KSCE Journal of Civil Engineering (2017) 21(4):1410-1416

DOI 10.1007/s12205-016-2045-5

Structural Engineering

pISSN 1226-7988, eISSN 1976-3808
www.springer.com/12205

Optimum Design of RC Continuous Beams Considering Unfavourable Live-Load Distributions

Sinan Melih Nigdeli* and Gebrail Bekdaş**

Received December 14, 2015/Revised March 22, 2016/Accepted May 25, 2016/Published Online July 7, 2016

Abstract

The cross-sections of Reinforced Concrete (RC) members are assumed in the preliminary design. With optimization, optimum dimensions of cross-sections providing required security measures can be obtained. Although, the area of the reinforcement bars is a computable value according to the cross-section, the amount and size of bars are also design variables in order to ensure the placement of bars providing adherence and other physical conditions. In this paper, the optimum design of RC continuous beams is presented by considering design constraints given in ACI-318 (Building Code Requirements for Structural Concrete). The most critical stress resultants of continuous beams are computed for all live load distribution patterns using three moment equations and were used in the analysis of continuous beam. A Random Search Technique (RST) is developed in order to minimize the material cost of the continuous beam. The RST is employed in different stages of optimization process such as cross-section and reinforcement bar optimization. The approach is effective to find the detailed optimum design of RC continuous beams with minimum cost.

Keywords: *RC continuous beam, random search technique, optimization, Three moment equations, ACI-318, live load distribution patterns*

1. Introduction

In the design of Reinforced Concrete (RC) structures, there are several uncertain variables and influences. For that reason, several design codes have been proposed in order to analyze RC structures. Also, the optimum design of RC members is important to find the best solution with minimum cost because concrete and reinforcement steel bars are extremely different in price and the amount of reinforcement bars is depending to the cross-section of RC members. A balance between the costs of these materials can be found by using an effective optimization technique considering safety conditions in design codes and physical conditions in the production of RC members.

The optimization of RC members or structures has been investigated by several academic papers. Coello *et al.* (1997) optimized RC beam members by employing Genetic Algorithm (GA) in order to obtain more economic costs than conventional iterative methods depending to decisions of a designer. Also, the optimum design of reinforced concrete biaxial columns was investigated with the use of GA by Rafiq and Southcombe (1998). Koumousis and Arsenis (1998) employed GA for the detailed optimum design of RC members including deep beams. The shapes of RC cross-sections were optimized by using the Sequential Quadratic Programming (SQP) technique in order to modify the initial rectangular shape of the member. For the cost optimization, GA was employed to find optimum diameter and number of main reinforcement bars (Rath *et al.*, 1999). Camp *et al.* (2003) optimally designed RC flexural frames according to ACI318 (2005) by using GA. The optimal design of T-shaped RC beams according to various design codes was conducted by Ferreira *et al.* (2003). Two metaheuristic algorithms, GA and the simulated annealing (SA) algorithm, were used together in order to find optimum values of continuous steel reinforced beam by Leps and Sejnoha (2003). Barros *et al.* (2005) developed the optimum expressions of bending moment, steel area and ratio for the cost optimization of singly and doubly RC beams. Using GA, Govindaraj and Ramasamy (2005) investigated the detailed optimum design of RC continuous beams by considering different groups of reinforcements in order to minimize the cost of RC beams. Also, Govindaraj and Ramasamy (2007) developed an optimum design approach for the optimization of RC frames. Guerra and Kiousis (2006) optimized multi-story and multi-bay RC frames by using SQP algorithm. Gil-Martin *et al.* (2010) investigated the optimum of required reinforcements in RC columns under biaxial bending. Barros *et al.* (2012) proposed the optimum depth and area of the reinforcement bars for rectangular

*Ph.D., Dept. of Civil Engineering, Faculty of Engineering, Istanbul University, 34320 Avcılar, Istanbul, Turkey (E-mail: melihnig@istanbul.edu.tr)
**Associate Professor, Dept. of Civil Engineering, Faculty of Engineering, Istanbul University, 34320 Avcılar, Istanbul, Turkey (Corresponding Author, E-mail: bekdas@istanbul.edu.tr)

RC sections for ultimate bending strength. Fedghouche and Tiliouine (2012) optimized singly reinforced T-shaped RC beams by employing GA and using the design procedures of Eurocode2. Camp and Akin (2012) employed Big Bang-Big Crunch Optimization technique for cantilever retaining RC walls in order to minimize cost or weight per unit length. Bekdas and Nigdeli (2012) optimally designed T-shaped RC beams under flexural effects by using Harmony Search (HS) algorithm.

In this paper, continuous RC beams are optimized. The objective of the optimization is to find economically optimum cross-section dimensions and amount of the reinforcement by considering the design constraints given in ACI318 (Building Code Requirements for Structural Concrete). A Random Search Technique (RST) is developed in order to reach this aim. Three moment equations (Timoshenko, 1953; Dias da Silva, 2006) developed by using Clapeyron's theory is employed to find most critical stress resultants in critical sections of continuous beams by considering the most unfavourable solution between all live-load distribution patterns.

2. Random Search Technique (RST)

A Random Search Technique (RST) was developed for the optimization of RC continuous beams. The methodology of RST can be seen in flowchart given in Fig. 1. The developed program has the ability to find internal forces of the RC continuous beam for all possible live-load patterns. The methodology of the approach can be explained in six steps.

i. First, the properties of the continuous beams are defined to the program. These properties are the number of the spans, their lengths and loading conditions of the spans. The program has ability to solve trapezoid loads for live and dead loads. Also, ranges of the design variables are defined.

ii. After the definition of a continuous beam, breadth (B) and height (H) of the RC cross-sections are randomly assigned for all spans of the continuous beam. A possible solution range for the cross-section dimensions is used by considering architectural conditions and restriction given in design codes. Also, all optimum dimensions assigned from the program are the multiples of 10 mm for practical use in production. This value can be changed by user.

iii. After the cross-sections dimensions are defined, internal forces including design flexural moments (M_d) and shear forces (V_d) can be analysed for the critical sections of the beam. Three moment equations are employed in the analyses. Also, all possible live-load combinations are considered and the most unfavourable solution is found for all spans and supports.

iv. After the internal forces are found, the required flexural and shear reinforcements are designed according to the constraints given in ACI-318.

For flexural reinforcements, the positioning of the steel bars is an important issue and depth of the beam (d) is depended to the design. Also, it is not possible to ensure the exact reinforcement area with steel bars produced with constant bar diameter. For that reason, the reinforcing bars are randomly chosen by considering a user defined bar diameter range. With this procedure, the placement of the bars is controlled according to a design constraint in order ensure adherence between steel and concrete. D_{max} and $\phi_{avarage}$ are the biggest aggregate size and average of the size of the reinforcement bars, respectively. Also, the program can place reinforcement bars in two lines if it is needed. The clear distance between the bars (a_ϕ) must ensure the following condition.

$$g_1 : a_\phi > \begin{cases} \phi_{\text{avarage}} \\ 25mm \\ \frac{4}{3}D_{\max} \end{cases} \tag{1}$$

An equivalent rectangular compressive stress distribution is used in the study as described in ACI 318 code. The moment capacity of the cross-sections can be defined as

$$M_n = \phi A_s f_y\left(d - \frac{a}{2}\right) \tag{2}$$

where ϕ, the strength reduction factor is equal to 0.9. A_s and f_y are the area of longitudinal tension reinforcements and the yield strength of reinforcement, respectively. The ratio of the longitudinal reinforcements to the effective area of the cross section (ρ) must be less than

$$g_2 : \rho \le 0.75\left(0.85\beta_1\frac{f_c}{f_y}\left(\frac{600}{600+f_y}\right)\right) \tag{3}$$

and more than

$$g_3 : \rho \ge \max\left(0.25\frac{\sqrt{f_c}}{f_y}, \frac{1.4}{f_y}\right) \tag{4}$$

In the equations, the depth of equivalent rectangular stress block (a) is obtained by multiplying β_1 with the depth of the neutral axis in compression section (c). According to compressive strength of concrete (f_c), β_1 is equal to the following expressions.

$$\begin{cases} \beta_1 = 0.85 \text{ for } 17\ MPa < f_c \le 28\ MPa \\ \beta_1 = \max[0.85 - 0.0071428(f_c - 28) \text{ and } 0.65] \text{ for } f_c > 28\ MPa \end{cases} \tag{5}$$

If the maximum reinforcement area is exceeded for the singly reinforcement beam, the fracture of the beam will be brittle. In that case, the compressive stress on the concrete can be reduced by adding reinforcements bar in the compressive section of the beam. In that case, doubly reinforced beams are taken into consideration during the optimization process and reinforcements are also randomized for the compressive section of the beam. If the beam is doubly reinforced, the ratio of the reinforcement bars is reduced by considering the stress of the reinforcement bars of compression section for Eq. (3).

In the preliminary design, depth of the beam is assumed by considering concrete cover, minimum stirrup diameter and minimum longitudinal reinforcement. After reinforcement bars are randomly chosen, the exact value of the depth is calculated and the required reinforcement area is updated. In design of stirrups, the size and distance between the bars (s) are randomly assigned according to the following constraints.

$$g_4 : A_v \geq \frac{1}{3}\frac{Bs}{f_y} \tag{6}$$

$$g_5 : \begin{cases} s \leq \dfrac{d}{2} & \text{if} \quad V_s \leq 0.33\sqrt{f_c'}b_w d \\ s \leq \dfrac{d}{4} & \text{if} \quad V_s \geq 0.33\sqrt{f_c'}b_w d \end{cases} \tag{7}$$

$$g_6 : V_d \geq 0.75\left[\frac{\sqrt{f_c'}}{6}Bd + \frac{A_v f_y d}{s}\right] \tag{8}$$

The area of stirrups is defined with A_v in the equations. At the end of the design process, the flexural moment capacity is controlled as the last constraints (g_7).

$$g_7 : M_d \geq M_n \tag{9}$$

The randomizing of the steel reinforcement bars (the process in this step) is repeated until several conditions are met. There are three conditions in order to reach the optimum solution.

a. All seven constraints must be provided.

b. The total area of the reinforcement bars must not exceed the required reinforcement area not more than 5% of the required.

c. If the beam is doubly reinforced, the total area of the reinforcement of compression section must be less than the bars at the tensile section.

v. In fifth step, the total material cost of continuous beam is calculated. The objective function of the optimization is to minimize the total material cost. This function ($f(x)$) is given as Eq. (10).

$$f(x) = C_c V_c + C_s W_s \tag{10}$$

In this equation, C_c, C_s, V_c and W_s are cost of concrete per volume, cost of steel bars per weight, volume of concrete and weight of reinforcement bars, respectively.

vi. In this stage, the iteration number of the optimization is checked. Number of iterations can be defined by user according to the convergence of the optimum cost. If the iteration number is satisfied, the program output the results. Otherwise, the range of the cross-section dimensions are modified according to the best solution of previous iterations and the process is repeated from the second step. The upper or the lower bounds of the range are updated with the best cross-section dimensions of the span with the smallest values of cross sections. This modification is done with 50% probability. Thus, the optimum solution can be rapidly found. The ranges defined by user are stored and used with 50% probability. By this operation, entrapping to a local optimum is neglected.

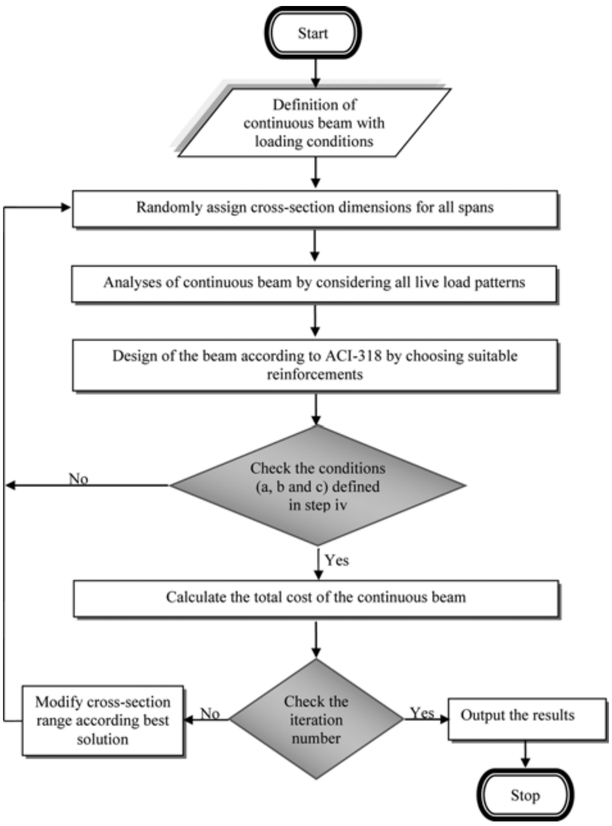

Fig. 1. Flowchart of Program

3. Numerical Examples

In this section, a two-span and five-span RC continuous beams were optimized. The two-span RC continuous beam is a symmetric structure and the other one is not symmetric. The cost of concrete and steel was taken as 40\$/m^3 and 400\$/ton, respectively. These costs may be changed according to regions. The major factor is the ratio of concrete and steel costs which is 1/10 in the numerical examples.

3.1 Two-span Symmetric Continuous Beam

The approach was numerically investigated on a two-span symmetric continuous beam with 6 m span lengths. The spans of the beam are loaded with trapezoid loads with maximum 15 kN/m dead load (D) and maximum 5 kN/m live load (L) as seen in Fig. 2.

In the optimization, two different cases of cross-section dimension limits were used. In the first case, breadth and height of the beams were searched for the ranges 200 mm – 400 mm and 300 mm – 600 mm, respectively. A smaller range than case 1 was used in case 2 in order to see the convergence to optimum

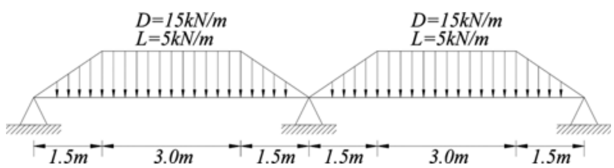

Fig. 2. Two-span Symmetric Continuous Beam

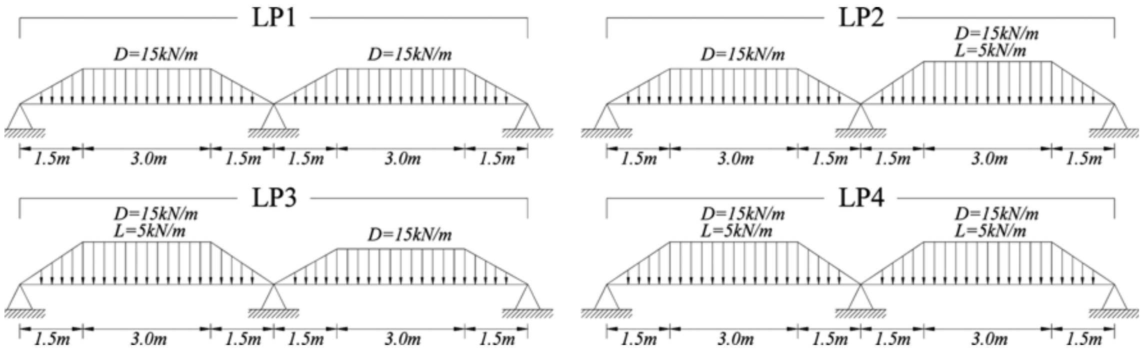

Fig. 3. Loading Conditions for Live-load Patterns

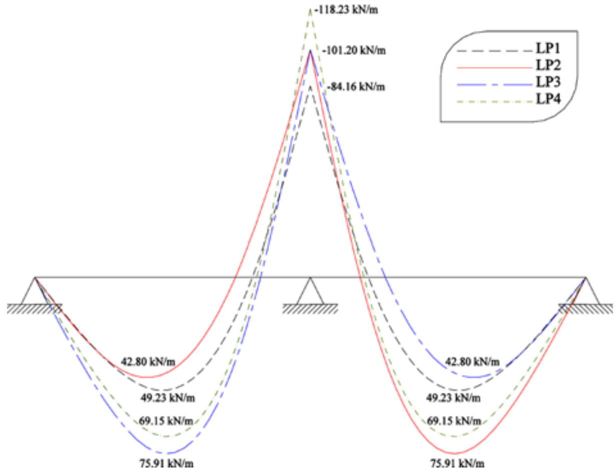

Fig. 4. Moment Diagram of the Two-span Symmetric Continuous Beam for Case 1

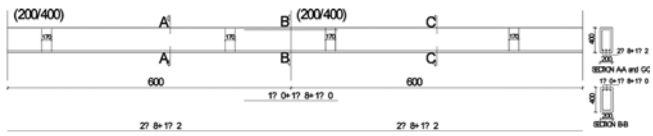

Fig. 5. The Optimum Design of Two-span Symmetric Continuous Beam

solution. Respectively, the range of case 2 for breadth and height of the beam are 200 mm – 300 mm and 300 mm – 400 mm. The diameter ranges of longitudinal reinforcements and stirrups are between 10 mm – 36 mm and 8 mm – 14 mm, respectively for all cases. The concrete cover of reinforcement and the maximum size of aggregate used in concrete are taken as 30 mm and 16 mm, respectively for the placements of the reinforcements in cross-section.

The loading conditions for live-load patterns are given in Fig. 3. For the optimum results of case 1, the moment diagram of the beam can be seen for different loadings in Fig. 4. As expected, the maximum bending moment in the spans occurred under LP1 and LP3 loadings and the maximum moment at the middle support is originated from LP4 loading.

In Table 1, the optimum results of two-span beam for case 1 can be seen. The optimum cost of the beam is 75.44 $. Since the beam is symmetric, the optimum results of spans are the same. The drawing for the optimum design is given as Fig. 5.

The optimum results of case 2 are given in Table 2. The cost of the beam is only 0.02 $ (0.03%) lower than case 1 although the optimum height of the beam is 390 mm (10 mm lower than case 1). Under the loading conditions given in Fig. 3, it is not necessary to use reinforcement in compressive section and reinforcements in tensile section can be places in a single line. The convergence to the optimum results for case 1 and 2 can be seen in the graphics of total cost vs. iteration (Fig. 6). Between 100 and 200 iterations, the optimum results can be nearly found.

In design, the cross-section dimensions are generally assumed according to the experience of the designer. For that reason, the designer can assume a value within a defined range. The range of

Table 1. The Optimum Results of Two-span Symmetric Continuous Beam for Case 1

	First span	Middle support	Second span
Cross-section (B/h) (mm)	200/400	-	200/400
First line of tensile section	2Φ18+1Φ12	1Φ30+1Φ10+1Φ18	2Φ18+1Φ12
Stirrup steel diameter /distance (mm)	Φ8/170	-	Φ8/170
Optimum Cost ($)		75.44	

Table 2. The Optimum Results of Two-span Symmetric Continuous Beam for Case 2

	First span	Middle support	Second span
Cross-section (B/H) (mm)	200/390	200/390	200/390
First line of tensile section	1Φ24+1Φ12+1Φ10	1Φ32+1Φ16+1Φ10	1Φ24+1Φ12+1Φ10
Stirrup steel diameter /distance (mm)	Φ8/170	-	Φ8/170
Optimum Cost ($)		75.42	

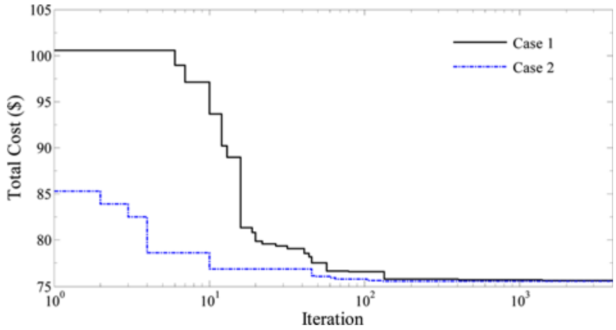

Fig. 6. Total Cost vs. Iteration for Case 1 and 2 (five-span beam)

the case 1 used in this study may be too big for a designer but the second one may be suitable. The program used in this study randomly assigns cross-section dimensions and the best solution is updated according to iterations. In the first three iterations, the costs of the beam for case 2 are 85.34 $, 83.89 $ and 82.55 $. The designer can also find these values by assuming cross-section dimensions. For that reason, the optimum results of the present approach can be compared with these values. The optimum results of case 1 and 2 are 8.61% more economic than the first trials of optimization for case 2 or a designer. If the first trials of case 1 are compared with the optimum results, the present approach is 24.97% more economic than the first trials of optimization for case 1.

In order to examine the success of the present approach for doubly reinforced concrete, the dead and live loads were increased by multiplying with 1.6. Also, the breadth of the beam was taken as a constant value (200 mm) and the height of the beam was search between 300 mm and 400 mm (case 3). Thus, a doubly reinforced design is needed for the middle support because this situation was provided by the increase of the load. A doubly reinforcement may be needed for the critical sections of the spans according to the cross-section dimensions of the beam. According to results given in Table 3, the present approach is effective under the maximum flexural moments given in Fig. 7.

According to optimum results of case 3, doubly reinforcement design is only needed for the middle support. The optimum height of the beam is the limit of the range. Thus, the optimum

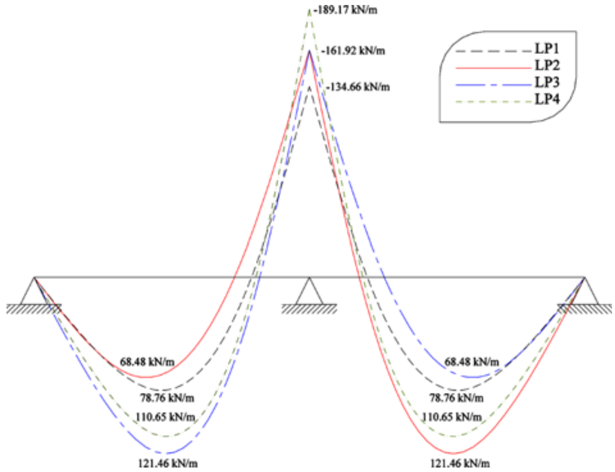

Fig. 7. Moment Diagram of the Two-span Symmetric Continuous Beam for Case 3

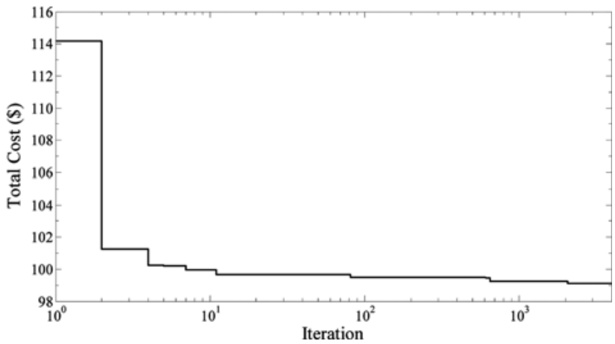

Fig. 8. Total Cost vs. Iteration for Case 3 (five-span beam)

design is escaping from the doubly reinforcement deign if it is not necessary. In Fig. 8, total cost vs. iteration graphic is given. According to first iteration, the optimum result of case 3 is 13.18% more economic.

3.2 Five-span Continuous Beam

A five-span continuous beam with different span lengths was investigated for the optimum design (Fig. 9). The same ranges for cases 1 and 2 with the two-span example were used. The

Table 3. The Optimum Results of Two-span Symmetric Continuous Beam for Case 3

	First span	Middle support	Second span
Cross-section (B/H) (mm)	200/400	-	200/400
First line of compressive section	-	2Φ10	-
First line of tensile section	1Φ24+2Φ20	2Φ34	1Φ24+2Φ20
Stirrup steel diameter /distance (mm)	Φ8/170	-	Φ8/170
Optimum Cost ($)		99.10	

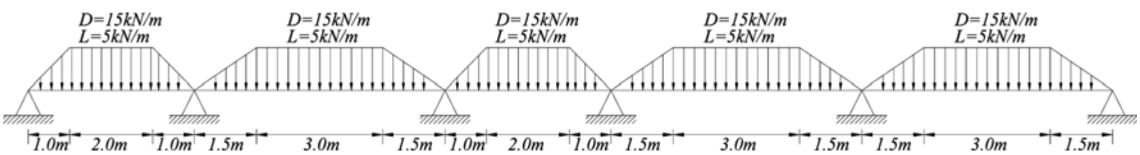

Fig. 9. Five-span Continuous Beam

Table 4. The Optimum Results of Five-span Symmetric Continuous Beam for Case 1

	First span	Second support	Second span	Third support	Third span	Fourth support	Fourth span	Fifth support	Fifth span
Cross-section (b_w/h) (mm)	200/350	-	200/360	-	210/350	-	200/370	-	200/370
First line of tensile section	1Φ12+ 1Φ14	1Φ20+ 1Φ24	2Φ14+ 1Φ18	1Φ12+ 1Φ26	1Φ10+ 1Φ12	1Φ14+ 1Φ22	2Φ12+ 1Φ18	1Φ12+ 1Φ18+ 1Φ30	1Φ14+ 1Φ16+ 1Φ22
Stirrup steel diameter/distance (mm)	Φ8/150	-	Φ8/150	-	Φ8/150	-	Φ8/160	-	Φ8/160
Optimum Cost ($)	147.89								

Table 5. The Optimum Results of Five-span Symmetric Continuous Beam for Case 2

	First span	Second support	Second span	Third support	Third span	Fourth support	Fourth span	Fifth support	Fifth span
Cross-section (b_w/h) (mm)	200/340	-	200/330	-	200/300	-	200/390	-	200/390
First line of tensile section	1Φ10+ 1Φ16	1Φ24+ 2Φ16	1Φ26+ 1Φ12	1Φ26+ 1Φ18	2Φ10	1Φ12+ 1Φ16+ 1Φ18	1Φ12+ 1Φ22	1Φ16+ 1Φ32	1Φ26+ 1Φ14
Stirrup steel diameter/distance (mm)	Φ8/140	-	Φ8/140	-	Φ8/120	-	Φ8/170	-	Φ8/170
Optimum Cost ($)	146.63								

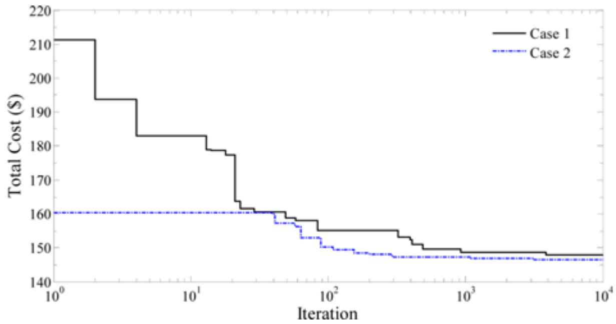

Fig. 10. Total Cost vs. Iteration for Case 1 and Case 2 (five-span beam)

optimum results for case 1 and case 2 can be seen in Table 4 and Table 5, respectively. The optimum preference of the program is to use single reinforcement design for both cases.

As seen from the total cost vs. iteration graph for five-span beam (Fig. 10), the optimum results of the cases are nearly equal to each other at 300^{th} iteration. After this iteration, convergence to the optimum results for case 2 is better than case 1 as expected. When comparing the optimum costs with the first three iterations of case 1, a 24% reduction of the cost is obtained. This reduction is 8% for the comparison with the first three iterations of case 2.

4. Conclusions

A Random Search Technique (RST) is proposed to find optimum cross-section dimensions and reinforcements of continuous RC beams. Internal forces of the RC beam are solved by using three moment equations for all iterations of RST. The design constraints given in ACI-318 are checked and the detailed reinforcement bar design is done by considering placement of the bars providing adherence. If the placements of bars are not considered during optimization, the area of reinforcement bars is a calculable value but it cannot be exactly provided in production. Also, the sizes (diameter) of reinforcement bars are effective on the depth of the bars.

The present approach was demonstrated with two numerical examples. The optimum results were obtained for different cross-section dimension ranges. Cross-section dimension range of the second case is smaller than the first one. The first example is a two-span symmetric continuous beam. Under maximum internal forces obtained by considering all live load distributions, singly reinforced concrete design is optimum for case 1 and case 2. By increasing the amount of loading, a third case with constant breadth was used in the optimization. Thus, the success of the optimization on doubly reinforced concrete was evaluated. Comparing to design process of designers, the present approach is approximately 9% more economic for two-span example.

The second example is a five-span RC continuous beam. The same range limits of two-span beam were used for case 1 and case 2. The optimum results of five-span continuous beam are approximately 8% more economic than a possible design.

The RST proposed in the present study is effective on rapidly finding optimum design variables of continuous beam with minimum material cost by considering ACI-318 design procedure, placement of the reinforcements in cross-section and live load distribution patterns.

In the design methodology, the optimum design for spans and supports are consequently done. When nonlinear behaviour is considered by using redistribution of the flexural moments, design of all spans and supports must be done before finishing the iterative optimization process of a span or support. In that case, the optimization process will be complex and a global optimum solution cannot be found since the probability to find all parts of the beam as optimum is low for an iteration. In the future works, the redistribution of the flexural moments as defined in design codes will be considered by developing a new and modified methodology.

References

Coello, C. C., Hernandez, F. S., and Ferrera, F. A. (1997). "Optimal design of reinforced concrete beams using genetic algorithms." *Expert Syst. Appl.*, Vol. 12, No. 1, pp. 101-108, DOI: 10.1016/S0957-4174(96)00084-X.

Rafiq, M. Y. and Southcombe, C. (1998). "Genetic algorithms in optimal design and detailing of reinforced concrete biaxial columns supported by a declarative approach for capacity checking." *Comput. Struct.,* Vol. 69, No. 4, pp. 443-457, DOI: 10.1016/S0045-7949(98)00108-4.

Koumousis, V. K. and Arsenis, S. J. (1998). "Genetic algorithms in optimal detailed design of reinforced concrete members." *Comput-Aided Civ. Inf.*, Vol. 13, No. 1, pp. 43-52, DOI: 10.1111/0885-9507.00084.

Rath, D. P., Ahlawat, A. S., and Ramaswamy, A. (1999). "Shape optimization of RC flexural members." *J. Struct. Eng.-ASCE.*, Vol. 125, No. 12, pp. 1439-1446, DOI: 10.1061/(ASCE)0733-9445(1999)125:12(1439).

Camp, C. V., Pezeshk, S., and Hansson, H. (2003). "Flexural design of reinforced concrete frames using a genetic algorithm." *J. Struct. Eng.-ASCE.*, Vol. 129, No. 1, pp. 105-11, DOI: 10.1061/(ASCE)0733-9445(2003)129:1(105).

ACI 318M-05 (2005). *Building code requirements for structural concrete and commentary,* American Concrete Institute.

Ferreira, C. C., Barros, M. H. F. M., and Barros, A. F. M. (2003). "Optimal design of reinforced concrete T-sections in bending." *Eng. Struct.*, Vol. 25, No. 7, pp. 951-964, DOI: 10.1016/S0141-0296(03)00039-7.

Leps, M. and Sejnoha, M. (2003). "New approach to optimization of reinforced concrete beams." *Comput. Struct.*, Vol. 81, Nos. 18-19, pp. 1957-1966, DOI: 10.1016/S0045-7949(03)00215-3.

Barros, M. H. F. M., Martins, R. A. F., and Barros, A. F. M. (2005). "Cost optimization of singly and doubly reinforced concrete beams with EC2-2001." *Struct. Multidiscip. O.*, Vol. 30, No. 3, pp. 236-242, DOI: 10.1007/s00158-005-0516-2.

Govindaraj, V. and Ramasamy, J. V. (2005). "Optimum detailed design of reinforced concrete continuous beams using Genetic Algorithms." *Comput. Struct.*, Vol. 84, Nos. 1-2, pp. 34-48, DOI: 10.1016/j.compstruc.2005.09.001.

Govindaraj, V. and Ramasamy, J. V. (2007). "Optimum detailed design of reinforced concrete frames using genetic algorithms." *Eng. Optimiz.*, Vol. 39, No. 4, pp. 471-494, DOI: 10.1080/03052150601180767.

Guerra, A. and Kiousis, P. D. (2006). "Design optimization of reinforced concrete structures." *Comput Concrete*, Vol. 3, No. 5, pp. 313-334, DOI: 10.12989/cac.2006.3.5.313.

Gil-Martin, L. M., Hernandez-Montes, E., and Aschheim, M. (2010). "Optimal reinforcement of RC columns for biaxial bending." *Mater. Struct.*, Vol. 43, No. 9, pp. 1245-1256, DOI: 10.1617/s11527-009-9576-x.

Barros, A. F. M., Barros, M. H. F. M., and Ferreira, C. C. (2012). "Optimal design of rectangular RC sections for ultimate bending strength." *Struct. Multidiscip. O.*, Vol. 45, No. 6, pp. 845-860, DOI: 10.1007/s00158-011-0717-9.

Fedghouche, F. and Tiliouine, B. (2012). "Minimum cost design of reinforced concrete T-beams at ultimate loads using Eurocode2." *Eng. Struct.*, Vol. 42, pp. 43-50, DOI: 10.1016/j.engstruct.2012.04.008.

Camp, C. V. and Akin, A. (2012). "Design of retaining walls using big Bang–Big crunch optimization." *J. Struct. Eng.-ASCE*, Vol. 138, No. 3, pp. 438-448, DOI: 10.1061/(ASCE)ST.1943-541X.0000461.

Bekdaş, G. and Nigdeli, S. M. (2012). "Cost optimization of t-shaped reinforced concrete beams under flexural effect according to ACI 318." *In: 3rd European Conference of Civil Engineerin*g, December 2-4 2012, Paris, France.

Timoshenko, S. P. (1953). *History of strength of materials*, McGraw-Hill, New York.

Dias da Silva, V. (2006). *Mechanics and strength of materials*, Springer, Berlin Heidelberg New York.