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A Mathematical Programming Model for Solving Cost-Safety Optimization (CSO) Problems in the Maintenance of Structures

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Abstract

Cost-Safety tradeoff analysis is one of the most challenging tasks of structural maintenance. Undoubtedly, developing an economic and efficient schedule for structural maintenance and rehabilitation is highly acknowledged. While meta-heuristic optimization algorithms have been used widely to determine the best maintenance strategies to provide more economical structures, we present a mathematical programming model to overcome the limitations of previous studies. In this paper a Mixed Integer Non-linear Programming (MINLP) has been presented to find the optimal time of applying maintenance intervention in a deteriorating structure. While, considering the time value of money, postponing the maintenance actions will be more economic, this postponement may cause a decrease in the safety of structures. Due to this contradictory relation between the objectives, it is vital to find a reasonable trade-off between cost-safety. Our proposed approach considers different values of the discount rate of money. We apply our mathematical programming model to solve two optimization examples, which are found in the structural maintenance literature. It is shown that our proposed model is able to determine the optimal time of applying maintenance intervention to the structures with less total life cycle cost, and higher level of safety.

Keywords: *structural safety, non-linear programming, optimization, reliability profile, life-cycle cost, maintenance*

1. Introduction

Nowadays, the maintenance of deteriorating structures and infrastructure assets is a significant concern (Hong *et al.*, 2013; Barone *et al*., 2014). Due to the limitations of allocated funds to the maintenance of a structure or infrastructure, the decision makers should choose the most economic time of intervention (Okasha and Frangopol, 2009). It is also crucial to maintain the structure in an adequate level of safety to avoid the loss of financial resources and, in more extreme cases, human lives due to the collapse of structures. According to the US Federal Highway Agency (FHWA), in 2005, 28% of their 595 000 bridges are rated as being deficient, about 15% of them are structurally deficient (Wenzel, 2009). These huge numbers can unfold the importance of of structural maintenance. Consequently, it is important to manage the lifetime costs of a structure or infrastructure rather than the initial cost of design or construction.

Increasing the safety will cause an increase in the costs of the structure, which is not desired by the owners. The limited budget motivates the owners to develop an economic and efficient schedule to determine the best time for the application of maintenance interventions with a reasonable cost and level of safety. As a result, it is necessary to find the optimal trade-off between these two contradictory objectives while trying to determine the optimal time of intervention.

As the regular inspections of the structures and infrastructures have their own costs, especially in the conditions that the probability of the repair is low, and the exact methods such as structural health monitoring are still pretty expensive and complicated, developing a reasonable deterioration model to consider the uncertainties and predict the disfunctions of structures will be highly rewarding (Estes, 1997; Wenzel, 2009). It is also necessary to present an effective and efficient optimization model that would be able to find the optimal answers in a reasonable time.

Regarding the importance of the safety issue in the structures and their stochastic nature, it is incumbent up on researchers to use a safety measure that could entail the uncertainties properly. The traditional safety measures, such as factor of safety and load factor have a deterministic approach and cannot incorporate the uncertainties of loads and resistances; they also have significant pitfalls such as lack of invariance (Melchers, 2002). But, probabilistic safety measures can quantify both aleatory and epistemic uncertainties. Nowadays, structural reliability is widely used (Cho *et al.*, 2004) as a robust probabilistic safety measure, which does not have the mentioned drawbacks; it can take into account the

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uncertainties of loads and resistances. This measure is extensively accepted among researchers and engineers; furthermore, a number of computer programs have been developed for structural reliability analyses over the last decades (Frangopol and Maute 2003).

2. Research Background

Many studies have tackled the problem of life-cycle cost optimization of the structures considering the structural safety or performance with different deterioration models (Hong *et al.*, 2013). Thompson (1993) and Hawk and Small (1998) tried to model the deterioration of structures considering transitions between discrete condition states as Markovian processes; however, as this method disregards most of the history of deterioration and of maintenance actions, other researchers have used the safety and condition profiles of the structures (Neves and Frangopol, 2005). By modeling the bridge as a series-parallel combination of failure modes, Estes and Frangopol (1999) tried to optimize the repair costs of the bridge while maintaining the reliability of the system in a prescribed level. Frangopol and Maute (2003) wrote an encyclopedic review about reliability-based optimization of structural systems. Morcous and Lounis (2005) tried to optimize the costs of infrastructure networks using Genetic Algorithm. Using Monte Carlo method and the idea of condition and safety profiles of the structure, Neves and Frangopol (2005) tried to optimize the life-cycle costs of the bridges and make a comparison between preventive and essential maintenance actions. Neves *et al.* (2006) applied a Multi-objective Genetic Algorithm (MOGA) to the problem of the cost optimization in bridges, with just one maintenance type, considering safety and performance; they also used a Latin Hypercube Sampling technique to incorporate the uncertainty. In another study, they also tried to solve the problem with a combination of maintenance types (Neves *et al.*, 2006a). Furuta *et al*. (2006) used a MOGA to optimize the life-cycle cost of the structures considering the seismic risk. By combining the reliability and condition profiles of structures, Frangopol *et al.* (2009) tried to optimize the life-cycle cost of the structures. Okasha and Frangopol (2009) optimized the life-cycle cost of structures considering reliability and redundancy of the system by a MOGA.

Notwithstanding the variety of the studies in this area, most researchers have tried to find the optimal time of applying a specific maintenance action to a structure (Okasha and Frangopol, 2009). In other words, the main decision variable in most of the former works is the time of applying the maintenance intervention. The bulk of researchers have used multi-objective genetic algorithms for optimization (Neves *et al*., 2006; 2006a; Frangopol and Liu, 2007; Okasha and Frangopol, 2009), and the probabilistic characteristics of profiles of deteriorating structures were calculated by Monte-Carlo simulation (Neves *et al.*, 2005; Frangopol *et al*., 2009; Jahani *et al*., 2013). Admittedly, Monte Carlo simulation has a high computational demand due to its random nature (Melchers, 2002).

Meta-heuristic algorithms became so popular in the last decades, and a number of new meta-heuristic algorithms are proposed in the last few years (Kaveh *et al.*, 2012; Kaveh, 2014). One of the main advantages of the meta-heuristic algorithms–e.g. GAs- is that, they can find approximate answers for the problems with high computational complexity such as NP-complete problems. On the other hand, they have significant drawbacks such as high computational cost (Okasha and Frangopol, 2009) and finding approximate answers rather than the exact ones. If problems could be modified in a way that their computational complexity would be simplified to easier problems with polynomial time, the application of heuristic and meta-heuristic algorithms will not be desirable anymore, because the exact algorithms can solve such problems with much better efficiency and effectiveness. Therefore, using a mixed integer non-linear programming rather than a genetic algorithm is one of the main ideas of this paper.

Furthermore, it is vital to consider the most realistic and also critical states by finding the best probability distributions that are assigned to the basic variables in the problems of structural reliability. This point is also included in this paper and will be elaborated in the numerical examples. Finally, to emphasize the significance of time value of money in the problem of structural maintenance, the problem is solved with different values of the discount rate of money and the results are compared to see the effect of this parameter on the optimal time of intervention.

2.1 Structural Safety

Nowadays, reliability based methods gain a universal acceptance to address the safety of structures; however, some other methods are available in the literature to define the performance or safety criteria for the structures. One of these performance indicators is the condition index which is a visual criteria and is obtained by visual inspections. The best value of this index for instance would be 1 for no sign of corrosion on the structure and the worst would be 5 for severe loss of section due to corrosion (Neves *et al.*, 2005). It is clear that this index just gives a superficial estimate about the performance of the structure. For example, the condition index of a corroded concrete bridge will improve dramatically (from 5 to 1) with a minor concrete repair that covers the bars, while the corroded bars are not repaired (Neves *et al*., 2005). The standard of Highways Agency of United Kingdom (2001) defines the safety index of a bridge as the ratio of available to required live load capacity. Many researchers (Cho *et al*., 2004; Choi *et a*l., 2007; Okasha and Frangopol, 2009; Morcous *et al.*, 2010; Kim *et al.*, 2013; Barone *et al*., 2014) used reliability as a suitable measure to represent the safety of structures and infrastructures. The reliability of a system can be associated with the probability that a failure occurs. Considering the stochastic properties of loads and resistances and all of the possible failure modes of the structure, reliability is a more viable indicator for structural safety. The probability of failure may be defined as follows:

$$P_f = P\{R < S\} \tag{1}$$

where P_f is the probability of failure, R is the resistance; and S is the load effect. P_f can be evaluated from the following convolution integral in the cases of two explicit variables:

$$P_f = \int_0^{\infty} F_R(S) f_s(S) dS \tag{2}$$

where F_R is the cumulative distribution function of R; and $f_s(S)$ is the Probability Density Functions (PDF) of S. This convolution integral has a closed-form solution if both, R and S, are normal (Gaussian). For independent normally distributed variables the probability of failure is:

$$P_f = 1 - \phi(\beta), \beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3}$$

where ϕ is the CDF of the standard normal distribution; and β is the reliability index or the safety index; μ_R and μ_S denote the mean values of R and S and σ_R^2 and σ_R^2 are the variances of R and S.

However, in more general cases where the loads and resistances are not explicitly expressed, the probability of failure is formulated as follows:

$$p_f = P[G(X) \leq 0] = \int \dots \int f_X(X) dX \tag{4}$$

where X is the vector of the basic variables, $G(X)$ is the limit state function, and the integral of Eq. (4) is taken over $G(X) \leq 0$ which is called the failure domain. Performing this integral, which is possible analytically only in rare cases, is the subject of the theory of structural reliability (Melchers, 2002; Frangopol and Maute, 2003; Nowak and Collins, 2012)

The changes in the condition or reliability index over time can be represented by the condition and reliability profiles over the life of the structure (Fig. 1 and Fig. 2). As shown in Fig. 1 and 2, these deterioration profiles may be assumed bi-linear under no maintenance (Neves *et al*., 2005). However, this assumption will be verified in this paper by numerical examples. The bi-linear reliability profile under no maintenance can be expressed as:

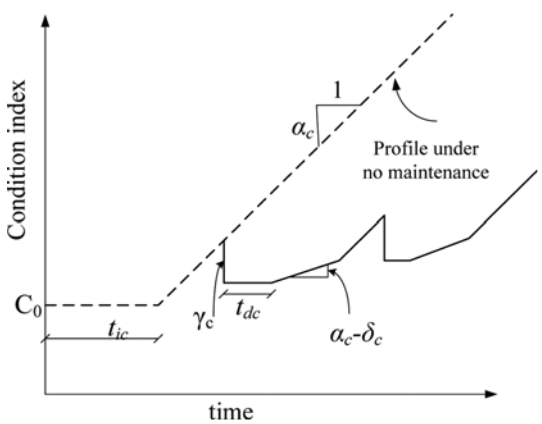

Fig. 1. The Condition Profile

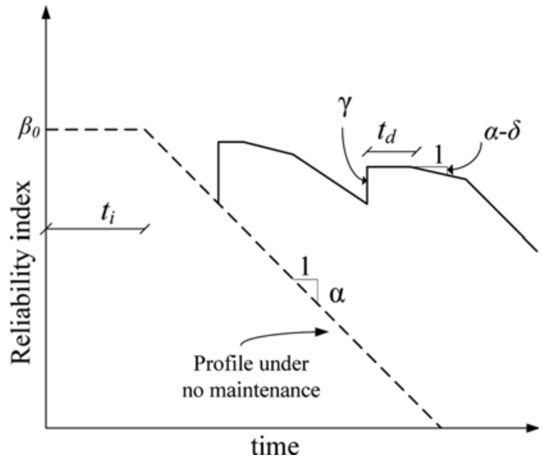

Fig. 2. Reliability profile (or safety profile) of the Structure

$$\beta = \begin{cases} \beta_0 & t < t_i \\ \beta_0 - \alpha(t - t_i) & t \geq t_i \end{cases} \tag{5}$$

A schematic reliability profile is drawn under two maintenance actions in Fig. 2. At first, for a specific time (t_i) the structure has no deterioration and will not require any maintenance. Then the reliability index decreases linearly with a rate α named deterioration rate. The maintenance interventions can increase reliability index immediately after application by γ, suppress the deterioration in reliability index during a time interval after application (t_d) and reduce the deterioration rate reliability index by δ during a time interval after application (Frangopol *et al.*, 2009).

2.2 Life-cycle Cost

Studying all costs of a structure during its lifetime and trying to minimize it is highly rewarding. Both construction and maintenance costs are taken into account by considering life-cycle costs of a structure. The costs of applying maintenance actions to a structure or infrastructure can be classified as direct and indirect costs. The direct costs are imposed on the owners of the structure, while the indirect costs are due to losses that the users should burden. For instance, in the case of a bridge, direct costs are the costs of material and labor, and indirect costs may be due to the increase in travel time, vehicle depreciation and accidents. Obviously, the collapse of a structure will result in tremendous costs that every owner tries to avoid them. Determining the indirect costs of maintenance actions and the costs of collapse of structure is harder than finding the direct costs (Estes and Frangopol, 1999; Chang and Shinozuka, 1996).

From another point of view, the costs of a structure can be divided to two parts. First, the initial cost that consists of the construction costs, and second the costs of the maintenances during the life of the structure. Since the maintenance is done over a long period of time, the time value of money must be taken into consideration. The Life-cycle Cost (*LCC*) of a structure is calculated for the problem of structural maintenance as follows (Okasha and Frangopol, 2009):

$$LCC = C_0 + \sum_k^n \frac{C_{mk}}{(1+\nu)^{tk}} \qquad (6)$$

where C_0 is the initial cost, C_{mk} is the cost of applying maintenance intervention k, ν is the discount rate of money, t_k is the time of applying maintenance intervention k; and n is the number of interventions. It could be simplified as Eq. (7) in the case of single maintenance type with a maintenance cost equals to C_m.

$$LCC = C_0 + \frac{C_m}{(1+\upsilon)^t} \qquad (7)$$

It is not easy to predict the discount rate of money, as it may have fluctuation with changes in economic condition during the structure life. It certainly has different amounts in different countries as well. As an example, this value is 6% for investment in bridge construction industry in the United Kingdom (Neves *et al.*, 2005).

2.3 Model Formulation

The objective of this paper, as mentioned above, is to determine the optimal time of the application of a prescribed maintenance intervention so that the costs would be minimized and the safety would be maximized simultaneously. In this paper only one type of maintenance is applied to the system named replacing all steel elements. A weighted non-linear programming is used to simultaneously optimize the two main objectives, where each of the optimization objectives are first normalized (Marler and Arora, 2004) and then their weighted algebraic sum is minimized. It is not economic to apply any maintenance action in the first years and last years of the life of the structure. Also, an allowable probability of failure should be considered to define the minimum acceptable level of safety. Actually, this parameter represents the level of risk that is tolerable by the facility manager (or the owner). Obviously, different structures, with different functionalities and levels of importance, must have different allowable probabilities of failure (Estes and Frangopol, 1999; Okasha and Frangopol, 2009). Based on these explanations, the model may be formulated as follows:

$$Minimize\left(W_{P_f} \times \frac{P_f - P_{fmin}}{P_{fmax} - P_{fmin}} + W_{LCC} \times \frac{LCC - LCC_{min}}{LCC_{max} - LCC_{min}}\right) \qquad (8)$$

S.T.

$$P_f \le P_{f,all} \qquad (9)$$

$$t_{min} \le t \le t_{max} \qquad (10)$$

$$W_{Pf} + W_{LCC} = 1 \qquad (11)$$

where W_{Pf} is the relative weight of the probability of failure, W_{LCC} is the relative weight of life-cycle cost, P_f is the probability of failure, P_{fmax} and P_{fmin} is the maximum and minimum values of P_f among the optimal solutions respectively, LCC is life-cycle cost of the structure as defined in Eq. (7), t is the time of applying maintenance actions; t_{min} and t_{max} are the minimum and maximum economic time of intervention. W_{Pf} and W_{LCC} are determined based on the importance of cost or safety objectives for the owner. It should be noted that both of P_f and β are used in the literature, and they are equivalent. In other words, a safer structure is one which has a greater reliability index and a smaller probability of failure. The weight of $\beta(W\beta)$ will be multiplied by−1, if P_f is substituted with β in Eq. (8). The reliability index is a function of time and is defined by a formula like Eq. (5). This kind of formulation can be handled easily in programming with a binary variable named y that takes a value 1 while $t \le t_i$, otherwise takes 0. Therefore, Eq. (5) may be rewritten as:

$$\beta = \beta_0 y + [\beta_0 - \alpha(t - t_i)](1-y), \qquad y \in \{0, 1\} \qquad (12)$$

2.4 Numerical Examples

2.4.1 Example I

The first example is a one-story steel truss, which is taken from Okasha and Frangopol (2009) as a validation for the presented model and assumptions (Fig. 3). Therefore, all numbers in the problem statement including loads, resistances and boundary conditions are identically taken from Okasha and Frangopol (2009). The yield stress of the material is assumed to be lognormal with a mean of 250 and 125 MPa for tension and compression, respectively. An initial cross-sectional area of 300 mm^2 and an initial coefficient of variation of 10% of the initial resistances of all bars are assumed. The time-variant applied load is also assumed as a lognormal variable with an initial mean of 20 kN and initial standard deviation of 4 kN.

The mean of the resistances of the bars decreases annually by the deterioration rate DRi = 0.03% per year, due to a continuous section loss over time, while its standard deviation increases by the same rate. The mean of the applied load increases annually by a rate of LIR = 1% per year. The coefficient of variation of the applied load is COV(P) = 20%, which is assumed to be constant throughout the life of the structure. The load and the resistances of bars themselves are taken statistically independent. Okasha and Frangopol (2009) conducted the structural reliability analysis using CalREL (Der Kiureghian *et al.*, 2005). Considering only one type of maintenance, that is replacing all bars, and applying

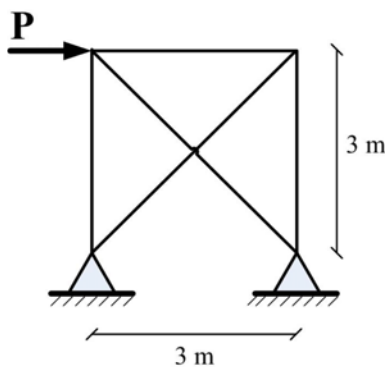

Fig. 3. The Geometry and Loading of the Truss in Example I

Table 1. The Parameters of the Problem

ν	t_{min} (year)	t_{max} (year)	$P_{f,all}$	C_0 ($)
2%	5	45	0.005	5250

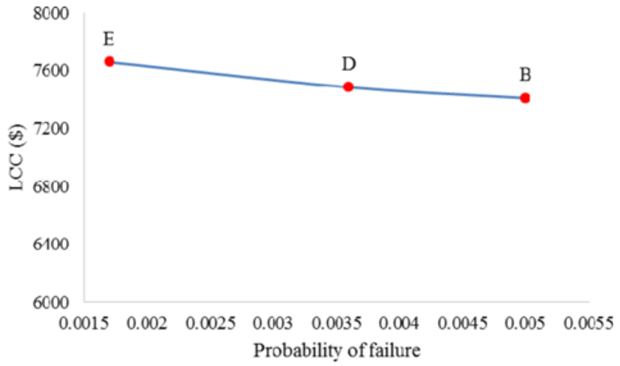

Fig. 4. Optimal Pareto Solutions Presented by Okasha and Frangopol (2009)

Table 2. Comparison of Selected Optimal Pareto Solutions

Pareto solution	*t* (year)	P_f	*LCC* ($)
B	44.7	0.005	7410
D	42.96	0.0036	7485
E	39.22	0.0017	7658

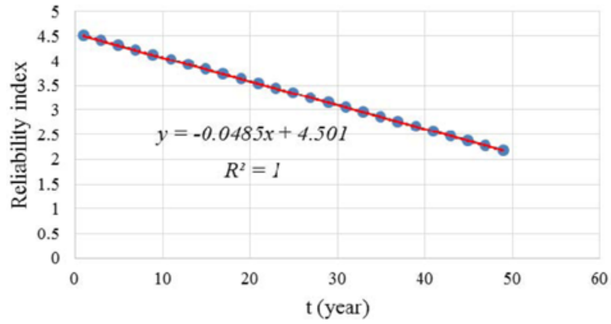

Fig. 5. The Linear Trend of the Reliability Index During Life of the Structure

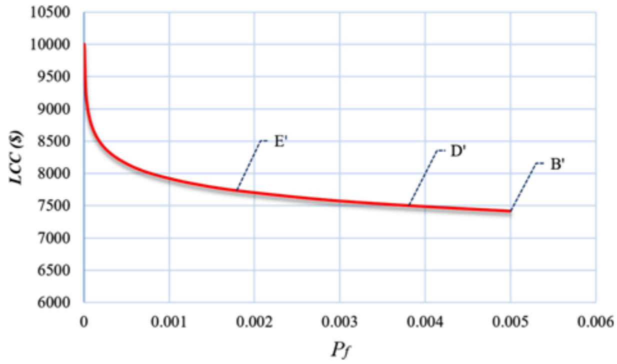

Fig. 6. Pareto Optimal Solutions of the Proposed Model

an NSGA-II to the problem with the parameters of Table 1, they extracted the Pareto-optimal of Fig. 4 for the safety and cost.

Three points are highlighted on the Pareto-optimal (B, D and E). Each one indicates an optimal time of maintenance application with a minimized cost and maximized level of safety. The values of probability of failure and life-cycle cost are shown for these non-dominant solutions in Table 2.

This example was identically solved by the model presented in Eq. (8) to (11). In this example, we took the reliability profile bilinear under no maintenance. This assumption is not a limitation for the presented model, and the model works for non-linear reliability profiles as well; it easily can be implemented by robust programming tools such as GAMS and LINGO. This reasonable assumption is useful to avoid the time variant structural reliability analysis and the heuristic and meta-heuristic optimization algorithms, which both are highly time consuming. To prove this premise in this example, a line was fitted to the resulted data from the time-variant structural reliability analysis (Fig. 5). In this paper, structural reliability analysis was performed using FERUM (Der Kiureghian *et al*., 2005), which is an open source MATLAB-based software of structural reliability. In addition to its open license which makes FERUM easy to use, this software has cutting-edge libraries for structural reliability analysis. The resulted coefficient of correlation was ρ = 1, which shows the linear trend of the data; however, this perfect linear relationship in this example could be the result of linear changes in the loads and resistances, and the authors understand that the relationship may not be linear in general. But as previously mentioned, the non-linear deterioration models could be also handled by the model. The non-linear profiles could be whether smoothed by fitting a polyline or be treated as a non-linear function in the model. Based on the given explanations and Fig. 5, the reliability profile of the structure is formulated as follows:

$$\beta(t) = \begin{cases} 4.501 & t \le 5 \\ 4.501 - 0.0485(t-5) & t > 5 \end{cases} \tag{13}$$

The model expressed by Eq. (8) to (12) was implemented by LINGO 11.0 with all of the parameters of Table 1. By using 5% increments in the relative weights of Eq. (8) a set of optimal trade-offs between maximizing safety and minimizing life-cycle cost was found (Fig. 6). The curve of Fig. 6 has a very similar trend to the Pareto-optimal of Fig. 4, clearly stating that the trade-off between the objectives is similar using both mathematical and meta-heuristic methods. This close affinity between the results of the two optimization methods proves the effectiveness of the proposed model.

However, an evident difference between the Pareto-optimals of Fig. 6 and Fig. 4 is the steep rise of the left side of the curve of Fig. 6. This steepness in the curve is the result of $W_{LCC} = 0$, which is equal to the intervention in the fifth year of the life of the truss. Evidently, this decision is very conservative and is not practical at all, but it was included in the Pareto-optimal as one of theoretical unique trade-offs to show the true performance of the model in limit states. It means that when W_{Pf} approaches 0 the optimal time of maintenance application approaches t_{max}, and

Table 3. Comparison of Selected Solutions from the Optimal Set of Mathematical Method

Pareto solution	*t* (year)	P_f	*LCC* ($)
B’	44.73	0.005	7414
D’	42.77	0.0038	7500
E’	37.8	0.00179	7733

when W_{Pf} approaches 1 the optimal time of applying the maintenance intervention approaches t_{min}. As a result, the first and last points on the Pareto could be dropped and ignored by decision makers in real-world projects. For a more detailed comparison of solutions of two methods three solutions are selected from Fig. 6, and the value of their objectives are compared in Table 3. The solutions are obviously non-dominant, and an improvement in one of the objectives will make the other one worse. Another point worth noting is that even by replacing the whole structure the reliability index is not restored to its initial value. This is because of the increase in loads that affects the reliability index.

2.4.2 Example II

The second example will be solved in order to show the effectiveness of the proposed model and, more importantly, to elaborate further on the problem. The second example is a two-story steel truss, shown in Fig. 7(a). The truss elements are made of IPE 160 profiles from ST-37 steel. The elasticity modulus of the truss elements has a lognormal distribution (Hess *et al.*, 2002; Alpsten, 1972) with a mean of and a standard deviation of 2.1×10^5. A time variant point load *F* is applied to the top of the structure with an initial mean of 200 kN and a standard deviation of 50 kN with an extreme value Type I (Gumbel) distribution. The Gumbel distribution was assigned to *F*, since it is a lateral load and can be supposed as the wind load. The maximum wind velocity and load per year might be represented by Gumbel distribution, as this is based on an underlying wind phenomenon which is described as normal in probability distribution (Melchers, 2002; Nowak and Collins, 2012). In addition to the physical reasoning and probability theory that admit the Gumbel distribution for *F*, it will be shown that assigning the Gumbel distribution to the lateral load will have more critical results in comparison to other distributions such as lognormal that have been used in example I by Okasha and Frangopol (2009).

The structural reliability analysis, consisting of computing the reliability index and the probability of failure were conducted using FERUM. The limit state function can be specified in FERUM either by analytical expression or as a displacement limit from finite element analysis. In this example, the second method is chosen and the displacement limit is equal to 15 mm for the top of the truss which is the sum of the allowable drifts of the stories (AISC, 2003). All variables (resistances and the load) are considered statistically independent.

In Table 4, the initial safety index and the probability of failure are calculated for two different scenarios. The only difference between these scenarios is that different probability distributions are assigned to the lateral load. In the first case, a Gumbel distribution and in the second a lognormal distribution is assigned to *F*. It is clear from Table 4 that the load with a Gumbel distribution has more critical results in comparison to a lognormal distribution.

Before conducting the reliability analysis over time for the truss of Fig. 7(a) and solving the optimization problem, it is worth to bring up a particular point. Although this study focuses

Table 4. A Comparison between the Two Assigned Distributions to the Load

Number	Assigned distribution to *F*	Safety index (β)	Probability of failure (P_f)
1	Gumbel (EV I)	3.8197	0.00007
2	Lognormal	4.2344	0.00001

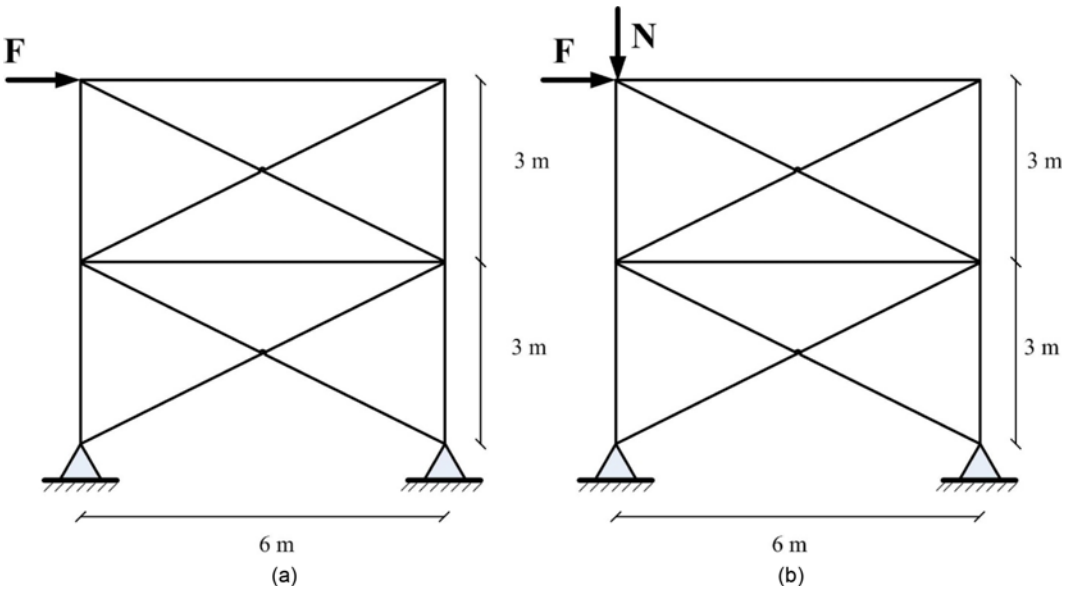

Fig. 7. The Proportions and Loading of the Steel Truss

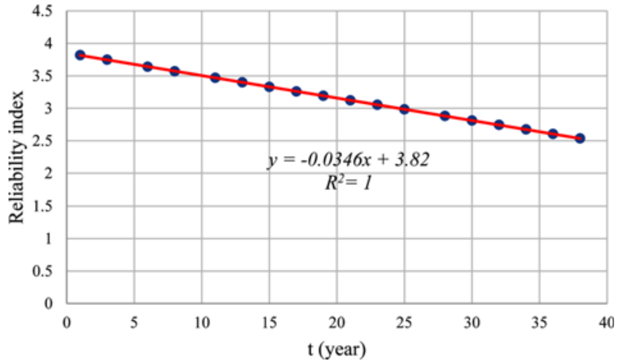

Fig. 8. Fitting a Line to the Data of Structural Reliability Analysis in Example II

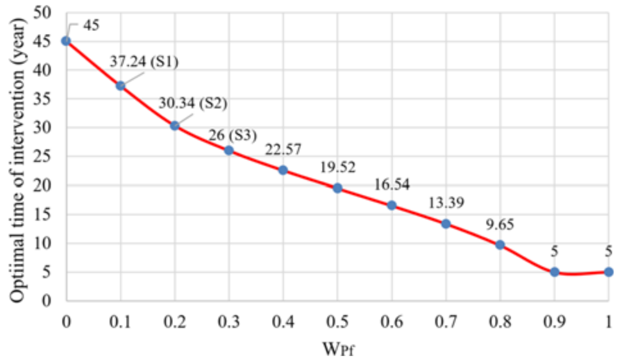

Fig. 9. The Optimal Time of Application of Maintenance Intervention vs W_{Pf}

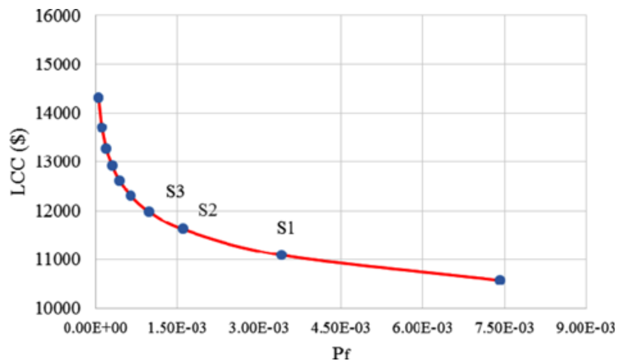

Fig. 10. Optimal Pareto Solutions in Example II

on lateral loads, which usually have more uncertainty than vertical loads (Melchers, 2002), but one might reasonably argue that structures are always exposed to vertical loads; and what will be the results if the vertical loads are considered as well? To respond to this question, a concentrated downward vertical load *N* was added to the top of the truss (Fig. 7(b)). This vertical load is normally distributed with a mean of 70 kN and a standard deviation of 10 kN. The reliability analysis was performed one more time. Applying this vertical load decreased the initial reliability index from 3.82 to 3.69, and increased the initial probability of failure from 0.00007 to 0.00011.

The changes in resistances and the load over time are considered as example I. Again a line was fitted to the data of time-variant structural reliability analysis (done by FERUM) and a completely linear trend was observed (Fig. 8). Trying to formulate the safety index over time with a bi-linear equation similar to Eq. (5), the following equation was resulted:

$$\beta(t) = \begin{cases} 3.82 & t \le 5 \\ 3.82 - 0.0346(t-5) & t > 5 \end{cases} \tag{14}$$

The model expressed by Eqs. (8) to (10) was implemented by LINGO 11.0. The assumed parameters are presented in Table 5. The parameter in this table is the discount rate of money, which was previously used and described in Eqs. (6) and (7). The effect of this parameter on the final answer will be investigated in this example. The maintenance strategy is a single maintenance, which is replacing all bars. The cost of this maintenance is assumed to be equal to C0 for simplicity (Okasha and Frangopol, 2009). It is worth pointing out that all costs of this example were calculated according to the Iranian Unit Cost of Building, which is published by Management and Planning Organization of Iran (2014). Evidently, in the weighting method the optimal answer depends on the weights of objectives. For parameters of Table 5, the optimal answers are shown against the relative weight of safety (W_{Pf}) in Fig. 9. Again, two theoretically interesting points in this Figure may be W_{Pf} = 1 and W_{Pf} = 0, which are the safest and the least safe decisions.

Table 5. Parameters of Example II

ν	t_{min} (year)	t_{mac} (year)	C_m($)	C_0 ($)	$P_{f,all}$
2%	5	45	7500	7500	0.01

In Fig. 10 a Pareto-optimal is drawn for two objectives by 10% increments in relative weights. Each point on this Pareto-optimal represents an optimal answer from Fig. 9. However, the increments in relative weights are highlighted on both figures for more clarification. In addition, three of the optimal solutions named S1, S2 and S3 are selected from the Pareto-optimal of Fig. 10, and the value of the objectives are presented for them in Table 6. Again, the selected solutions prove the non-dominancy of the solutions on the Pareto front.

Both of the above examples were solved with a discount rate of 2%. To see the effect of this parameter on the results, example II has been solved with three different discount rates (1%, 2% and 3%), and the optimal times of application of the maintenance intervention have been drawn versus the relative weight of the safety objective in Fig. 11. To compare the results, the values of the optimal time of maintenance application are written on the curves. The curves with larger discount rate of money are on top of those ones that have a less discount rate. Therefore, Fig. 11 clearly states that it is more economic to postpone the time of intervention in

Table 6. Comparison of Three Optimal Solutions

Optimal answer	Time of intervention (year)	P_f	*LCC* ($)
S1	37.24	0.00342	11087
S2	30.34	0.00162	11613
S3	26	0.00099	11981

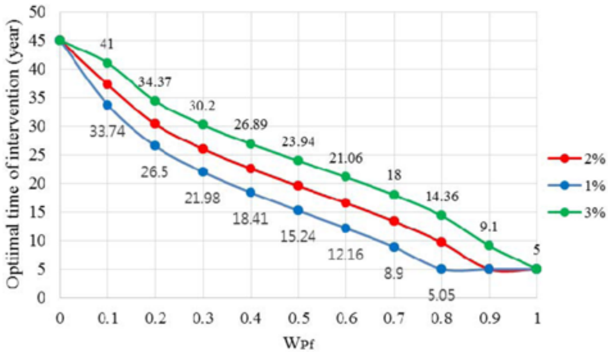

Fig. 11. Optimal Time of Applying the Maintenance Intervention for Different Discount Rates

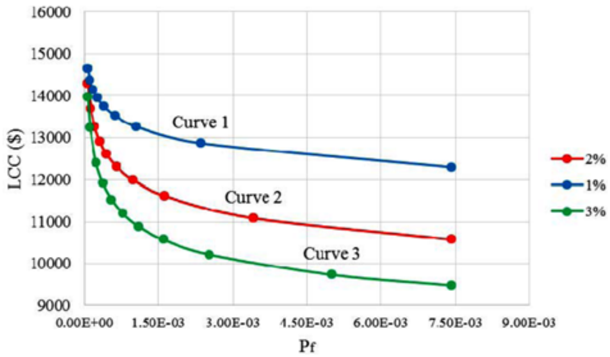

Fig. 12. Optimal Pareto Solutions for Different Discount Rates in Example II

an economical condition that the discount rate of money is high.

Another comparison between the results of different amounts of discount rate of money is shown in Fig. 12. In this Figure, three Pareto-optimals are drawn for three amounts of discount rate. As expected the curve with a larger discount rate of money has less life-cycle costs. Another crucial point that could be understood from Fig. 12 is that the diversity of the costs of the optimal trade-offs increases dramatically when the discount rate increases. This causes more difference between the costs of the optimal solutions while the discount rate has a larger value. For example, the difference between the costs of the most conservative and most economic answers is 2343 $ for curve 1 (discount rate of 1%), while this difference is 4487 $ for curve 3 (discount rate of 3%). This point is also understood from the configuration of the highlighted points on the curves that each one represents a 10% increment in the relative weights. These points are closer in curve 1 and their distance increases from curve 1 to curve 3. A decision maker may select the most conservative answer (W_{Pf}= 1) on curve 1, since it has the least probability of failure and the difference between the costs of this answer and other similar answers –e.g. W_{Pf}= 0.1, 0.2,..., 0.7- may be negligible. While this difference is more substantial on curves 2 and 3, and increases for larger values of the discount rate. It is worthy to note that a large range of discount rate may be found in the construction industry of different countries. For example, as mentioned above, 6% is recorded in UK, while this rate may be more than 30% in Iran (Central Bank of IRI, 2013).

3. Conclusions

In this paper a mixed integer non-linear programming model was developed to solve the problem of finding the optimal time of applying a maintenance intervention to a deteriorating structure. This optimization was done by finding an optimal trade-off between the safety of the structure and the costs of maintenance. Weighting method was used to articulate the preference of objectives; reliability and life-cycle cost were used to address the safety and the costs of the structures respectively. Using an exact optimization algorithm was one of the important points of this paper. Admittedly, the exact algorithms have substantial advantages over the heuristic and meta-heuristic algorithms. By developing a bi-linear reliability profile for the structure, solving the problem by the exact algorithms became possible. However, the proposed linear reliability profile was compared with the results of time-variant structural reliability analysis, and the results were almost the same in both solved examples.

To prove the valid performance of the presented model an example from literature (that has been solved by a MOGA) was solved by the proposed model, and the results were quite satisfactory. For more elaboration, another example was defined and solved. The optimal time of maintenance application and the optimal trade-off between the objectives were identified. Additionally, it was shown that the types of the probability distributions that are assigned to the basic variables have a significant effect on the results of the reliability analysis. The reliability index of the structure of example II was calculated a couple of times with two different distributions for the lateral load. The results of the Gumbel distribution were more critical in comparison to the lognormal distribution.

To see the effect of the discount rate of money on the problem, example II was solved with three different values of this parameter. The results were substantially affected by changing this parameter. The optimal time of intervention has been increased by increasing the discount rate of money. As a result, we recommend that owners or facility managers try to have enough information about the discount rate of money and its possible changes over the life of the structure, because this parameter has different amounts in different countries and also may have significant changes during the lifetime of the structure.

This paper only studies one type of maintenance, however the model can be generalized and be employed for different types of maintenance. Furthermore, weighting method may be a straightforward and popular method to articulate the objective preferences, but misinterpretation of the practical meaning of the weights can make the process of intuitively selecting the weights an inefficient chore.

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