x_0)(y-x_0)$ and (4) is true.

If $y \in [b,c]$, since $g(b)=f(b)$ and f,g are differentiable and convex over $[a, b]$ and $[b, c]$ respectively, we have

$$\begin{aligned} H(y)-H(x_0) &= g(y)-f(x_0) = g(y)-g(b)+g(b)\\ &\quad -f(x_0) \ge g'(b)(y-b)+f'(x_0)(b-x_0)\\ &= g'(b)(y-b)+f'(x_0)(y-x_0+b-y)\\ &= f'(x_0)(y-x_0)+(g'(b)-f'(x_0))(y-b). \end{aligned}$$

Since $g'(b) \ge f'(x_0)$ and $y \ge b$, we have

$$f'(x_0)(y-x_0)+(g'(b)-f'(x_0))(y-b) \ge f'(x_0)(y-x_0).$$

As a result, $H(y)-H(x_0) \ge f'(x_0)(y-x_0)$ always holds, which means that (2.4) is always satisfied by letting $\xi = f'(x_0)$

Case 2: $x_0 \in (b,c]$. In a similar way to Case 1, we can show (2.4) for this case.

Case 3: $x_0 = b$. Since $f(b)=g(b)$, we have

$$H(y)-H(x_0)=H(y)-f(b)=H(y)-g(b).$$

Since both f and g are differentiable and convex, we have

$$H(y)-H(x_0)=H(y)-f(b)=f(y)-f(b) \ge f'(b)(y-b)$$

for $y \in [a,b]$ and $H(y)-H(x_0) = H(y)-g(b) = g(y)-f(b) \ge g'(b)(y-b)$ for $y \in [b,c]$. This means that (4) is satisfied when $\xi \in [f'(b), g'(b)]$. In consequence, $H(x)$ is convex over $[a,c]$.

By Lemma 2.2, we have the following theorem for the new piecewise impedance function (2).

Theorem 2.3 *Suppose that* $\alpha>0$, $\beta \ge 1$, *and* $0<\varepsilon \le Z_a - C_a$. Then we have the following statements.

(i) *The function* $t_a(x_a)$ *is continuous over* $[0, Z_a+\varepsilon]$.

(ii) *The function* $t_a(x_a)$ *is differentiable everywhere over* $[0, Z_a+\varepsilon]$ *except the points* Z_a *and* C_a.

(iii) *The functions* $t_a(x_a)$ *and* $t_a(x_a)x_a$ *are both convex over* $[0, Z_a+\varepsilon]$.

Proof. The statements (i) and (ii) are easy to verify. Note that, by direct calculation, we have

$$t_a'(x_a)=\begin{cases} 0, & 0 \le x_a \le C_a,\\ t_a^0\left(\dfrac{1}{Z_a-C_a}+\dfrac{\alpha\beta}{Z_a-C_a}\left(\dfrac{x_a-C_a}{Z_a-C_a}\right)^{\beta-1} e^{\alpha\left(\frac{x_a-C_a}{Z_a-C_a}\right)^{\beta}}\right), & \\ \quad C_a < x_a < Z_a, & \\ t_a^0\left(\dfrac{1}{\varepsilon}+\dfrac{\alpha\beta}{\varepsilon}\left(\dfrac{x_a-Z_a+\varepsilon}{\varepsilon}\right)^{\beta-1} e^{\alpha\left(\frac{x_a-Z_a+\varepsilon}{\varepsilon}\right)^{\beta}}\right), & \\ \quad Z_a \le x_a \le Z_a+\varepsilon & \end{cases}$$

and

$$(t_a(x_a)x_a)'=\begin{cases} t_a^0, & 0 \le x_a \le C_a,\\ t_a^0\left(\dfrac{1}{Z_a-C_a}+\dfrac{\alpha\beta}{Z_a-C_a}\left(\dfrac{x_a-C_a}{Z_a-C_a}\right)^{\beta-1} e^{\alpha\left(\frac{x_a-C_a}{Z_a-C_a}\right)^{\beta}}\right)x_a & \\ \quad +t_a^0\left(\dfrac{x_a-C_a}{Z_a-C_a}+e^{\alpha\left(\frac{x_a-C_a}{Z_a-C_a}\right)^{\beta}}\right), & \\ \quad C_a < x_a < Z_a, & \\ t_a^0\left(\dfrac{1}{\varepsilon}+\dfrac{\alpha\beta}{\varepsilon}\left(\dfrac{x_a-Z_a+\varepsilon}{\varepsilon}\right)^{\beta-1} e^{\alpha\left(\frac{x_a-Z_a+\varepsilon}{\varepsilon}\right)^{\beta}}\right)x_a & \\ \quad +t_a^0\left(\dfrac{x-Z_a+\varepsilon}{\varepsilon}+e^{\alpha\left(\frac{x_a-Z_a+\varepsilon}{\varepsilon}\right)^{\beta}}\right), & \\ \quad Z_a \le x_a \le Z_a+\varepsilon. & \end{cases}$$

It is obvious that $t'_a(x_a)$ is increasing in $[0, C_a]$, $[C_a, Z_a]$ and $(Z_a, Z_a+\varepsilon]$ respectively, $t'_{a-}(C_a)<t'_{a+}(C_a)$, $t'_{a-}(Z_a)<t'_{a+}(Z_a)$, $t'_{a-}(C_a)C_a<t'_{a+}(C_a)C_a$, and $t'_{a-}(Z_a)Z_a<t'_{a+}(Z_a)Z_a$, where $t'_{a-}(x_a)$ and $t'_{a+}(x_a)$ denote the left and right derivatives at x_a respectively. By Lemma 2.2, we can show that $t_a(x_a)$ is convex over $[0, Z_a+\varepsilon]$. The convexity of $t_a(x_a)x_a$ over $[0, Z_a+\varepsilon]$ can be shown in a similar way.

Figure 3 Displays the Function $t_a(x_a)x_a$ in Details. Here, All Parameters are the Same as in Fig. 2

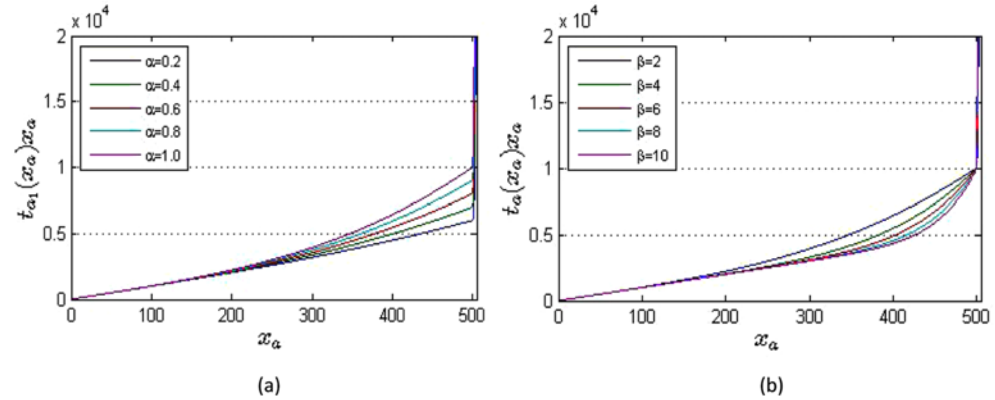


Fig. 3. The Function $t_a(x_a)x_a$: (a) Different Values of α, (b) Different Values of β

Since the main goal in modeling for transportation networks is to avoid traffic jams, Theorem 2.3 will be used to ensure the model (P1) established in the next section to be a convex but nonsmooth optimization problem.

3. Problem Formulation

Consider a mixed traffic network with GVs and BEVs, which are distinguished by two factors, that is, driving distance limits and travel costs. As stated as by Jiang and Xue (2014), lots of families may wish to own both GVs and BEVs simultaneously in transition periods to enjoy the lower operating cost and avoid the anxiety of distance limits of BEVs. For this reason, we make the following assumptions:

Assumption 3.1 In the network, all drivers can select GVs or BEVs freely without any extra cost.

Assumption 3.2 All drivers are rational, that is, all drivers choose vehicles and routes according to their minimal travel costs.

Assumption 3.3 There is no traffic incident in the network.

In addition, the notations used in our model are given in Table 1.

Table 1. Notations

Notations	Meaning
Sets	
A	set of all links in the traffic network.
K	set of O-D pairs.
k_g	O-D pair with the flow of GVs.
k_e	O-D pair with the flow of BEVs.
Variables	
$\omega_{k_g}^a$	traffic flow of GVs on link a between O-D pair k.
ω_{k_e}	traffic flow of BEVs on link a between O-D pair k.
x_{a_g}	total traffic flow of GVs on link a.
x_{a_e}	total traffic flow of BEVs on link a.
Parameters	
E_{a_g}	dot-arc matrix of all feasible paths for GVs.
E_{a_e}	dot-arc matrix of all feasible paths for BEVs.
d_{a_g}	original outflow or terminal inflow of GVs between O-D pair k.
d_{a_e}	original outflow or terminal inflow of BEVs between O-D pair k.
p	value of time.
c_g	operating cost per mile of GVs.
c_g	operating cost per mile of BEVs.
l_a	length of link $a \in A$.

3.1 Problem Formulation

In general, travelers' route choices may be influenced by many factors such as travel time, reliability, safety, etc. Here, we consider a route choice model based on the perceived travel time of each route by travelers. If the instantaneous travel time of each route is provided to travelers, the perceived travel time is assumed to be the same as the instantaneous travel time. If not, the perceived travel time is assumed to be free flow travel time, in other words, the travel time is equal to t_a^0 on link a. The corresponding minimization problem can be formulated as

$(P1)$

$$\min_{\omega_{k_g}^a,\omega_{k_e}^a} f(\omega)=\sum_{a\in A}\left(pt_a\left(x_{a_e}+x_{a_g}\right)\cdot\left(x_{a_e}+x_{a_g}\right)+\left(c_e x_{a_e}+c_g x_{a_g}\right)l_a\right) \tag{5}$$

$$\text{s.t. } E_{k_g}\omega_{k_g}^a = i_{k_g} d_{k_g}, \forall k_g \in K, a \in A \tag{6}$$

$$E_{k_e}\omega_{k_e}^a = i_{k_e} d_{k_e}, \forall k_e \in K, a \in A \tag{7}$$

$$x_{a_g} + x_{a_e} \le Z_a, \forall a \in A \tag{8}$$

$$x_{a_g} = \sum_{k\in K}\omega_{k_g}^a, \forall a \in A \tag{9}$$

$$x_{a_e} = \sum_{k\in K}\omega_{k_e}^a, \ \forall a \in A \tag{10}$$

$$\omega_{k_g}^a \ge 0,\ \omega_{k_e}^a \ge 0,\ \forall a \in A,\ k \in K \tag{11}$$

where t_a is defined as in (2) and can be written as

$$t_a(x_{a_g}+x_{a_e}) = \begin{cases} t_a^0, & 0 \le x_{a_g}+x_{a_e} \le C_a \\ t_a^0\left(\dfrac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}+e^{\alpha\left(\frac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}\right)^\beta}\right), & C_a < x_{a_g}+x_{a_e} < Z_a \\ t_a^0\left(\dfrac{x_{a_g}+x_{a_e}-Z_a+\varepsilon}{\varepsilon}+e^{\alpha\left(\frac{x_{a_g}+x_{a_e}-Z_a+\varepsilon}{\varepsilon}\right)^\beta}\right), & Z_a \le x_{a_g}+x_{a_e} \le Z_a+\varepsilon \end{cases} \tag{12}$$

$\beta \ge 1$ and $\alpha > 0$ are two parameters, the vectors $i_{k_g} = (1, 0, ..., 0-1)^T$ and $i_{k_e} = (1, 0, ..., 0, -1)^T$ with suitable dimensions, where the elements 1 correspond to the originals of k_g and k_e, the elements -1 correspond to the terminals of k_g and k_e. The objective function (5) denotes the total travel cost of the networks, which includes two parts: the total travel time costs and the operating costs of GVs and BEVs. Both (6) and (7) are flow balance constraints related to GVs and BEVs on each link between every O-D pair. Constraint (8) ensures the total traffic flow on each feasible link no more than the threshold value. Constraints (9) and (10) are definitions of flows of GVs and BEVs on link a respectively, and (11) requires that all variables are nonnegative.

On the other hand, since two classes vehicles are considered in the model, the problem also belongs to the multiclass traffic assignment. Comparing with other multiclass traffic assignment models (Dafermos, 1972; Nagurney and Dong, 2002; Yang and Huang, 2004; Xu *et al*., 2014; Levin and Boyles, 2016), there are certain differences of the model as follows: (1) BEV is often limited the travel distance; (2) A new impedance function is utilized; (3) Form Assumption 3.1, the users can select GVs or BEVs freely.

By Theorem 2.3, the objection function (5) is a convex function but not continuously differentiable everywhere and so the above problem (P1) is a convex but nonsmooth optimization problem. Thus, the classical optimization algorithms may not be utilized to solve (P1) effectively.

In general, since jam situation does not occur based on the minimization of the sum of time cost and operating costs, the last piece in (12) is not considered in this paper. To deal with the piecewise smooth function t_a, we introduce a binary variable δ_a for each $a \in A$ and rewrite the function involved as

$$t_a(x_{a_e}+x_{a_g}) = (1-\delta_a)t_a^0 + \delta_a t_a^0 \left(\frac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}+e^{\alpha\left(\frac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}\right)^\beta}\right) \tag{13}$$

$$\delta_a C_a \le x_{a_e}+x_{a_g} \le \delta_a Z_a + (1-\delta_a)C_a \tag{14}$$

$$\delta_a \in \{0,1\},\ \forall a \in A \tag{15}$$

We have the following result immediately.

Theorem 3.1 *Conditions* (13)-(15) *are equivalent to*

$$t_a(x_{a_g}+x_{a_e}) = \begin{cases} t_a^0, & 0 \le x_{a_g}+x_{a_e} \le C_a, \\ t_a^0\left(\dfrac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}+e^{\alpha\left(\frac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}\right)^\beta}\right), & C_a \le x_{a_g}+x_{a_e} \le Z_a. \end{cases}$$

Then, problem (P1) can be reformulated as a mixed integer nonlinear programming problem. By rewriting $\delta_a \in \{0, 1\}$ as $\delta_a(\delta_a - 1) = 0$, we can obtain the following equivalent smooth optimization problem:

($P2$)

$$\min_{\omega_{k_g}^a,\omega_{k_e}^a} f(\omega) = \sum_{a\in A} p\left((1-\delta_a)t_a^0 + \delta_a t_a^0\left(\frac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}+e^{\alpha\left(\frac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}\right)^\beta}\right)\right)\cdot(x_{a_e}+x_{a_g}) + \sum_{a\in A}(c_e x_{a_e}+c_g x_{a_g})l_a \tag{16}$$

$$\text{s.t. } E_{k_g}\omega_{k_g}^a = i_{k_g}d_{k_g},\ \forall k_g \in K, a \in A \tag{17}$$

$$E_{k_e}\omega_{k_e}^a = i_{k_e}d_{k_e},\ \forall k_e \in K, a \in A \tag{18}$$

$$x_{a_g} = \sum_{k\in K}\omega_{k_g}^a,\ \forall a \in A \tag{19}$$

$$x_{a_e} = \sum_{k\in K}\omega_{k_e}^a,\ \forall a \in A \tag{20}$$

$$\delta_a C_a \le x_{a_e}+x_{a_g} \le \delta_a Z_a + (1-\delta_a)C_a, \qquad \forall a \in A \tag{21}$$

$$\delta_a(\delta_a-1) = 0, \qquad \forall a \in A \tag{22}$$

$$\omega_{k_g}^a \ge 0,\ \omega_{k_e}^a \ge 0,\ \forall a \in A,\ k \in K \tag{23}$$

Note that the objective function of the above optimization problem is a simple extension of the Beckmann's transformation (Beckmann *et al.*, 1956) and so we may utilize some classical optimization algorithms to solve the problem.

3.2 Optimality Conditions

The optimality conditions of (16)-(23) can be stated as follows:

$$\begin{aligned}
&\nabla_\omega L(\omega,\mu,\lambda,\gamma,\upsilon,\tau) = 0 \\
&x_{a_g}+x_{a_e}-\delta_a Z_a-(1-\delta_a)C_a \le 0,\ \lambda_a \ge 0,\ \lambda_a \\
&\quad \left(x_{a_g}+x_{a_e}-\delta_a Z_a-(1-\delta_a)C_a\right) = 0 \\
&x_{a_g}+x_{a_e}-\delta_a C_a \ge 0,\ \gamma_a \ge 0,\ \lambda_a\left(x_{a_g}+x_{a_e}-\delta_a C_a\right) = 0 \\
&\omega_{k_g}^a \ge 0,\ \upsilon_{k_g}^a \ge 0,\ \omega_{k_g}^a\upsilon_{k_g}^a = 0 \\
&\omega_{k_e}^a \ge 0,\ \upsilon_{k_e}^a \ge 0,\ \omega_{k_e}^a\upsilon_{k_e}^a = 0 \\
&E_{k_g}\omega_{k_g}^a = i_{k_g}d_{k_g},\ E_{k_e}\omega_{k_e}^a = i_{k_e}d_{k_e} \\
&x_{a_g} = \sum_{k\in K}\omega_{k_g}^a,\ x_{a_e} = \sum_{k\in K}\omega_{k_e}^a,\ \delta_a(\delta_a-1) = 0 \\
&\forall k_g \in K, k_e \in K, \forall a = (a_i,a_j) \in A
\end{aligned}$$

where a_i, a_j are the origin and tail of link a respectively and

$$L(\omega,\mu,\lambda,\gamma,\upsilon,\tau)=\sum_{a\in A} p\left(\begin{array}{c}(1-\delta_a)t_a^0+\delta_a t_a^0\\ \left(\dfrac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}+e^{\alpha\left(\frac{x_{a_g}+x_{a_e}-C_a}{Z_a-C_a}\right)^{\beta}}\right)\end{array}\right)\cdot(x_{a_e}+x_{a_g})$$
$$+\sum_{a\in A}(c_e x_{a_e}+c_g x_{a_g})l_a-\sum_{a\in A}\sum_{k\in K}\mu_{k_g}^a(E_{k_g}\omega_{k_g}^a-i_{k_g}d_{k_g})$$
$$-\sum_{a\in A}\sum_{k\in K}\mu_{k_e}^a(E_{k_e}\omega_{k_g}^e-i_{k_e}d_{k_e})+\sum_{a\in A}\lambda_a(x_{a_g}+x_{a_e}-Z_a-(1-\delta_a)C_a)$$
$$-\sum_{a\in A}\gamma_a(x_{a_g}+x_{a_e}-\delta_a C_a)-\sum_{a\in A}\sum_{k_g\in K}\upsilon_{k_g}^a\omega_{k_g}^a-\sum_{a\in A}\sum_{k_e\in K}\upsilon_{k_e}^a\omega_{k_e}^a$$
$$-\sum_{a\in A}\tau_a\delta_a(\delta_a-1)$$

denotes the Lagrangian function of problem (16)-(23). We can observe the following properties:

(1) Waiting cost on link can be denoted a by the multiplier λ_a. We have $\lambda_a=0$ if $x_{a_g}+x_{a_e}<Z_a$, that is, the flow on link a does not increase the traffic jam level, which is consistent with the fact that users do not need to wait. Conversely, if $x_{a_g}+x_{a_e}=Z_a$, namely, the flow on link a reaches the maximum capacity of link a, the waiting time λ_a is generally greater than zero.

(2) If there is no path feasible for the O-D pair $k_e\in K$ or $k_g\in K$, there is no flow of BEVs or GVs correspondingly.

(3) Suppose that there are some paths feasible for the O-D pair $k_e\in K$ and $k_g\in K$ at the same time. Then the unit operating costs c_e and $c_g$decide how to choose vehicles. Under the condition $x_{a_g}+x_{a_e}<Z_a$, we have $\omega_{k_g}=0$ if $c_e<c_g$, which means that all drivers choose BEVs on these paths, and we have $\omega_{k_e}=0$ in the case of $c_e>c_g$, which means that all drivers choose GVs on these paths.

4. Numerical Analysis

In this section, we use two different-size numerical examples to illustrate rationality and efficiency of the model given in the previous section. We first recall the popular Sequential Quadratic Programming (SQP) algorithm for nonlinear optimization problems and then report our numerical experiences.

4.1 SQP Algorithm

The SQP algorithm was firstly presented by Wilson (1963). With the development of quasi-Newton methods, several different quasi-Newton SQP algorithms have attracted more and more attention. Compared with other optimization algorithms, the SQP algorithms have good convergence, high computational efficiency, and strong border search capability. The SQP algorithm employed here was proposed by Fletcher and Powell (1963), which can be summarized as follows.

Set the distance limits of BEVs and find some feasible paths of BEVs and GVs for all O-D pairs.

Choose an initial feasible flow $\omega_0=\{\omega_{k_g 0}^a,\omega_{k_e 0}^a: k_g\in K, k_e\in K, a\in A\}$, such that the strict inequalities in (21) and (23) hold. Take a tolerance $\varepsilon>0$ and let $\mathbf{H}_0$ be the unit matrix. Set $n=0$.

Solve the approximation quadratic programming problem

$$\min \frac{1}{2}d^T\mathbf{H}_n d+\nabla f(\omega_n)^T d \quad \text{s.t.}:(21)-(23)$$

to get the Lagrange multiplier λ_n and the search direction $d_n=\omega_{n+1}-\omega_n$.

Calculate the new iteration point $\omega_{n+1}=\omega_n+\alpha_n d_n$, where α_n is a stepsize calculated by line search.

If $\|\omega_{n+1}-\omega_n\|\le\varepsilon$, stop. Otherwise, update the Hessian matrix $\mathbf{H}_n$ by BFGS algorithm, let $n=n+1$, and go to Step 2.

4.2 Numerical Analysis

In this subsection, we try to use some numerical examples to evaluate the inference on the network from the distance limit of BEVs and the unit price cost of BEVs and GVs, to check the efficiency of the SQP algorithm, and to verify the proprieties given at the end of Section 3.

4.2.1 Nguyen-Dupuis' Network

The network shown in Fig. 4 was originally used by Nguyen and Dupuis (1984). It includes 19 links, 13 nodes, and 4 O-D pairs. Nodes 1 and 4 represent origin zones and nodes 2 and 3 represent destination zones. In our test, the supply and demand information were the same as in Nguyen and Dupuis (1984). The free-flow travel time of each link was used as a proxy of the link length and additional parameter values were given by the value of time $p=20$, $\alpha=3$, and $\beta=100$. We enumerated all paths and identified the distance feasibility of these paths, as given in Table 2. Such information is very useful to understand the combined mode-route choice results under the driving distance limit. 0.05 times of link length was taken as free-flow travel time of each link and the travel demand of each O-D pair was set to be 400. In addition, the unit operating costs of GVs and BEVs and distance limit of BEVs were treated as variables. Our attention was paid on how driving distance limit and unit operating cost affect the mode-route choices and network flows.

4.2.2 Changing the unit Operating Cost of BEVs

We set the unit operation costs c_g to be 3 and the distance limit

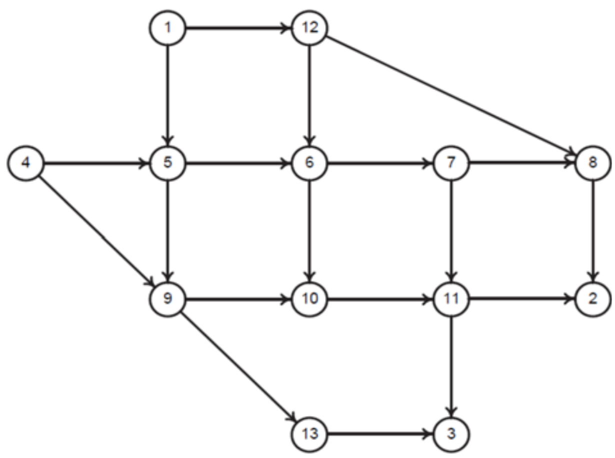


Fig. 4 Nguyen-Dupuis' Network

Table 2. Analysis Results for Nguyen-Dupuis' Network

O-D	Route	Node sequence	Length	O-D	Route	Node sequence	Length
(1, 2)	1	1-5-6-7-8-2	33	(1, 3)	9	1-5-6-7-11-3	39
	2	1-5-6-7-11-2	38		10	1-5-6-10-11-3	44
	3	1-5-6-10-11-2	43		11	1-5-9-13-3	39
	4	1-12-8-2	31		12	1-5-9-10-11-3	49
	5	1-5-9-10-11-2	48		13	1-12-6-7-11-3	52
	6	1-12-6-7-8-2	46		14	1-12-6-10-11-3	57
	7	1-12-6-7-11-2	51				
	8	1-12-6-10-11-2	56	(4, 3)	20	4-5-6-7-11-3	41
(4, 2)	15	4-5-6-7-8-2	35		21	4-5-9-13-3	41
	16	4-5-6-7-11-2	40		22	4-9-13-3	36
	17	4-5-6-10-11-2	45		23	4-5-6-10-11-3	46
	18	4-9-10-11-2	45		24	4-5-9-10-11-3	51
	19	4-5-9-10-11-2	50		25	4-9-10-11-3	46

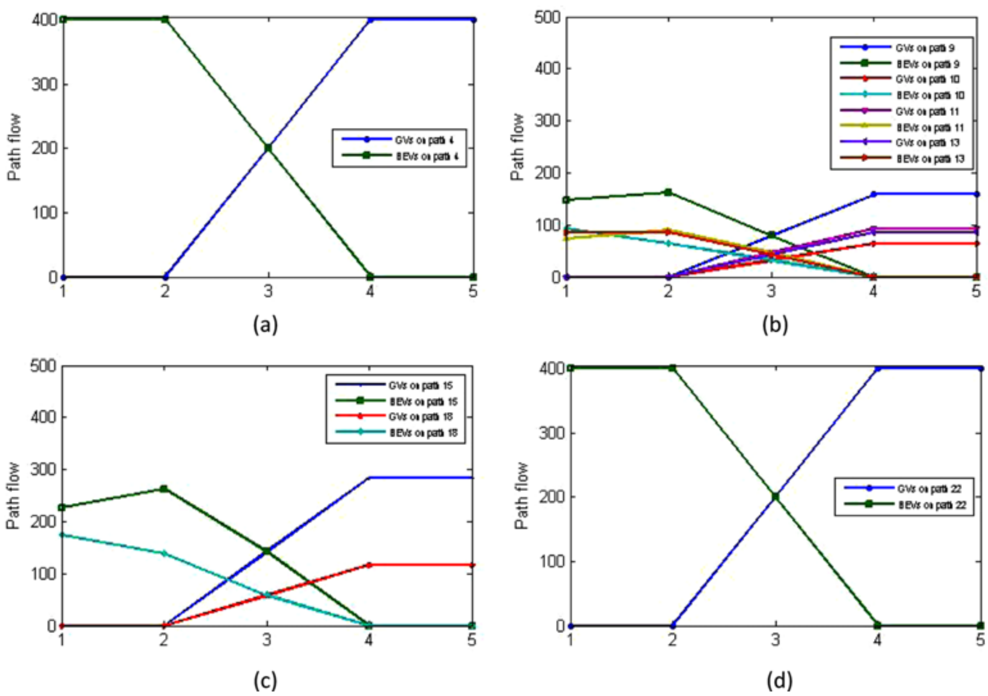


Fig. 5. Changing Unit Operating Cost of BEVs in Nyuyen-Dupuis' Network: (a) O-D Pair (1, 2) (b) O-D pair (1, 3), (c) O-D Pair (4, 2) (d) O-D Pair (4, 3)

of BEVs to be 0 and chose c_e from 1 to 5. The numerical results are shown in Fig. 6 and Fig. 6 (a).

Figure 6(a) shows that, if the unit operating cost of c_e is smaller than 3 (i.e., $c_e < c_g$), the traffic management department should lead users to choose BEVs; when the unit cost of c_e is equal to 3 (i.e., $c_e = c_g$), the utility efficiency of BEVs and GVs are equivalent; when the unit operating cost of c_e is bigger than 3 (i.e., $c_e > c_g$), the traffic management department should lead users to choose GVs. From Fig. 5, we see that, from a personal point of view, when the unit operating cost of $c_e < 3$, users should choose the BEVs; when $c_e = 3$, users may choose BEVs or GVs freely; when $c_e > 3$, users should choose the GVs.

4.2.3 Changing the Unit Operating Cost of GVs

We set the unit operation costs c_e to be 1 and the distance limit of BEVs to be 0 and chose c_g from 1 to 5. The numerical results are shown in Fig. 6(b) and Fig. 7.

Figure 6(b) shows that, when the unit operating cost of c_g is bigger than 1 (i.e., $c_e < c_g$), the traffic management department should lead users to choose BEVs; when the unit operating costs of c_g is equal to 1 (i.e., $c_e = c_g$), the utility efficiency of BEVs and GVs are equivalent. In Fig. 7, all O-D pairs have the same results. This means that, when the unit operating costs of BEVs and GVs are equivalent, the users can choose them freely and, when $c_e < c_g$, the users should choose BEVs.

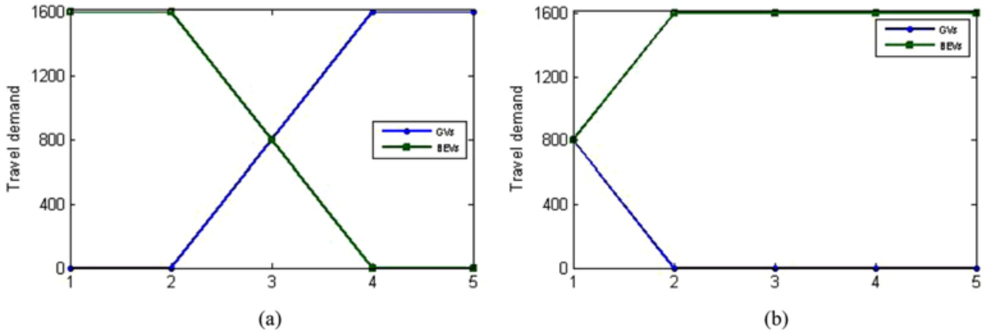


Fig. 6. Relationship between Mode Split and Unit Operating Cost in Nguyen-Dupuis' Network: (a) Unit Operating Cost of BEVs, (b) Unit Operating Cost of GVs

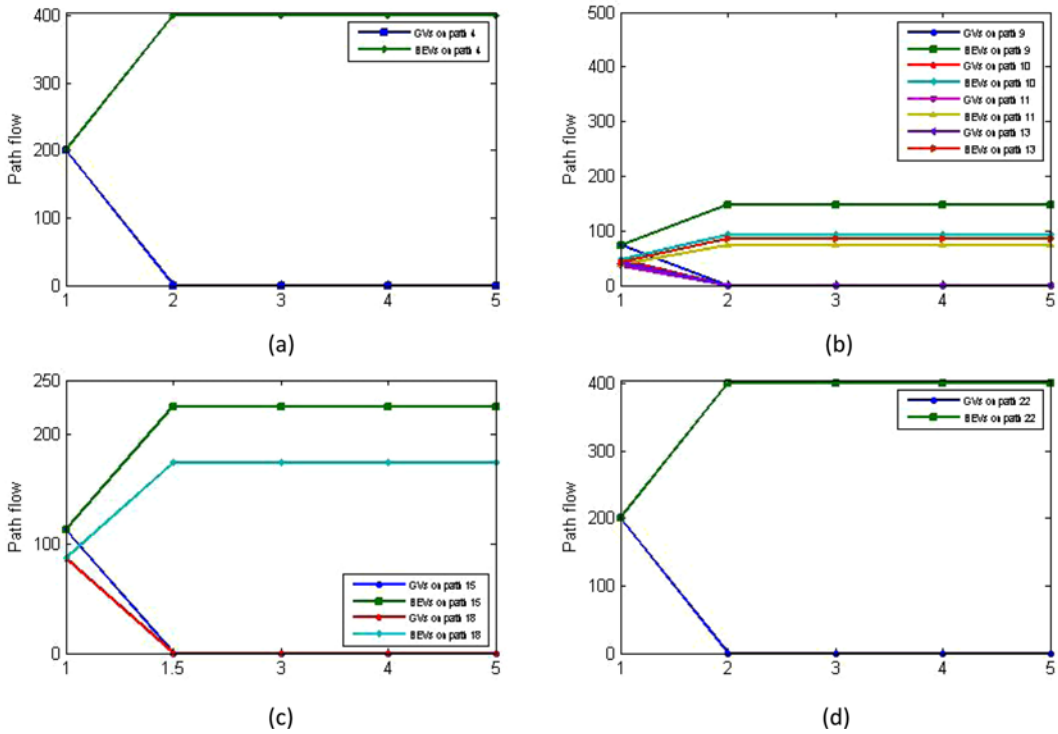


Fig. 7. Changing Unit Operating Cost of GVs in Nyuyen-Dupuis' Network: (a) O-D Pair (1, 2) (b) O-D Pair (1, 3), (c) O-D pair (4, 2) (d) O-D Pair (4, 3)

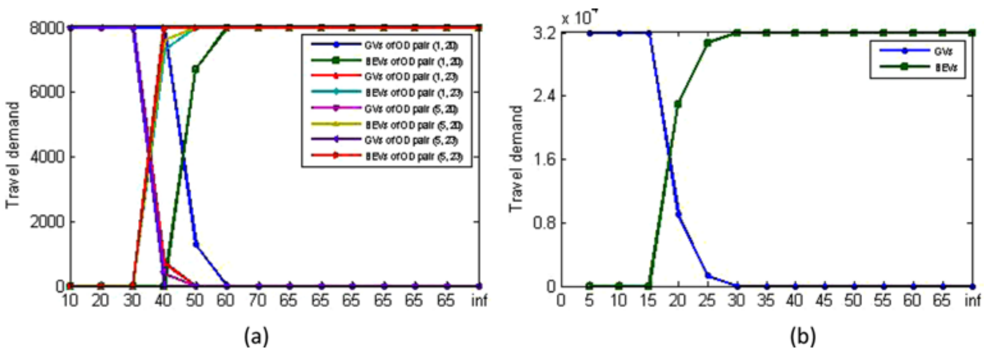


Fig. 8. Changing Distance Limit of BEVs in Sioux Falls' Network: (a) Each OD Pair, (b) All OD Pairs

4.2.4 Summary

Form Subsections 4.2.2 and 4.2.3, if the unit operating cost of BEVs is smaller than GVs, the traffic management department should lead users to choose the BEVs. In the network, the higher the unit operating cost of BEVs, the less the BEVs chosen by drivers, which can be seen from Fig. 6(a). Similar result for GEs

Table 3. Traffic Flow of Each Link_1

	5		10		15		20		25		30		35		40		45		50		55		60		65		inf	
	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs
1-2	5198	0	5198	0	5198	0	5891	0	4613	0	1600	4508	0	5946	0	6005	0	6009	0	6008	0	6010	0	6010	0	6002	0	6002
1-3	14802	0	14802	0	14802	0	6735	7374	0	15387	2400	11492	0	12454	0	13995	0	13991	0	13992	0	13990	0	13990	0	13998	0	13998
2-1	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400
2-6	4798	0	4798	0	4798	0	5491	0	4213	0	1200	4508	0	5946	0	5605	0	5609	0	5608	0	5610	0	5610	0	5602	0	5602
3-1	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	0	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
3-4	4097	0	4097	0	4097	0	3568	0	0	4344	0	3809	0	3790	0	3769	0	3750	0	3752	0	3750	0	3750	0	3803	0	3803
3-12	12305	0	12305	0	12305	0	4767	7374	400	12243	2400	9283	0	10264	0	10226	0	10241	0	11840	0	11840	0	11840	0	11796	0	11796
4-3	1600	0	1600	0	1600	0	1600	0	400	1200	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
4-5	800	0	800	0	800	0	800	0	800	0	0	800	0	800	0	800	0	800	0	800	0	800	0	800	0	800	0	800
4-11	11133	0	11133	0	11133	0	3568	7662	2613	9122	2400	8312	0	8643	0	11066	0	11048	0	11050	0	11048	0	11048	0	11089	0	11089
5-4	7036	0	7036	0	7036	0	0	7662	1413	5978	0	6903	0	7252	0	7297	0	7298	0	7298	0	7298	0	7298	0	7286	0	7286
5-6	5554	0	5554	0	5554	0	2000	2852	2187	3610	3600	951	0	4769	0	4712	0	4708	0	4709	0	4708	0	4708	0	4710	0	4710
5-9	7810	0	7810	0	7810	0	2715	5171	800	6412	0	8946	0	6379	0	6391	0	8394	0	8394	0	8394	0	8394	0	8404	0	8404
6-2	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	0	2400	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000
6-5	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
6-8	7552	0	7552	0	7552	0	4691	2852	3600	3610	2000	5458	0	6715	0	6717	0	7517	0	7517	0	7518	0	7518	0	7511	0	7511
7-8	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	0	0	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200
7-18	2562	0	2562	0	2562	0	3891	338	41	2410	0	4046	0	2759	0	3553	0	3565	0	3565	0	3563	0	3563	0	3608	0	3608
8-6	800	0	800	0	800	0	800	0	800	0	800	0	0	0	0	0	0	800	0	800	0	800	0	800	0	800	0	800
8-7	2962	0	2962	0	2962	0	4291	338	441	2410	400	4046	0	2759	0	3953	0	3965	0	3965	0	3963	0	3963	0	4008	0	4008
8-9	4000	0	4000	0	4000	0	4000	0	4000	0	4000	0	0	4000	0	4000	0	4000	0	4000	0	4000	0	4000	0	4000	0	4000
8-16	6990	0	6990	0	6990	0	2800	2514	5559	1200	0	5413	0	5957	0	5964	0	5952	0	5952	0	5955	0	5955	0	5903	0	5903
9-5	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	0	0	0	0	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000
9-8	4000	0	4000	0	4000	0	4000	0	4000	0	0	4000	0	4000	0	4000	0	4000	0	4000	0	4000	0	4000	0	4000	0	4000
9-10	7010	0	7010	0	7010	0	1915	5171	0	6412	3200	4946	0	6379	0	7591	0	7594	0	7594	0	7594	0	7594	0	7604	0	7604
10-9	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	0	0	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200
10-11	2000	0	2000	0	2000	0	2000	0	141	1859	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000
10-15	6600	0	6600	0	6600	0	4715	0	2772	3353	3200	3346	0	7638	0	7667	0	7647	0	7646	0	7645	0	7645	0	7622	0	7622
10-16	3211	0	3211	0	3211	0	0	5171	3087	0	4400	0	0	2740	0	2724	0	2747	0	2748	0	2749	0	2749	0	2783	0	2783
10-17	3200	0	3200	0	3200	0	3200	0	0	3200	0	3200	0	3200	0	3200	0	3200	0	3200	0	3200	0	3200	0	3200	0	3200
11-4	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	0	0	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400
11-10	2000	0	2000	0	2000	0	2000	0	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000
11-12	2400	0	2400	0	2400	0	2400	0	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400
11-14	10814	0	10814	0	10814	0	3026	7662	1954	9855	1600	8814	0	9211	0	10810	0	10811	0	10811	0	10811	0	10811	0	10811	0	10811
12-3	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	0	0	0	0	0	0	0	1600	0	1600	0	1600	0	1600	0	1600
12-11	2881	0	2881	0	2881	0	2658	0	0	3273	0	2902	0	2968	0	2944	0	2963	0	2961	0	2963	0	2963	0	2922	0	2922
12-13	11423	0	11423	0	11423	0	4109	7374	0	11370	2000	8781	0	9695	0	10882	0	10878	0	10879	0	10877	0	10877	0	10873	0	10873

can be observed from Fig. 6(b). Moreover, the changing trend of the flow on each path is similar to that of each O-D pair and, with the in crease of the unit operating cost of BEVs, the number of BEVs decreases, while the number of GVs has opposite tendency, as shown in Figs. 5 and 7. Therefore, at least in our cases, when a path is feasible, drivers' choosing strategies depend on the unit operating cost, namely, the lower the unit operating cost, the greater the chance of the corresponding vehicles chosen by drivers.

4.3 Sioux Falls' Network

The Sioux Falls' network was used by Suwansirikul *et al.* (1987); see Fig. 9(a) for the real traffic network and Fig. 9(b) for a simulated image of the network.

In our test, four OD pairs (1, 20), (1, 23), (5, 20), (5, 23) were chosen and each pair's traffic demand was set to be 8000. Traffic capacity of each link was assumed to be smaller than 16000. Fig. 8(a) reveals that, when the distance limit of BEVs is shorter than all feasible paths' length, all drivers should choose GVs and, when the distance limit of BEVs is bigger than 30, all drivers can select BEVs. The total changes of BEVs and GVs in the entire traffic network is shown in Fig. 8(b). It can be seen that, when the distance limit of BEVs is smaller than 20, all drivers should choose the GVs and, when the distance limit of BEVs is bigger than 30, all drivers should choose the BEVs.

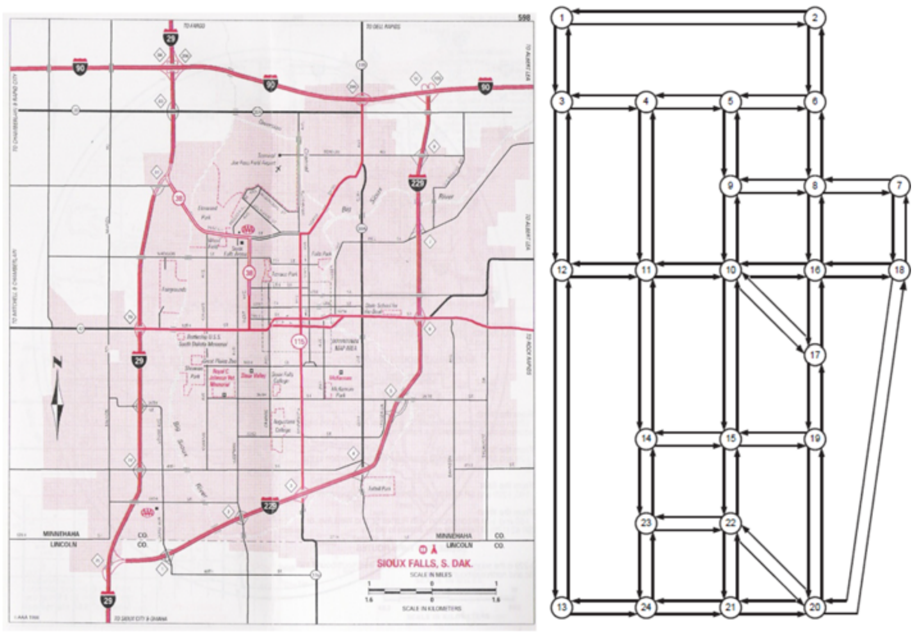
The link results are given in Tables 3 and 4, from which we can observe that, if the range is no more than 15, all links have no BEVs and, if the range is no less than 35, all links have no GVs.

5. Conclusions

This paper focuses a mixed traffic network containing BEVs and GVs. A more reasonable piecewise impedance function has been presented and, based on this new impedance function, a

Table 3. Traffic Flow of Each Link_2

	5		10		15		20		25		30		35		40		45		50		55		60		65		inf	
	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs	GVs	BEVs
13-12	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	0	0	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200
13-24	11823	0	11823	0	11823	0	4509	7374	400	11370	2400	8781	0	9695	0	11282	0	11278	0	11279	0	11277	0	11277	0	11273	0	11273
14-11	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	0	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	16W00
14-15	2000	0	2000	0	2000	0	2000	0	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000
14-23	10814	0	10814	0	10814	0	3026	7662	1954	9855	1600	8814	0	10811	0	10810	0	10811	0	10811	0	10811	0	10811	0	10811	0	10811
15-10	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400
15-14	2000	0	2000	0	2000	0	2000	0	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000
15-19	3948	0	3948	0	3948	0	2937	0	1354	857	800	1792	0	3494	0	3487	0	3498	0	3497	0	3495	0	3495	0	3493	0	3493
15-22	2651	0	2651	0	2651	0	1778	0	218	3696	0	3954	0	4144	0	4180	0	4149	0	4148	0	4150	0	4150	0	4128	0	4128
16-8	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000
16-10	1600	0	1600	0	1600	0	1600	0	1600	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
16-17	7400	0	7400	0	7400	0	0	7685	5846	1200	2400	4613	0	5897	0	5887	0	5899	0	5900	0	5904	0	5904	0	5886	0	5886
16-18	1200	0	1200	0	1200	0	1200	0	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200
17-10	3200	0	3200	0	3200	0	3200	0	3200	0	3200	0	0	3200	0	3200	0	3200	0	3200	0	3200	0	3200	0	3200	0	3200
17-16	800	0	800	0	800	0	800	0	800	0	0	800	0	800	0	800	0	800	0	800	0	800	0	800	0	800	0	800
17-19	7400	0	7400	0	7400	0	0	7685	2646	4400	0	7013	0	5897	0	5887	0	5899	0	5900	0	5904	0	5904	0	5886	0	5886
18-7	800	0	800	0	800	0	800	0	800	0	800	0	0	0	0	800	0	800	0	800	0	800	0	800	0	800	0	800
18-16	1200	0	1200	0	1200	0	1200	0	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200
18-21	3362	0	3362	0	3362	0	4691	338	841	2410	800	4046	0	4359	0	4353	0	4365	0	4365	0	4363	0	4363	0	4408	0	4408
19-15	1200	0	1200	0	1200	0	1200	0	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200
19-17	800	0	800	0	800	0	800	0	800	0	800	0	0	800	0	800	0	800	0	800	0	800	0	800	0	800	0	800
19-20	10948	0	10948	0	10948	0	2537	7685	4800	4057	1600	7605	0	8991	0	8975	0	8997	0	8997	0	8999	0	8999	0	8979	0	8979
20-19	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
20-21	2400	0	2400	0	2400	0	2400	0	2400	0	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400	0	2400
20-22	2000	0	2000	0	2000	0	2000	0	2000	0	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000	0	2000
21-18	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
21-20	7600	0	7600	0	7600	0	9200	0	400	9120	0	8441	0	8065	0	8045	0	8054	0	8054	0	8051	0	8051	0	8092	0	8092
21-24	1200	0	1200	0	1200	0	862	338	1241	1289	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200
22-15	1200	0	1200	0	1200	0	1200	0	1200	0	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200	0	1200
22-20	3451	0	3451	0	3451	0	2578	0	800	2823	0	4354	0	4944	0	4980	0	4949	0	4948	0	4950	0	4950	0	4928	0	4928
22-23	1600	0	1600	0	1600	0	1600	0	218	2473	0	2000	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
23-14	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
23-22	1600	0	1600	0	1600	0	1600	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
23-24	800	0	800	0	800	0	800	0	800	0	0	800	0	800	0	800	0	800	0	800	0	800	0	800	0	800	0	800
24-13	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	0	0	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600	0	1600
24-21	4638	0	4638	0	4638	0	4571	0	0	8000	800	3195	0	4106	0	4092	0	4089	0	4089	0	4088	0	4088	0	4084	0	4084
24-23	7586	0	7586	0	7586	0	0	7712	841	4659	0	7586	0	7589	0	7590	0	7589	0	7589	0	7589	0	7589	0	7589	0	7589



(a) (b)

Fig. 9.Sioux Falls' Network: (a) Sioux Falls' Network (b) Simulated image of Sioux Falls' Network

new traffic network flow assignment model has been constructed. The new model is actually a convex nonsmooth optimization problem. By introducing a binary variable, the problem has been rewritten a smooth optimization problem. Two numerical examples have been used to verify the efficiency and reasonability of the model. As a future work, we will consider the mixed traffic networks with some uncertainties (weather, traffic accidents, etc.) and analyze how to determine the operating cost of BEVs in the next step.

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