KSCE Journal of Civil Engineering (2017) 21(4):1076-1086

DOI 10.1007/s12205-016-0933-3

TECHNICAL NOTE

Construction Management

pISSN 1226-7988, eISSN 1976-3808
www.springer.com/12205

Exploring Wavelet Applications in Civil Engineering

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Received May 4, 2016/Accepted May 23, 2016/Published Online July 25, 2016

Abstract

Although use of wavelet transform has significantly increased in civil engineering, selection of a proper wavelet type for a particular civil engineering application remains to be a question. The purpose of this study is to provide a guideline for the proper selection of wavelet type in civil engineering applications. A total of 64 articles were reviewed with an emphasis on the use of wavelet transform and the choice of a mother wavelet. A survey of the literature shows that wavelet transform is useful to mathematically address a range of problems, with the specific applications of the wavelet transform in civil engineering being six-fold: denoising, discontinuity detection, feature extraction, frequency identification, system modeling, and data compression. The result of the study is expected to help civil engineers to choose the right type of wavelet transform for the particular field of civil engineering.

Keywords: *wavelet transform, civil engineering, exploratory study*

1. Introduction

The wavelet transform is a mathematical tool with which a signal can be reconstructed from wavelet representations (Mallat, 1989). The fundamental idea behind the wavelet transform is that phenomena that appear complex are often relatively simple in reality. For example, a piece of music, with melodies and rhythms, is in fact no more than a combination of sound signals with different spectra. A painting may be reduced from a work of art to a mixture of basic colors of different wavelengths.

The wavelet transform is theoretically rooted in mathematics, physics, and electric/electronic engineering. Each wavelet transform begins with a Fourier transform. The underlying mathematical technique of the wavelet transform has strong similarities to those of Fourier transforms. Both types of transform decompose signals into groups of linear combinations and are conducted using basis functions that are compared to signals of interest to compute coefficients that assess the similarity between them. However, the wavelet transform may be fundamentally differentiated from the Fourier transform because the wavelet transform has an infinite set of possible basis functions, while the Fourier transform has a single set of basis functions that includes only sine and cosine functions (Graps, 1995).

The features of multiple basis functions for wavelet transforms have long been considered superior to those of Fourier transforms. Unlike Fourier transforms, wavelet transforms allow the user to choose the kind of wavelet (basis function) that is best suited for a particular application. The specific wavelet chosen or tailored for a particular application is therefore ideal in terms of the accuracy and efficiency of the analysis. However, this freedom to choose has also been considered burdensome to users. The wrong choice of wavelet may easily lead to inaccurate or substandard results.

The wavelet transform has been applied to signals and images in civil engineering for uses including denoising, discontinuity detection, feature extraction, frequency identification, data compression, and system modeling. Selecting optimal basis functions is important for the successful application of the wavelet transform to real-life problems. However, although wavelet applications have been successful in civil engineering, no general guidelines exist identifying wavelets that are suitable for specific types of civil engineering data.

The main objective of this research was to provide a guideline involving wavelet selection and applications in civil engineering. As previously mentioned, no clear-cut answer exists as to what wavelet analysis is used for and what specific wavelet type needs to be chosen for a particular domain of civil engineering. This paper overviews wavelet theory, focusing on the major wavelet

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properties that need to be considered for successful application of wavelet analysis. For each application area of civil engineering, an in-depth discussion is presented to explain the proper choice of wavelet type. Limitations of wavelet-based applications in civil engineering are also discussed in the context of comparison to other types of signal processing.

2. Overview of Wavelet Analysis

2.1 Basic Concepts

This section outlines the conceptual theory of wavelet transform, based on the works of Daubechies (1992), Rao and Bopardikar (1998), and Mallat (1989). Wavelet Transform (WT) is a means of obtaining representations of both time and frequency content for a signal. A wavelet is a basis function used in an integral transform. The Continuous Wavelet Transform (CWT) of a function *f(t)* is given by Eqs. (1) and (2):

$$W(a, \tau) = \int_{-\infty}^{+\infty} f(t)\,\psi_{a,\tau}(t)dt \quad (1)$$

$$\psi_{a,\tau}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-\tau}{a}\right) \quad (2)$$

Equation (1) indicates that *f(t)* is converted to wavelet coefficients $W(a, \tau)$, where a is the dilation factor (a real and positive number) and τ is the translation parameter (a real number), dilating and translating the mother wavelet $\psi(t)$, respectively. These parameters are often discretized, leading to the Discrete Wavelet Transform (DWT). In general, the scaling is discrete and dyadic with $a = 2^{-j}$. The translation is discretized with respect to each scale by using $\tau = k2^{-j}$. After discretization, the wavelet function is defined as Eq. (3):

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^j t - k) \quad (3)$$

In Eq. (3), j and k are the integer scale and translation factors. The wavelet coefficients are also given by the inner product between the signal and the wavelet, as shown in Eq. (4).

$$d(j, k) = \langle f(t), \psi_{j,k}\rangle \quad (4)$$

2.2 Properties of Wavelets

A range of wavelet types have been used for a multitude of engineering problems. The Haar, Daubechies, Coiflets, Morlet, and Mexican Hat (also known as Ricker) types have been used successfully. It is even possible to create new wavelets that are customized to satisfy specific conditions. A mother wavelet can be designed to obtain desired characteristics associated with particular functions. However, the diverse group of already existing wavelets is generally sufficient for the proper selection of wavelets for particular applications. Thus, it is important to understand the basic wavelet properties to consider when selecting the proper wavelet. In this section, three crucial major wavelet properties–vanishing moment, filter length, and regularity–are reviewed based on the works of Hubbard (1998) and Mathworks (2010).

2.3 Vanishing Moment

The vanishing moment is a criterion for the sparse representation of piecewise smooth functions. The j^{th} moment of a wavelet function can be defined by Eq. (5):

$$m_j(t) = \int_{-\infty}^{+\infty} t^j\,\psi(t)dt \quad (5)$$

If all of the moments, m_j (with j being 0 to k), vanish, then the wavelet is considered to have vanishing moments of order k+1. In the wavelet transform, the k vanishing moment allows for the suppression of polynomial signals that have degrees less than or equal to k (Hubbard, 1998). The more vanishing moments a mother wavelet has, the more the wavelet is capable of capturing high frequencies. Furthermore, the wavelet coefficients of a piecewise smooth function decay faster as the scale j goes to infinity.

2.4 Size of Support

Another important factor for the choice of optimal wavelets is the size of support of the mother wavelet. The size of support indicates the length of the convolution filter associated with the wavelet transform. To analyze a signal with a high degree of polynomials or differentiability, the wavelet transform requires a filter with a high level of support size. The minimum size of support for a given number of vanishing moments, equal to p, is $2p$-1 (Daubechies, 1992). Naturally, there is a close relationship between two important properties of wavelet bases, the number of vanishing moments and the size of support. Higher vanishing moments requires larger support. The size of support is also linked to the computational efficiency of the wavelet transform; a larger size of support increases the amount of computations required. Therefore, the selection of a wavelet involves a tradeoff between the size of support and the vanishing moment.

2.5 Regularity

Regularity is another important property of wavelet bases. The order of regularity refers to the number of continuous derivatives (Mathworks, 2010), and conceptually indicates how smooth the wavelet is. If $f(t)^{(m)}$ resembles $|x - x_0|^r$ near x_0, the regularity s of a signal f(t) is defined as $s = m + r$ with $0 < r < 1$ (Mathworks 2010). Regularity has significant influence on the error introduced by thresholding or quantizing the wavelet coefficients (Mathworks, 2010). If a wavelet has n number of continuous derivatives, then the wavelet must have vanishing moments of an order at least k+1. This affects the quality of reconstruction, especially the image compression. For example, when a smooth image is transformed with a Haar wavelet that is not smooth and not continuous, the reconstructed image from wavelet coefficients tends to have many edges that did not exist in the original image (i.e., it is discontinuous).

For these reasons, each wavelet property is strongly related to the accuracy and efficiency of wavelet-based analyses. By considering these properties, the choice of a wavelet can affect the accuracy and efficiency of analyses.

3. Wavelet Applications in Civil Engineering

The literature reviewed in this study encompasses all disciplines of civil engineering and was mainly selected from journals published by the American Society of Civil Engineers (ASCE). Articles were retrieved as follows: 1) the abstract/title/keywords of the articles were searched by the keyword, "wavelet" in the ASCE library and subsequently 63 articles altogether were scanned with the keyword; 2) three articles were excluded because they did not address wavelet applications; 3) three articles were excluded as the results of the wavelet approaches were not superior to those of others. Thus, 57 articles were selected for review. In addition, to strengthen the area of wavelet application in hydrology, seven randomly chosen articles, including one review article, were added to the list of articles, resulting in a total of 64 articles. Six broad categories of wavelet applications were defined according to purpose as follows:

1) Denoising: the use of wavelets to attenuate white noise in a signal.
2) Discontinuity detection: the use of wavelets to detect discontinuities in a signal.
3) Feature extraction: the use of wavelets to derive mid-level data that are fed into a reasonably complex decision making process.
4) Frequency identification: the use of wavelets to identify the main frequency component in a signal.
5) System modeling: the use of wavelets for system prediction.
6) Data compression: the use of wavelets to represent a signal with fewer coefficients.

The wavelet transform has been used for different purposes in various fields of signal processing in civil engineering. Table 1 shows the classification of the 63 articles reviewed (excluding the review article), by the six purposes. Since two articles showed successful application of wavelet analysis using two mother wavelets, these articles were double counted. Ten applications were classified under denoising, nine articles under discontinuity detection, 18 articles under feature extraction, 13 articles under frequency identification, three articles under data compression, and 12 articles under system modeling. Another way to categorize the 63 articles is by application area: structural engineering, transportation engineering, water engineering, and other. The following sections detail how wavelet analysis was used for each study according to these four areas of engineering.

3.1 Structural Engineering

Thirty articles published by ASCE journals were reviewed in the field of structural engineering. Table 2 lists the 31 articles describing the particular types of wavelet transforms used in each study.

3.1.1 Denoising

Adeli and Kim (2004) explained their use of the wavelet transform in structural engineering to eliminate dynamic environmental disturbance signals, or the lower frequency components, from ground acceleration signals of a civil engineering structure, using Daubechies wavelets with three vanishing moments. Adeli and Jiang (2006) described a signal processing method they developed to smooth contaminated data in acceleration responses of the structure under earthquakes, based on the discrete wavelet packet transform using a Daubechies wavelet of order 4. Jiang and Mahadevan (2008) also employed the same methodology to remove noise from signals for the nonparametric identification of structures. Rizzo *et al.* (2005) explained the use of the discrete wavelet transform (db40) for signal denoising in the toneburst signals of very small structures with dimensions less than 1mm. The reason why this study employed a high order wavelet, as opposed to the previous two studies, was that it had a more narrowband character that could match the toneburst signals.

3.1.2 Discontinuity Detection

Liew and Wang (1998) adopted a harmonic wavelet transform, which combines the advantages of the short-time Fourier transform and the continuous wavelet transform, to identify the location of cracks in structures without mathematical and computational burdens. Their primary concern was to overcome inaccuracies in the high-order mode of eigenvalue analysis. Hou *et al.* (2000) and Hera and Hou (2004) used the Daubechies wavelet with four vanishing moments to detect a sudden structural damage and its location, as indicated by a peak in the high-resolution wavelet details of the acceleration response data. Melhem and Kim (2003) used the Daubechies wavelet with four vanishing moments to detect structural damage or a change in a concrete structure based on the wavelet contour map of the acceleration response data.

Table 1. Summary of Literature on Wavelet Applications

	Denoising	Discontinuity detection	Feature extraction	Frequency identification	Data compression	System modeling	Total
Daubechies	8	5	8	1		5	27
Morlet		1	4	9		2	16
Haar		1	1		3	1	6
Littlewood-Paley				1		2	3
Mexican hat			1	1			2
Others	2	2	4	1		2	11
Total	10	9	18	13	3	12	65

Table 2. Applications of the Wavelet Transform in Structural Engineering

Area	Journal	Author(s) (year)	Category	Wavelet
Structure	JSE	Adeli and Kim (2004)	Denoising	Db03
	JEM	Jiang and Mahadevan (2008)	Denoising	Db04
	JSE	Adeli and Jiang (2006)	Denoising	Db04, Packet
	JEM	Rizzo *et al.* (2005)	Denoising	Db40
	JEM	Liew and Wang (1998)	Discontinuity Detection	Harmonic
	JEM	Hou *et al.* (2000)	Discontinuity Detection	Db04
	JEM	Hera and Hou (2004)	Discontinuity Detection	Db04
	JEM	Melhem and Kim (2003)	Discontinuity Detection	Db06
	JEM	Marwala (2000)	Feature Extraction	Db02
	JEM	Hajj (1999)	Feature Extraction	Db04
	JSE	Ren *et al.* (2008)	Feature Extraction	Db05
	JSE	Sun and Chang (2002)	Feature Extraction	Db15, Packet
	JSE	Sun and Chang (2004)	Feature Extraction	-, Packet
	JIS	Park *et al.* (2008)	Feature Extraction	Morlet
	JEM	Yoon *et al.* (2000)	Frequency Identification	Morlet
	JEM	Gurley *et al.* (2003)	Frequency Identification	Morlet
	JEM	Ji and Chang (2008)	Frequency Identification	Morlet
	JEM	Kijewski-Correa and Kareem (2007)	Frequency Identification	Morlet
	JSE	Chakraborty and Basu (2010)	Frequency Identification	Littlewood-Paley
	JEM	Basu and Gupta (1998)	System Modeling	Littlewood-Paley
	JEM	Chakraborty and Basu (2008)	System Modeling	Littlewood-Paley
	JEM	Kitada (1998)	System Modeling	Db02
	JEM	Basu and Nagarajaiah (2010)	System Modeling	Db02
	JEM	AL-Qassab and Nair (2003)	System Modeling	Db04
	JEM	Law *et al.* (2006)	System Modeling	Db04
	JEM	Spanos and Rao (2001)	System Modeling	Biorthogonal spline
	JEM	Spanos and Failla (2004)	System Modeling	Morlet
	JSE	Honda and Ahmed (2011)	System Modeling	Morlet
	JSE	Grant (2010)	System Modeling	-
	JCCE	Zhang and Li (2006)	Data Compression	Haar
	JIS	Mizuno *et al.* (2008)	Data Compression	Haar

Note: JSE=*Journal of Structural Engineering*; JEM=*Journal of Engineering Mechanics*; JIS=*Journal of Infrastructure Systems*; JCCE=*Journal of Computing in Civil Engineering*.

3.1.3 Feature Extraction

Hajj (1999) explained the use of the wavelet transform to quantify the level of intermittency of energy-containing scales in the surface layer of the atmospheric boundary layer. Marwala (2000) described the use of wavelet spectrum as input variables for a neural network model to identify damage in a population of cylindrical shells.

Sun and Chang (2002) developed a feature extraction methodology to characterize the damage feature based on wavelet packet transform, which enables focusing on any part of the time-frequency domain by providing a level-by-level decomposition of a signal (Mallat, 1989). Sun and Chang (2004) also adopted the wavelet packet transform to extract minute abnormalities from vibration signals for structural health monitoring. Both studies used Daubechies wavelets as the mother wavelet.

Ren *et al.* (2008) adopted the wavelet packet transform to characterize damage features as energy components to identify their locations using the Daubechies wavelet with five vanishing moments. Park *et al.* (2008) developed a piezoelectric sensor-based health monitoring technique using a support vector machine classifier; the Morlet wavelet was used to extract the features that were later fed into the classifier.

3.1.4 Frequency Identification

The Morlet wavelet or the complex modified has been widely used as a mother wavelet to identify the characteristic features on the spectral or probabilistic structure of the analyzed signals (Yoon *et al*., 2000; Gurley *et al.*, 2003; Ji and Chang, 2008). Using graphical representations, such as the scalogram, which describe the signal energy on a domain, previous studies revealed characteristics about non-stationary processes in structural responses. The Morlet wavelet was employed, owing to its good localization in time-frequency domains. Kijewski-Correa and Kareem (2007) evaluated the performances of two different transforms, empirical mode decomposition with Hilbert transform and wavelet transform, when extracting signals embedded in noise. They adopted the Morlet wavelet due to its shape like a frequency-modulated wave. Chakraborty and Basu (2010) adopted a modified form of the Littlewood-Paley basis function to examine the time varying frequency content in the response of primary-secondary systems. Their continuous wavelet analysis clearly showed that the response of the secondary system contained strong frequency non-stationarity.

3.1.5 System Modeling

Using the Daubechies wavelets, measured signals can be represented as a series expansion of wavelets (mathematical functions) for the identification of a nonlinear structural dynamic (Kitada, 1998; AL-Qassab and Nair, 2003; Law *et al*., 2006; Basu and Nagarajaiah, 2010); the computational features of the Daubechies wavelets are recursive, mutually orthogonal, and compactly supported. Conversely, Basu and Gupta (1998) employed a modified form of the Littlewood-Paley wavelet to model ground motions as non-stationary processes in terms of both amplitude and frequency nonstationarity because this mother wavelet has an excellent frequency localization despite poor localization in time. Furthermore, it provides numerical computational advantages by enabling the energy computation of any signal in nonoverlapping frequency bands. Spanos and Rao (2001) described their use of a non-orthogonal spline wavelet basis function for the representation of random fields due to its simplicity and properties as rational filter coefficients. Spanos and Failla (2004) developed a wavelet-based mathematical model of random evolutionary processes, as a finite series with time-dependent coefficients, using the Morlet wavelet. Chakraborty and Basu (2008) modeled non-stationary motion as a summation of orthogonal stochastic processes at different frequency scales in the wavelet domain using a modified form of the Littlewood-Paley basis, which offers nonoverlapping frequency bands for wavelet bases at different scales. Grant (2011) added a set of wavelet-based adjustment functions to a ground-motion of time history for the modification of the spectral response. This

Table 3. Applications of the Wavelet Transform in Engineering Fields Other Than Structural Engineering

Area	Journal	Author(s) (year)	Category	Wavelet
Transportation	JTE	Adeli and Karim (2000)	Denoising	Db04
	JTE	Karim and Adeli (2002)	Denoising	Db04
	JTE	Karim and Adeli (2003)	Denoising	Db04
	JTE	Wei *et al.* (2005)	Discontinuity Detection	Db03
	JTE	Vlahogianni *et al.* (2008)	Discontinuity Detection	-
	JTE	Adeli and Ghosh-Dastidar (2004)	Feature Extraction	Coifman
	JTE	Jiang and Adeli (2005)	Feature Extraction	Mexican hat
Water	JWPCOE	Goring (2008)	Denoising	Db05
	JHE	Kişi (2009)	Denoising	-
	JHLE	Ferrante *et al.* (2007)	Discontinuity Detection	Db01
	JWPCOE	Özger (2011)	Discontinuity Detection	Morlet
	JEM	Stamos and Hajj (2001)	Feature Extraction	Morlet
	JH	Badrzadeh *et al.* (2015)	Feature Extraction	Db03
	JH	Nourani *et al.* (2015)	Feature Extraction	Db04
	JH	Shoaib *et al.* (2015)	Feature Extraction	Meyer
	JHE	Mwale *et al.* (2007)	Feature Extraction	Morlet
	JHE	Mwale and Gan (2010)	Feature Extraction	Morlet
	JHE	Kim and Valdeœ (2003)	Feature Extraction	-
	JEM	Addison *et al.* (2001)	Frequency Identification	Db02, Mexican hat
	JEM	Ma *et al.* (2010)	Frequency Identification	Morlet
	JH	Duvert *et al.* (2015)	Frequency Identification	Morlet
	JH	Fang *et al.* (2015)	Frequency Identification	Morlet
	JH	Yu and Lin (2015)	Frequency Identification	Morlet
	JHE	Mwale *et al.* (2011)	Frequency Identification	Morlet
	JWPCOE	Woo and Liu (2004)	Frequency Identification	-
	JHLE	Sattar *et al*. (2009)	System Modeling	Db03, Haar
Geotech	JCCE	Chandan *et al.* (2004)	Feature Extraction	Db07, 09
	JCCE	Kim *et al.* (2004)	Feature Extraction	Db02
	JCCE	Shin and Hryciw (2004)	Feature Extraction	Haar
Survey	JSVE	Satirapod *et al.* (2003)	Denoising	-
	JSVE	Ogaja *et al.* (2003)	Discontinuity Detection	Haar
	JCCE	Wu *et al.* (2002)	Data Compression	Haar

Note: JTE=*Journal of Transportation Engineering*; JHLE=*Journal of Hydraulic Engineering*; JHE= *Journal of Hydrologic Engineering*; JCCE=*Journal of Computing in Civil Engineering*; JSVE=*Journal of Surveying Engineering*; JWPCOE=*Journal of Waterway, Port, Coastal, and Ocean Engineering*; JH=*Journal of Hydrology* approach was conducted for a response spectral matching of two horizontal ground-motion components. Honda and Ahmed (2011) illustrated the use of the Morlet wavelet to amplify the time-frequency characteristics of ground motion for the development of feature indexes.

3.1.6 Data compression

Wavelet analysis, while not yet widely used in the field of structural engineering, obtained some exposure in data compression analysis (Zhang and Li, 2006). This exposure used the Haar wavelet as a mother wavelet to minimize the computational power required. Mizuno *et al.* (2008) also applied the Haar wavelet to transient waves from vibration responses for fast damage detection in civil structures. The thresholding and quantization process was able to compress the raw data into what was referred to as 'signature.'

3.2 Transportation Engineering

Table 3 lists the seven articles describing wavelet transform application under transportation. In transportation engineering, wavelet transforms have been predominantly used to attenuate undesirable fluctuations or enhance desirable features in traffic data as a preprocessing method (Adeli and Karim, 2000; Karim and Adeli, 2002; Karim and Adeli, 2003). In these studies, it was difficult to accurately distinguish between outliers and traffic incidents in traffic flow data; traffic incidents are uniquely nonrecurrent and pseudorandom. Moreover, noise should be effectively removed in traffic data in order to detect traffic incidents with accuracy. Attempts were made to solve these problems by using the 4^{th} order Daubechies wavelet.

Wei *et al.* (2005) utilized wavelet analysis to detect physical phenomenon such as pavement surface distress, not in the frequency-time domains, but in the frequency-distance domain using the 5^{th} order Daubechies wavelet. This study calculated wavelet energies each year for the same road section for trends of pavement roughness deterioration. As a result, detailed roughness features of interest could be obtained together with summary roughness statistics.

Adeli and Ghosh-Dastidar (2004) employed a wavelet transform to directly extract features from traffic flow data using the Coifman wavelet. Their study used the orthogonal wavelets as filters to enable a scale-invariant interpretation of the traffic flow data. By comparison, Jiang and Adeli (2005) selected the Mexican hat wavelet (non-orthogonal differentiable wavelet) to extract traffic flow features to forecast traffic flow using mathematical reason; their approach required derivatives of the wavelet function.

3.3 Water Engineering

Table 3 lists the 19 articles describing wavelet transform application in water engineering. In the field of water engineering, one of the most widely used mother wavelets is the Morlet's wavelet. According to Massel's (2001) proof, the Morlet's wavelet is suitable for processing ocean wave data due to its oscillatory nature. Stamos and Hajj (2001) employed the Morlet wavelet to separate the two parts of a wave record–incident and reflected wave components. Mwale *et al.* (2007) and Mwale and Gan (2011) also used the Morlet wavelet to extract non-stationary characteristics in regional rainfall and runoff data. Ma *et al.* (2010) adopted the same mother wavelet to identify wave characteristics using the wavelet power spectrum, where the magnitude of the wavelet coefficients represented the surface displacement. Mwale *et al.* (2011) illustrated their use of wavelet transform to find the dominant spatial and temporal features of a streamflow time series; the structure of the Morlet wavelet resembles the measured data. Özger (2011) developed a wavelet-based method to detect the temporal characteristics of a fluctuating significant wave height time series. Duvert *et al.* (2015) demonstrated the usefulness of the Morlet's wavelet to assess the relationships between different climatic and hydrological records. Fang *et al.* (2015) applied the Morlet's wavelet to spatio-temporal validation of long-term hydrological simulations of a forested catchment. Yu and Lin (2015) and used the same mother wavelet to analyze the space-time non-stationary patterns of rainfall-groundwater interactions.

Other mother wavelets were also used for the specific research purposes of each study. Addison *et al.* (2001) illustrated their use of wavelet transform to identify flow features in fluid velocity time signals using the Mexican hat (continuous) wavelet and Daubechies (discrete) wavelet. The redundant nature of the continuous wavelet can be effective in feature extraction, whereas the zero redundancy of the discrete wavelet transform can be effective in a statistical analysis of the signal. Woo and Liu (2004) used wavelet transform to analyze transient features of harbor resonance problems such as wave energy trapping and amplification in a semienclosed water body.

Another application of the wavelet transform was the detection of discrete leakages or blockages in pressure signals using the Daubechies wavelet (Ferrante *et al.,* 2007; Sattar *et al.*, 2009). Goring (2008) and Kişi (2009) applied wavelet transform to denoise a signal obtained by the sensors. Kim and Valdeś (2003) illustrated the use of wavelet transform to capture helpful information on various resolution levels for drought forecasting.

In the field of hydrology, use of wavelet transform in conjunction artificial intelligence (AI) has gained momentum due to its apt capability to model complex phenomena of hydrology. Nourani *et al.* (2014) conducted a thorough review on hybrid models that combine wavelet and AI for hydrologic applications. They chose six hydro-climatologic applications of wavelet-AI models, such as precipitation modeling, flow forecasting, rainfall-runoff modeling, sediment modeling, groundwater modeling, and other hydro-climatologic applications, and found the advantages and potential of the hybrid approach in hydrology. In a similar line of study, Badrzadeh *et al.* (2015), Shoaib *et al*. (2015), and Nourani *et al.* (2015) combined wavelet transform with certain types of AI. Badrzadeh *et al.* (2015) suggested wavelet-based AI models such as wavelet neural networks and hybrid adaptive neuro-fuzzy inference systems for real time runoff forecasting. They confirmed that the proposed models with the use of Daubechies order three (db3) wavelet outperform the traditional neural network or neuro-fuzzy approaches. Shoaib *et al.* (2015) used gene expression programming with wavelet transform to forecast the runoff. They concluded that the hybrid wavelet gene expression programming approach performs well with the discrete approximation of meyer wavelet. Nourani *et al.* (2015) presented a wavelet transform-feed forward neural network model for groundwater level modeling. They used Daubechies-4 (db4) mother wavelet to capture the features of ground water level time series.

3.4 Other Civil Engineering Fields

Table 3 lists the six articles describing wavelet transform application in other civil engineering fields. In geotechnical engineering, wavelet transform has been used to extract the geometric properties of soil or aggregate particles from images. Shin and Hryciw (2004) used wavelet transform to determine the average grain size from images of fairly uniform particle size soil masses using the Haar wavelet. Kim *et al.* (2004) applied wavelet transform to stone aggregates images to extract features indicative of the material gradation using the Daubechies wavelet. Chandan *et al*. (2004) conducted image analysis to characterize the texture, angularity, and form of aggregate particles using a wavelet transform with the Daubechies wavelet.

In surveying engineering, Satirapod *et al*. (2003) described the use of wavelet transform when extracting bias components at high frequencies from Global Positioning System (GPS) measurements in order to attenuate the noise. Ogaja *et al.* (2003) explained their use of wavelet transform to make enlarged significant peaks and reduced random noise from GPS measurements using the Haar wavelet. Wu *et al.* (2002) illustrated the use of wavelet transform in minimizing the computational power required while maximizing the operation speed.

4. Lessons Learned and Implications for Future Research

Although the 64 papers reviewed were a small portion of the growing field of wavelet application in civil engineering, it could play the role of statistically meaningful sample for the whole population. Based on the review of the 64 papers and wavelet property theory, the following recommendation was derived for proper selection of mother wavelet.

4.1 Choice of Wavelets

Wavelet analysis has proven to be an extremely powerful tool for extracting information from civil engineering data. However, the selection of mother wavelets for wavelet analyses of particular signal is still difficult. For example, if a study attempts to use the wavelet transform for data compression with a certain wavelet basis, the choice of the wavelet forces a tradeoff between compression rate and computational speed. Thus, each study constitutes a different specific context for the wavelet transform. Therefore, there is no universal recommendation as to which type of wavelet should be used that will apply to all studies. However, a certain framework for wavelet selection can be developed based on important factors and the application purposes. Therefore, this study attempted to provide guidelines for the selection of wavelets for particular applications using the classification proposed in this paper. Fig. 1 illustrates the guideline linking "application purposes" to "factors to consider" and to "suggested wavelets" in civil engineering. The suggested mother wavelets in Fig. 1 do not mean an impeccable solution to every civil engineering problem, but it could easily be one of the optimal bases for successful wavelet application.

4.1.1 Denoising

Some coefficients expressed as zero or close to zero can be ignored if they are lower than a particular threshold value as long as they do not affect the main features of the data. Denoising can therefore be conducted without changing important features of the original data. Almost all of the studies reviewed here employed the Daubechies family to select mother wavelets for denoising. The 4th order Daubechies was mainly used for non-stationary signals (Adeli and Karim, 2000; Karim and Adeli, 2002; Karim and Adeli, 2003; Adeli and Kim, 2004; Adeli and Jiang, 2006; Goring, 2008; Jiang and Mahadevan, 2008), whereas the 40th order was used for stationary signals (Rizzo *et al.,* 2005). Therefore, it is recommended that the lower orders of

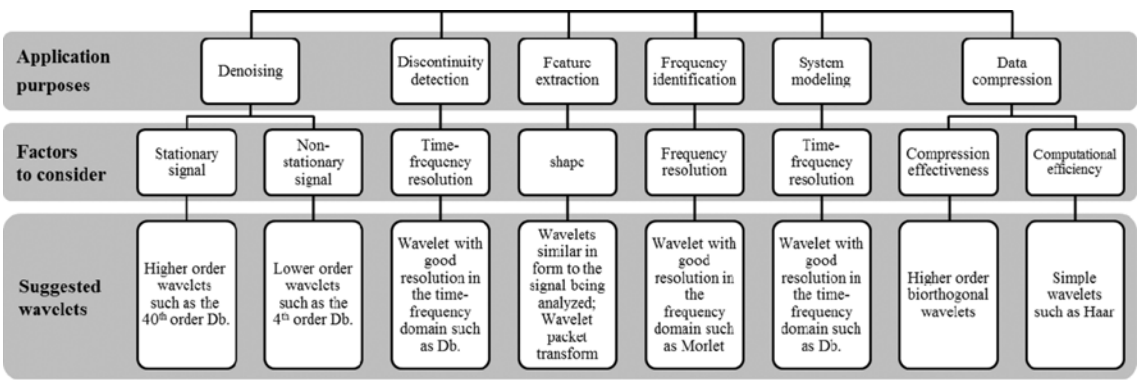

Fig. 1. Guidelines for Wavelet Selection.

wavelets, such as the 4th order Daubechies, seem to be much more effective for non-stationary signals, while the higher order wavelets, such as the 40th order Daubechies, are suitable for stationary signals (Abdullah *et al.,* 2008).

4.1.2 Discontinuity Detection and System Modeling

The majority of studies employed the Daubechies family for discontinuity detection, which allows well-balanced resolution in both time and frequency domains (Hou *et al*., 2000; Melhem and Kim, 2003; Hera and Hou, 2004; Wei *et al*., 2005; Ferrante *et al*., 2007). The Daubechies family was also selected as the mother wavelet for applications of system modeling due to its mathematically exact time-localization properties (Kitada, 1998; AL-Qassab and Nair, 2003; Law *et al.,* 2006; Basu and Nagarajaiah, 2010). It must be emphasized that the trade-off between time resolution and frequency resolution is well-balanced in the use of wavelets for discontinuity detection & system modeling. In this regard, the Daubechies family is one of the suitable bases, with an emphasis on both domains.

4.1.3 Frequency Identification

The continuous wavelet transform has been widely used for frequency identification. Unlike the discrete wavelet transform, the continuous wavelet transform can change the location and dilation of the mother wavelet in a smooth rather than dyadic manner. This enables the extraction of redundant but rich information from the signal of interest, which makes it well suited for frequency identification.

In frequency identification, the dominant choice of the studies reviewed was the Morlet wavelet, which is known to have a good frequency resolution, although with a less precise temporal resolution (Yoon *et al.*, 2000; Gurley *et al.*, 2003; Kijewski-Correa and Kareem, 2007; Ji and Chang, 2008; Ma *et al*., 2010; Mwale *et al.*, 2011; Duvert *et al.*, 2015; Fang *et al*., 2015; Yu and Lin, 2015). Therefore, wavelets with good resolution in the frequency domain, such as the Morlet, should be used for frequency identification applications.

4.1.4 Feature Extraction

Feature extraction was addressed in the largest number of articles reviewed in this study, approximately 29 percent. Various kinds of mother wavelets were selected because they matched the shapes of the measured signals for feature extraction (Torrence and Compo, 1998). An empirical example is the use of the Morlet wavelet for a wide variety of purposes in water engineering due to the similarity between the shape of the mother wavelet and the shape of the measured signals. It is also worth noting that wavelet transform tends to work well with artificial intelligence in its role of feature extraction (Nourani *et al.,* 2014; Badrzadeh *et al*., 2015; Nourani *et al*., 2015; Shoaib *et al.,* 2015).

Another interesting finding in the feature extraction application is the use of the wavelet packet transform. Sun and Chang (2002), Sun and Chang (2004), and Ren *et al.* (2008) adopted the wavelet packet transform to capture high-quality features representative of the raw data. The packet transform allows users to select the optimum decomposition for their particular applications; this flexibility can easily lead to better features.

4.1.5 Data Compression

For data compression applications, the proper selection of mother wavelets requires two main factors: computational efficiency and compression effectiveness. Computational efficiency is the computational speed or computational power required, with the former inversely proportional to the latter. Compression effectiveness is a concept that combines the compression rate (ratio of the compressed data amount to the original data amount) and compression quality (the level of similarity between the compressed data and the original data). Simple wavelet bases can achieve high computational efficiency, but may only achieve low levels of compression effectiveness. Wu *et al.* (2002), Zhang and Li (2006), and Mizuno *et al.* (2008) chose the Haar wavelet as the optimum wavelet basis because their application required fast data processing. However, in cases where compression effectiveness is more important, higher order biorthogonal wavelets are recommended. The symmetric nature of the biorthogonal wavelets are also known to reduce the boundary artifacts.

4.2 Limitations of Wavelet-Based Applications in Civil Engineering

Even though the wavelet transform is a powerful tool for signal processing, several studies have outlined its drawbacks compared to other signal processing methodologies. Fenton (1999) developed a stochastic model to represent a soil property using a variety of data transform methods, including periodogram and wavelet based approaches. They demonstrated the same abilities for discerning between finite-scale and fractal process but the wavelet transform was inferior to the periodogram approach because it required unnecessarily complicated analysis. Kijewski-Correa and Kareem (2006) conducted a comparative study of Hilbert and wavelet transforms for a time-frequency analysis. For the non-stationary and nonlinear signals, the wavelet transform detected supercyclic frequency modulations. Conversely, the Hilbert transform captured subcyclic information, as well as supercyclic frequency characteristics. To provide additional subcyclic information, the wavelet transform must rely on the instantaneous bandwidth of a signal. Liang *et al.* (2007) estimated the evolutionary power spectral density function from a ground motion time-history record using three different transforms: Short-time Fourier Transform (STFT), wavelet transform, and Hilbert-Huang Transform (HHT). The first two methods represented the energy-time-frequency distribution using analyzing functions, whereas the HHT replaced the evolutionary response spectrum with an adaptive representation. The spectra were well estimated by all the methods. However, a comparison of the results showed that the HHT gave more concentrated energy when compared with those given by the STFT and the wavelet transform.

5. Conclusions

This study explored the wavelet transform and its uses in civil engineering. A total of 64 wavelet transform articles were reviewed in this paper. The emphasis was put on the uses of the wavelet transform and the choices of mother wavelets for each purpose.

To demonstrate the practical uses of the wavelet transform, the articles were classified into six categories based on the use of the wavelet transform: denoising, discontinuity detection, feature extraction, frequency identification, system modeling, and data compression. The articles were also classified by the mother wavelet used in each study. One of the most extensively used mother wavelets was the Daubechies wavelet family, because their computational features are recursive, mutually orthogonal, and compactly supported. The articles were also categorized by the specific fields of civil engineering, including: structural, transportation, water resources, and geotechnical engineering. In the field of water engineering, the great majority of civil engineering applications of wavelet transform was based on the Morlet's wavelet, due to its resemblance to the measured data.

A survey of literature shows that wavelet transform has the potential to address a range of problems in civil engineering from a mathematical point of view. The need to deal with extracting characteristics in measured data for signal processing was the fundamental justification for the use of wavelet transform. The specific advantages of using the wavelet transform are sixfold: 1) to attenuate white noise in a signal; 2) to detect discontinuities in a signal; 3) to derive mid-level data, which are fed into a reasonably complex decision making process; 4) to identify the major frequency components after obtaining the wavelet-domain information; 5) to reduce the size of the original data; and 6) to use wavelets as a component for system predictions. This study also provided a guideline for the proper selection of a wavelet for a particular application using the purpose classification outlined.

Further research is necessary to bring the level of wavelet applications in civil engineering to the next stage. Although the wavelet community has developed more advanced techniques, such as complex wavelets and framelets, for the last decade, not much effort have been made to utilize those tools in the civil engineering community, as move that would provide opportunities for improved analyses of civil engineering data.

Acknowledgements

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP; Ministry of Science, ICT & Future Planning) (NRF-2014R1A2A1A11052499, No.2011-0030040, and NRF-2016R1C1B1009711).

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