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A Simplified Solution for Stresses Around Lined Pressure Tunnels Considering Non-radial Symmetrical Seepage Flow

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Abstract

In this paper, an analytical method, for calculating seepage induced stresses and displacements in underwater lined circular pressure tunnels, is developed on the basis of a generalized effective stress law. The problem is considered as axisymmetric, and the lining and rock mass are assumed to be elastic, homogeneous and isotropic. The solution accounts for the seepage forces with the steady-state flow and hydro-mechanical pressures between adjacent zones. The proposed method can be applied for the analysis and design of pressure tunnels with concrete lining, prestressed concrete lining, grouted rock mass, as well as for the analysis of pressure tunnels considering the effects of the surrounding fractured or damaged zone. Illustrative examples are given to demonstrate the performance of the proposed method, and also to examine the effect of seepage forces on stability of pressure tunnels. It is concluded that, the classic solutions (Lame's solution), that is based on considering the internal pressure as a mechanical load applied to the tunnel surface, is not applicable to pervious media and results in unsafe designs.

Keywords: *axisymmetric problem, effective stress, excavation damaged zone, lined pressure tunnel, seepage forces*

1. Introduction

Pressure tunnels are excavated as conduits which convey pressurized water. They may be either unlined or provided with pervious or impervious lining systems. In an impervious lined pressure tunnel, water pressure is applied as a direct load, uniformly distributed and perpendicular to the tunnel surface. On the other hand, in unlined and pervious lined pressure tunnels, water infiltrates cavities (pores, cracks, fissures, and so on) and the resulted seepage forces act on the lining and each zone of the rock mass. In these tunnels, as a result of induced stress concentrations by loads from the ground or induced disturbances by Tunnel Boring Machine (TBM) or blasting during the excavation stage (Brown and Bray, 1982; Kelsall *et al*., 1984; Moore, 1989; Pusch, 1989; Bai *et al.*, 1999; Tsang *et al.*, 2005; Fahimifar and Zareifard, 2009; Saiang and Nordlund, 2009; Fernandez and Moon, 2010; Butscher *et al*., 2011a, b) or hydrofracturing phenomenon during the operation stage (Schleiss, 1986; Deere and Lombardi, 1989), a damaged zone with altered hydro-mechanical characteristics may be developed in the surrounding rock mass. This may result in excessive leakage from the tunnel, excessive pore pressure or increasing the portions of loads carried by the lining (thus, increasing the possibility of cracks development) or other undesirable effects. In this regard, grouting is specified as means of stiffening and decreasing permeability of the surrounding rock mass. The grout must fill, on the one hand, the gap between the lining and the rock, and on the other hand, the fractures and large pores in the rock mass. Furthermore, grouting can be used as a passive prestressing, individually or in combination with active prestressing produced by tendons, for avoiding cracks development in the lining. Therefore, around a pressure tunnel, there may be zones with different mechanical and hydraulic properties (the lining, the unaltered zone and the altered zone including the grouted or damaged zones). At the boundary between these zones, hydraulic and mechanical boundary pressures will be applied.

Despite the engineering advances, the analysis of pressure tunnels is usually based on considering the internal pressure as a mechanical load. Several reports of unsatisfying pressure tunnels show that in most cases, failures have occurred during the first water filling and operation or shortly afterward, because either a fundamental mode of failure had not been recognized at the design stage, or the design had not been carried out based on a powerful analysis.

In this regard, by using numerical methods, implemented in computer programs, which model the flow of fluid (e.g., groundwater) through the pervious media, geotechnical structures, such as pressure tunnels, can be analyzed hydro-mechanically. In these methods, all complexities can be covered, ideally. However, the numerical analysis of pressure tunnels is a relatively laborious and complicated process, and does not easily permit designers to identify key variables controlling the tunnel

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behavior, as well as, the sensitivity of the tunnel behavior to one or several of these key variables.

No closed-form analytical solutions exist that include the full complexity of such a problem.

Analytical solutions of simpler cases, if found, are preferred for the preliminary design or controlling the final design of pressure tunnels and have the potential of providing insight into the problem. For instance, the analysis of a circular pressure tunnel excavated in an elastic, homogeneous and isotropic rock mass, under simplified seepage flows and boundary conditions provide the means to obtain estimates of stresses and strains in the lining and the rock mass, quickly. Analytical solutions, however, are limited because they usually require a number of assumptions and simplifications that often apply to the problems with limited practical interest. Nevertheless, the advantages of having a closed-form solution often outweigh the limitations.

The effect of seepage forces on mechanical responses of traditional tunnels has investigated by a number of researchers (Brown and Bray, 1982; Lee and Nam, 2001; Fahimifar and Zareifard, 2009, 2014; Carranza-Torres and Zhao, 2009; Bobet, 2010; Shin *et al.,* 2010, 2011). On the other hand, limited effort has been dedicated to the analysis of pressure tunnels under seepage forces. Analytical solutions for the problem of pressure tunnels presented by Bouvard and Pinto (1969), Brown and Bray (1982); Schleiss (1986); Fernandez and Alvarez (1994); Fernandez (1994); Schleiss (1997); Bobet and Nam (2007) and Fahimifar and Zareifard (2013) reveal that the calculation of stresses and strains in the lining and the rock mass must be carried out by considering the seepage forces.

In this paper, a simplified closed-form analytical solution for lined underwater pressure tunnels is presented. The model consists of three zones, which interact with each other through boundary mechanical and hydraulic pressures: a. lining, b. altered ground, and c. unaltered ground.

For the development of the new solution the following assumptions are made: (1) the lining and the altered zone are hollow thick-walled cylinders; (2) the tunnel is deep enough below the *ground surface*; (3) the axisymmetric conditions are utilized.

2. Problem Description

It is generally recognized that, the pore pressure has different effects on deformations, and initiation of cracks and fractures (Terzaghi, 1923; Skempton, 1961; Garg and Nur, 1973 Jaeger and Cook, 1979; Paterson and Wong, 2005). Both the theoretical analyses and the experimental observations show that, provided that the rock or concrete contain connected system of pores, the initiation of a crack or fracture is controlled by the Terzaghi effective stress σ', defined as:

$$\sigma' = \sigma - p_w \tag{1}$$

On the other hand, deformations are controlled by the Biot effective stress σ'' as:

$$\sigma'' = \sigma - \beta p_w \tag{2}$$

where σ is the induced total stress; p_w is the induced pore water pressure, and β is the Biot-Willis coupling poroelastic constant (by convention, compressive stress is positive). The Biot-Willis coupling poroelastic constant β depends on the bulk modulus of both solid matrix and solid grain material, as described by the following equation (Biot and Willis, 1941; Nur and Byerlee, 1971).

$$\beta = 1 - \frac{K}{K_s} \tag{3}$$

where K and K_s are the bulk modulus of the matrix material and the solid constituent, respectively. In a saturated porous rock, the value of may be expected as low as 0.5 (Skempton, 1961). The Biot coefficient can be estimated from laboratory tests. Different researchers have attempted to obtain β values for different rocks (Terzaghi, 1923; Skempton, 1961; Garg and Nur, 1973; Berryman, 1993 ; Gurevich, 2004; Paterson and Wong, 2005). For instance, Berryman (1993) derived effective stress coefficients β for various rocks composed of a number of mineral constituents.

In the polar coordinates (r, θ), the induced stress field for each element of the ground and lining (see Fig. 1) has to fulfill the equilibrium equations as (Timoshenko and Goodier, 1982):

$$\frac{\partial \sigma_r''}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta r}}{\partial \theta} - \frac{\sigma_\theta'' - \sigma_r''}{r} + F_r = 0 \tag{4}$$

$$\frac{\partial \sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta''}{\partial \theta} + \frac{\sigma_{r\theta} + \sigma_{\theta r}}{r} + F_\theta = 0 \tag{5}$$

where F_r and F_θ are the seepage body forces in the radial, and circumferential directions, respectively, and are given as

$$F_r = \beta \frac{\partial p(r, \theta)}{\partial r} \tag{6}$$

$$F_\theta = \frac{\beta \partial p_{w(r,\theta)}}{r \, \partial \theta} \tag{7}$$

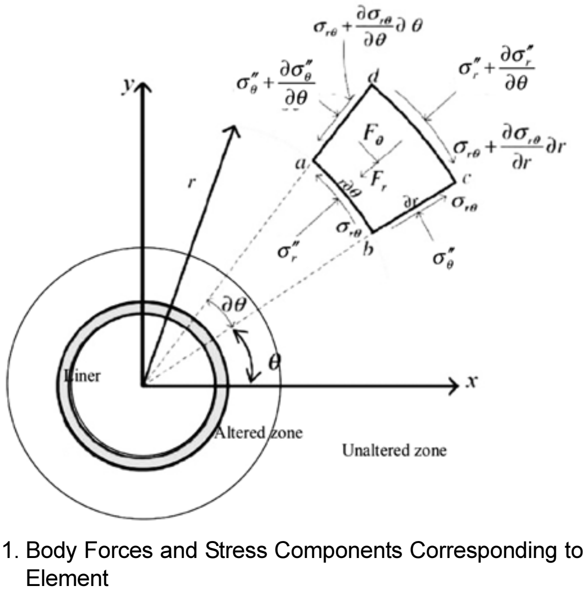

Fig. 1. Body Forces and Stress Components Corresponding to an Element

where $p_{w(r,\theta)}$ is the induced pore pressure at (r, θ), and β is the Biot-Willis coupling poroelastic constant.

Obtaining an analytical solution for the problem is impossible without using appropriate simplifying assumptions. For instance, in the majority of solutions for tunnels in high or low groundwater tables, the governing equilibrium equation is solved utilizing the radial flow pattern with a specific radius of seepage affected zone (Bouvard and Pinto, 1969; Brown and Bray, 1982; Schleiss, 1986-1997). In such radial-flow-based solutions, the variations of pore pressure in different directions are ignored. This assumption may be only applicable for tunnels in high *groundwater table*, namely $h_1 \gg r_o$ (h_1 is depth of the tunnel from groundwater table) . Therefore, the use of radial flow pattern for tunnels at *low depth* below groundwater surface is open to question. However, the radial-flow-based solutions are practically used for the analysis of tunnels in both high and low *groundwater levels*.

For instance in the radial-flow-based solutions presented by Schleiss (1986-1997) the distance of the tunnel from groundwater level is considered as the radius of seepage affected zone. In these solutions, the internal radius (the tunnel perimeter), the external radius (the radius of the seepage affected zone = h_1) and the induced pore pressures at both radii are the controlling variables for the calculation of the seepage pressure and the acting mechanical boundary pressure at the interface between the different zones. This has a slight influence on the seepage losses out of the tunnel calculated by Schleiss even if for its calculation the non-radial symmetrical solution is used.

In the present work, the analyses are performed for each direction based on the axisymmetric conditions; thus, the stress state at a distance r is defined by the radial stress σ_r and the circumferential stress σ_θ. Therefore, the term $\frac{1}{r}\frac{\partial \sigma_{\theta r}}{\partial \theta}$ will be cancelled from Eq. (4) and the radial components of the seepage force are only taken into account. Then, the circumferential components are neglected. It means that the analysis through each direction can be carried out independently of other directions. By conducting numerical analyses for cases of tunnels in high and low *groundwater levels*, Fernandez and Alvarez (1994) and Fahimifar and Zareifard (2012) showed that the shear stresses

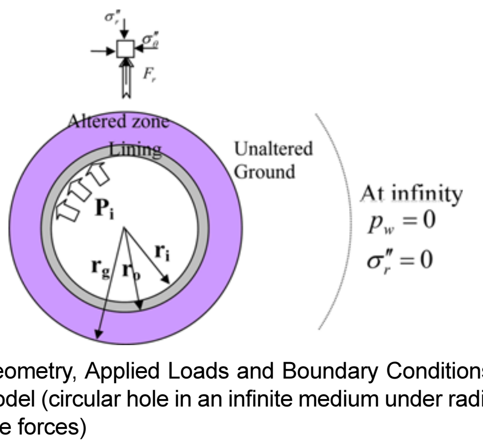

Fig. 2. Geometry, Applied Loads and Boundary Conditions for the Model (circular hole in an infinite medium under radial seepage forces)

σ_{θ_r} are negligible, in contrast to σ_r'' and σ_θ'' , even for tunnels in very low groundwater levels.

Thus, the solution for each direction simplifies to an axisymmetric solution which only depends on the seepage forces through this direction independently of other directions. The resulting model is shown in Fig. 2.

In this way, the system of partial differential equations governing the stresses and strains in each zone will be reduced to an ordinary differential equation as

$$\frac{d\sigma_r''}{dr} - \frac{\sigma_\theta'' - \sigma_r''}{r} + F_r = 0 \tag{8}$$

$$\varepsilon_\theta = \frac{u_r}{r} \tag{9}$$

$$\varepsilon_r = \frac{du_r}{dr} \tag{10}$$

where F_r is the applied radial seepage force, and it depends on the pore pressure gradient through the considering direction, u_r is the radial component of displacement; and ε_θ and ε_r are the circumferential and radial strains, respectively.

The relationships between the induced strains, ε_r and ε_θ, and induced Biot effective stresses σ_r'' and σ_θ'' in each zone are given by Hooke's law for the plane-strain condition (Timoshenko and Goodier, 1982):

$$\sigma_r'' = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_r + \nu\varepsilon_\theta] \tag{11}$$

$$\sigma_\theta'' = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_\theta + \nu\varepsilon_r] \tag{12}$$

where and E and ν are elasticity modulus and Poisson' s ratio of the considering zone, respectively.

Equilibrium Eq. (8), strain-displacement Eqs (9) and (10), and Hooke's law, i.e. Eqs. (11) and (12), must be satisfied for all zones. In this manner, substituting Eqs. (11) and (12) into Eq. (8) and applying Eqs. (9) and (10) gives the following equation for the unknown induced radial displacement u_r.

$$\frac{u_r}{r^2} - \frac{1}{r}\frac{du_r}{dr} - \frac{d^2u_r}{dr^2} = \beta\frac{dp_w}{dr}\frac{(1+\nu)(1-2\nu)}{E(1-\nu)} \tag{13}$$

where u_r is the induced radial displacement, and p_w is the induced pore pressure at the radial distance, r, and E and ν are elasticity modulus and Poisson's ratio of the considering zone, respectively.

In Eq. (13), the induced pore pressure gradient through the considering direction is utilized, as mentioned previously.

The seepage flow in a medium with homogeneous and isotropic permeability can be formulated in terms of a function of complex variables named characteristic function, in which its real part represents the equipotential lines, and its imaginary part represents the flow lines. In the case of a pressurized cylinder, at a constant hydraulic head, beneath a horizontal groundwater table in a homogeneous and isotropic permeable medium, Kolymbas and Wagner (2006) derived the characteristic function of the non-radial seepage flow. In the proposed solution, the pore pressure $p_{w(r,\theta)}$ induced by the applied internal pressure P_i in the unaltered zone is estimated using the flow pattern proposed by Kolymbas and Wagner (2006) as.

$$p_{w(r,\theta)} = \frac{\gamma_w q}{2\pi k_r}\log\left(\frac{c^2r^2\cos^2\theta+(cr\sin\theta-c^2r_g)^2}{c^2r^2\cos^2\theta+(cr\sin\theta-r_g)^2}\right) \quad r\geq r_g \tag{14}$$

$$c = \frac{h_1-\sqrt{h_1^2-r_g^2}}{r_g} \tag{15}$$

where q is seepage flow rate, k_r is the permeability of the unaltered rock mass, r_g is the radius of the altered zone, h_1 is depth of the tunnel from the *groundwater surface*.

On the other hand, the radial flow is utilized for estimation of the induced pore pressure in the lining and the altered zone. Thus, using Darcy's equation, pore pressures in the concrete lining and the altered zone are obtained from Eqs. (16) and (17), respectively.

$$p_{w(r,\theta)} = \frac{\gamma_w q}{2\pi k_c}\log\left(\frac{r}{r_i}\right)+p_{w(r_i)} \qquad r_i\leq r\leq r_o \tag{16}$$

$$p_{w(r,\theta)} = \frac{\gamma_w q}{2\pi k_g}\log\left(\frac{r}{r_o}\right)+p_{w(r_o)} \qquad r_o\leq r\leq r_g \tag{17}$$

where r_o and r_i are the external and internal radii of the lining, respectively, and k_g and k_c are the permeability of the altered zone and concrete lining, respectively. Furthermore, the induced pore pressure should satisfy the following boundary conditions at the internal radius of the lining and at an infinite radius, respectively:

$$p_{w(r_i,\theta)} = P_i-\gamma_w h_1 \tag{18}$$

$$p_{w(r=\infty,\theta=0)} = 0 \tag{19}$$

where $P_i = H_i\gamma_w$ is the final internal water pressure and H_i is the final internal water head.

At the interfaces between the lining and altered zone and between the altered and unaltered zones, the pore pressure calculated in the external zone must be identical to that in the internal zone, since, the pore pressure must be continuous over the boundary. Consequently, Eq. (14) must be equal to Eq. (17) at $r = r_g$, and Eq. (16) should be equal to Eq. (17) at $r = r_o$. The following equation for the seepage flow rate, q is derived from these continuity equations by taking into account the boundary conditions (18) and (19):

$$q = 2\pi\frac{p_i-\gamma_w h_i}{\gamma_w}\frac{1}{-\frac{1}{k_c}\log\left(\frac{r_o}{r_i}\right)-\frac{1}{k_g}\log\left(\frac{r_g}{r_o}\right)+\frac{1}{k_r}\log(c)} \tag{20}$$

The governing differential Eq. (13) is a linear equation; thus, the principle of superposition can be applied to solve it for each zone.

For the saturated region, using the superposition concept, the total stresses, strains and displacements are divided into the following components:

a. Components Corresponding to the Induced Seepage Forces.
b. Components corresponding to the induced boundary pressures.

3. Induced Stresses and Strains Corresponding to the Seepage Forces

3.1 Unaltered Zone

In this section, the solution is presented for pore pressure distribution through the horizontal direction, as an example.

Solving the differential Eq. (13) using the induced pore pressure gradient through the horizontal direction $\frac{dp_{w(\theta=0)}}{dr}$, gives analytical expressions for the induced circumferential strain $\varepsilon^{SE}_{\theta(r)}$, and the induced radial strain $\varepsilon^{SE}_{r(r)}$, corresponding to the seepage forces at any radius r (Fahimifar and Zareifard, 2012) :

$$\varepsilon^{SE}_{\theta(r)} = C_1+\frac{1}{r^2}C_2 - \frac{1}{4}\frac{(1+\nu_r)(1-2\nu_r)}{E_r c^2 r^2(1-\nu_r)}\frac{\beta_r p_{w(r_g)}}{\log(c)}\left[\begin{array}{l} r_g^2c^4\left(\log\left(r^2+c^2r_g^2\right)-1\right) \\ -r_g^2\left(\log\left(r_g^2+c^2r^2\right)-1\right) \\ +c^2r^2\log\left(\frac{r^2+c^2r_g^2}{c^2r^2+r_g^2}\right)\end{array}\right] \tag{21}$$

$$\varepsilon^{SE}_{r(r)} = C_1-\frac{1}{r^2}C_2 - \frac{1}{4}\frac{(1+\nu_r)(1-2\nu_r)}{E_r c^2 r^2(1-\nu_r)}\frac{\beta_r p_{w(r_g)}}{\log(c)}\left[\begin{array}{l} \left(2r^2c^2-r^2c^2\right)\log\left(\frac{r^2+c^2r_g^2}{c^2r^2+r_g^2}\right) \\ +r_g^2\log\left(c^2r^2+r_g^2\right) \\ -r_g^2c^4\log\left(r^2+c^2r_g^2\right) \\ \frac{2r^2r_g^2c^4}{r^2+c^2r_g^2}+\frac{2r^2r_g^2c^2}{c^2r^2+r_g^2} \\ +r_g^2\left(c^4-1\right) \\ +2r^3c^2\left(\frac{2r}{r^2+c^2r_g^2}-\frac{2c^2r}{c^2r^2+r_g^2}\right)\end{array}\right] \tag{22, 23}$$

$$C_1 = -\frac{(1+\nu_r)(1-2\nu_r)}{2E_r(1-\nu_r)}\beta_r p_{w(r_g)}$$

$$C_2 = -\frac{r_g^2(1+\nu_r)}{4E_r c^2(1-\nu_r)}\frac{p_{w(r_g)}}{\log(c)}\left[\begin{array}{l}\beta_r(1-2\nu_r)(c^4-1)\log(r_g^2(1+c^2)) \\ +\log(c)c^2(\beta_r+2(1-\beta_r)(1-\nu_r))\end{array}\right] \tag{24}$$

where c is obtained from Eq. (15).

In this paper, superscript SE refers to the quantities corresponding to the seepage forces, subscript r refers to quantities corresponding to the unaltered rock mass, subscript g refers to quantities corresponding to the altered rock mass, and subscript c refers to quantities corresponding to the concrete lining.

Using Eqs. (21), (22), (11), (12), (1) and (2) gives the values of the strains and stresses corresponding to the seepage forces, i.e:, $\varepsilon^{SE}_{\theta(r)}$, $\varepsilon^{SE}_{r(r)}$, $\sigma''^{SE}_{r(r)}$, $\sigma''^{SE}_{\theta(r)}$, $\sigma'^{SE}_{r(r)}$ and $\sigma'^{SE}_{\theta(r)}$, respectively.

The analytical expressions for the induced circumferential and radial strains corresponding to the seepage forces through the other directions can be found in Fahimifar and Zareifard (2012). It should be notified that for directions within the limits of

$(0^{\circ} < \theta < 180^{\circ})$, there exist both dry and saturated zones. In this case, the boundary pressures between theses zones should be considered.

3.2 Altered Zone

Solving the differential Eq. (13) using the induced pore pressure gradient obtained from Eq. (17), gives the induced circumferential strain $\varepsilon_{\theta(r)}^{SE}$, and the induced radial strain $\varepsilon_{r(r)}^{SE}$, corresponding to the seepage forces at any radius r:

$$\varepsilon_{\theta(r)}^{SE} = C_1' + \frac{1}{r^2}C_2' - \frac{1}{4}\frac{\left(p_{w(r_g)} - p_{w(r_o)}\right)\left(1-2\nu_g\right)\left(1+\nu_g\right)\left(2\log(r)-1\right)\beta_g}{\log\left(\frac{r_g}{r_o}\right)E_g\left(1-\nu_g\right)} \tag{25}$$

$$\varepsilon_{r(r)}^{SE} = C_1' - \frac{1}{r^2}C_2' - \frac{1}{4}\frac{\left(p_{w(r_g)} - p_{w(r_o)}\right)\left(1-2\nu_g\right)\left(1+\nu_g\right)\left(2\log(r)+1\right)\beta_g}{\log\left(\frac{r_g}{r_o}\right)E_g\left(1-\nu_g\right)} \tag{26}$$

where C_1' and C_2' are integration constants, and are obtained from the boundary conditions $\sigma''^{SE}_{r(r_o)} = (1-\beta_g)p_w(r_o)$ and $\sigma''^{SE}_{r(r_g)} = (1-\beta_g)p_w(r_g)$ as follows:

$$C_1' = \frac{1}{4}\frac{(1-2\nu_g)(1-\nu_g)}{(r_g^2-r_o^2)E_g(1-\nu_g)}\left(\begin{array}{c}\dfrac{(p_{w(r_g)}-p_{w(r_o)})(r_g^2-r_o^2)(1-2\nu_g)\beta_g}{\log\left(\frac{r_g}{r_o}\right)} \\ 2\beta_g(p_{w(r_g)}-p_{w(r_o)})r_g^2 \\ +4(1-\nu_g)(1-\beta_g)(p_{w(r_g)}r_g^2-p_{w(r_o)}r_o^2)\end{array}\right) \tag{27}$$

$$C_2' = \frac{1}{2}\frac{r_g^2 r_o^2(1+\nu_g)(p_{w(r_g)}-p_{w(r_o)})(2(1-\nu_g)(1-\beta_g)+\beta_g)}{E_g(r_g^2-r_o^2)(1-\nu_g)} \tag{28}$$

Similar equations are utilized for the analysis of the pervious lining, with the internal and external radii r_i and r_o; the internal and external induced pore pressures $p_{w(r_i)} = P_i - \gamma_w h_1$ and $p_{w(r_o)}$ and elasticity modulus E_c and Poisson's ratio ν_c.

The permeability and elasticity modulus of the altered zone can be variable. Derivation of the induced stresses and strains for this case is presented in Appendix A.

4. Induced Stresses and Strains Corresponding to the Boundary Pressures

4.1 Unaltered Ground

The components corresponding to the induced boundary pressure $\sigma_r'(r_g)$ applied to $r = r_g$, are obtained by solving Eq. (13), considering $\frac{dp_w}{dr} = 0$:

$$\varepsilon_{\theta(r)}^{BU} = -\frac{1+\nu_r}{E_r}\sigma'_{r(r_g)}\frac{r_g^2}{r^2} \tag{29}$$

$$\varepsilon_{r(r)}^{BU} = \frac{1+\nu_r}{E_r}\sigma'_{r(r_g)}\frac{r_g^2}{r^2} \tag{30}$$

$$\sigma''^{BU}_{\theta(r)} = -\sigma'_{r(r_g)}\frac{r_g^2}{r^2} \tag{31}$$

$$\sigma''^{BU}_{r(r)} = -\sigma'_{r(r_g)}\frac{r_g^2}{r^2} \tag{32}$$

where the superscript BU refers to the quantities corresponding to the boundary pressures.

4.2 Altered Ground

The components corresponding to the induced boundary pressures $\sigma'_{r(r_o)}$ applied to $r = r_o$, and $\sigma'_{r(r_g)}$ applied to $r = r_g$ by solving differential Eq. (13), considering $\frac{dp_w}{dr} = 0$:

$$\varepsilon_{\theta(r)}^{BU} = \frac{1+\nu_g}{E_g}\left(\sigma'_{r(r_g)} - \sigma'_{r(r_o)}\right)\frac{r_o^2}{r_g^2 - r_o^2}\left(\left(1-2\nu_g\right) + \frac{r_g^2}{r^2}\right) + \frac{1+\nu_g}{E_g}\sigma'_{r(r_o)}\left(1-2\nu_g\right) \tag{33}$$

$$\varepsilon_{r(r)}^{BU} = \frac{1+\nu_g}{E_g}\left(\sigma'_{r(r_g)} - \sigma'_{r(r_o)}\right)\frac{r_o^2}{r_g^2 - r_o^2}\left(\left(1-2\nu_g\right) - \frac{r_g^2}{r^2}\right) + \frac{1+\nu_g}{E_g}\sigma'_{r(r_o)}\left(1-2\nu_g\right) \tag{34}$$

$$\sigma''^{BU}_{\theta(r)} = \left(\sigma'_{r(r_g)} - \sigma'_{r(r_o)}\right)\frac{r_o^2}{r_g^2 - r_o^2}\left(1 + \frac{r_g^2}{r^2}\right) + \sigma'_{r(r_o)} \tag{35}$$

$$\sigma''^{BU}_{r(r)} = \left(\sigma'_{r(r_g)} - \sigma'_{r(r_o)}\right)\frac{r_o^2}{r_g^2 - r_o^2}\left(1 - \frac{r_g^2}{r^2}\right) + \sigma'_{r(r_o)} \tag{36}$$

Similar equations are utilized for the analysis of the pervious lining, with the internal and external radii r_i and r_o; the internal and external mechanical pressures $\sigma'_{r(r_i)} = 0$ and $\sigma'_{r(r_o)}$ and elasticity modulus E_c and Poisson's ratio ν_c.

The induced displacements, strains and stresses are obtained from the sum of two principal components as follows:

$$\varepsilon_{\theta(r)} = \varepsilon_{\theta(r)}^{SE} + \varepsilon_{\theta(r)}^{BU} \tag{37}$$

$$\varepsilon_{r(r)} = \varepsilon_{r(r)}^{SE} + \varepsilon_{r(r)}^{BU} \tag{38}$$

$$\sigma''_{\theta(r)} = \sigma''^{SE}_{\theta(r)} + \sigma''^{BU}_{\theta(r)} \tag{39}$$

$$\sigma''_{r(r)} = \sigma''^{SE}_{r(r)} + \sigma''^{BU}_{r(r)} \tag{40}$$

4.3 Boundary Pressures

At radius r_o, a mechanical boundary pressure $\sigma'_{r(r_o)}$ will be applied between the altered zone and the lining; and at radius r_g, a mechanical boundary pressure $\sigma'_{r(r_g)}$ will be applied between the altered zone and the unaltered rock masses. These boundary pressures are obtained from compatibility conditions at these radii, i.e., $\varepsilon_{\theta(r_o)}^{liner} = \varepsilon_{\theta(r_o)}^{altered}$, and $\varepsilon_{\theta(r_g)}^{altered} = \varepsilon_{\theta(r_g)}^{unaltered}$:

$$\sigma'_{r(r_g)} = \frac{\alpha_5\alpha_6 - \alpha_2\alpha_3}{\alpha_1\alpha_3 - \alpha_4\alpha_6} \tag{41}$$

And:

$$\sigma'_{r(r_o)} = \alpha_3\left(\alpha_1\sigma'_{r(r_g)} + \alpha_2\right) \tag{42}$$

$$\alpha_1 = r_o^2\left(\left(1+\nu_g\right)E_r - \left(1+\nu_r\right)E_g\right) - r_g^2\left(\left(1+\nu_g\right)\left(1-2\nu_g\right)E_r + \left(1+\nu_r\right)E_g\right) \tag{43}$$

$$\alpha_2 = E_g E_r\left(r_g^2 - r_o^2\right)\left(\varepsilon_{\theta(r_g)}^{SE(altered)} - \varepsilon_{\theta(r_g)}^{SE(unaltered)}\right) \tag{44}$$

$$\alpha_3 = \frac{1}{2E_r r_o^2(1-\nu_g^2)} \tag{45}$$

$$\sigma_4 = 2r_g^2(1-\nu_g^2) \tag{46}$$

$$\alpha_5 = E_g\left(r_g^2 - r_o^2\right)\left(\varepsilon_{\theta(r_o)}^{SE(altered)} - \varepsilon_{\theta(r_o)}^{SE(liner)}\right) \tag{47}$$

$$\alpha_6 = \frac{\left(r_o^2 - r_i^2\right)E_c}{\begin{pmatrix} E_g\left(1+\nu_c\right)\left(-4\nu_c r_i^2 + 2\nu_c r_o^2 - r_o^2 + 3r_i^2\right)\left(r_o^2 - r_g^2\right) \\ + \left(r_o^2 - 2\nu_g r_o^2 + r_g^2\right)\left(1+\nu_g\right)\left(r_o^2 - r_i^2\right)E_c \end{pmatrix}} \tag{48}$$

where $\varepsilon_{\theta(r_o)}^{SE(liner)}$, $\varepsilon_{\theta(r_o)}^{SE(altered)}$, $\varepsilon_{\theta(r_g)}^{SE(altered)}$ and $\varepsilon_{\theta(r_g)}^{SE(unaltered)}$ are the circumferential strains corresponding to the seepage forces in the lining at radius r_o, in the altered zone at radius r_o, in the altered zone at radius r_g, in the unaltered ground at radius r_g, respectively. These strains are calculated from the formulations presented in Section 3.

It should be notified that, when there is not an altered zone around the tunnel, the boundary pressure between the ground and the lining is obtained from the compatibility condition at radius r_o($\varepsilon_{\theta(r_o)}^{liner} = \varepsilon_{\theta(r_o)}^{unaltered}$) as:

$$\sigma'_{r(r_o)} = -\frac{\left(\varepsilon_{\theta(r_o)}^{SE(unaltered)} - \varepsilon_{\theta(r_o)}^{SE(liner)}\right)\left(r_o^2 - r_i^2\right)E_c E_r}{r_i^2\left(\left(1-\nu_r\right)E_c - \left(1-\nu_c\right)E_r\right) - r_o^2\left(\left(1+\nu_c\right)\left(1-2\nu_c\right)E_r + \left(1+\nu_r\right)E_c\right)} \tag{49}$$

In many cases, the lining tends to separate from the rock. High tensile stresses will be transmitted to the rock, if the lining is prestressed by grouting the gap between the lining and the rock mass at high pressure. Otherwise, at the lining- rock boundary a gap would open (Thurnherr and Uherkovich, 1978). In this case, the boundary pressure between the ground and the lining reduces to zero, i.e., $\sigma'_{r(r_o)} = 0$, which may result in diminishing the tensile stresses in the lining. Therefore, the opening of the gap can has a beneficial effect on avoiding new longitudinal cracks. In practice, the gap can open if the tunnel is pressurized slowly. Additionally, the gap opening can be facilitated by spraying a bond breaker of white wash or similar on the rough rock surface be concreting the lining (Schleiss, 1986).

5. Comparison with Other Solutions

The results obtained by the proposed solution are compared with those obtained by the other closed-form solutions (the radial-flow-based solution and Lame's solutions) using a typical data set shown in Table 1 (data set 1). In contrast to the radial-flow-based solution, in the proposed solution, the axisymmetric conditions are only assumed for mechanical analysis. As shown by Fernandez and Alvarez (1994) and Fahimifar and Zareifard (2012), this assumption is valid even for pressure tunnels in low groundwater tables, namely for $\frac{h_1}{r_o} \geq 5$.

The results obtained by the proposed solution are compared with the radial-flow-based solution for the considering pressure tunnel (Data set 1 in Table 1). In the radial-flow-based solutions, the reach of seepage flow is assumed to be equal to the distance of the tunnel from the *groundwater surface* as proposed by Schleiss (1986-1997).

A tunnel of radius $r_o = 2.0$ m is excavated at depth 100 m below a horizontal *ground surface*, where horizontal minor and vertical major principal in-situ stresses $\sigma'^0_{\min} = 1$ MPa and $\sigma'^0_{\max} = 2.5$ MPa are initially applied. The surrounding rock medium has elastic constants E_r = 10 GPa and $\nu_r = 0.2$, and Biot-Willis coupling poroelastic constant $\beta_r = 0.8$. The tunnel is lined with a concrete lining with an internal radius $r_i = 1.75$ m, an external radius $r_o = 2.0$ m; a tensile strength σ_t = 1 MPa; elastic constants E_c = 25 GPa and ν_c = 0.25; Biot-Willis coupling poroelastic constant β_c = 0.8; and permeability $k_c = 0.1k_r$.

In addition, depth of the tunnel from a horizontal *groundwater table* is h_1 = 20 m, and an internal water head of H_i = 50 m is applied to the internal surface of the lining under the operational conditions. This produces induced boundary mechanical and hydraulic pressures $\sigma'_{r(r_o)}$ = -0.00454 MPa and $p_{w(r_o)}$ = 0.203 Mpa. The circumferential stresses at the inner surface of the lining and the rock mass are $\left|\sigma'_{\theta(r_i)}\right| = 0.784$ MPa$<\sigma_t$ (because the circumferential tensile stress is smaller than tensile strength of concrete, tensile cracks will not develop) and $\left|\sigma'_{\theta(r_o)}\right|$ = 0.28 MPa, respectively, and the induced radial displacement at the rock - lining interface is $u_{r(r_o)}$ = -0.0477 mm.

Figure 3 show a comparison between the stress and pore water pressure distributions obtained from different analytical solutions (Lame's solution, proposed solution, and the radial-flow-based solution). The results indicate that the seepage induced pore pressure and tensile stresses estimated from the radial-flow-based solution are generally lower than those obtained by the proposed solution. The same trend is observed for the results obtained by the proposed solution through the tunnel crown, to some extent. However, the proposed solutions for the other directions generally show different trends.

Comparison of the results indicates that the induced pore pressure and tensile stresses obtained through the tunnel floor are higher than those obtained through the tunnel crown, and that the induced pore pressure and tensile stresses obtained through the tunnel spring line are between them. In addition, the rate of decrease in pore pressure, radial and circumferential stresses, through the tunnel crown, is considerably faster than the tunnel spring line and floor, which is of practical importance for

Table 1. Data Set

r_o = 2.0 m, r_i = 1.75 m, E_r = 10 GPa, ν_r = 0.2, β_r = 0.8, r_i = 1.75 m, σ_t = 1 Mpa, E_c = 25 GPa, ν_c = 0.25, β_c = 0.8, k_c = 0.1 k_r	Set 1	Homogenous
	Set 2	Prestressed: $\sigma_{p(r_o)}$ = 0.3 MPa, $\sigma_{p(r_o)}$ = 0.3 MPa, $\sigma_{p(r_g)}$ = 0.5 MPa, r_g = 3 m, k_g = 0.2 k_r, ν_g = 0.2, E_g = 15000 Mpa, β_g = 0.8
	Set 3	With a damaged zone: r_g = 5 m, ν_g = 0.2, β_g = 0.8, $k_{g(r_o)} = 100k_r \xrightarrow{\text{linear variation}} k_{g(r_g)} = k_r$ $E_{g(r_o)} = 0.3E_r \xrightarrow{\text{linear variation}} E_{g(r_g)} = E_r$

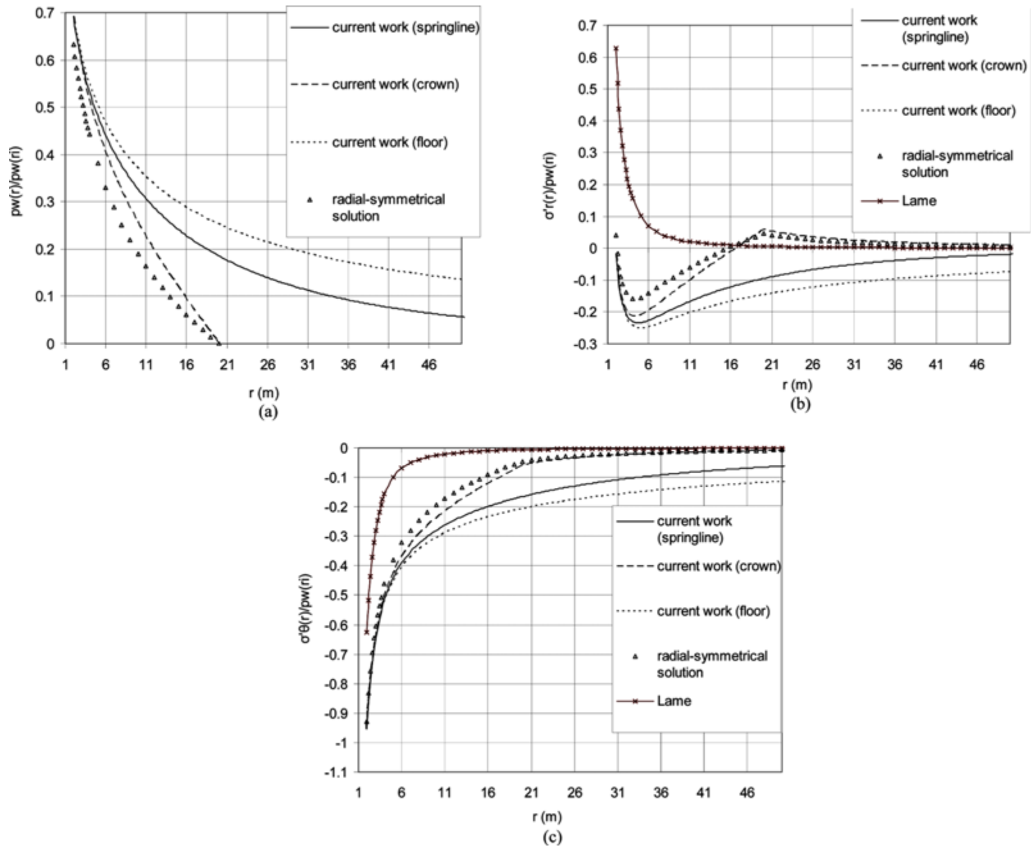

Fig. 3. Comparison of the Results Obtained by Different Solutions for the Lined Pressure Tunnel (Data set 1, H_i = 50 m, h_1 = 20 m): (a) Distribution of the Induced Pore Pressure, (b) Radial Stress, (c) Circumferential Stress

controlling the stability of the valleys located in the vicinity of pressure tunnels. The results obtained from the proposed solution can be used for evaluation of hydrojacking and hydrofracturing potentials in the surrounding rock mass by comparing the induced stresses and pore pressures with the initial or in-situ stresses (hydrofracturing is the event that produces fractures in a sound rock due to water pressure, while hydrojacking is just the opening of the existing cracks or joints due to water pressure (Deere and Lombardi, 1989). In this way, Fernandez and Alvarez (1994) suggested that for evaluating the values of the safety factor against hydrojacking of planes parallel to the *ground surface*, the induced stresses perpendicular to that plane should be compared with the minor principal in-situ stress $\sigma'^{0}_{\min}$. For the considering tunnel, the factor of safety against hydrojacking of the plane parallel to the *ground surface* passing through the tunnel centerline at the side walls is obtained as $FS_{sides} = \dfrac{\sigma'^{0}_{\min}}{\sigma'_{\theta(r_o)}} = 3.57$.

The results of Lame's solution, which assumes the surrounding rock mass to be impervious and the internal pressure to act as a mechanical surface load, are also shown in these figures. As observed, the results for Lame's solution are generally different from the other solutions. In addition, in contrast to the case in which the internal pressure is treated as a mechanical load (i.e. impervious pressure tunnels); seepage affects a wide zone of the rock mass surrounding a pervious pressure tunnel. It should be noted that, the results obtained are only valid for a certain concrete lining permeability compared to the rock permeability ($\frac{k_c}{k_r} = 0.1$); and for smaller permeability ratios $\frac{k_c}{k_r}$, the results become closer to Lame's solution.

As seen in Fig. 3, the stresses and pore pressure for different directions are not the same; thus, radial-flow-based solution which provides the same variations for all directions is not realistic. To compare the proposed solution with the radial-flow-based solution, the differences in percentage terms between the results (the induced radial and circumferential stresses) obtained by these solutions (at radial distances r = 10 m and r = 2 m from the tunnel centerline through the side walls, crown and bottom) for varying $\frac{h_1}{r_o}$ are plotted in Fig. 4. It is observed that at the tunnel perimeter the results obtained by the proposed solution for different directions are exactly the same. The results show that the difference in circumferential stresses at the tunnel perimeter (r = 2 m) is always below 10% even for tunnels in low *groundwater tables*. On the other hand, away from the tunnel, the difference in the circumferential stresses is above 10% even for tunnels in high *groundwater tables*. However, the differences decrease with increasing in $\frac{h_1}{r_o}$. In contrast, no general trend can be found in the variation of the induced radial stresses. Nevertheless, as observed, at both radii (r = 10 m and r = 2 m), the difference in

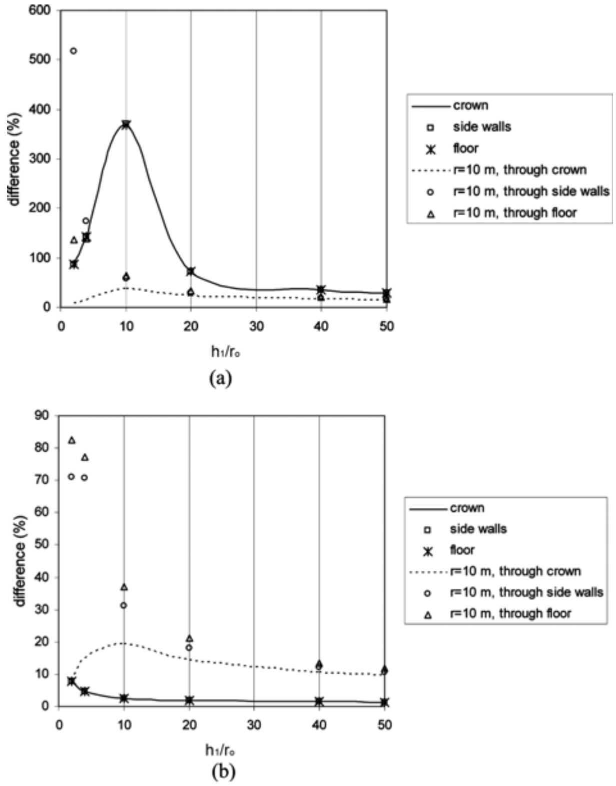

Fig. 4. Differences between the Induced Stresses Obtained by the Proposed Solution and the Radial-flow-based Solution for Different Tunnel Depths *from the Groundwater Table* for the Lined Tunnel (Data set 1, H_i = 50 m), where: *difference*(%) = $(\sigma'^{proposed\ solution} - \sigma'^{radial\ symmetrical\ solution})/\sigma'^{proposed\ solution} \times 100$: (a) Radial Stress, (b) Circumferential Stress

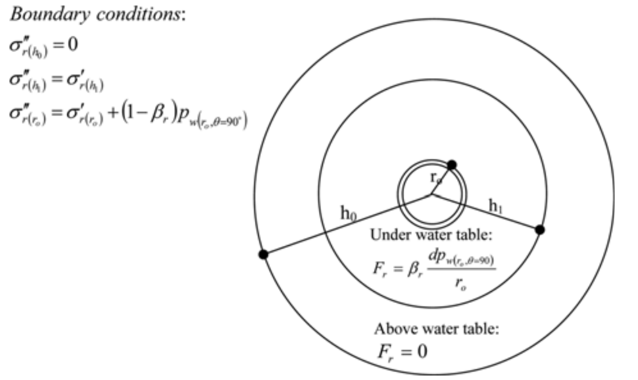

Fig. 5. Depth-dependent Model: Thick Walled-cylinder under Radial Seepage Forces with Internal Radius r_o and External Radius h_0

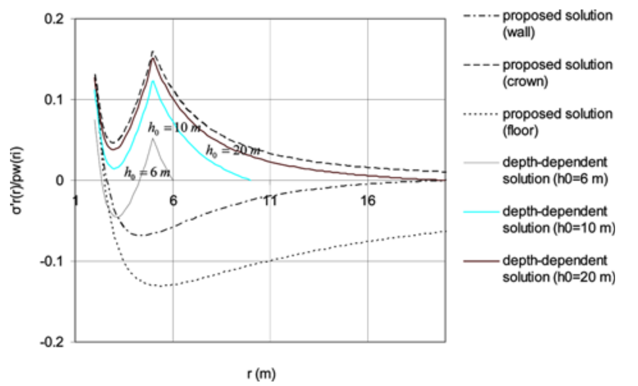

Fig. 6. Effect of Depth from free Ground Surface h_0 on the Induced Radial Stress for the Lined Tunnel (Data set 1, H_i = 50 m, h_1 = 5 m)

the radial stresses is above 10% even for tunnels in high *groundwater tables*.

6. Effect of the Free Ground Surface

In the proposed solution the effect of free ground surface is not taken into account. In this regard, as shown in Fig. 2, it is assumed that the tunnel is deep enough. Thus, for very shallow tunnels, ignoring the free *ground surface* may result in large differences.

Here, the effect of depth of pressure tunnel from the *ground surface* is investigated. In this regard, comparing the results obtained from the proposed solution with those obtained from a depth-dependent solution, the degree of accuracy of the proposed solution in approximating stresses and displacements for shallow tunnels can be evaluated.

If a model is considered which has a free external boundary with radius equal to h_0, an implicitly depth-dependent solution is attained (shown in Fig. 5). In this case, compared to the model introduced in Fig. 2, the geometry, boundary condition and applied loads through the tunnel crown is generalized to all directions. Thus, as shown in Fig. 5, the problem is converted to a thick-walled cylinder with an internal radius r_o and an external radius of the tunnel depth h_0.

Figure 6 shows a comparison between the radial stresses distributions obtained by the proposed solution (Fig. 2) and the depth-dependent solution (Fig. 5) for different depths of the tunnel from the *ground surface* h_0 (Data set 1 in table 1 for case h_1 = 5 m). The differences between the depth-dependent and proposed solutions for different values of the tunnel depth in approximating the induced circumferential and radial stresses at the tunnel radius (i.e, $\sigma'_{\theta(r_o)}$ and $\sigma'_{r(r_o)}$) are plotted in Fig. 7. It is observed that in the vicinity of the tunnel, the results obtained by the proposed solution for different directions are identical, indicating that the utilized simplification, i.e. generalizing pore pressure distribution to the all directions, has negligible effect on the induced stresses near the tunnel walls. Comparing the results obtained for different depths based on the depth-dependent solution with those obtained by the proposed solution (through the tunnel crown) shows that by increasing h_0 the results obtained by the proposed solution converge to those obtained by the depth-dependent solution, and that for h_0 > 20 m ($\frac{h_0}{r_o} > 10$) the effect of free surface on the results becomes inconsiderable. It can be concluded that, although the proposed solution is initially derived for deep tunnels, it provides sufficiently accurate results for relatively shallow tunnels.

7. Stresses and Strains Induced by Prestressing

Cracks will develop in the concrete lining, due to applying the internal pressure, if the circumferential stresses at its internal radius exceed the tensile strength of the concrete. The cracks can be controlled to some extent by reinforcing. On the other hand, cracks can be prevented either by reducing loads within the lining (by increasing the thickness of the lining, rock grouting, and drainage system) or by increasing the resistance of the lining using prestressing. At high pressure heads, cracks are only avoided by prestressing, or using steel linings; while, prestressing is a more popular option, especially for shortening the construction period, increasing in durability and reducing the costs. Different prestressing methods have been developed: prestressing by grouting (rock grouting and gap grouting), or passive prestressing and prestressing by tendons or active prestressing (Schleiss, 1986). In the passive case, the prestressing is applied by grout pressure acting in a cavity between the lining and the surrounding rock. In this method, an adequate strength or adequate overburden depth is required for the permanent maintenance of prestress. On the other hand, in the active prestressing, prestress is produced by individual tendons running around the concrete lining. These two methods are usually used together to attain the best performance by composite action of the rock and the lining. Furthermore, in this case, a uniform prestressing pressure around the entire surface will be induced (Thurnherr and Uherkovich, 1978).

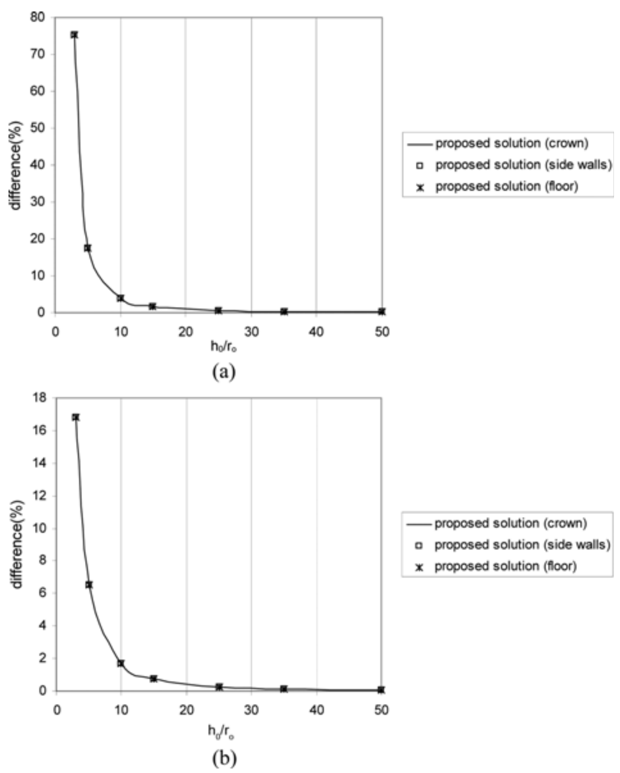

Fig. 7. Differences between the Induced Stresses at the Tunnel Radius Obtained by the Proposed Solution and the Depth-dependent Solution for Different Tunnel Depths from the Ground Surface for the Lined Tunnel (Data set 1, H_i = 50 m, h_1 = 5 m), where, *difference*(%) = ($\sigma'_{(r_o)}{}^{proposed\ solution}$ – $\sigma'_{(r_o)}{}^{depth\text{-}dependent\ solution}$)/ $\sigma'_{(r_o)}{}^{depth\text{-}dependent\ solution}$ × 100: (a) Radial Stress, (b) Circumferential Stress

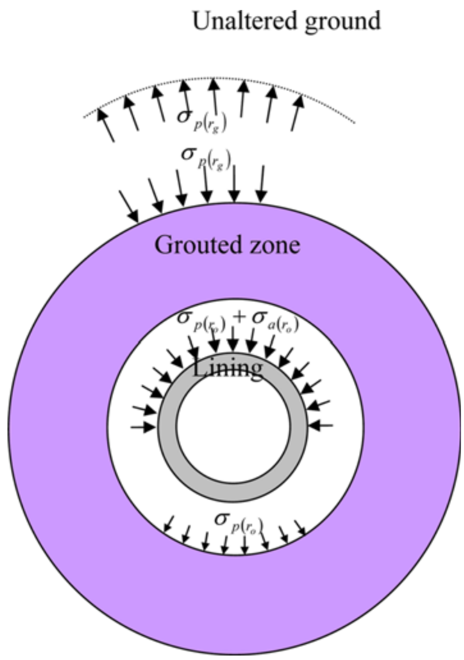

Fig. 8. Boundary Prestressing Pressures Applied to Different Zones

In this paper, the effect of passive prestressing is considered by uniform boundary pressures $\sigma_{p(r_o)}$ and $\sigma_{p(r_g)}$ applied to the internal and external radii of the grouted zone i.e. r_o and r_g; and the effect of active prestressing is considered by a uniform boundary pressure $\sigma_{a(r_o)}$ applied to the external radius of the lining(see Fig. 8). Using superposition concept, the stresses induced by these prestressing pressures are added to the seepage induced stresses in the lining, the altered zone (here, the grouted zone) and the unaltered zone.

The stresses induced by the prestressing pressures in each zone are obtained by solving Eq. (13) taking into account the boundary conditions shown in Fig. 8, namely,

In the lining:

$$\sigma'^{PR}_{r(r)} = \left(\sigma_{a(r_o)} + \sigma_{p(r_o)}\right)\frac{r_i^2}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right) + \sigma_{a(r_o)} + \sigma_{p(r_o)} \tag{50}$$

$$\sigma'^{PR}_{\theta(r)} = \left(\sigma_{a(r_o)} + \sigma_{p(r_o)}\right)\frac{r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right) + \sigma_{a(r_o)} + \sigma_{p(r_o)} \tag{51}$$

In the grouted zone:

$$\sigma'^{PR}_{r(r)} = \left(\sigma_{p(r_g)} - \sigma_{p(r_o)}\right)\frac{r_o^2}{r_g^2 - r_o^2}\left(1 - \frac{r_g^2}{r^2}\right) + \sigma_{p(r_g)} \tag{52}$$

$$\sigma'^{PR}_{\theta(r)} = \left(\sigma_{p(r_g)} - \sigma_{p(r_o)}\right)\frac{r_o^2}{r_g^2 - r_o^2}\left(1 + \frac{r_g^2}{r^2}\right) + \sigma_{p(r_g)} \tag{53}$$

$$\sigma'^{PR}_{r(r)} = \sigma_{p(r_g)}\left(\frac{r_g^2}{r^2}\right) \tag{54}$$

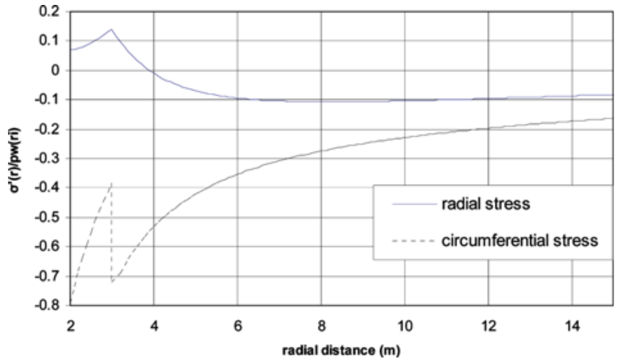

Fig. 9. The Induced Stresses in the Rock Mass under Seepage Forces and Presstressing Pressures Around the Tunnel (Data set 2, H_i = 250 m, h_1 = 20 m)

$$\sigma'^{PR}_{\theta(r)} = -\sigma_{p(r_g)}\left(\frac{r_g^2}{r^2}\right) \tag{55}$$

As an example, for the considering tunnel in the previous sections (Data set 1 in Table 1 for case h_1 = 20 m), if an internal water head of H_i = 250 m is applied to the internal surface of the lining, the circumferential stress equal to $\sigma'_{\theta(r_i)}$ = - 6.01 MPa will be induced in the inner surface of the lining. In this case, the boundary stress between the lining and the rock mass is a tensile stress; thus, the lining tends to separate from the rock. If, a gap is opened at the lining- rock boundary, the induced circumferential stress will reduce to $\sigma'_{\theta(r_i)}$ = -5.72 MPa.

It is observed that in both cases (with or without the gap opening), tensile cracks will develop. Therefore, in order to avoid cracks development in the lining, a combination of active and passive prestressing methods with the following boundary pressures is used (Data set 2 in Table 1 for case h_1 = 20 m).

$\sigma_{a(r_o)}$ = 1.2 MPa, $\sigma_{p(r_o)}$ = 0.3 MPa, $\sigma_{p(r_g)}$ = 0.5 MPa

The properties of the grouted zone are listed as:

r_g = 3 m, $k_g = 0.2 \times k_r$, ν_g = 0.2, E_g = 15000 MPa, β_g = 0.8

In this case, the calculated circumferential stress at the internal surface of the lining is equal to $\sigma'_{\theta(r_i)}$ = 10.03 MPa. Fig. 9 shows the calculated induced stresses in the rock mass. It should be notified that, the magnitude of induced stresses in the rock mass must be smaller than the initial in-situ stresses in the rock mass, to prevent instability of the tunnel.

8. Effect of the Excavation Damaged Zone

In this section, the effect of a damaged zone, which may be developed around tunnels, is investigated. The tunnel presented in section 5 is considered for this purpose; except that, in this case, a damaged zone is developed around the tunnel. A damaged zone can be defined as a rock zone around a tunnel where the rock permeability and stiffness parameters have changed due to the processes related to the excavation and operation phases. Different mechanisms are related to the development of a damaged zone. Major factors related to the development of a damaged zone are (a) excavation impact; (b) stress redistribution after excavation; (c) hydrofracturing phenomenon due to the internal pressure. Both the theoretical analyses and the experimental observations has shown that the thickness of the zone of damaged rock can range from few centimeters up to several meters. It should be recognized that, in the damaged zone, neither the stiffness nor the permeability are constant for any rock unit.

In the method proposed in this paper, the effect of the damaged zone on the stresses and displacements around pressure tunnels is considered. Here the effects of the damaged zone are examined. As an illustrative example, it is assumed that a damaged zone with radius r_g = 5 m, is developed around the considering tunnel (Data set 3 in Table 1).

For the considering tunnel, it is assumed that the permeability and the elastic modulus of the damaged zone have linear variations between r_o and r_g, i.e. (Data set 3):

$$k_{g(r_o)} = 100k_r \xrightarrow{\text{linear variation}} k_{g(r_g)} = k_r$$
$$E_{g(r_o)} = 0.3E_r \xrightarrow{\text{linear variation}} E_{g(r_g)} = E_r$$

Furthermore, Poisson's ratio and Biot-Willis constant of the damaged zone are ν_g = 0.2 and β_g = 0.8, respectively. Depth of the tunnel from a horizontal *groundwater table* is h_1 = 20 m, and an internal water head of H_i = 50 m is applied to the internal surface of the lining under the operational conditions.

The high changes in the permeability and stiffness chosen for the damaged zone are not unlikely (Lanyon, 2011). However, this case are not common in pressure tunnels; if required precautionary measures in the excavation stage (using a controlled excavation procedure) or in the design stage (e.g., using continuous injection) are taken. This case is considered to examine the maximum possible effect of the alterations in parameters on the predicted induced stresses, and to illustrate the different models, which can be considered.

The analyses are carried out through the horizontal direction for the tunnel (Data set 3) based on the following models:

Case 1: effect of the damaged zone is neglected, and $r_g = r_o$ (rock mass is considered as homogenous).

Case 2: effect of the damaged zone is neglected, but in a cylindrical zone with an external radius r_g, the analysis is carried out based on the radial flow pattern (namely, the formulation presented in 4.2 and 5.2.)

Case 3: effect of the permeability variation in the damaged zone is considered; therefore, in a cylindrical zone with an external radius r_g, the analysis is carried out on the basis of the formulation presented in 5.2 and Appendix A.1.

Case 4: effect of the elasticity modulus variation in the damaged zone is considered; therefore, in a cylindrical zone with an external radius r_g, the analysis is carried out based on the formulation presented in Appendix A.2.

Case 5: effect of the permeability and elasticity modulus variations in the damaged zone is considered; therefore, in a cylindrical zone with an external radius r_g, the analysis is carried out based on the formulation presented in Appendix A.3.

Case 5 (by numerical method): *FLAC V.4* program (Itasca Consulting Group, 1997), as a Finite Difference Method (FDM), was selected for numerical analysis. In the problem of an underwater pressure tunnel, seepage flow will be developed in a wide zone. In this case, the free ground surface is considered at the groundwater table $(h_0 = h_1 = 20\text{ m})$. The numerical model encompasses a large area around the tunnel (200 m through the tunnel springline and the tunnel floor), in order to avoid any influence of the model boundaries on the results of analysis. In addition, as the lining is permeable and seepage flow is developed through it, the lining must be considered as a separate zone with a much finer mesh. This increases the required runtime for the numerical program. A finer mesh is created in the region adjacent to the tunnel surface, and a coarse grid extends away from the tunnel region. The fine-mesh region extends from -20.0 m to +20.0 m in the x-direction and from -20.0 m to +20.0 m in the y-direction. The induced stresses and pore water pressure are fixed to zero at the boundaries of the model. The initial gravitational water head is reduced. Finally, the induced internal pressure is applied to the internal surface of the lining.

The stress distribution in the surrounding rock mass calculated for different cases are plotted in Fig. 10. As can be seen, there is good agreement between the analytical solution and the FLAC2D solution for case 5. Fig. 10 also shows a decrease in both radial and circumferential effective stresses with a decrease of the stiffness parameter in the damaged zone. In this case, a higher tensile radial effective stress will be applied at the boundary of the opening. Therefore, the loads transferred to the lining will increase. This can only occur when the induced tensile stresses between the lining and the rock mass doesn't exceed the boundary tensile strength, which rarely occurs in practice unless the liner–rock connection is specifically designed to sustain tension; otherwise, the rock will detach from the liner.

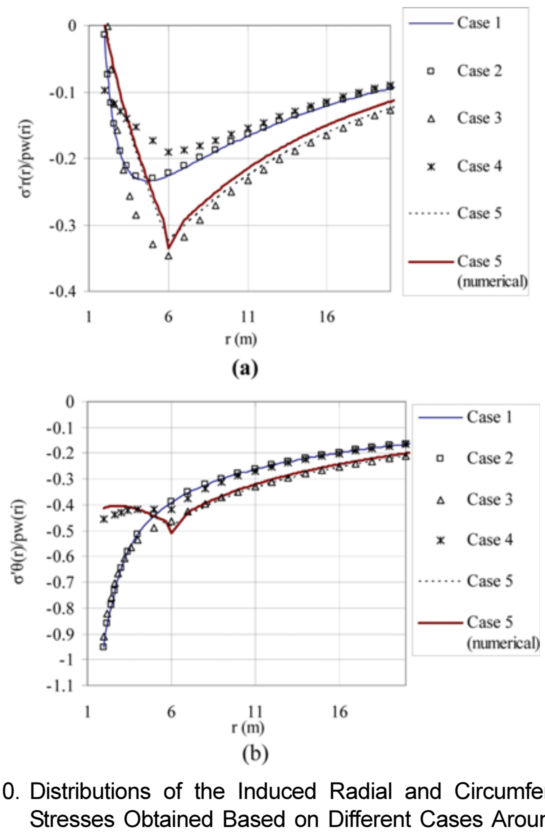

Fig. 10. Distributions of the Induced Radial and Circumferential Stresses Obtained Based on Different Cases Around the Tunnel (Data set 3, H_i = 50 m, h_1 = 20 m): (a) Radial Stress, (b) Circumferential Stress

On the other hand, in most areas of the rock mass stresses increase with increasing the permeability of the damaged zone, which can increase the hydrojacking and hydrofracturing potentials in the rock mass. In this respect, the radial distance at which the maximum radial tensile stress is developed transfers to r_g and the magnitude of the maximum stress increases.

The circumferential tensile stresses at the inner surface of the lining $\sigma'_{\theta(r_i)}$ for cases 1, 2, 3, 4, and 5 are -0.784 MPa, -0.784 MPa, -0.783 MPa, -0.995 MPa, -0.993 MPa, respectively.

The danger of excessive seepage flow, which in most cases depend upon the hydrojacking and hydrofracturing phenomena, may be the most important issue in evaluating the stability of pressure tunnels. For the cases with the increased permeability in the damaged zone, the evaluated seepage flow rate is equal to 1.32 times greater than that for the cases with constant permeability, which seems to be insignificant.

It is observed that, the results of cases 1 are close to those for case 2; thus, the effect of assuming the radial flow pattern in the damaged zone is found to be insignificant. On the other hand, utilizing the other models will lead to different results. Therefore, when a damaged zone is developed around the tunnel its effect cannot be neglected. However, the effect of the permeability variation in the damaged zone on the induced stresses in the lining seems to be negligible.

It is observed that, the permeability and stiffness of the damaged zone are the most important parameters. However, measurement of these parameters is difficult, especially in the early design of tunnels. Hence, incorporating the damaged zone in the early design stage of pressure tunnels is often neglected. One of the main advantages of the proposed solution is that, in contrast to standard FDM or FEM-based codes, it permits performing parametric studies more conveniently. Parametric studies are usually very recommendable in such cases where a certain deal of uncertainties are always expected. This kind of analysis is very helpful for controlling uncertainties encountered in the quantification of the damaged zone.

In this way, Figs. 11 illustrates the effect of the minimum elasticity modulus ratio $\frac{E_{g(r_o)}}{E_r}$ of the damaged zone at the tunnel periphery on the maximum tensile effective stresses in the lining for different radii of the damaged zone. As illustrated, this value is effective in the stability of the pressure tunnel. Cases 4 has been used for this purpose. These results, which can be useful from a practical design consideration, are obtained with a simple code in a fast and easy way, whereas obtaining the same

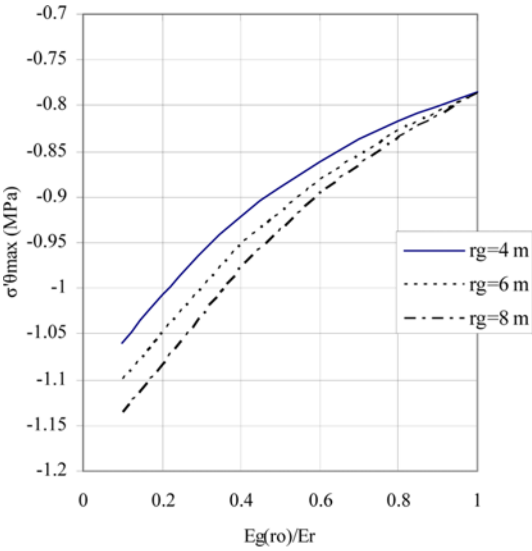

Fig. 11. Influence of the Stiffness of the Damaged Zone in the Maximum Circumferential Effective Stresses in the Lining (Data set 3, H_i = 50 m, h_1 = 20 m)

information by means of a standard numerical code, i.e., FLAC-2D, would take much longer.

9. Conclusions

In this paper, based on a generalized effective stress law, a closed-form analytical solution was introduced, for approximation of seepage induced stresses and strains around a circular lined pressure tunnel excavated below a horizontal groundwater table. In this solution, elastic responses for the lining and the rock mass are assumed.

Admittedly, the utilized simplifications are limitations of the proposed solution, since, in most cases, are not realistic. However, this kind of solution is still valuable which provide the means to quickly obtain estimates of the induced stresses and displacements; and can be used for the preliminary design of tunnels or as a first step for more elaborate numerical models.

The solution accounts for the hydraulic and mechanical boundary pressures between the lining, the altered rock mass (damaged or grouted zone) and the unaltered surrounding rock mass, as a result of seepage forces. It is relatively simple, easy to use, and clearly indicates the sensitivity of the chosen solution through a range of various the grounds and lining parameters.

This solution was compared with the more simplified solutions. It was shown that, the proposed solution provides more accurate results, in contrast to the more simplified solutions (the Lame's solution and the radial-flow-based solution) by considering the variation of the seepage body forces through the different directions. The results clearly show that the groundwater flow has a significant effect on the stresses and strains in all zones. It is concluded that, the approximations made in derivation of the closed-form solution have insignificant effects on the results in most cases; therefore, it can be used for the design of pervious pressure tunnels, safely.

Notations

E= Elastic modulus
F_r= Induced radial seepage body forces
F_θ= Induced circumferential seepage body forces
H_i= Final internal head at the internal radius of the lining
h_0= Depth of the tunnel from ground surface
h_1= Depth of the tunnel from groundwater surface
K= Bulk modulus of matrix material
K_s= Bulk modulus of solid material.
P_i= Final internal pore-pressure at internal radius of the lining
p_w= Induced pore-water pressure
r= Radial distance from the center of the tunnel
r_i= Internal radius of the lining
r_o= External radius of the lining
r_g= External radius of the altered zone
u_r= Induced radial displacement
(x,y)= Cartesian coordinates
β= Biot-Willis coupling poroelastic constant
ε_θ= Induced circumferential strain
ε_r= Induced radial strain
γ_w= Water specific gravity
ν= Poisson's ratio
θ= Angle measured clockwise from the horizontal direction
σ' = Induced Terzaghi effective stress
σ'' = Induced Biot effective stress
σ_θ= Induced circumferential stress
σ_r= Induced radial stress
$\sigma_{\theta r}$= Induced shear stress in plane θr
σ_a= Active prestressing pressure
σ_p= Passive prestressing pressure
Subscript c= Refers to quantities corresponding to the concrete lining
Subscript r= Refers to quantities corresponding to the unaltered rock mass
Superscript BU=Refers to the quantities induced by the boundary pressures
Superscript SE= Refers to the quantities induced by the seepage forces
Superscript PR= Refers to the quantities induced by the prestressing pressures
Subscript (r)= Refers to quantities corresponding to the radius r.

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Appendix. Analysis of Thick-walled Cylinder with Varied Properties

Varied Permeability

In order to consider the effect of the varied permeability in the altered zone, it can be assumed that k_g varies linearly with radius from $k_{g(r_o)}$ at r_o to $k_{g(r_g)}$ at r_g as:

$$k_{g(r)} = ar + b \tag{56}$$

$$a = \frac{k_{g(r_g)} - k_g(r_o)}{r_g - r_o} \tag{57}$$

$$b = \frac{k_{g(r_g)} r_g - k_{g(r_o)} r_o}{r_g - r_o} \tag{58}$$

In this case, the induced pore pressure in the altered zone is calculated from an analytical integration of the pore pressure increments obtained from Darcy's equation as:

$$p_{w(r,\theta)} = \frac{\gamma_w q}{2\pi}\frac{1}{b}\log\frac{r(ar_o + b)}{r_o(ar + b)} + p_{w(r_o)} \tag{59}$$

The seepage flow rate q is calculated from the continuity condition as:

$$q = 2\pi\frac{p_i - \gamma_w h_1}{\gamma_w}\frac{1}{-\frac{1}{k_c}\log\left(\frac{r}{r_i}\right) + \frac{1}{k_0}\log(c) + \frac{1}{b}\log\frac{r_o(ar_g + b)}{r_g(ar_o + b)}} \tag{60}$$

Solving the differential Eq. (13) considering the pore pressure variation (Eq. 59), gives the induced circumferential strain $\varepsilon^{SE}_{\theta(r)}$, and the induced radial strain $\varepsilon^{SE}_{r(r)}$ as follows:

$$\varepsilon^{SE}_{\theta(r)} = C'_{11} + \frac{1}{r^2}C'_{12} + \frac{\beta_g \gamma_w q(1-2\nu_g)(1+\nu_g)\begin{pmatrix} 2\log(ar+b)(a^2r^2 - b^2) \\ -2a^2r^2\log(r) + 3b^2 + 2abr \end{pmatrix}}{8\pi E_g(1-\nu_g)a^2br^2} \tag{61}$$

$$\varepsilon^{SE}_{r(r)} = C'_{11} - \frac{1}{r^2}C'_{12} - \frac{\beta_g \gamma_w q(1-2\nu_g)(1+\nu_g)\begin{pmatrix} 2(ar+b)\left(-a^2r^2\log\frac{ar+b}{r} - 2b^2\log(a+b)\right) \\ +5ab^2r + 3b^2 \end{pmatrix}}{8\pi E_g(1-\nu_g)a^2br^2(ar+b)} \tag{62}$$

where C'_{11} and C'_{12} are integration constants, and are obtained from the boundary conditions $\sigma''^{SE}_{r(r_o)} = (1-\beta_g)p_{w(r_o)}$ and $\sigma''^{SE}_{r(r_g)} = (1-\beta_g)p_{w(r_g)}$ as follows:

$$C'_{11} = + \frac{(1+\nu_g)(1-2\nu_g)}{4E_g(1-\nu_g)a^2b(r_g^2 - r_o^2)}\begin{pmatrix} abQ(1-2\nu_g)(r_g - r_o) \\ 2a^2b(1-\nu_g)(1-\beta_g)(p_{w(r_g)}r_g^2 - p_{w(r_o)}r_o^2) \\ -Qb^2\beta_g(1-2\nu_g)\log\left(\frac{ar_g + b}{ar_o + b}\right) \\ -Qr_g^2a^2\log\left(\frac{ar_g + b}{r_g}\right) + Qr_g^2a^2\log\left(\frac{ar_o + b}{r_o}\right) \end{pmatrix} \tag{63}$$

$$C'_{12} = \frac{(1+\nu_g)}{4E_g(1-\nu_g)a^2b(r_g^2 - r_o^2)}\begin{pmatrix} 2Qr_g^2r_o^2a^2\beta_g\log\frac{r_g}{r_o} \\ -2Qr_o^2\log(ar_g + b)(b^2(1-2\nu) + r_g^2a^2) \\ +2Qr_g^2\log(ar_o + b)(b^2(1-2\nu) + r_o^2a^2) \\ -bQ(1-2\nu_g)(r_g - r_o)(3b(r_o + r_g) + 2ar_or_g) \\ +4a^2br_o^2r_g^2(1-\beta_g)(1-\nu_g)(p_{w(r_g)} - p_{w(r_o)}) \end{pmatrix} \tag{64}$$

$$Q = \frac{\beta_g \gamma_w q}{2\pi} \tag{65}$$

Varied Elasticity Modulus

The same procedure can be used for deriving the induced stresses and strains, when elasticity modulus is variable in the altered zone. In order to consider the effect of variable elasticity modulus within the altered zone, it can be assumed that E_g varies linearly with radius from $E_{g(r_o)}$ at r_o to $E_{g(r_g)}$ at r_g as:

$$E_{g(r)} = a_0 r + b_0 \tag{66}$$

$$a_0 = \frac{E_{g(r_g)} - E_{g(r_o)}}{r_g - r_o} \tag{67}$$

$$b_0 = \frac{E_{g(r_g)} r_g - E_{g(r_o)} r_o}{r_g - r_o} \tag{68}$$

For this case, solving the differential Eq. (13), gives the induced circumferential strain $\varepsilon^{SE}_{\theta(r)}$, and the induced radial strain $\varepsilon^{SE}_{r(r)}$ as follows:

$$\varepsilon^{SE}_{\theta(r)} = C'_{21} + \frac{1}{r^2}C'_{22} + \frac{\beta_g\left(p_{w(r_g)} - p_{w(r_o)}\right)(1-2\nu_g)(1+\nu_g)\begin{pmatrix} \log(a_0r + b_0)(a_0^2r^2 - b_0^2) \\ -a_0^2r^2\log(r) + a_0b_0r \end{pmatrix}}{2\log\left(\frac{r_g}{r_o}\right)(1-\nu_g)a_0^2b_0r^2} \tag{69}$$

$$\varepsilon^{SE}_{r(r)} = C'_{21} - \frac{1}{r^2}C'_{22} + \frac{\beta_g\left(p_{w(r_g)} - p_{w(r_o)}\right)(1-2\nu_g)(1+\nu_g)\begin{pmatrix} a_0^2r^2\log\left(\frac{a_0r + b_0}{r}\right) \\ +b_0^2\log(a_0r + b_0) - a_0b_0r \end{pmatrix}}{2\log\left(\frac{r_g}{r_o}\right)(1-\nu_g)a_0^2r^2b_0} \tag{70}$$

where C'_{21} and C'_{22} are integration constants and obtained from the boundary conditions at radii r_o and r_g:

$$C'_{21} = -\frac{(1+\nu_g)(1-2\nu_g)}{2E_{(r_o)}E_{(r_g)}(1-\nu_g)a_0^2 b_0(r_g^2-r_o^2)}\begin{pmatrix} a_0 b_0 Q_0(1-2\nu_g)(r_g-r_o) \\ +2a_0^2 b_0(1-\nu_g)(1-\beta_g)(p_{w(r_g)} r_g^2 E_{(r_o)} - p_{w(r_o)} r_o^2 E_{(r_g)}) \\ -Q_0 b_0^2(1-2\nu_g)\log\left(\frac{a_0 r_g + b_0}{a_0 r_o + b_0}\right) \\ -Q_0 r_g^2 a_0^2 \log\left(\frac{a_0 r_g + b_0}{r_g}\right) \\ +Q_0 r_o^2 a_0^2 \log\left(\frac{a_0 r_o + b_0}{r_o}\right) \end{pmatrix} \tag{71}$$

$$C'_{22} = \frac{(1+\nu_g)}{2E_{(r_o)}E_{(r_g)}(1-\nu_g)a_0^2 b_0(r_g^2-r_o^2)}\begin{pmatrix} r_g r_o a_0 b_0 Q_0(1-2\nu_g)(r_g-r_o) \\ +Q_0 r_g^2 r_o^2 a_0^2 \beta_g \log\frac{r_g}{r_o} \\ -Q_0 r_o^2 \log(ar_g+b_0)(b_0^2(1-2\nu_g)+r_g^2 a_0^2) \\ +Q_0 r_g^2 \log(ar_o+b_0)(b_0^2(1-2\nu_g)+r_o^2 a_0^2) \\ +4a_0^2 b_0 r_o^2 r_g^2(1-\beta_g)(1-\nu_g)(p_{w(r_g)}-p_{w(r_o)}) \end{pmatrix} \tag{72}$$

And:

$$Q_0 = \frac{\beta_g \gamma_w q E_{(r_o)} E_{(r_g)}}{2\pi k_g} \tag{73}$$

In this case, the induced strains corresponding to the boundary pressures can be obtained by solving Eq. (13) considering the boundary conditions $\sigma''^{BU}_{r(r_o)} = \sigma'_{r(r_o)}$ and $\sigma''^{BU}_{r(r_g)} = \sigma'_{r(r_g)}$:

$$\varepsilon^{BU}_{\theta(r)} = \frac{(1+\nu_g)}{E_{(r_o)}E_{(r_g)}r^2(r_g^2-r_o^2)}\begin{pmatrix} r_g^2 E_{(r_o)}\sigma'_{r(r_g)}((1-2\nu_g)r^2-r_o^2) \\ -r_o^2 E_{(r_g)}\sigma'_{r(r_o)}((1+2\nu_g)r^2-r_g^2) \end{pmatrix} \tag{74}$$

And:

$$\varepsilon^{BU}_{r(r)} = \frac{(1+\nu_g)}{E_{(r_o)}E_{(r_g)}r^2(r_g^2-r_o^2)}\begin{pmatrix} r_g^2 E_{(r_o)}\sigma'_{r(r_g)}((1-2\nu_g)r^2-r_o^2) \\ -r_g^2 E_{(r_g)}\sigma'_{r(r_g)}((1+2\nu_g)r^2+r_g^2) \end{pmatrix} \tag{75}$$

Varied Permeability and Elasticity Modulus

If both permeability and elasticity modulus have linear variations in the altered zone, the induced radial displacement corresponding to the seepage forces $u^{SE}_{r(r)}$ can be obtained as:

$$u^{SE}_{r(r)} = C'_{31} r + \frac{1}{r}C'_{32} + \frac{\beta_g \gamma_w q(1-2\nu_g)(1+\nu_g)\begin{pmatrix} \log(a_0 r+b_0)(a_0^2 r^2 ab - ab_0^2) \\ -\log(a_0 r+b_0)(a^2 r^2 a_0 b_0 - b^2 b_0^2 a_0) \\ +a^2 a_0 r^2(a_0 b - b_0 a)\log(r) \end{pmatrix}}{4\pi(1-\nu_g)aa_0 bb_0(a_0 b - b_0 a)r} \tag{76}$$

The integration constants C'_{31}, and C'_{32}, and the induced strains for this case can be derived by using the same procedure as presented in the previous sections.

Finally, using Eqs. (11) and (12) gives the values of the induced stresses corresponding to the seepage forces $\sigma''^{SE}_{r(r)}$ and $\sigma''^{SE}_{\theta(r)}$. In this case, the induced strains corresponding to the boundary pressures are obtained from Eqs. (74) and (75).