KSCE Journal of Civil Engineering (2017) 21(2):488-500

DOI 10.1007/s12205-017-1488-7

Design Optimization and Applications in Civil Engineering

pISSN 1226-7988, eISSN 1976-3808
www.springer.com/12205

Multi-objective Optimization of Structural Steel Buildings under Earthquake Loads using NSGA-II and PSO

Manuel Barraza*, Edén Bojórquez, Eduardo Fernández-González***, and Alfredo Reyes-Salazar******

Received August 31, 2016/Revised October 31, 2016/Accepted November 2, 2016

Abstract

The aim of this study is to illustrate and compares the use of Genetic Algorithms (GAs) and Particle Swarm Optimization (PSO) for multi-objective optimization of two and three dimensional moment resisting steel structures subjected to earthquake loads. For this purpose, steel buildings with different characteristics are designed under earthquakes using the Non-dominated Sorting Genetic Algorithm (NSGA-II) and PSO as a tool to achieve the best structure in terms of: minimize the total structural weight (which is directly related with the costs), control of the maximum inter-story drift, and to satisfy the strength requirements of the AISC-LRFD specification. It is considered that all the steel structures are constituted by elements with W section (256 in total) taken from the LRFD-AISC Database. Although, the GAs and PSO are applied for moment resisting steel structures, the concepts can be extended for other structural systems. It is concluded that the use of NSGA-II and PSO reduce the structural weight and they are a very useful tools to improve the structural performance of the buildings. Finally, the structural buildings obtained via PSO are in general better solutions in comparison with the NSGA-II approach.

Keywords: *genetic algorithms, particle swarm optimization, multi-objective optimization, steel structures, earthquake loads*

1. Introduction

Currently, because the scientific and technologic advances in the computational areas, new optimization approaches based on artificial intelligence and bio-inspired have been proposed, such as: genetic algorithms, particle swarm optimization, ant colony optimization, bee colony optimization, tabu search, firefly algorithm, harmony search algorithm among others. All these techniques known as meta-heuristics have been used to solve several engineering optimization problems (Yang, 2010); in particular, the most used procedures around the word are: genetic algorithms and particle swarm optimization (Eslami *et al*., 2012). Genetic algorithms is a technique inspired in natural selection (Holland, 1975), which is gaining a growing for solving engineering, social, physical and other problems. Although the first ideas to use the natural selection as a tool to solve engineering optimization problems were developed in the 50's and 60's, the GAs were essentially invented by John Holland in the 60's (Coley, 1999). In the field of earthquake and structural engineering, the GAs have been used in several applications such as the optimum design of structural steel and reinforced concrete frames (Camp *et al*., 1998; Rajeev and Krishnamoorthy, 1998; Pezeshk and Camp, 2000; Lee and Ahn, 2003; Kripakaran *et al*., 2011), record selection for seismic performance and design of buildings (Naeim *et al*., 2004; Bojórquez *et al*., 2013), in particular most of the studies focused on the structural optimization using evolutionary techniques as genetic algorithms have been developed for the optimum design of steel trusses (Dede *et al*., 2011; Sonmez, 2011; Prendes *et al*., 2011). In the case of GAs for the structural analysis and design of planar or spatial steel frames only one objective function is considered in several studies, usually the total weight of the building. Since the seismic design of buildings required the control of various parameters, such as the cost, the performance of the building, among others. It is important to consider more objective functions for the design of complex systems as in the case of 3D steel framed buildings under earthquake loads though GAs. By the other hand, Eslami *et al.* (2012) present a state of the art of particle swarm optimization

*Ph.D. Student, Faculty of Engineering, Universidad Autónoma de Sinaloa, Calzada de las Américas y Boulevard Universitarios S/N, Ciudad Universitaria, Culiacán, Sinaloa, México, C.P. 80040 (Corresponding Author, E-mail: mb12_05@hotmail.com)
**Professor, Faculty of Engineering, Universidad Autónoma de Sinaloa, Calzada de las Américas y Boulevard Universitarios S/N, Ciudad Universitaria, Culiacán, Sinaloa, México, C.P. 80040 (E-mail: eden@uas.edu.mx)
***Professor, Faculty of Engineering, Universidad Autónoma de Sinaloa, Calzada de las Américas y Boulevard Universitarios S/N, Ciudad Universitaria, Culiacán, Sinaloa, México, C.P. 80040 (E-mail: eddyf@uas.edu.mx)
****Professor, Faculty of Engineering, Universidad Autónoma de Sinaloa, Calzada de las Américas y Boulevard Universitarios S/N, Ciudad Universitaria, Culiacán, Sinaloa, México, C.P. 80040 (E-mail: reyes@uas.edu.mx)

and its application on various optimization problems arising in different fields. They demonstrated among the meta-heuristic procedures that *the particle swarm optimization and its modified versions have widespread application in complex optimization domains, and is currently a major research topic, offering an alternative to the more established evolutionary computation techniques that may be applied in many of the same domains*. In addition, as in the case of GAs, the applications to solve structural engineering problems are usually discussed for the design of trusses (Hosseini *et al*., 2015), without discussion of the solution as a viable option in real structural engineering practice. Motivated by the need to obtain economic and safety buildings with the tools known as GAs and PSO for multi-objective optimization of steel buildings subjected to earthquakes. The aim of this study is to illustrate the application of GAs and PSO for multi-objective optimization of 2D and 3D moment resisting steel structures under earthquake loads. For this aim, several steel buildings with different characteristics are designed through the Non-dominated Sorting Genetic Algorithm and PSO considering two objective functions. While the first objective function is to minimize the total structural weight (a key parameter related with the total cost), the second objective function is the control of the maximum inter-story drift which is the main parameter suggested by most of the seismic design codes to guarantee a satisfactory earthquake-resistant design. In addition, the strength requirements of the AISC-LRFD regulations are satisfied as constraint variables. For the present study, all the structural steel buildings are constituted by elements with W shape (256 in total) taken from the LRFD-AISC Database. The sections are represented by binary codification of eight bits in the case of GAs while for the case of PSO is used decimal encoding, and the earthquakes are simulated as horizontal loads located in the floors of the buildings which are the place where the mass is concentrated. Finally, it is important to say that in the present study a constraint was considered to achieve a correct assemble of beam-column joints, which is not commonly used in structural optimization studies that involve the weight minimization of the system. Thus, the steel buildings designs obtained here constitute real structural options that can be built into the engineering practice. In the following, a brief description of genetic algorithms including NSGA-II and particle swarm optimization is provided.

2. Genetic Algorithms

Genetic algorithms are heuristic methods used to solve optimization problems, which are based on the principles of natural selection postulated by Darwin (Holland, 1975; Goldberg, 1989; Kuri-Morales and Galaviz-Casas, 2002). The main characteristic of GAs lies in the principles of survival and adaptation of the ablest organisms. An advantage of the application of GAs is working on a population of individuals consisting of feasible solutions to the optimization problem specific, where each individual is defined by a chain, and each chain is composed of a series of characters, can be binary, decimal, numbers and so on., representing a particular solution. The technique of GAs is to randomly generate a population of individuals or possible solutions to a given problem, and assign a rating to each individual depending on their ability to adapt to the solution of the problem. For example, in a maximization problem of a function, individuals with higher rating will be those with the highest value of the function; in addition, the higher the adaptation of an individual to the problem, the greater the chance that it will be selected to move on to the next generation as in the case of natural selection. Thus a new population of individuals or possible solutions occurs, which replaces the previous one and contains a better proportion of good features compared to the preceding populations. Therefore, along the good characteristics generations propagate through the population. The three main parts of a genetic algorithm are: encoding and decoding variables or individuals in chain or arrangements; adaptation assessment of each possible solution or chain, and finally, the application of genetic operators to produce the next generation of possible solutions (Camp *et al.,* 1998).

Most genetic algorithms are variations of the simple genetic algorithm proposed by Goldberg (1989), which consists of three basic genetic operators: reproduction, crossover and mutation The reproduction is based on the selection mechanisms of Darwin's theory; that is, the fittest individuals survive (Koza, 1992). The objective of crossing is to create variations in new populations of individuals, producing new solutions consist of strings other parts taken from parent solutions known chains. Mutation introduces random changes in the population of a generation; in general, the mutation may be beneficial as it allows to introduce diversity in a population.

The evolutionary heuristic methods such as genetic algorithms have proven to be powerful to handle the exponential complexity of problems and the lack of "noble" mathematical properties of the functions of the problem and circumvent local solutions converging to areas close to the global optimum (Deb, 2001; Coello *et al.,* 2002). In particular, the well-known Non-dominated Sorting Genetic Algorithm has provided very well results for solving multi optimization problems. This is the first selected technique used herein for the multi-objective optimization of structural steel buildings under earthquake loads. In the following, a short description of the NSGA-II approach is described.

3. Non-dominated Sorting Genetic Algorithms (NSGA-II)

The technique NSGA-II proposed by Deb (2001) and Deb *et al.* (2002) is used in this study for the multi-objective optimization design of structural steel buildings under seismic forces. The main idea of the NSGA-II approach is to find non-dominated solutions all of which represent a Pareto frontier. For example, let suppose that it is necessary to minimize all objective functions in a multi-objective problem. Fig. 1, shows all the feasible solutions of the optimization problem, note that the non-dominated solutions corresponds to those which are not worse

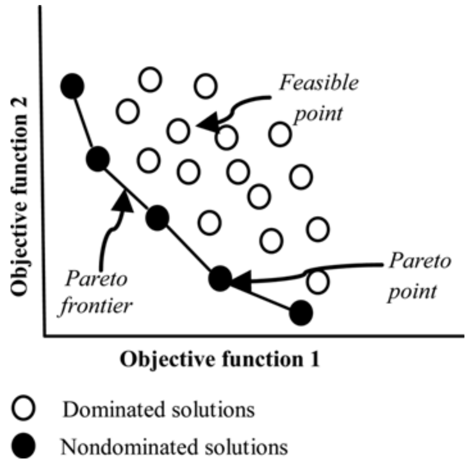

Fig. 1. Pareto Frontier

that other solution by considering all the objectives, or if the solution is better than other in at least on objective function, these solutions represent the Pareto frontier. In general, the NSGA-II is implemented with an effective sorting method based on individual ranking by non-dominated sorting and a crowded distance sorting which evaluates the population density of solutions in the same rank. The typical steps of the NSGA-II approach are:

1. An initial parent population P_0 is randomly generated, and the non-dominated sorting is implemented on P_0 where each individual is ranked based on the dominance relation in the objective space.
2. Individual within each rank is sorted again based on the crowded distance where the population density is evaluated. For further information about the crowded distance see Deb (2001).
3. Individuals selected by a tournament selection are stored in an intermediate mating pool which has a high probability for occurrence of better ranked and less crowded solutions.
4. In the mating pool, genetic operations such as crossover and mutation generate the child populations Q_t where subscript "t" denotes the number of generations.
5. An integrated population Rt is created by combining Pt and Q_t, and fitness values are assigned to all individuals by the non-dominated sorting and crowded distance sorting.
6. Finally, individuals with better fitness are selected by elitist sorting and these become the parent individuals P_{t+1}.
7. Steps 2–6 are repeated (while t <NG); where NG represent the number of generations required.
8. Individuals with rank one among parents at Ptmax are Pareto optimal solutions.

4. Particle Swarm Optimization (PSO)

The particle swarm optimization is a population-based technique to solve several optimization problems, which was proposed by Kenney and Eberhart (1995). The main characteristic of this approach is the simulation of the social behavior for example to represent the movement of organisms in a bird flock. As it is indicated by Ruan (2010) and Eslami *et al.* (2012), in a PSO system, multiple candidate solutions coexist and collaborate simultaneously. Each solution named a particle, flies in the problem search space looking for the optimal position to land. A particle during the generations modify its position according to its own experience as well as the experience of neighboring particles. The PSO approach combines local search method (through self experience) with global search methods (through neighboring experience), attempting to balance exploration and exploitation. For the sake of brevity, only the conceptual algorithm for basic PSO is illustrated in Fig. 2 according to Cazacu and Grama (2013); nevertheless, its implementation for the earthquake-resistant design of steel frames is discussed in chapter 6. In Fig. 2 *Pbest* is the best known position of the particle and *Gbest* is the swarm best historical position.

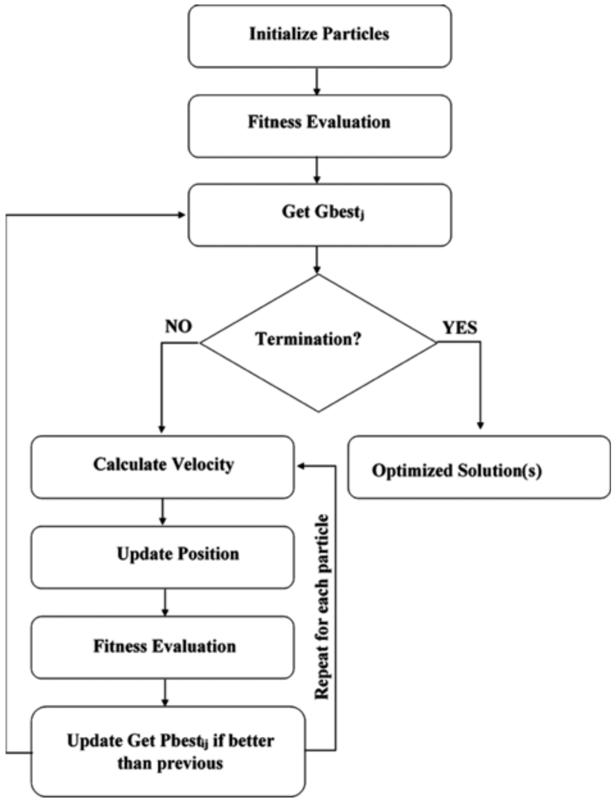

Fig. 2. Conceptual flow chart for basic PSO (Cazacu and Grama, 2013)

In the following, the methodologies for the structural design of steel framed buildings based on the NSGA and PSO approaches are illustrated.

5. Structural Design of Steel Buildings Under Earthquake Loads using NSGA-II: Methodology

The potential of the NSGA-II approach as a tool for the optimum seismic design of structural steel frames can be

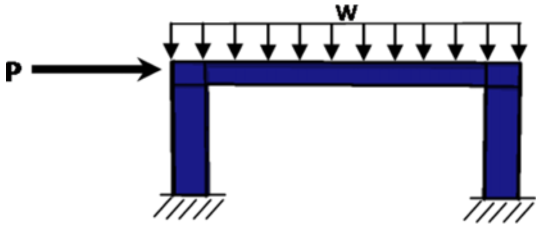

Fig. 3. Typical One Story One Bay Steel Frame under Gravity and Earthquake Load

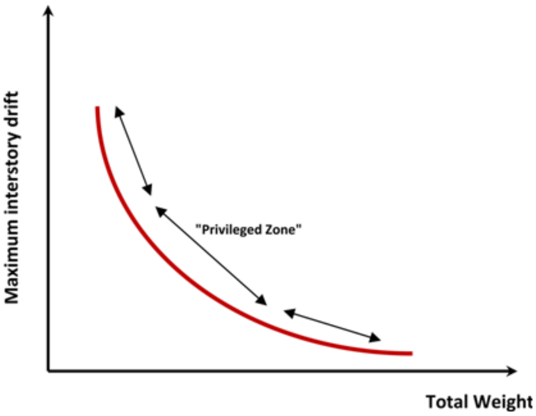

Fig. 4. Surface of All Possible Solutions of a Structural Frame at the Pareto Frontier

illustrated as following: let suppose that we want to design a structural steel planar frame of one story and one bay subjected to the loads p and w (see Fig. 3). The design objectives are to minimize the total structural weight W_T and the maximum interstory drift. For this case, let suppose that a total of 256 different types of steel cross sections W can be used for the elements (beams and columns). If there are a total of 256 different outcomes for each structural element constituting the frame, that means there are a total of 256^3(16777216) possible solutions; even if we consider that the columns are similar, there are a wide number of possible solutions to the problem. Thus the selection of the frame with the less weight and interstory drift requires several computational analysis despite this is one of the simplest structural frames. In general, for a frame consisting of N elements (beams and columns), the number of possible solutions will be 256^N. Of course, not all solutions are satisfactory, in fact, from the entire space of solutions, some of them will be dominated by heavy structures and small maximum interstory drift demand, which are optimal in terms of displacements; moreover, if higher demands of drifts are presented with very small structural weight; these solution will be optimal in terms of weight. The structural engineering is not interested in any of these solutions but rather in those in which there is a balance between economy (lightweight structures) and structural performance (controlling of the maximum interstory drift, mechanical elements, and so on) or in other words solutions that are in the "privileged zone" of the Pareto frontier as illustrated in Fig. 4.

It is clear that all solutions of the red line named "Pareto frontier" of Fig. 4 are "non- dominated" solutions. This means that marks the front of the best solutions found by the algorithm in the surface of all the possible solutions. In the present work, the Pareto frontier is obtained by considered two objectives, the total structural weight and the control of a maximum interstory drift of 0.01, this value may vary depending on the requirements of seismic design codes and the structural performance of interest. Bojórquez *et al.* (2011) recommend a maximum interstory drift of 0.01 for steel frames considering cumulative damage and a ductility behavior factor of 2. Finally, in this paper the structures are designed to also satisfy the requirements of AISC-LRFD resistance; moreover, a correct assemble of beam-column joints which is not commonly used in structural optimization studies is considered.

The procedure used for the seismic design of structural steel frames is described using GAs through the NSGA-II approach:

1. *Initial population*: the first generation or initial population is defined randomly of individuals and possible solutions of structural steel frames "P_0" of size "N" individuals, to be parents according to the NSGA-II method. Each individual consists of a structural steel frame constituted by W steel section elements taken from a total of 256 sections from the LRFD (AISC) Database. Binary encoding to represent each section is used, which means that a total of 8 bits are needed to codified the entire space of W sections. For example, the binary number 00000000 will represent the first section of the database, while the last section is 11111111 see Fig. 5. Therefore, an individual will consist of a binary encoding of $8 \times N$ bits, where N represents the total number of elements in each frame, or alternative each structure is formed by a vector of N elements each of 8 bits. In other words, the frame of Fig. 3 is a vector of three parameters (see Fig. 6),

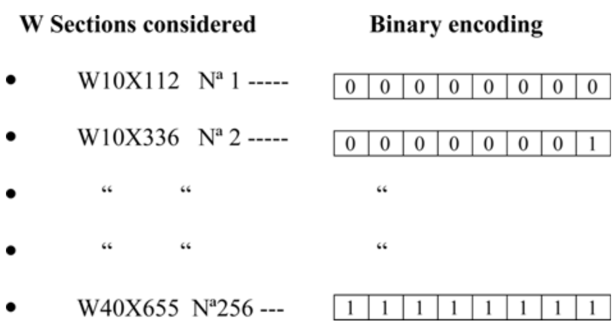

Fig. 5. Binary Codification of W Sections

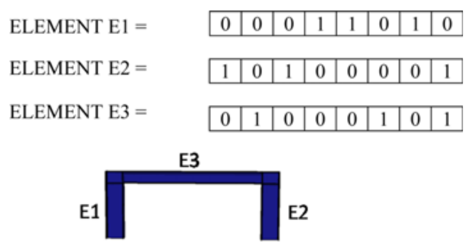

Fig. 6. Binary Codification of a Steel Frame

where each parameter is an 8-bit binary number representing a specific part of the structure. Finally, we consider the number of individuals or structural frames unchanged from generation to generation, and a total of 100 individuals are used by each generation for this work, which is within the typical recommended values b etween 20 and 1000 individuals for generation (Coley, 1999).

2. *Analysis of the structural steel frame buildings*: Gravity and earthquake loads are considered for the purpose of the structural analysis, where the earthquake loads are simulated as horizontal forces located in each of the floor of a specific steel structure. Firstly, the structural analysis of each of the 100 possible solutions (N individuals) is computed.
3. *Objective functions or adaptive*: (formulation problem) The mathematical expressions that are used to measure the adaptability of an individual or the two objective functions under consideration are the following:

$$\text{Minimize } F_{O1} = W_T \times F_P \tag{1}$$

$$\text{Minimize } F_{O2} = |\gamma_C - P_1 \times \gamma_D| \times F_P \tag{2}$$

Subject to:

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0 \quad \text{For } \frac{P_r}{P_c} \ge 0.2 \tag{3}$$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0 \quad \text{For } \frac{P_r}{P_c} < 0.2 \tag{4}$$

$$\text{where } W_T = \sum_{i=1}^{i=N} \rho_i L_i A_i \tag{5}$$

In Eqs. (1)-(5) F_{O1} is the objective function related to the structural weight; F_{O2} is the objective function related to the maximum interstory drift; F_P is a penalty function for those solutions who violate the limitation of the maximum interstory drift and strength requirements provided in Eqs. (3) and (4); ρ_i is the density of the steel; L_i the element length i; A_i section area i; γ_c is capacity of maximum interstory drift equal to 0.01. Finally γ_D is demand of interstory drift, P_r and M_r are the required strengths and flexural stress, P_C and M_C are the available strengths and flexural stress. The steel structural frames selected will be those with the lowest values o f the objective functions given in Eqs. (1) and (2). Finally, $P_1 = 1$ if $\gamma_D \le \gamma_C$ and $P_1 = 50$ if $\gamma_D > \gamma_C$.

The penalty function F_P is estimated as follows:

$$F_P = F_{Rb} \times F_{Rc} \times P_k \times [1 + (P_v - 1)] \tag{6}$$

where $P_K = 2$ if $kL/r > 200$ and $P_K = 1$ if $kL/r \le 200$

kL/r is called the slenderness ratio of the column; the length kL is known as the effective length of the column. The dimensionless coefficient K is called the effective length factor and r is the radius of gyration of the cross section about the axis of bending. For F_{Rc} and F_{Rb} the maximum value of the resistance factor of each elements (beams and columns) is checked. Firstly, F_{Rc} (for columns) and F_{Rb} (for beams) are equal to one, if the element satisfies with ($0.5 \le F_{Rb} \le 1$ or $0.5 \le F_{Rc} \le 1$) they are equal to one, on the contrary, they are penalized.

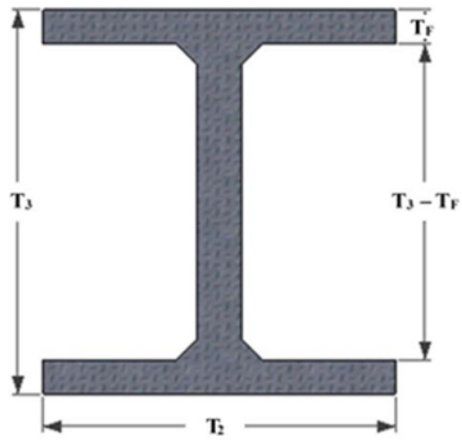

Fig. 7. Transversal Section of a Typical W Shape

Finally, P_V is used to penalize those steel frame buildings that do not satisfy the beam-colum joint for the engineering practice. Thus, if the assemble is acceptable to be built, a value equals with one is selected. To further illustrate this, the geometrical characteristics of the transversal area of the W sections to be used as beam or column are shown in Fig. 7.

where $\mathbf{T_3}$= Depth; $\mathbf{T_f}$ = Flange thickness; $\mathbf{T_2}$ = Flange width and $\mathbf{T_3 - T_f}$ = the web of the section.

Correct beam-column joint for planar (2D) steel framed structures

In planar steel frames, the only constraint to get a correct assemble of beams and columns is that the width of the flange of the beam T_{2b} be smaller than the width flange of the column T_{2c}, that is: if $T_{2c} \ge T_{2b}$ the joint is adequate, on the contrary, the steel frame is penalized through the equation $P_v = P_v + (T_{2b} - T_{2c})$.

Correct beam-column joint for spatial (3D) steel framed structures

In 3D steel frames, in addition to the constraint of planar frames. The beam must be correctly join to the web of the column. In other words, the width of the beam T_{2b} have to be smaller than the depth of the web column $T_{3c} - 2 \times T_{fc}$. If $T_{2c} \ge T_{2b}$ and $T_{3c} - 2 \times T_{fc} \ge T_{2b}$ the joint is adequate, on the contrary, the steel frame is penalized.

4. *Classification:* The population is sorted into different layers or classes (first non-dominated individuals); first class C_1, usually consists of non-dominated solutions or minor weakness P_0. The second layer C_2 contains the non-dominated solutions or minor weakness in individuals of $\{P_0\} - \{C_1\}$ and so on until classify all the individuals P_0 (see Fig. 8). Each solution is assigned a rank equal to the number of layer in which they are located, considering they are better individuals who were first selected.
5. *Selection*: The selection of parents is based on the binary tournament named “Crowded Tournament Selection Operator” which consists in the selection of two individuals randomly, which compete with each other (binary tournament,

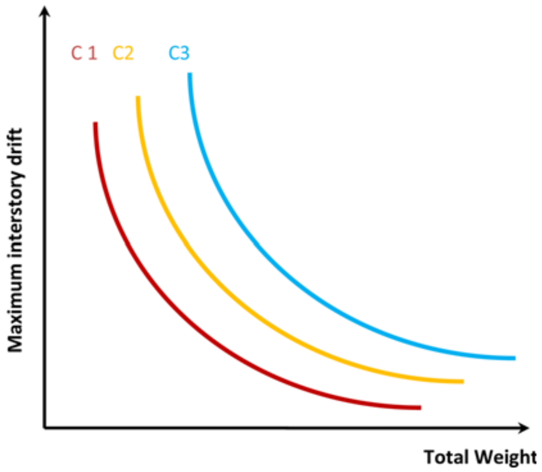

Fig. 8. Ranking in Layers or Classes of the Population Parent "N"

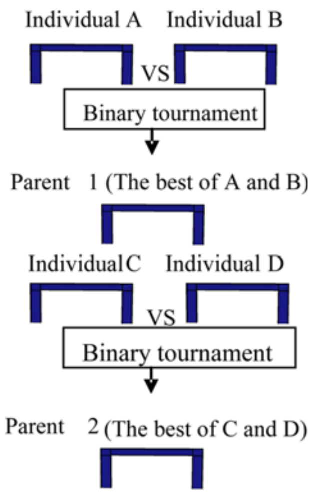

Fig. 9. Binary Tournament (Crowded Tournament Selection Operator) of the Structural Frame

see Fig. 9); the best individual win the tournament and form the first parent. This tournament is repeated to find the second and the following parents. Once the parents are found, the algorithm proceed to create the offspring through the process of crossover and mutation. In this case, two children are obtained from two parents. In our problem from the population of 100 parents (N individuals) a population of 100 offspring will be created.

6. *Crossover*: At this stage of the evolutionary process, two selected individuals (parents) are joined to form another pair of individuals (offspring) with the characteristics of the parents. At the end of 100 new structural frames we will have a new generation of structural steel frames constituted by the same number of individuals of the previous generation. A single point crossing for each element is used; that is, the crossing occurs in a portion of each individual element or structural frame.
7. *Mutation*: It is used to ensure a good diversity of the structural steel frames. This process is applied to each new gener-

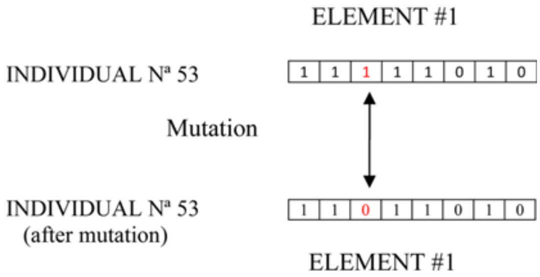

Fig. 10. Illustrative Example of a Typical Mutation

ation completely, and very occasionally is to invest in the position of each bit a 1 for a 0 or the inverse, with a very small probability of mutation P_M, to have a wider range of possible solutions. Typical values of P_M are in a range from about 0.001 (Coley, 1999), which means that one in a thousand will be mutated a bit, this value of probability of mutation was used for the present study. It is important to say that special careful must be the selection of the mutation probability, since very high values c ould extend the convergence to the optimal solution. Fig. 10 shows an example of an individual mutated.

8. *Combination*: After Parent populations (initial population) of size N are joined to the population of offspring of size N, a new population Pt' of size 2N (200 individuals) is obtained.
9. *Classification:* It is performed in the same way as step 4 but with twice of individuals 2N (200 individuals), this means that there will be changes in classification as there are new individuals (offspring) with different features.
10. *Creation of the new population:* As the algorithm works with a population of N individuals a new population P_{t+1} of size "N" is created by taking the best individuals of the layers contained in P_t' (in the previous step). Usually the first layer passes complete to P_{t+1} (implementation of elitism), then the fittest individuals (in smaller class rank) of the following layers are taken. If all individuals have the same fitness, a the niche technique is used to maintain the diversity of the population. As it was discussed before, NSGA-II uses a niche technique based on a measure of "crowding" (crowding distance). This measure is the density of the solutions in some region of non-dominated layers. The solutions in areas of less density of solutions are preferred because they help to keep the population diversity and "spread" the population more evenly on the Pareto front.

6. Structural Design of Steel Buildings under Earthquake Loads using PSO: Methodology

The steps of the methodology for seismic design of steel frame buildings are in general similar to the NSGA-II approach. For example, the same objective functions and constrains have been used. The first difference between NSGA-II and PSO applied to seismic design of steel structures corresponds to the encoding,

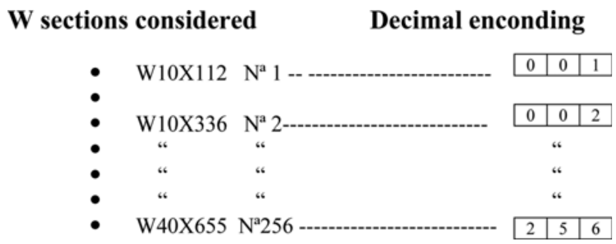

Fig. 11. Binary Codification of W Sections

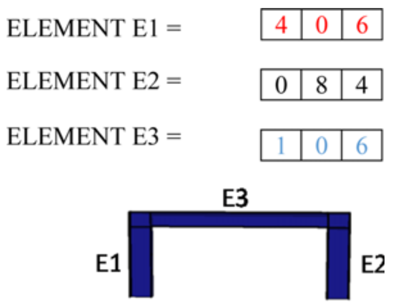

Fig. 12. Decimal Codification of a Steel Frame

which in the case of PSO is decimal for the structural members and steel frames as indicated in Figs. 11 and 12. The main difference corresponds to the method itself, which is described in the next paragraph.

In an N-dimensional space search, every particle swarm i (in this case a steel building model) know its current position $X_{ij} = [X_{i1}, X_{i2}, \ldots, X_{iN}]$, the speed $V_{ij} = [V_{i1}, V_{i2}, \ldots, V_{iN}]$ with which it has reached that position and the best position $P_{ij} = [P_{i1}, P_{i2}, \ldots, P_{iN}]$ in which it has been found (personal best position). In addition, all the particles know the best among all personal best positions in the swarm, which is called best global position $G_j = [G_1, G_2, \ldots, G_N]$. At each iteration t of the algorithm, each component j (section of the steel frame) of the speed and position of each particle swarm i is updated according to the following equations:

$$V_{ij}[t+1] = w \times V_{ij}[t] + C_1 \times Rand(.) \times (Pbest_{ij}[t] - X_{ij}[t]) + C_2 \times rand(.) \times (Gbest_j[t] - X_{ij}[t]) \quad (7)$$

$$X_{ij}[t+1] = X_{ij}[t] + V_{ij}[t+1] \quad (8)$$

where $V_{ij} = [V_{i1}, V_{i2}, \ldots, V_{iN}]$ is called the velocity of the particle i, which represents the travel distance of the particle from the current position $X_{ij} = [X_{i1}, X_{i2}, \ldots, X_{iN}]$; *Pbest* represents the best previous position of the particle i (ie, the set not mastered the best positions experience); *Gbest* represents all not dominated the top positions among all *Pbest* in the population (best overall position); w is the inertia parameter; C_1 is the cognitive parameter; C_2 is the social parameter; finally, *Rand*(.) and *rand*(.) are functions that returns a random number in the interval [0, 1]. In the present study, a value of 2 was used for C_1 and C_2, and w decreased linearly from 0.9 to 0.4 during a run as suggested by Eberhart and Shi (2000).

Equation (7) computes a new velocity vector for the *i-th* particle from its current speed, the Euclidean distance personal best position and the Euclidean distance to the best global position. In Eq. (8) the components of the position vector of the *i-th* particle are updated according to each component of the new speed.

7. Numerical Examples

In the previous sections, the approaches for the structural design of steel buildings under earthquake loads were described using the NSGA-II and PSO. In this part of the study, the procedure is applied for the structural design of a 2D and a 3D structural steel frame building. The first application consists in a 2D framed structure of four-stories and two bays and the second is a 3D frame of three levels with three bays in X direction and three bays in orthogonal Y direction.

7.1 EXAMPLE 1: Structural Design of a 2D Steel Framed Building of 4 Stories and 2 bays

7.1.1 Application with NSGA-II

Figure 13 shows the first example of a structural design which is a steel frame with 4 story levels and two bays named F4-2D. The geometrical characteristics of the steel structure are illustrated in Fig. 9. Note that to simulate the effect of the earthquake, lateral seismic loads were placed in each floor increasing according to the height and corresponding to 3, 6, 9 and 12 tons (T) respectively. The gravity loads corresponds to distributed forces of 2.5 T/m.

The steps for the structural design of steel buildings under earthquake loads using the NSGA-II are the following:

1. The first step of the procedure is to randomly generate the initial population (P_0) of "N" structural steel frames (in this study 100 steel frames were used) which will be the "parents" according to procedure. For this frame, four different W sections were used; 2 for columns (elements C1 to C6 = column 1 and C7 to C12 = column 2) and 2 for the beams (V1 to V4 = beam 1 and from V5 to V8 = beam 2) therefore,

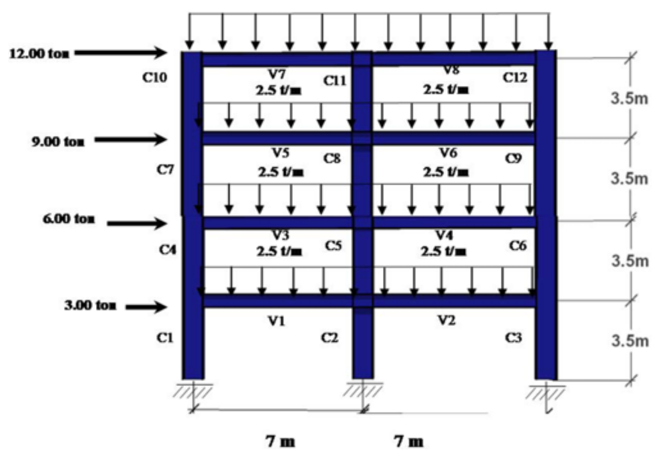

Fig. 13. Geometric Characteristics and Lateral Loads for the Structural 2D Steel Frame (F4-2D) of the First Design Example

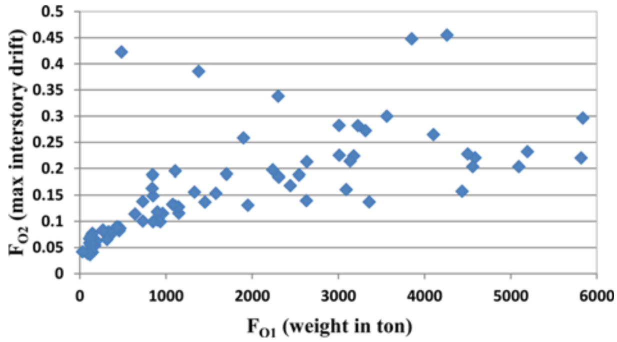

Fig. 14. Estimation of the Objective Functions F_{O1} and F_{O2} for the First Generation of Randomly Created Steel Structures

32 bits are used for the binary codification.

2. *Analysis of the structural frame*: The structural analysis of the 2D steel frame under the gravity and lateral loads was performed for each of the 100 possible solutions or individual, in other words for each structural steel frame model.
3. In these steps, the objective functions are evaluated for each individual or steel frame see Eq. (1)-(5). The results are shown in Fig. 14 where each point represents a structural frame. While the horizontal axis indicates the value of the objective function 1 (F_{O1}) related to the structural weight (see Eq. 1), the vertical axis represents the objective function 2 (F_{O2}) based on the maximum interstory drift (Eq. 2). It can be seen that no trend is observed in both cases corresponding to the weight and drift objective functions since all the steel frames were randomly generated. In addition, the space of solution is large, which is good to explore a wide range of the total space of solutions.
4. After the objective functions are evaluated for each individual, the total population is classified into layers (frontiers), according to the approach described before.
5. The parent population is established through the binary tournament described before.
6. After the binary tournament the new population of offspring of size "N" is created based on the crossover of the parents.
7. Parents are crossed and the new population has "N" individuals (100 frames) that will be the future offspring, but not without before be carried out the mutation process. The process is explained in detail in the previous chapter.
8. Once the population of "N" offspring (100 steel frames) was established its joins to the population of "N" parents to create a new population Pt' of "2N" individuals with 200 steel frames.
9. The classification of the new population will take place in the same way that the population of Parent; with the only difference that now the population is of size "2N".
10. Once the classification frontiers of Pt population was obtained with size of "2N", the new parent population of size "N individuals" (100 frames) will be the second generation, so the best 100 individuals of the 200 structural steel frames will be selected. Usually the first layer passes complete to P_{t+1} (implementation of elitism), then the fittest individuals (in smaller class rank) of the following layers are taken. If all individuals have the same fitness, a niche technique is used to maintain the diversity of the population as it was described before.

Notice that individuals which were not selected will be completely discarded to the next generation because their properties are not good compared to the properties of those individual selected. The 100 individuals will be the new parents in the next generation.

Once the new population of parents is obtained, the procedure is repeated until the phenomenon of convergence, which means that both objectives functions cannot continue being improved simultaneously in subsequent generations. Fig. 15 shows the evolutionary procedure for the example under consideration and objective function 1 (related with the weight). Each point represent a solution (steel frame) in a specific generation. It is observed that the objective function based on the weight tend to decreases with

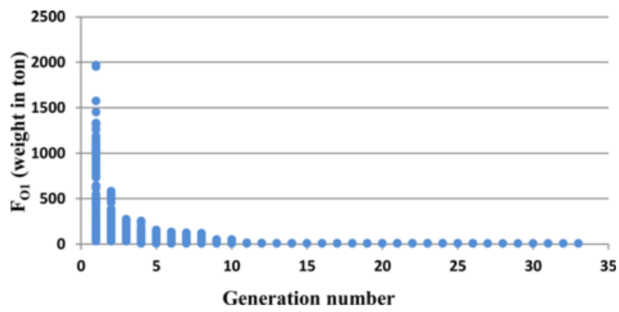

Fig. 15. Evolution of Objective 1 Based on the Structural Weight in Term of the Number of Generation for the F4-2D Structure

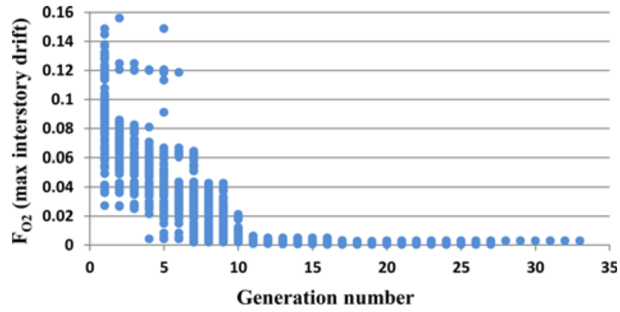

Fig. 16. Evolution of Objective 2 Based on Maximum Interstory Drift in Term of the Number of Generation for the F4-2D Structure

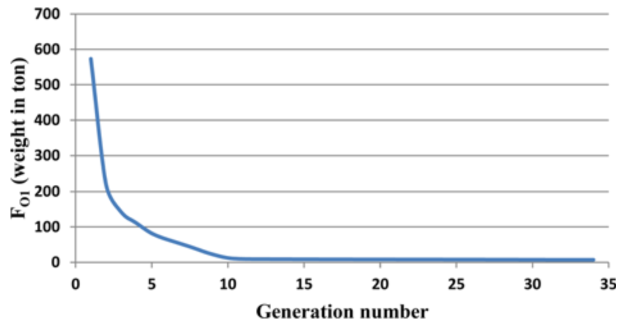

Fig. 17. Average values of F_{O1} for the F4-2D Structure

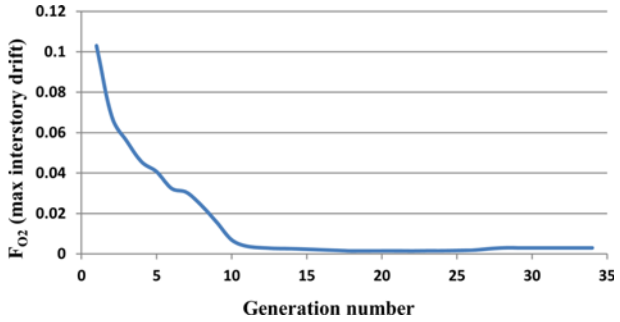

Fig. 18. Average Values of F_{O2} for the F4-2D Structure

the number of generations and in general tends to converge to a specific value. Similar results are valid for the objective function 2 in terms of the maximum interstory drift (see Fig. 16). This illustrates that the NSGA-II technique is quite useful in order to obtain steel framed buildings designs that minimizes the objective functions of interest based on weight and structural performance in terms of the maximum interstory drift. In addition, Figs. 17 and 18 illustrate the average results by generation of both objective functions (weight and interstory drift). The results show that the objective functions decrease as the number of generations increases. In other words, the average of the structural steel frame buildings tend to improve its characteristics as the number of generation increase.

7.1.1.1 Structural Solution for the Steel Frame

The final solution of the NSGA-II procedure after 30 generation is: Column 1 = W21 × 62; Column 2 = W12 × 45; Beam 1 = W21 × 48 and Beam 2 = W12 × 45). It is important to say that the beams can be perfectly assemble in the columns. Further, the constraint given by Eqs. (3) and (4) are satisfactory since the resistance factor of each element does not exceeds the unity. In other words, the design is acceptable in terms of strength and safety since the elements used on average between 80% and 96% of its resistance capacity, which is a good indicator of a correct design. This indicates that the final design obtained is quite adequate in terms of cost and performance. Table 1 provide the elements, sections, total structural weight and maximum interstory drift obtained for the final solution. Note that in this case only one solution was obtained at the end of the evolutionary procedure; however, with the aim to compute the Pareto frontier which incorporate several possible solutions, it is necessary to repeat the procedure several times. This is illustrated in Fig. 19 which

Table 1. Final Results for W Shapes, Weight and Interstory Drift for the Structure F4-2D Estimated by Means of the NSGA-II Approach

Element	Section	Total Structural Weight (ton)	Max interstory drift
Column 1	W21×62	7.19	0.0082
Column 2	W12×45		
Beam 1	W21×48		
Beam 2	W12×45		

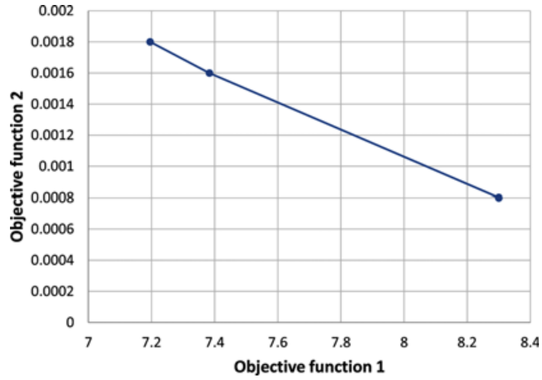

Fig. 19. Pareto Frontier for the F4-2D Steel Building Obtained Via NSGA-II

shows three solutions for the steel frame building able to control the maximum interstory drift and the strength requirements of the design codes. Due to the nature of the multi-objective optimization problem, several solutions can be obtained as adequate, herein the structural engineer or a decision maker will choose that solution more appropriated for the specific problem. It is observed that the structure with less total weight corresponds to that indicated in Table 1.

7.1.2 Application with PSO

The particle swarm optimization also is used to obtain the best steel frame building according to the chapter 6 described before. Nevertheless, for the sake of brevity, only the Table 2 which provide the elements, sections, total structural weight and maximum interstory drift obtained for the final solution, and the

Table 2. Final Results for W Shapes, Weight and Interstory Drift for the Structure F4-2D Estimated by Means of the PSO Approach

Element	Section	Total Structural Weight (ton)	Max interstory drift
Column 1	W21×55	6.65	0.0063
Column 2	W16×40		
Beam 1	W21×48		
Beam 2	W16×40		

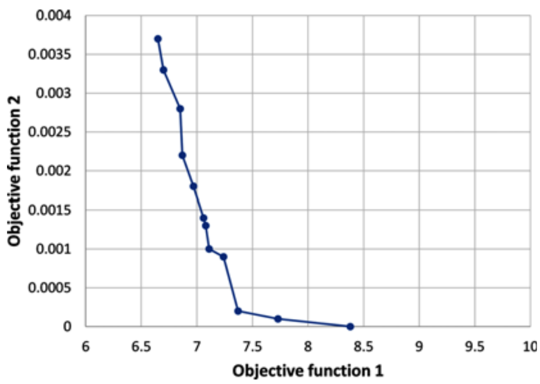

Fig. 20. Pareto Frontier for the F4-2D Steel Building Obtained Via PSO

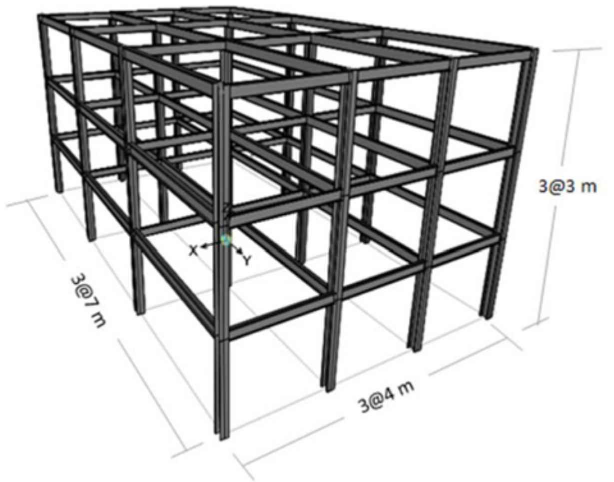

Fig. 21. Geometric Characteristics of the 3D Structural Steel Frame with Three Story Levels

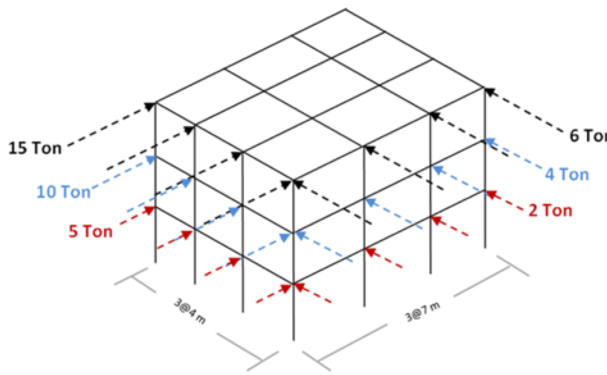

Fig. 22. Lateral Loads on the 3D Structural Steel Frame

Pareto frontier (see Fig. 20) are shown. In the Pareto frontier it can be observed that a structure with less total weight does not necessarily corresponds to a steel building with the large interstory drift (larger displacement). In fact, the structure with the less total weight equals to 6.65 ton is that with the smallest peak drift value. It is important to say that the results obtained via NSGA-II and PSO for the 2D and 3D steel framed buildings are compared below.

7.2 EXAMPLE 2: 3D Frame of 3 Story Levels with 3 bays in Both Directions

In Fig. 21 the geometrical characteristics of the 3D structural steel frame building consisting in 3 story levels of 3 m, and 3 bays (for both directions) is illustrated. While the dimensions of the bays in the large direction is 7 m, in the short direction are 4 m. To simulate the effect of an earthquake, lateral loads of 5, 10 and 15 ton were located in the nodes of the large direction and 2, 4 and 6 ton for the short direction (see Fig. 22); in addition, a distributed load of 2.5 T/m in each beam was employed to represent the vertical loads.

7.2.1 Application with NSGA-II

For the structural system, three types of W shapes have been selected. One for the columns, the second to represent the beams in the large direction and finally another to the beams in the short direction. In such a way that a 24 bits were necessary to codify the steel building.

Figs. 23 and 24 show the evolutionary procedure for the 3D structural steel frame and the final results of each generation for both objectives functions (in terms of weight and peak drift). The same trend observed in the first example is concluding here. The objective functions tend to decrease with the number of generations. As in case of the previous example the phenomenon of convergence can be observed after few generations for both objective functions.

The final solution after several generations the structural steel frame obtained corresponds to (Column = W18 × 86, Beams for short direction = W12×19 and Beams for large direction = W16 × 40; note that the beams assemble perfectly to the columns. In general, the elements used an average of 85% to 100% of its total strength capacity. It is concluded that the design is adequate. Table 3 shows the elements, sections, total structural weight and maximum interstory drift obtained for the final solution.

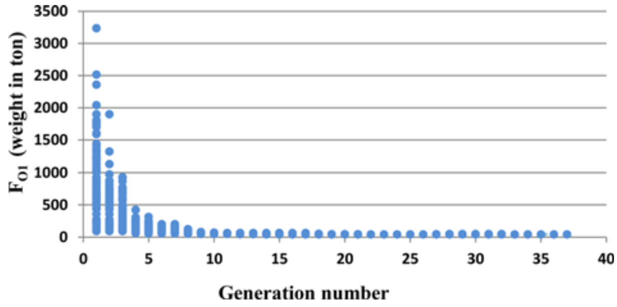

Fig. 23. Evolution of the Objective Function one Based on the Structural Through All the Generations

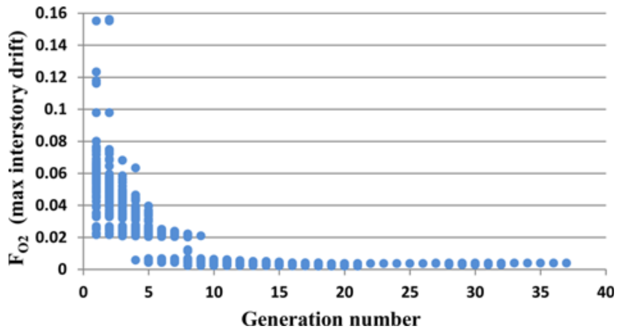

Fig. 24. Evolution of the Objective Function Two Based on the Structural Through All the Generations

Table 3. Final Results for W Shapes, Weight and Interstory Drift of the 3D Frame with 3 Story Levels and Two Bays Trough the NSGA-II Method

Element	Section	Total Structural Weight (ton)	Max interstory drift
Column	W18×86	40.64	0.0071
Beam short direction	W12×19		
Beam large direction	W16×40		

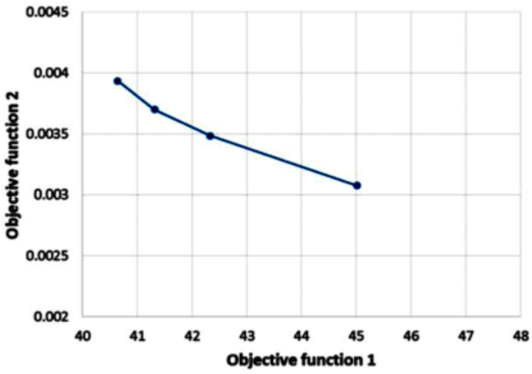

Fig. 25. Pareto Frontier for the 3D Steel Building Via NSGA-II

The above procedure can be executed several times in order to observe the effects of the randomness of the initial population where it is possible to find different solutions, which allows to select the most appropriate. In other words, this can be done to obtain the Pareto frontier. In Fig. 25 the 3D structural steel frames were obtained after 12 different runs. The figure clearly shows that an initial frontier marked by dots and blue line corresponds to the Pareto frontier, note that some solutions were repeated. Due to the nature of the problems multi-objective, a single solution is not obtained but a set of feasible solutions is provided. In such a way that a structural designer have to decide which of the feasible solutions is better according with the W shapes available or for engineering practical purposes. However, any solution adopted of the Pareto frontier is suitable.

7.2.2 Application with PSO

The results obtained for the 3D steel building are shown in Table 4, which indicates the elements, sections, total structural weight and maximum interstory drift obtained for the final solution. In addition, Fig. 26 shows the Pareto frontier, note that PSO provide more solutions for the Pareto optimal in comparison with the NSGA-II procedure. A direct comparison of the results via NSGA-II and PSO via the Pareto frontier is illustrated in the following subsection.

Table 4. Final Results for W Shapes, Weight and Interstory Drift for the 3D Steel Structure Estimated by Means of the PSO Approach

Element	Section	Total Structural Weight (ton)	Max interstory drift
Column	W12×79	40.4	0.0067
Beam short direction	W14×26		
Beam large direction	W16×40		

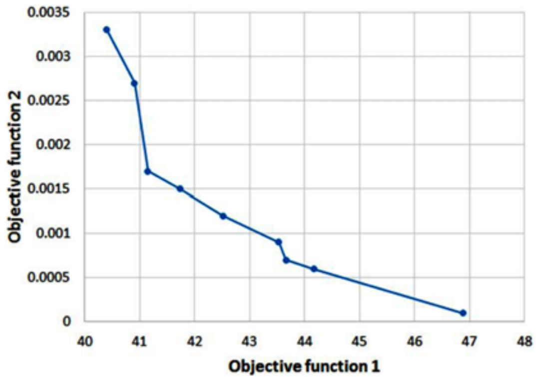

Fig. 26. Pareto Frontier for the 3D Steel Building Obtained Via PSO

7.2.2.1 Comparison of the Pareto Frontier Obtained with NSGA-II and PSO: Summary of the Results

In Fig. 27 the Pareto frontiers computed with the two meta-heuristics method (NSGA-II and PSO) and for the two selected steel frame buildings design examples are compared. The figure shows that for both steel structures, the Pareto frontier of the PSO approach dominate the Pareto frontier obtained via the NSGA-II. Furthermore, the Pareto frontier are defined with more precision and points in the case of PSO. In such a way the correct Pareto optimal tend to be that computed with the PSO. For this reason, it is concluded that the structural steel buildings obtained via PSO method are in general better solutions in comparison with the NSGA-II approach for the structures under consideration and the selected parameters of both procedures.

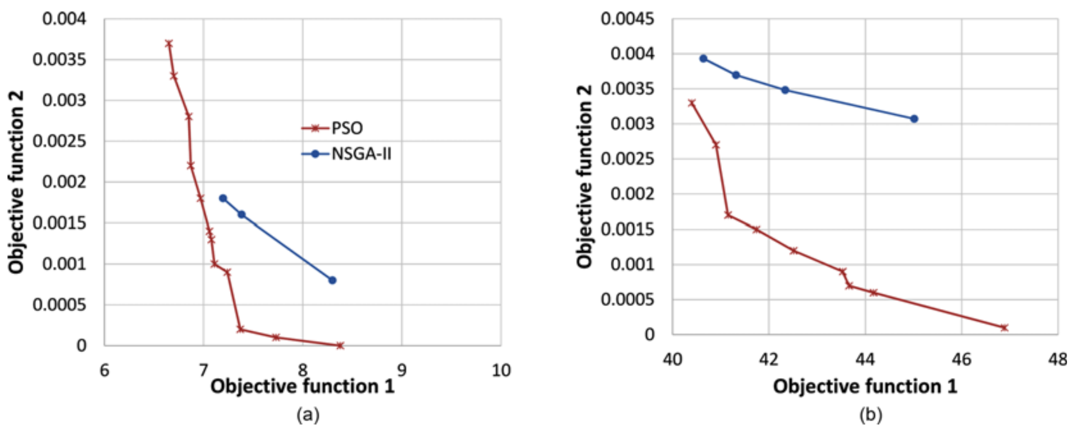

Fig. 27. Comparison of the Pareto Frontier Computed with NSGA-II and PSO for the Selected: (a) 2D Steel Framed Building, (b) 3D Steel Framed Building

8. Conclusions

This study shows the great potential of multi-objective genetic algorithms (through the NSGA-II approach) and particle swarm optimization for the seismic design of 2D and 3D steel frame buildings. It is concluded that the use of genetic algorithms and PSO are very useful tools for finding solutions to problems of structural optimization. It is noteworthy that the two most important variables for seismic design purposes were used as part of the objective functions. Thus, the structural weight related with the cost and the maximum interstory drift related with the structural performance were minimized. Moreover, the control of the strength requirements of the elements of the buildings was considered. Although several other variables can be considered, note that in the present work the problem of finding buildings that are able to build in the professional engineering practice is solved, which is not commonly discussed in problems related to structural optimization. Finally, the structural buildings obtained via PSO are in general better solutions in comparison with the NSGA-II approach for the examples under consideration.

Acknowledgements

The authors wish to express their gratitude to the two anonymous reviewers that helped to improve the quality of the paper. The scholarship for PhD studies given by El Consejo Nacional de Ciencia y Tecnología to the first author and the support under grant CB-01-167419 to the second author is appreciated. Financial support also was received from the Universidad Autónoma de Sinaloa under grant PROFAPI.

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