KSCE Journal of Civil Engineering (2017) 21(3):782-791

DOI 10.1007/s12205-016-1126-9

Structural Engineering

pISSN 1226-7988, eISSN 1976-3808
www.springer.com/12205

# An Automated Approach for Optimal Design of Prestressed Concrete Slabs using PSOHS

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Received February 10, 2015/Revised May 29, 2015/Accepted July 11, 2015/Published Online May 10, 2016

## Abstract

In this paper a new automated approach is presented for optimal design of prestressed concrete slabs. To achieve this goal, the model of a slab is formed using SAP2000, and it is linked to a meta-heuristic code. This code utilizes the result of analyzed models in each iteration to provide new design parameters of the slab for the subsequent one. Canadian Standard Association requirements are met thoroughly to reach a safe code-based design. A recently enhanced version of Particle Swarm Optimization (PSO), so-called PSOHS, is employed, and it is shown that the latter is superior to the standard PSO. Results prove the efficiency of the PSOHS, indicating the possibility of its application in professional engineering. Furthermore, the comparison of the PSO and PSOHS shows that PSOHS is less parameter sensitive, and provides final designs with smaller cost functions.

Keywords: *prestressed concrete, slab, optimal design, PSO, PSOHS, Canadian Standard Association (CSA)*

## 1. Introduction

Prestressed concrete slabs are efficient systems for covering the long spans where placing columns interrupts the serviceability of the structure. For instance audiences, parking lots, hotels, airports, etc. are examples of such structures in which columns may cause problems for the users. Prestressed concrete slabs provide floor scheme with smaller thickness which not only reduces the cost of the structure, but also it decreases the mass of the structure; as a result, the earthquake effects can be decreased.

In recent decades, metaheuristic algorithms have emerged, and applied to many structural problems. In this way, optimal design of floor systems has drawn the attention of several researchers. Kaveh and Shakouri Mahmoud Abadi (2010) utilized IHS for optimal design of composite floor systems; in the case of optimal design of prestressed concrete floor systems, the work of Rozvany (1963) is one of the pioneering attempts; MacRae (1987) used a nonlinear programming and conjugate direction method as optimization algorithm along with equivalent load method as the analysis method to achieve this goal. Kuyucular's (1991) attempt was to minimize the weight of prestressing cables by considering several predefined cable profiles for each section. He also used a combined finite element method and equivalent load method for structural analysis. Lounis (1993) considered two objective functions to be minimized which were cost and initial camber. One of these functions was used as the objective function, and the other was treated as a constraint for ε-constraint approach. In sum, they employed a Projected Lagrangian algorithm for optimization, and a sectional stress analysis and force-in-tendon method for analysis of floor slabs. Based on the work of Semelawy (2012), a concrete slab was modeled using a consistent triangular shell element that was originally developed by Koziey (1997). Steel tendons are modeled as a discrete integral part of the shell element. Direct search methods, heuristic optimization techniques such as Genetic Algorithms, and multi-objective optimization techniques were considered.

As it is mentioned so far, a structural analysis method along with an optimization algorithm is necessary for optimal design of structures, and prestressed concrete slab is not an exception. Metaheuristic optimization algorithms, due to fewer limitations of application, have attracted many researchers; hence, these algorithms have been applied by Kaveh and Nasrollahi (2014a, b) in various structural optimization problems and showed effectiveness. One of the robust metaheuristic algorithms is Particle Swarm Optimization, PSO, proposed by Eberhart and Kennedy (1995). The capability of searching in a continuous feasibility space, easy implementation, and not being trapping in a local minimum are the main characteristics of this algorithm. However, the lack of balanced exploration and exploitation and a shortcoming in dealing with the violated particles from feasibility boundaries reduce its robustness significantly (Kaveh and Nasrollahi, 2014c, d). To remove these problems, Kaveh and Nasrollahi

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(2013) proposed a hybrid PSO and Harmony search algorithm called PSOHS. This algorithm showed a good performance in design optimization of RC shear walls (Kaveh and Nasrollahi 2014).

Despite of numerous researches in the field of structural optimization, the absence of an efficient approach which can be used by consultant engineers companies to use the concepts of optimization in the conventional methods and apply the optimization algorithms in commercial design software motivated us to present this study. The main objective of the present study is to introduce an automated method for optimal design of prestressed concrete slabs. For the purpose of analysis, a commercial SAP2000 analysis package is employed to facilitate the analysis procedure. Furthermore, to examine efficiency of the improvements, both PSO and PSOHS are applied to a problem of optimal long span prestressed concrete slab. Canadian Standard Association requirements are met thoroughly to reach a safe code-based design. Simplicity of implementation and practicality are two main characteristics of the method which have not been considered in the studies carried our thus far.

## 2. Hybrid PSO and HS Optimization Algorithm

### 2.1 Standard PSO

Particle Swarm Optimization, PSO, is a multi-agent meta-heuristic optimization algorithm introduced by Eberhart and Kennedy (1995). It makes use of a velocity vector to update the current position of each particle in the swarm. The velocity vector is updated using a memory in which the best position of each particle and the best position among all particles are stored. This can be considered as an autobiographical memory. Therefore, the position of each particle in the swarm adapts to its environment by flying in the direction of the best position of whole particles and the best position of particle itself, and this mechanism provides the search of the PSO. The position of the $i^{th}$ particle at iteration $k$+1 can be calculated as

$$x_{k+1}^i = x_k^i + v_{i+1}^i \cdot \Delta t \tag{1}$$

Where, $x_{k+1}^i$ is the new position; $x_k^i$ stands for the position at iteration $k$; $v_{k+1}^i$ represents the updated velocity vector of the $i^{th}$ particle; and $\Delta t$ is the time step which is considered as unity. The velocity vector of each particle is determined as:

$$v_{k+1}^i = w \cdot v_k^i + c_1 \cdot r_1 \cdot \frac{(p_k^i + x_k^i)}{\Delta t} + c_2 \cdot r_2 \times \frac{(p_k^g - x_k^i)}{\Delta t} \tag{2}$$

Where, $v_k^i$ is the velocity vector at iteration $k$; $r_1$ and $r_2$ are two random numbers between 0 and 1; $p_k^i$ represents the best ever position of $i^{th}$ particle, *local best*; $p_k^g$ is the *global best* position in the swarm up to iteration $k$; $c_1$ is the cognitive parameter; $c_2$ is the social parameter; and $w$ is a constant named inertia weight.

With the above description of PSO, the algorithm can be summarized as follow:

1. Initialization

Initial position, $x_0^i$, and velocities, $v_0^i$, of particles are distributed randomly in feasible search space.

$$x_0^i = x_{min} + r \cdot (x_{max} - x_{min}) \tag{3}$$

$$v_0^i = \frac{x_{min} + r \cdot (x_{max} - x_{min})}{\Delta t} \tag{4}$$

Where, $r$ is a random number uniformly distributed between 0 and 1; $x_{min}$ and $x_{max}$ are minimum and maximum possible variables for the $i^{th}$ particle, respectively.

2. Solution Evaluation

Evaluate the objective function value for each particle, $f(x_k^i)$, using the design variables correspond to iteration $k$.

3. Updating Memory

Update the local best of each particle, $p_k^i$, and the global best, $p_k^g$, at iteration $k$.

4. Updating Positions

Update the position of each particle utilizing its previous position and updated velocity vector as specified in Eqs. (1) and (2).

5. Stopping Criteria

Repeat steps 2~4 until the stopping criteria is met.

### 2.2 Harmony Search

Harmony Search (HS) is a metaheuristic algorithm based on natural musical performance that occurs when a musician searches for a better state harmony, such as jazz improvisation. This algorithm was presented by Geem (2001) and works as: the engineers seek for a global optimum of an objective function, just like the musicians seek to find a musical pleasing harmony as determined by aesthetics. This seeking for a new improvised harmony is a search which if can be regulated in optimization; it can find the global minimum of the objective function.

*HS* algorithm includes a number of optimization operators, such as the harmony memory ***HM*** which is a memory that some best so far results are saved in it and if, in a stage, better solution is obtained, it is saved in ***HM*** and the worst one is excluded from it; Harmony memory size *HMS*, which is the number of solution vectors saved in ***HM***; Harmony memory considering rate *HMCR* varying between 0 and 1 sets the rate of choosing a value in the new vector from the historic values stored in the ***HM***; and the pitch adjusting rate *PAR*. The pitch adjusting process is performed only after a value is chosen from ***HM*** and sets the rate of choosing a value from neighboring of the best vector. Concept of HS is as follows:

A new harmony vector is improvised from the ***HM*** based on *HMCR* and *PAR*. With the probability of HMCR the new vector is generated from ***HM*** and with the probability of $(1 - HMCR)$ the new vector is generated randomly from possible ranges of values. The pitch adjusting process is performed only after a value is selected from ***HM***. The value $(1 - PAR)$ sets the rate of doing nothing. A *PAR* of 0.25 indicates that the algorithm will select a neighboring value with 0.2 × *HMCR*. It is recommended not to set *HMCR* as 1.0 because it is probable that the global minimum does not exist in ***HM***. With the aforementioned, the search of ***HM*** is summarized in Eq. (5). In which the term "w.p."

represents "with the probability".

If the generated harmony vector is better than a harmony vector in HM, judged in terms of the objective function value, the new harmony is included in ***HM*** and the worst one is excluded from it.

$$x_{i,j}=\begin{cases} w.p.\ HMCR \rightarrow \begin{cases} select\ a\ value\ for\ the\ varialbe\ freom \\ \quad HM\ w.p.\ (1-PAR) do\ nothing \\ w.p.\ PAR\ choose\ a\ neighboring\ value \end{cases} \\ w.p.\ (1-HMCR) \rightarrow select\ the\ varialbe\ randomly \end{cases} \tag{5}$$

The hybrid PSO and HS, called PSOHS, is proposed by Kaveh and Nasrollahi (2013). To elaborate modification applied in PSOHS, it is necessary to explain why such modifications are performed. There are two main problems associated with PSO: first, the lack of balance between exploration and exploitation; second, having no proper idea to control the violating variables from feasible search space. For definition of the first problem it should be mentioned that in metaheuristic optimization algorithms, there should be a balance between exploration and exploitation in a way that at initial iteration, the algorithm should have a global search and this search should cover the whole search space in a wise manner to locate the position of global minimum approximately. In this stage, some points which are expected to be near the global minimum of the cost function are found. Then at the latest iterations, the algorithm should perform a local search using the solution vectors found thus far. As it is apparent from Eq. (2), the definition of velocity vector for PSO, which is the search engine of the algorithm, has not this feature, and at initial iteration is the same to latest iterations; as a result, in PSO the lack of balance exists between exploration and exploitation.

This problem has been solved using dynamic variation of inertia weight by linearly decreasing $w$ with each iteration of the algorithm presented by Shi and Eberhart (1998) as shown in the following:

$$w_{k+1}=w_{max}-\frac{w_{max}-w_{min}}{k_{max}}k \tag{6}$$

Where, $w_{max}$ is the maximum considered inertia weight, $w_{min}$ is the minimum considered inertia weight, and $k_{max}$ is the number of iterations.

Utilizing Eq. (6), at initial iterations there will be a large value of inertia weight that provides a global search and by progression of the algorithm this value will reduce until at the latest iterations only local search will be performed based on the position of the best particle and the best ever position of the particles (see Eq. (2)).

The second problem existing in PSO is associated to the mechanism for controlling the violated particles from feasible search space. There are different methods to overcome this problem; one of the simplest approaches is utilizing the nearest limit values for the violating variable. Alternatively, one can force the violating particle to return to its previous position, or reduce the maximum value of the velocity to allow fewer particles to violate the variable boundaries. Although these approaches are simple, they are not sufficiently efficient and may lead to reduction of the exploration of the search space. This problem has previously been addressed and solved using the harmony search based handling approach. According to this mechanism, any component of the solution vector violating the variable boundaries can be regenerated from the ***HM*** using Eq. (5).This approach is an efficient one which improves the convergence rate of algorithm because of simultaneous action of two algorithms. If the particle is in the feasible search space, PSO will work and if violates from boundaries, HS will be activated. However, in the PSOHS it is necessary to have a memory in which the global best is stored to be extended and some of the best designed vectors stored. This memory can be used as ***HM*** when a particle violates and HS becomes active.

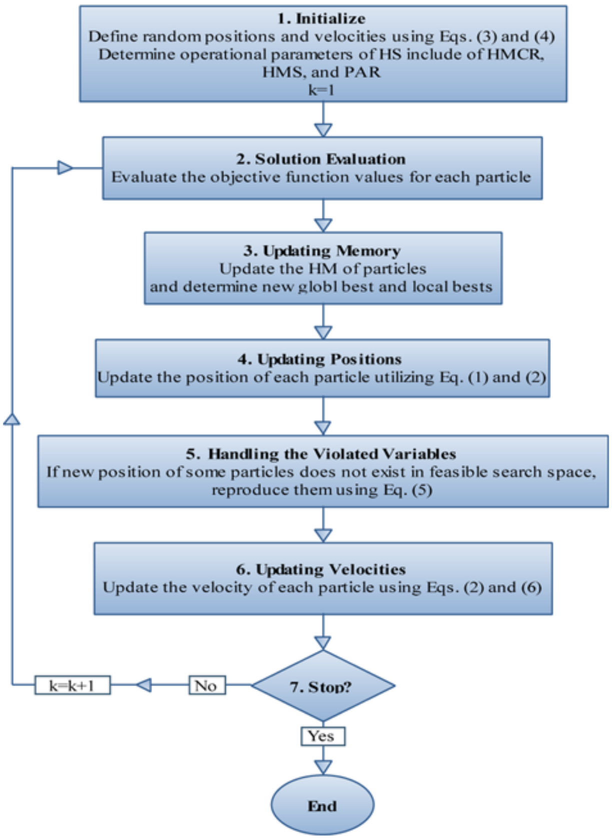

Fig. 1. Flowchart of PSOHS

Based on the abovementioned explanation, the steps of PSOHS are shown in the flowchart of Fig. 1.

## 3. Formulating the Optimal Design of Prestressed Concrete Slabs

In this research, cost was the main objective; however, in some other works, due to importance of other objectives, multi-objective functions, such as weight and deformation, have been considered. An example of multi-objective structural optimization is the work of Hosseini *et al*. (2015). In general, cost function, constraints, and feasible search space are three main components of an optimization problem. In optimal design of prestressed concrete slabs, it is desirable to minimize the cost of materials which in this paper consists of the costs of concrete and tendon. Therefore, the cost function is defined as:

$$F(X) + C_c \cdot V_c = \sum_{i=1}^{n} C_s \cdot L_s \tag{7}$$

Where $F(X)$ is the cost function; $C_c$ stands for the cost of concrete per volume; $V_c$ represents the total volume of the concrete; n is the number of tendons in both directions; $C_s$ is the cost of steel per unit meter; and $L_s$ is the total length of tendons.

In Eq. (7), $X$ is the design variable vector, and consists of: 1. Slab's thickness (t), 2. Number of tendons in x-direction ($N_x$), 3. Number of tendons in y-direction ($N_y$), 4. Diameter of tendons in the x-direction ($d_x$), 5. Diameter of tendons in y-direction ($d_y$), 6. Tendon eccentricity at one end of the slab ($e_1$), 7. Tendon eccentricity at the other end of the slab ($e_2$), 8. Tendon eccentricity at middle of the slab ($e_3$), 9. Allowable tensile stress of tendons ($S_{tendon}$). These variables are delineated in Fig. 2 and Fig. 3.

In order to meet CSA requirements, constraints are based on those provided in Table 1.

To meet the constraints, a penalty function approach is employed which is defined as

$$P(X) = (1 + \alpha \sum_{i=1}^{m} C_i)^{\beta} \tag{8}$$

Where $\alpha$ and $\beta$ are to constants, and in this paper, these values are $\alpha = 10$ and $\beta = 1$ ; $m$ is the total number of constraints; $C_i$ is the $i^{\text{th}}$ constraint in Table 1. To formulate the constraints in a normalized form, two instances are faced:

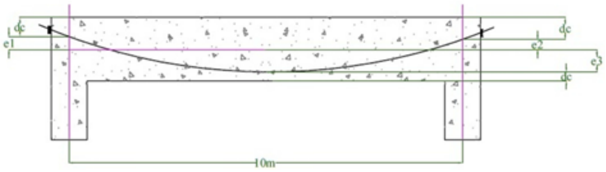


Fig. 2. Eccentricity of Tendons in the Considered Problem

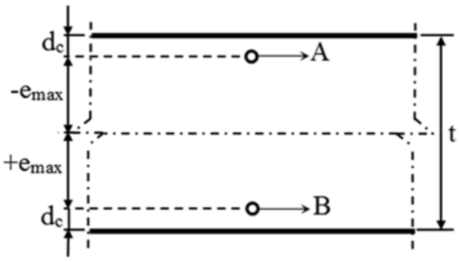


Fig. 3. Cross Section: Tendons Can be Located between A and B, and Cover dc Must be Considered

If the quantity obtained from analysis is less than its allowable value, $C_i$ is defined as:

*if the constraint is in the from of* $Q_i < Q_{iallowed} \Rightarrow$

$$C_i = \begin{cases} \dfrac{Q_i}{Q_{i_{allowed}}} - 1 & if \quad Q_i > Q_{i_{allowed}} \\ 0 & else \end{cases} \tag{9}$$

If the quantity obtained from analysis is greater than its allowable value, $C_i$ is defined as:

*if the constraint is in the form of* $Q_i > Q_{i_{allowed}} \Rightarrow$

$$C_i = \begin{cases} 1 - \dfrac{Q_i}{Q_{i_{allowed}}} - 1 & if \quad Q_i > Q_{i_{allowed}} \\ 0 & else \end{cases} \tag{10}$$

Where $Q_i$ and $Q_{i_{allowed}}$ are the $i^{\text{th}}$ quantity and its allowable value, respectively.

Then the penalty function is multiplied by the cost function to form

$$\varnothing(X) = F(X) \cdot P(X) \tag{11}$$

In the optimization algorithm, $\varnothing(X)$ is minimized; hence, if a

Table1. Design constraints of Prestressed Concrete Design Based on CSA

| Symbol | Constraint | clause (in CSA) | limit |
|---|---|---|---|
| $C_1$ | Stress in concrete (initial stage) | 18.3.1.1 | $-0.6f_c' < S_{conc} < 0.5\sqrt{f_c'}$ |
| $C_2$ | Stress in concrete (final stage) | 18.3.2 | $-0.6f_c' < S_{conc} < 0.5\sqrt{f_c'}$ |
| $C_3$ | Stress in tendons | 18.4 | $S_{tendon} < 0.7f_{pu}$ |
| $C_4$ | Ultimate bending moment | 18.6.2 (a) | $M_r > M_f$ |
| $C_5$ | Minimum factored resistance | 18.7 | $M_r > 1.2M_{cr}$ |
| $C_6$ | Punching shear[1] | 13.3 | $V_r > V_f$ |
| $C_7$ | Maximum/minimum eccentricity[2] | 7.9 & 6.6.6 | $\lvert e \rvert \le 1 - 2d_c/t$ |

Where $f_c'$ is the specified compressive strength of concrete; $S_{conc}$ is the stresses in concrete (obtained from SAP2000 analysis) for initial and final stages; $f_{pu}$ is the specified tensile strength of prestressing tendons; $S_{tendon}$ is the stresses in prestressing tendon (obtained from the algorithm written in Matlab); $V_r$ is the factored shear stress resistance; $V_f$ is the factored shear stress; $M_r$ is the factored moment resistance; $M_f$ is the factored moment; $M_{cr}$ is the cracking moment; $e$ is the eccentricity of the prestressing tendon at a specified key point – defined as a ratio to the thickness ranging from -1 to 1; $d_c$ is the distance from extreme fiber to the center of the longitudinal prestressing tendon located close to it (see Fig. 3) and $t$ is the thickness of the concrete slab.

[1]Punching shear will also depend on the column dimension, which was taken as a constant in this study.

[2]Directly related to the specified minimum concrete cover and tendon diameter (see Fig. 3).

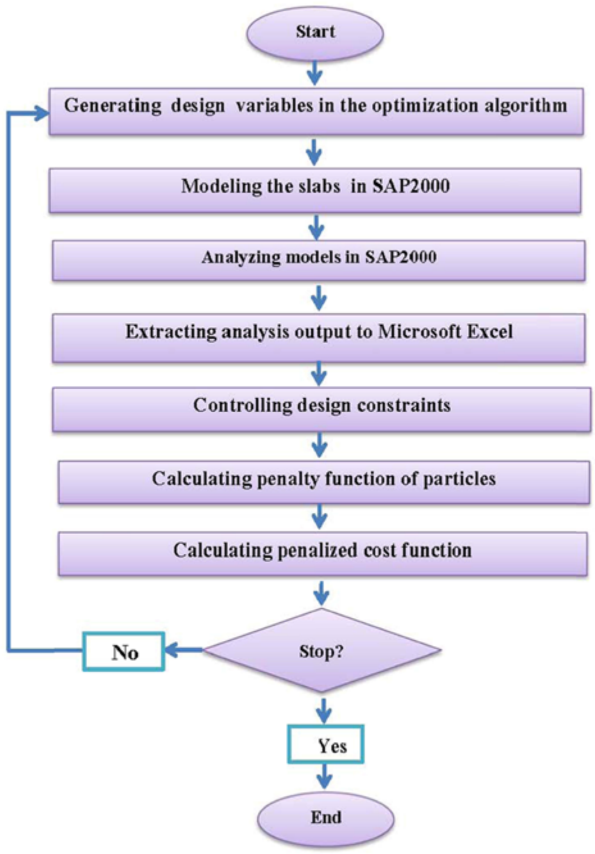


Fig. 4. Flowchart of Optimization Procedure

design variable vector violates the constraints, the penalty function increases its value; as a result, the algorithm will search for location of near variables which meet all requirements of the constraints.

The flowchart of implementing optimal design of prestressed concrete slabs is illustrated in Fig. 4.

## 4. Numerical Example

In order to show the effectiveness of the proposed formulation and approach, a real scale example is considered. This problem is solved using PSO and PSOHS to examine both the formulation and performance of the PSOHS. The considered problem is optimal design of a 10 m × 10 m prestressed concrete slab with an edge beam of 100 cm depth and 80 cm width. The slab is supported by columns at four corners whose dimensions are 80 cm × 80 cm as shown in Fig. 5. Also, the concrete cover on tendons is considered to be equal or more than 40 mm. The geometry of the slab is schematically delineated in Fig. 6. The applied loads are determined based on a typical structure as: slab and other elements self-weight made of reinforced concrete with specific weight of 24 kN/m$^3$, and a dead and a live surplus loads of 2.40 kN/m$^2$ imposed on the slab. The concrete compressive strength,

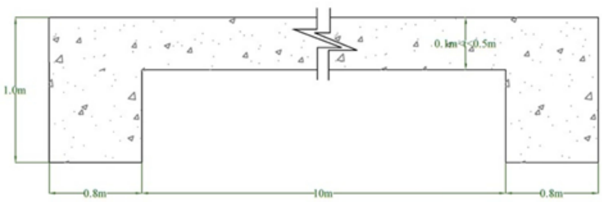


Fig. 5. Schematic View of Slab and Edge Beams

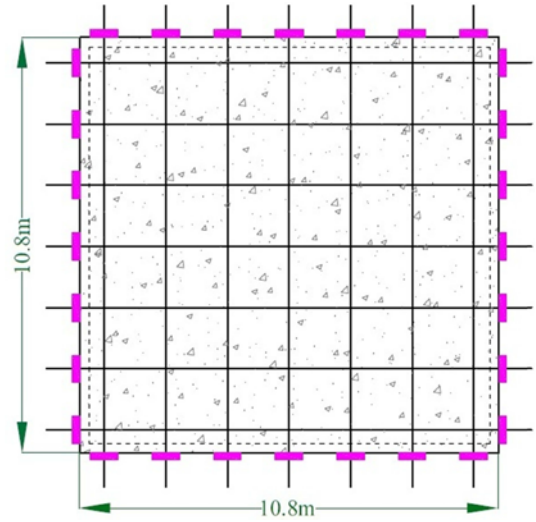


Fig. 6. Position of Tendons in the Considered Problem

Table 2. Cost of Materials

| Concrete Cost | | |
|---|---|---|
| 130 \$/m$^3$ | | |
| Prestressing tendons | | |
| Diameter(mm) | Area(mm$^2$) | cost \$/m |
| 20 | 314.16 | 2.363 |
| 21 | 346.36 | 2.56 |
| 22 | 380.13 | 2.757 |
| 23 | 415.48 | 2.954 |
| 24 | 452.39 | 3.151 |
| 25 | 490.87 | 3.348 |
| 26 | 530.93 | 3.545 |
| 27 | 572.56 | 3.742 |
| 28 | 615.75 | 3.939 |
| 29 | 660.52 | 4.136 |
| 30 | 706.86 | 4.333 |

Table 3. Upper and Lower Limit for Variables

| Variable | Lower limit | Upper limit |
|---|---|---|
| t | 100 mm | 500 mm |
| $N_x$ | 10 | 63 |
| $N_y$ | 10 | 63 |
| $d_x$ | 20 mm | 30 mm |
| $d_y$ | 20 mm | 30 mm |
| $e_1$ | No lower limit is considered[1] | No upper limit is considered[1] |
| $e_2$ | No lower limit is considered[1] | No upper limit is considered[1] |
| $e_3$ | No lower limit is considered[1] | No upper limit is considered[1] |
| $S_{tendon}$ | 400 MPa | 1860 MPa |

[1]The only considered constraint is the concrete cover (40 mm)

Table 4. The Best Three Results Obtained by PSO with w = 0.50

| | 1st Run | | 2nd Run | | 3rd Run | |
|---|---|---|---|---|---|---|
| Variable | Initial point | Solution | Initial point | Solution | Initial point | Solution |
| t | 221 | 180 | 131 | 150 | 144 | 183 |
| $N_x$ | 12 | 10 | 21 | 11 | 28 | 18 |
| $N_y$ | 12 | 10 | 21 | 11 | 28 | 18 |
| $d_x$ | 21 | 20 | 24 | 22 | 21 | 21 |
| $d_y$ | 21 | 20 | 24 | 22 | 21 | 21 |
| $e_1$ | -0.5928 | 0.5556 | 0.3893 | 0.4667 | 0.4444 | 0.5628 |
| $e_2$ | -0.5928 | 0.5556 | 0.3893 | 0.4667 | 0.4444 | 0.5628 |
| $e_3$ | 0.5113 | 0.5556 | 0.3893 | 0.4667 | 0.4444 | 0.5628 |
| Cost | 245029.356 | **54054.022** | 175714.65 | **66197.82** | 129282.20 | **34097.99** |
| Constraints | | | | | | |
| $C_1$ | 1.6551 | 1.5441 | 3.9649 | 2.1428 | 3.0510 | 1.6229 |
| $C_2$ | 1.3171 | 1.1925 | 3.1534 | 1.5239 | 2.4293 | 1.2102 |
| $C_3$ | 1.6457 | 1.8852 | 1.3877 | 1.6679 | 1.1308 | 0.9901 |
| $C_{4\text{-}X}$ | 0.2637 | 0.4245 | 0.3032 | 0.4118 | 0.2542 | 0.2332 |
| $C_{4\text{-}Y}$ | 0.6448 | 0.4245 | 0.3032 | 0.4118 | 0.2542 | 0.2332 |
| $C_{5\text{-}X}$ | 0.6749 | 0.3194 | 0.1036 | 0.2018 | 0.1257 | 0.1951 |
| $C_{5\text{-}Y}$ | 1.4003 | 0.3194 | 0.1036 | 0.2018 | 0.1257 | 0.1951 |
| $C_6$ | 0.0033 | 0.0022 | 0.0132 | 0.0032 | 0.0213 | 0.0069 |

$f_c'$, is 40 MPa , and tensile yielding stress of tendons, $f_y$, is 1860 MPa.The cost of concrete and tendons are included in Table 2; moreover, the feasible range of each variable is presented in Table 3.

Since all metaheuristic algorithms are sensitive to the constant parameters and random initialization, in order to reach the best result, several values are considered for inertia weight, *w*, which are 0.5, 1.0, 1.5, and 2. Also, both algorithms are run 20 times for each value to compensate the effect of random initialization.

Three best runs of the proposed approach are presented in Tables 4 thorough 11 for both PSO and PSOHS with different values of *w*. Also, convergence history curves of an average of 80 independent of such runs are depicted in the Figs. 7 and 8 for PSO and PSOHS, respectively. It can be concluded from these tables that, in this example, PSO has more diverse results than

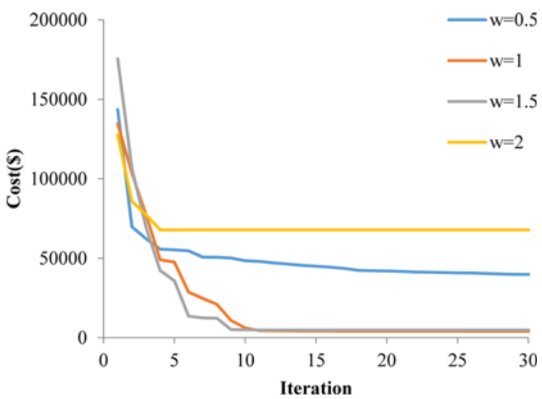


Fig. 7. Average of 80 Independent Runs of PSO with Different Values for w

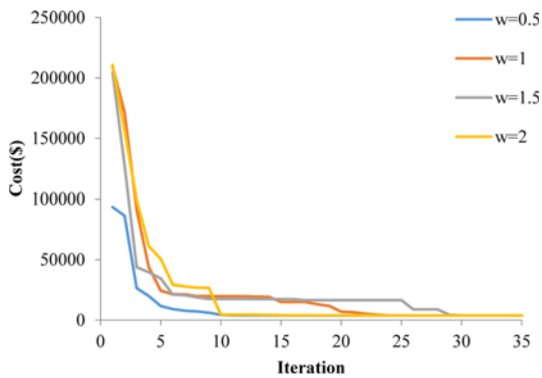


Fig. 8. Average of 80 Independent Runs of PSOHS with Different Values for w

PSOHS; therefore, it does not always lead to the best possible result. Furthermore, PSO does not always provide a safe design, and based on the tables, some final designs do not meet all the code requirements; on the contrary, PSOHS provides designs which all accord the CSA.

Considering Tables 4 through 13, the following conclusions can be drawn:

- The best result of PSO is attributed to the case when $w = 1.0$. In this case the obtained penalized cost was \$3998.80. For the PSOHS the best result obtained when $w = 1$ and 1.5, and the penalized cost was \$3787.60, which is 5.28% less than the best result of PSO.
- The ratios of penalized cost function to the best cost function of each algorithm are presented in Table 13 for different values of *w*. From this table, it can be concluded that the dependency of PSOHS on *w* is far less than the PSO; as a result,

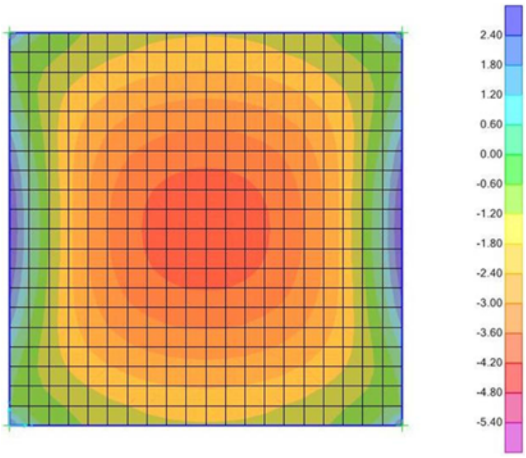


Fig. 9. x-direction Stress at the Final Stage

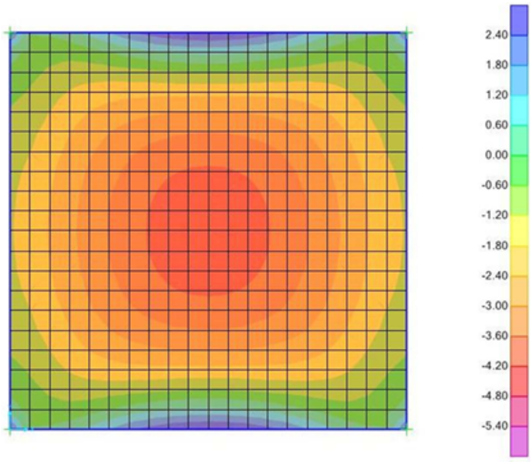


Fig. 10. y-direction Stress at the Final Stage

obtaining a reliable result by PSO necessitates examining the algorithm with different values for $w$ to ensure that the result is optimal.

The reasons for such observations can be referred to the shortcomings of the PSO and HS, and improvements applied to the PSOHS algorithm. In PSO when the value of w is big, the algorithm carries out only global search because the velocity of particles is too high to search near the optimal point; on the other hand, when a small value is assigned to $w$, the algorithm performs well only in the local search; therefore, it may either trapped in a local minimum or it may reach to the global minimum with a great error. In the HS algorithm, the parameter $b_w$, which is the radius of search around the variables stored in the HM, plays such role. Again, large values of this parameter lead to a global search only, and small values provide a local search; thus, a constant value cannot ensure a balanced search.

However, the value of w in PSOHS is large at beginning, and its value automatically decreases as the algorithm progresses. This mechanism provides PSOHS a global search at initial stages of the optimization process, and a local search at its latest stage; therefore, two defects, unbalanced search and no way to determine

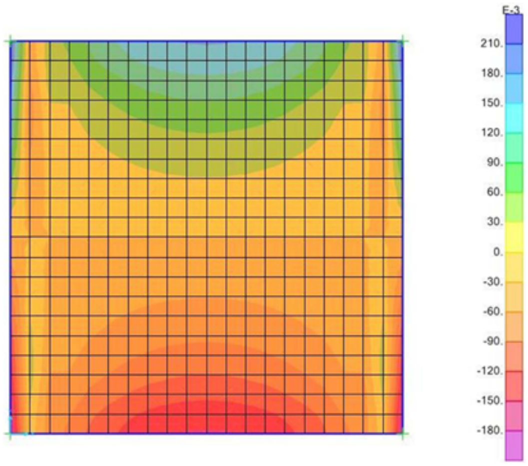


Fig. 11. Shear Stress in zy-direction at the Final Stage

$w$, are simultaneously removed.

The lack of a method to correct the position of violated particles is severe when the optimum point is located near the boundaries because the particles may cross sides when they are moving near them. To solve this problem, some approaches have been devised thus far such as fly-back (Li, 2007) or random reproduction in the feasible search space. However, none of them are as effective as a HS based algorithm because using a HS approach, provides a second robust search mechanism which has its beneficial characteristics such as searching near the best results, a mechanism to avoid trapping in a local minimum, etc. Therefore, when a particle swerves, a second optimization algorithm is activated, and optimization process is carried out without any halt.

## 5. Verification of the Design

Since the best obtained result is associated with 2nd run of the PSOHS with $w$ = 1.0, this model is checked for accuracy of the design. The load combinations are 1.25 $D$ + 1.5 and $D$ + $L$ for the initial and final stages, respectively. The maximum compressive stress must be checked with $0.6f_c'$, and maximum tensile stress must be checked with $0.5\sqrt{f_c'}$. The maximum values of SAP2000 outputs are presented in Table 14 and Figs. 9 to 11 based on these figures and table, it is observed that the maximum tensile stress is associated with initial stage with the value of 3.157 MPa and ratio of 0.9990 (for $f_r$ = 3.16 MPa in this study). For the case of initial stage, these values are 2.372 and 0.7507, respectively. The value od $C_3$ (refer to table ….) the tendons' prestressing forces are 623000 N, and stress ratio of 0.9916 (for $F_{pu}$ = 2000 MPa and tendons' diameter of 20 mm).

The bending strength of prestressed concrete sections is defined using Eq. (12):

$$M_r = (A_s \cdot \phi_s \cdot f_y - f_t) \times \left(d - \frac{a}{d}\right) \tag{12}$$

Table 5. The Best three Results Obtained by PSOHS with w = 0.50

| | 1st Run | | 2nd Run | | 3rd Run | |
|---|---|---|---|---|---|---|
| Vari-able | Initial point | Solution | Initial point | Solution | Initial point | Solution |
| t | 281 | 260 | 308 | 252 | 474 | 260 |
| $N_x$ | 32 | 12 | 22 | 10 | 14 | 10 |
| $N_y$ | 32 | 12 | 22 | 10 | 14 | 10 |
| $d_x$ | 28 | 20 | 29 | 21 | 23 | 20 |
| $d_y$ | 28 | 20 | 29 | 21 | 23 | 20 |
| $e_1$ | 0.7153 | 0.6923 | 0.7403 | 0.6825 | 0.8312 | 0.6923 |
| $e_2$ | 0.7153 | 0.6923 | 0.7403 | 0.6825 | 0.8312 | 0.6923 |
| $e_3$ | 0.7153 | 0.6923 | 0.7403 | 0.6825 | 0.8312 | 0.6923 |
| Cost | 105040 | **3950** | 3866.4 | **3788** | 24841.02602 | **3852.6** |
| Constraints | | | | | | |
| $C_1$ | 1.9552 | 0.9926 | 1.8019 | 0.9971 | 1.2554 | 0.9980 |
| $C_2$ | 1.5460 | 0.7482 | 1.4275 | 0.7476 | 0.9911 | 0.7526 |
| $C_3$ | 0.4546 | 0.7668 | 0.5884 | 0.9955 | 0.5703 | 0.8469 |
| $C_{4\text{-}X}$ | 0.0632 | 0.2113 | 0.0756 | 0.2475 | 0.1338 | 0.2479 |
| $C_{4\text{-}Y}$ | 0.0632 | 0.2113 | 0.0756 | 0.2475 | 0.1338 | 0.2479 |
| $C_{5\text{-}X}$ | 0.1122 | 0.4041 | 0.1415 | 0.4192 | 0.4454 | 0.4774 |
| $C_{5\text{-}Y}$ | 0.1122 | 0.4041 | 0.1415 | 0.4192 | 0.4454 | 0.4774 |
| $C_6$ | 0.0931 | 0.0022 | 0.0266 | 0.0016 | 0.0119 | 0.0016 |

Table 6. The Best Three Results Obtained by PSO with w = 1.0

| | 1st Run | | 2nd Run | | 3rd Run | |
|---|---|---|---|---|---|---|
| Vari-able | Initial point | Solution | Initial point | Solution | Initial point | Solution |
| t | 147 | 269 | 127 | 289 | 127 | 313 |
| $N_x$ | 18 | 6 | 36 | 5 | 57 | 12 |
| $N_y$ | 18 | 6 | 36 | 5 | 57 | 12 |
| $d_x$ | 25 | 23 | 23 | 28 | 21 | 21 |
| $d_y$ | 25 | 23 | 23 | 28 | 21 | 21 |
| $e_1$ | 0.4558 | 0.7026 | 0.3701 | 0.7232 | 0.3701 | 0.7444 |
| $e_2$ | 0.4558 | 0.7026 | 0.3701 | 0.7232 | 0.3701 | 0.7444 |
| $e_3$ | 0.4558 | 0.7026 | 0.3701 | 0.7232 | 0.3701 | 0.7444 |
| Cost | 81082.19 | **4003.88** | 129931.18 | **3998.80** | 208609.04 | 4047.20 |
| Constraints | | | | | | |
| $C_1$ | 2.5564 | 0.9212 | 2.9166 | 0.8456 | 3.5408 | 0.9221 |
| $C_2$ | 1.8455 | 0.7295 | 2.3227 | 0.6691 | 2.8245 | 0.7011 |
| $C_3$ | 0.8017 | 0.8361 | 0.7096 | 0.6290 | 0.6755 | 0.6525 |
| $C_{4\text{-}X}$ | 0.2517 | 0.1782 | 0.3527 | 0.1211 | 0.6909 | 0.1734 |
| $C_{4\text{-}Y}$ | 0.2517 | 0.1782 | 0.3527 | 0.1211 | 0.6909 | 0.1734 |
| $C_{5\text{-}X}$ | 0.1325 | 0.3423 | 0.1425 | 0.2755 | 0.2893 | 0.4300 |
| $C_{5\text{-}Y}$ | 0.1325 | 0.3423 | 0.1425 | 0.2755 | 0.2893 | 0.4300 |
| $C_6$ | 0.0086 | 0.0018 | 0.0403 | 0.0014 | 0.2043 | 0.0033 |

Table 7. The Best three Results Obtained by PSOHS with w = 1.0

| | 1st Run | | 2nd Run | | 3rd Run | |
|---|---|---|---|---|---|---|
| Vari-able | Initial point | Solution | Initial point | Solution | Initial point | Solution |
| t | 132 | 252 | 495 | 255 | 277 | 249 |
| $N_x$ | 14 | 10 | 13 | 10 | 35 | 10 |
| $N_y$ | 14 | 10 | 13 | 10 | 35 | 10 |
| $d_x$ | 24 | 21 | 26 | 20 | 27 | 22 |
| $d_y$ | 24 | 21 | 26 | 20 | 27 | 22 |
| $e_1$ | 0.3939 | 0.6825 | 0.8384 | 0.6863 | 0.7112 | 0.6787 |
| $e_2$ | 0.3939 | 0.6825 | 0.8384 | 0.6863 | 0.7112 | 0.6787 |
| $e_3$ | 0.3939 | 0.6825 | 0.8384 | 0.6863 | 0.7112 | 0.6787 |
| Cost | 155463.25 | **3788.00** | 360505.60 | **3787.60** | 125565.30 | **3788.40** |
| Constraints | | | | | | |
| $C_1$ | 4.4639 | 0.9980 | 1.2301 | 0.9990 | 1.9638 | 0.9917 |
| $C_2$ | 3.3194 | 0.7482 | 0.9699 | 0.7507 | 1.5517 | 0.7412 |
| $C_3$ | 0.5699 | 0.9939 | 0.5231 | 0.9916 | 0.4351 | 0.9997 |
| $C_{4\text{-}X}$ | 0.3665 | 0.2474 | 0.2208 | 0.2627 | 0.0622 | 0.2342 |
| $C_{4\text{-}Y}$ | 0.3665 | 0.2474 | 0.6306 | 0.2627 | 0.0622 | 0.2342 |
| $C_{5\text{-}X}$ | 0.1675 | 0.4193 | 0.6951 | 0.4665 | 0.1111 | 0.3784 |
| $C_{5\text{-}Y}$ | 0.1675 | 0.4193 | 1.8848 | 0.4665 | 0.1111 | 0.3784 |
| $C_6$ | 0.0058 | 0.0016 | 0.0211 | 0.0016 | 0.1279 | 0.0016 |

Table 8. The Best Three Results Obtained by PSO with w = 1.5

| | 1st Run | | 2nd Run | | 3rd Run | |
|---|---|---|---|---|---|---|
| Vari-able | Initial point | Solution | Initial point | Solution | Initial point | Solution |
| t | 203 | 317 | 410 | 266 | 177 | 387 |
| $N_x$ | 12 | 15 | 13 | 10 | 34 | 10 |
| $N_y$ | 12 | 15 | 13 | 10 | 34 | 10 |
| $d_x$ | 29 | 26 | 22 | 24 | 23 | 20 |
| $d_y$ | 29 | 26 | 22 | 24 | 23 | 20 |
| $e_1$ | -0.4680 | 0.7476 | 0.8049 | 0.6992 | 0.5480 | 0.7933 |
| $e_2$ | -0.4680 | 0.7476 | 0.8049 | 0.6992 | 0.5480 | 0.7933 |
| $e_3$ | 0.4680 | 0.7476 | 0.8049 | 0.6992 | 0.5480 | 0.7933 |
| Cost | 213351.51 | **5535.52** | 142775.76 | **4088.20** | 156658.61 | **5503.6** |
| Constraints | | | | | | |
| $C_1$ | 1.4499 | 1.0401 | 1.7940 | 0.8997 | 3.0266 | 0.8390 |
| $C_2$ | 1.1533 | 0.8216 | 1.4259 | 0.6774 | 2.4084 | 0.6416 |
| $C_3$ | 0.8345 | 0.5510 | 1.8413 | 0.7262 | 0.8907 | 0.7766 |
| $C_{4\text{-}X}$ | 0.1483 | 0.0877 | 0.2698 | 0.1753 | 0.1548 | 0.2400 |
| $C_{4\text{-}Y}$ | 0.3209 | 0.0877 | 0.2698 | 0.1753 | 0.1548 | 0.2400 |
| $C_{5\text{-}X}$ | 0.3940 | 0.2386 | 0.4451 | 0.3433 | 0.1136 | 0.6718 |
| $C_{5\text{-}Y}$ | 0.8998 | 0.2386 | 0.4451 | 0.3433 | 0.1136 | 0.6718 |
| $C_6$ | 0.0025 | 0.0073 | 0.0056 | 0.0015 | 0.0488 | 0.0028 |

where $\phi_s$ is the reinforcing steel resistance factor, and its value is 0.85; $f_t$ is the tendons's force; $d$ is the effective depth of the section, and $a$ is the depth of Witney's stress block. Accordint to Table 14, the value of $Mr$ for this section is 790 kN.m, and based on SAP2000 model, the value of existing bending moments are 185 kN.m and $1.2 \times 250 = 300$ kN.m for ultimate bending and minimum factored resistance, respectively. Hence, the ratios of existing moments to strength bending momens are 0.2342 and 0.3794 for these two stages, respectively.

The cracking moment ($M_{cr}$) can be calculated by using Eq. (13):

$$M_{cr} = \frac{f_r \cdot I_g}{y_t} - f_t \cdot T \cdot d_c \tag{13}$$

where $f_r = 0.6\sqrt{f_c}$; $I_g$ is the gross cross-secional moment of inertia around neutral axis, and $T$ is the total depth of the slab, and $d_c$ is the depth of concrete cover on the renforcing steels. The $M_{cr}$ is 250 kN.m which satisfies the $C_5$ in the constraint tables

Table 9. The Best three Results Obtained by PSOHS with w = 1.5

| | 1st Run | | 2nd Run | | 3rd Run | |
|---|---|---|---|---|---|---|
| Vari-able | Initial point | Solution | Initial point | Solution | Initial point | Solution |
| t | 460 | 256 | 156 | 255 | 397 | 252 |
| $N_x$ | 33 | 10 | 43 | 10 | 11 | 10 |
| $N_y$ | 33 | 10 | 43 | 10 | 11 | 10 |
| $d_x$ | 22 | 20 | 25 | 20 | 22 | 21 |
| $d_y$ | 22 | 20 | 25 | 20 | 22 | 21 |
| $e_1$ | 0.8261 | 0.6875 | 0.4872 | 0.6863 | -0.7481 | 0.6825 |
| $e_2$ | 0.8261 | 0.6875 | 0.4872 | 0.6863 | -0.7481 | 0.6825 |
| $e_3$ | 0.8261 | 0.6875 | 0.4872 | 0.6863 | 0.7481 | 0.6825 |
| Cost | 212655.41 | **3800.60** | 98290.62 | **3787.60** | 358461.76 | **3788.00** |
| Constraints | | | | | | |
| $C_1$ | 2.5295 | 0.9999 | 2.1292 | 0.9996 | 1.0701 | 0.9980 |
| $C_2$ | 1.9970 | 0.7520 | 1.6738 | 0.7514 | 0.8446 | 0.7482 |
| $C_3$ | 0.7218 | 0.9603 | 0.4654 | 0.9893 | 0.7687 | 0.9935 |
| $C_{4\text{-}X}$ | 0.1007 | 0.2597 | 0.2244 | 0.2626 | 0.2442 | 0.2474 |
| $C_{4\text{-}Y}$ | 0.1007 | 0.2597 | 0.2244 | 0.2626 | 0.7442 | 0.2474 |
| $C_{5\text{-}X}$ | 0.2148 | 0.4687 | 0.1428 | 0.4665 | 0.8943 | 0.4193 |
| $C_{5\text{-}Y}$ | 0.2148 | 0.4687 | 0.1428 | 0.4665 | 2.5329 | 0.4193 |
| $C_6$ | 0.1967 | 0.0016 | 0.0972 | 0.0016 | 0.0082 | 0.0016 |

Table 10. The Best three Results Obtained by PSO with w = 2.0

| | 1st Run | | 2nd Run | | 3rd Run | |
|---|---|---|---|---|---|---|
| Vari-able | Initial point | Solution | Initial point | Solution | Initial point | Solution |
| t | 116 | 134 | 147 | 147 | 124 | 166 |
| $N_x$ | 14 | 11 | 19 | 18 | 10 | 10 |
| $N_y$ | 14 | 11 | 19 | 18 | 10 | 10 |
| $d_x$ | 23 | 24 | 27 | 23 | 20 | 20 |
| $d_y$ | 23 | 24 | 27 | 23 | 20 | 20 |
| $e_1$ | 0.3103 | 0.4030 | 0.4558 | 0.4558 | 0.3548 | 0.5181 |
| $e_2$ | 0.3103 | 0.4030 | 0.4558 | 0.4558 | 0.3548 | 0.5181 |
| $e_3$ | 0.3103 | 0.4030 | 0.4558 | 0.4558 | 0.3548 | 0.5181 |
| Cost | 129558.17 | **67976.23** | 141880.93 | **71648.83** | 109059.12 | **63648.07** |
| Constraints | | | | | | |
| $C_1$ | 3.9551 | 2.7404 | 3.2394 | 2.3426 | 3.4077 | 1.7519 |
| $C_2$ | 2.8328 | 1.9476 | 2.5750 | 1.8566 | 2.4413 | 1.3961 |
| $C_3$ | 1.2493 | 1.4699 | 1.1424 | 1.1463 | 2.0827 | 2.1692 |
| $C_{4\text{-}X}$ | 0.4819 | 0.4176 | 0.2309 | 0.2793 | 0.6723 | 0.4670 |
| $C_{4\text{-}Y}$ | 0.4819 | 0.4176 | 0.2309 | 0.2793 | 0.6723 | 0.4670 |
| $C_{5\text{-}X}$ | 0.1320 | 0.1510 | 0.1036 | 0.1398 | 0.1975 | 0.2829 |
| $C_{5\text{-}Y}$ | 0.1320 | 0.1510 | 0.1036 | 0.1398 | 0.1975 | 0.2829 |
| $C_6$ | 0.0068 | 0.0036 | 0.0096 | 0.0086 | 0.0032 | 0.0024 |

(see Table 1).

The shear resistance of the prestressed section can be calculated using Eq. (14):

$$v_r = v_c + v_s \tag{14}$$

where $v_c$ is the shear resistance of concrete which is defined using Eq. (15); $v_s$ is the shear resistance of tendons which is defined using Eq. (16)

Table 11. The Best three Results Obtained by PSOHS with w = 2.0

| | 1st Run | | 2nd Run | | 3rd Run | |
|---|---|---|---|---|---|---|
| Vari-able | Initial point | Solution | Initial point | Solution | Initial point | Solution |
| t | 138 | 256 | 154 | 262 | 193 | 260 |
| $N_x$ | 19 | 10 | 52 | 10 | 11 | 10 |
| $N_y$ | 19 | 10 | 52 | 10 | 11 | 10 |
| $d_x$ | 20 | 20 | 21 | 20 | 25 | 20 |
| $d_y$ | 20 | 20 | 21 | 20 | 25 | 20 |
| $e_1$ | 0.4203 | 0.6875 | 0.4805 | 0.6947 | -0.5026 | 0.6923 |
| $e_2$ | 0.4203 | 0.6875 | 0.4805 | 0.6947 | -0.5026 | 0.6923 |
| $e_3$ | 0.4203 | 0.6875 | 0.4805 | 0.6947 | 0.5026 | 0.6923 |
| Cost | 90694.98 | **3800.60** | 302331.90 | **3813.60** | 239157.82 | **3800.60** |
| Constraints | | | | | | |
| $C_1$ | 2.8647 | 0.9993 | 4.6071 | 0.9382 | 1.2599 | 0.9964 |
| $C_2$ | 2.0612 | 0.7517 | 3.6746 | 0.7061 | 1.0018 | 0.7517 |
| $C_3$ | 1.1433 | 0.9619 | 0.9815 | 0.9627 | 0.9957 | 0.8514 |
| $C_{4\text{-}X}$ | 0.3579 | 0.2598 | 0.2137 | 0.2518 | 0.1928 | 0.2481 |
| $C_{4\text{-}Y}$ | 0.3579 | 0.2598 | 0.2137 | 0.2518 | 0.3949 | 0.2481 |
| $C_{5\text{-}X}$ | 0.1673 | 0.4687 | 0.1204 | 0.4779 | 0.4809 | 0.4773 |
| $C_{5\text{-}Y}$ | 0.1673 | 0.4687 | 0.1204 | 0.4779 | 1.0442 | 0.4773 |
| $C_6$ | 0.0103 | 0.0016 | 0.1983 | 0.0015 | 0.0017 | 0.0016 |

Table 12. The best Results Obtained by HS

| Variable | Initial point | Solution |
|---|---|---|
| t | 120 | 284 |
| $N_x$ | 14 | 10 |
| $N_y$ | 14 | 10 |
| $d_x$ | 21 | 29 |
| $d_y$ | 21 | 29 |
| $e_1$ | 0.3333 | 0.6429 |
| $e_2$ | 0.3333 | 0.6429 |
| $e_3$ | 0.3333 | 0.6429 |
| Cost | 132796.53 | **4678.51** |
| Constraints | | |
| $C_1$ | 4.1742 | 0.9232 |
| $C_2$ | 3.0310 | 0.8994 |
| $C_3$ | 1.2273 | 0.7961 |
| $C_{4\text{-}X}$ | 0.5136 | 0.1684 |
| $C_{4\text{-}Y}$ | 0.5136 | 0.1684 |
| $C_{5\text{-}X}$ | 0.1677 | 0.2063 |
| $C_{5\text{-}Y}$ | 0.1677 | 0.2063 |
| $C_6$ | 0.0065 | 0.0018 |

Table 13. The ratio of PSO/$PSO_{best}$ and PSOHS/$PSOHS_{best}$ for Different *w*

| w | PSO | | | PSOHS | | |
|---|---|---|---|---|---|---|
| 0.5 | 13.5176 | 16.5544 | 8.5271 | 1.0429 | 1.0001 | 1.0172 |
| 1 | 1.0013 | 1.0000 | 1.0121 | 1.0001 | 1.0000 | 1.0002 |
| 1.5 | 1.3843 | 1.0224 | 1.3763 | 1.0034 | 1.0000 | 1.0001 |
| 2 | 16.9992 | 17.9176 | 15.9168 | 1.0034 | 1.0069 | 1.0034 |

$$v_c = 0.2 \cdot \phi_c \cdot \sqrt{f_c'} \cdot b \cdot d \tag{15}$$

where $\phi_c$ is the concrete resistance factor, and its value is 0.6; and

Table 14. Checking Design Consideration of the Best Result Obtained by PSOHS

| Design Specification | Compressive Stress (MPa) | Tensile Stress (MPa) | Tendons' Stress (MPa) | Ultimate Bending Moment (kN.m) | Minimum Factored Resistance (kN.m) | Shear Force (N) | Eccentricity |
|---|---|---|---|---|---|---|---|
| Existing | 3.157 | 2.372 | 1983.2 | 185 | 300 | 0.18 | 0.343 |
| Resistance | 3.16 | 3.16 | 2000 | 790 | 790 | 112.5 | 0.686 |
| Ratio | 0.999 | 0.7507 | 0.9916 | 0.2342 | 0.3794 | 0.0016 | 0.5 |

$b$ is the width of the prestressed section.

$$v_s = A_T \cdot \phi_s \cdot f_y \cdot d/b \qquad (16)$$

where $A_T$ is the area of the tensons, and $f_y$ is the yiled stress of the tendons.

The ratio of the maximum shear force of the slab to its shearing strength yields the value of 0.0016 which shows that shear failure do not occur in the slab.

The maximum eccentricity constraint of the obtained tendon's profile is $e = \frac{(127.5-40)}{255} = 0.343$, and its acceptable value is $1-\frac{2\times 40}{255} = 0.686$; hence, the ratio of existing $e$ to its allowable value is 0.50.

Considering all aspects of the design which was explained above shows that the designed slab by PSOHS is acceptable, and none of the constraints has been violated.

## 6. Conclusions

In this study, optimal design of prestressed concrete slabs is performed using PSO, and its recently modified version PSOHS. The PSO lacks a balanced search mechanism, an approach is presented for determining the value of $w$, and an effective method is provided for correcting the position of the violated particles from feasibility search space. To examine the efficiency of the improvements applied to PSOHS, a large scale slab is considered for optimal design. The results show that the best result of the PSOHS is slightly better than that of PSO. Apart from this, the result of PSO is significantly varied with different values of $w$; while those of the PSOHS are hardly affected by $w$. Hence, for PSOHS a parameter adjustment is not necessary before performing the algorithm, and the algorithm provides a reliable result regardless of what w is assigned in the algorithm. Also, PSOHS makes use of HS characteristics to deal with the violated particles in the search space.

## References


Eberhart, R. and Kennedy, J. (1995). "A new optimizer using particle swarm theory." *Proceedings of the 6th International Symposium on Micro Machine and Human Science,* Nagoya, Japan.

Geem, Z., Kim, J. H., and Loganathan, G. V. (2001). "A new heuristic optimization algorithm: Harmony search." *Simulation,* Vol. 76, No. 2, pp. 60-68, DOI: 10.1177/003754970107600201.

Hosseini, S. S., Hamidi, S. A., Mansuri, M., and Ghoddosian, A. (2015). "Multi Objective Particle Swarm Optimization (MOPSO) for size and shape optimization of 2D truss structures." *Periodica Polytechnica, Civil Engineering*, Vol. 59, No. 1, p. 9.

Kaveh, A. and Nasrollahi, A. (2013). "Engineering design optimization using a hybrid PSO and HS algorithm." *Asian Journal of Civil Engineering,* Vol. 14, No. 2, pp. 201-223.

Kaveh, A. and Nasrollahi, A. (2014a). "Performance-based seismic design of steel frames utilizing charged system search optimization." *Applied Soft Computing,* Vol. 22, pp. 213-221.

Kaveh, A. and Nasrollahi, A. (2014b). "A new hybrid meta-heuristic for structural design: Ranked particles optimization." *Structural Engineering and Mechanics*,Vol. 52, No. 2, pp. 405-426.

Kaveh, A. and Nasrollahi, A. (2014c). "A new probabilistic particle swarm optimization algorithm for size optimization of spatial truss structures." *International Journal of Civil Engineering,* Vol. 12, No. 1, pp. 1-13.

Kaveh, A. amd Nasrollahi, A. (2014d). "Charged system search and particle swarm optimization hybridized for optimal design of engineering structures." *Scientia Iranica. Transaction A, Civil Engineering,* Vol. 21, No. 2, p. 295.

Kaveh, A. and Shakouri Mahmud Abadi, A. (2010). "Cost optimization of composite floor system using an improved harmony search algorithm." *Journal of Constructional Steel Research,* Vol. 66, No. 5, 664-669, DOI: 10.1016/j.scient.2012.04.001.

Kaveh, A., Zakizadeh, M. H., and Nasrollahi, A. (2014). "Performance-based optimal design of RC shear-walls utilizing pso and psohs metaheurisctic algorithms." *Asian Journal of Civil Engineering* (BHRC), Vol. 15, No. 5, pp. 683-704.

Koziey, B. and Mirza, F. A. (1997). "Consistent thick shell element." *Computers and Structures*, Vol. 65, No. 4, pp. 531-549, DOI: 10.1016/S0045-7949(96)00414-2 .

Kuyucular, A. (1991). "Prestressing optimization of concrete slabs." *Journal of Structural Engineering,* Vol. 117, No. 1, pp. 235-254, DOI: 10.1061/(ASCE)0733-9445(1991)117:1 (235).

Li, L., Huang, Z. B., Liu, F., and Wu, Q. H. (2007). "A heuristic particle swarm optimizer for optimization of pin connected structures." *Computers and Structures,* Vol. 85, No. 7, pp. 340-349, DOI: 10.1016/j.compstruc.2006.11.020.

Lounis, Z. and Cohn, M. Z. (1993). "Multiobjective optimization of prestressed concrete structure." *Journal of Structural Engineering,* Vol. 119, No. 3, pp. 794-808, DOI: 10.1061/(ASCE) 0733-9445 (1993)119:3(794) .

MacRae, A. and Cohn, M. Z. (1987). "Optimization of prestressed concrete flat plates." *Journal of Structural Engineering*, Vol. 113, No. 5, pp. 943-957, DOI: 10.1061/(ASCE)0733-9445(1987)113:5(943).

Rozvany, G. and Hampson, A. J. K. (1963). "Optimum design of prestressed plates." *Journal of the American Concrete Institute*, Vol. 60, No. 8, pp. 1065-1082, DOI: 10.14359/7891.

Semelawy, M. E., Nassef, A. O., and Damatty, A. A. El. (2012). "Design of prestressed concrete flat slab using modern heuristic optimization techniques." *Expert Systems and Applications*, Vol. 39, No. 5, pp. 5758-5766, DOI: 10.1016/j.eswa.2011.11.093.