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Optimization of Reinforced Concrete Retaining Walls Via Hybrid Firefly Algorithm With Upper Bound Strategy

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Abstract

This paper represents a novel hybrid optimization method that uses an improved firefly algorithm with a harmony search algorithm (IFA-HS), for optimizing the cost of reinforced concrete retaining walls. The IFA-HS is utilized to find an economical design adhering to ACI 318-05 provisions. Two design examples regarding retaining walls are optimized using the proposed hybrid method, and the optimization results confirm the validity and efficiency of the developed algorithm. The IFA-HS method offers improvements on the recently developed firefly algorithm. These improvements include utilizing the memory that contains information extracted online during a search, employing pitch adjusting operation of HS during firefly updates, and modifying the movement phase of the FA. Moreover, to decrease the computational effort of the IFA-HS, the upper bound strategy, which is a recently developed strategy for reducing the total number of structural analyses, is incorporated during the optimization process.

Keywords: *concrete retaining wall, metaheuristics, firefly algorithm, harmony search, hybrid optimization*

1. Introduction

Cantilever walls are the simplest and most common geotechnical structures used to support earth backfills. Their main representatives are retaining walls supporting deep excavations, bridge abutments, harbor-quay walls, and so forth. These types of walls are constructed of Reinforced Concrete (RC) and can be used in both cut and fill applications.

They have relatively narrow base widths and can be supported by both shallow and deep foundations. The position of the wall stem relative to the footing can be varied to accommodate right-of-way constraints. They are most economical at low to medium wall heights.

The cantilever walls generally consist of a vertical stem and a base slab. The base slab is made up of two distinct regions, i.e., a heel slab and a toe slab. All three components behave like one-way cantilever slabs: The stem acts as a vertical cantilever under lateral earth pressure; the “heel slab” and the “toe slab” act as a horizontal cantilever under the action of the resulting soil pressure.

Retaining walls are designed to withstand lateral earth and water pressures and for a service life based on consideration of the potential long-term effects of material deterioration on each of the material components that constitute the wall. Regular retaining walls should be designed for a minimum service life of fifty years, and temporary retaining walls should be designed for a minimum service life of five years (CalTrans, 2004).

On the other hand, most structural engineering design problems are nonlinear and highly constrained. Due to the limitations of exact methods in approaching such complex problems, more appropriate techniques are required. Evolutionary algorithms and, in particular, Swarm Intelligence (SI) provide a range of flexible and robust optimization methods capable of dealing with these types of problems.

The conventional design of RC retaining walls is highly dependent on the experience of engineers, in which the structure is defined on a trial-and-error basis. A tentative design must satisfy the limit states prescribed by concrete specification codes. This process leads to safe designs, however, the cost of the RC retaining walls is, consequently, highly dependent upon the experience of the designer. Therefore, to economize the cost of the RC retaining walls under design constraints, it is advantageous for the designer to consider the problem as an optimization problem.

Optimizing the design of retaining walls has been the subject of a number of studies. Sariba and Erbatur (1996) presented a detailed study of the optimization of RC cantilever retaining walls, using the cost and weight of the walls as objective

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functions. In their study, they controlled overturning failure, sliding failure, and shear and moment capacities of the toe slab, heel slab, and stem of the wall as constraints.

Sivakumar and Munwar (2008) introduced a target reliability approach for design optimization of retaining walls. Ceranic *et al*. (2001) reported the application of Simulated Annealing (SA) algorithm to minimize the design cost of RC cantilever retaining walls required to resist a combination of earth and hydrostatic loading using geometric design variables.

In addition, Yepes *et al.* (2008) utilized SA to optimize the design of RC cantilever retaining walls used in road construction. Ahmadi and Varaee (2009) proposed an optimization algorithm based on Particle Swarm Optimization (PSO) for finding the optimum design of the retaining walls. Khajehzadeh *et al.* (2010, 2011) developed a modified PSO to optimize the design cost of cantilever RC retaining walls.

Ghazavi and Bazzazian Bonab (2011) applied an Ant Colony Optimization (ACO) algorithm to obtain an optimum design of RC retaining walls. Also, Ghazavi and Salavati (2011) presented a bacterial foraging optimization algorithm for sensitivity analysis and optimization of RC walls.

The harmony search-based algorithms were proposed by Kaveh and Abadi (2010) for the least-cost design of RC walls. Donkada and Menon (2012) applied a Genetic Algorithm (GA) to minimize the design cost of three types of retaining walls: cantilever retaining walls, counterfort retaining walls, and retaining walls with relieving platforms.

Camp and Akin (2012) developed a procedure for designing low-cost or low-weight cantilever RC retaining walls using the big-bang-big-crunch algorithm (BB-BC). Kaveh *et al.* (2011) used the heuristic BB-BC for optimizing the seismic design of gravity retaining walls. The design of gravity retaining walls subject to dynamic loading was also optimized by Talatahari et al. (2012) and Talatahari and Sheikholeslami (2014) using the Charged System Search algorithm (CSS), where the Mononobe–Okabe method was used to determine dynamic earth pressures.

Pourbaba *et al.* (2013) optimized the design cost and analyzed the sensitivity of cantilever retaining walls in detail using the chaotic imperialist competitive algorithm. Papazafeiropoulos *et al.* (2013) applied the GA to optimize the design of cantilever retaining walls that are subject to earthquake loading and that respond in a linear elastic way.

In the present paper, a new meta-heuristic algorithm, the so-called hybrid improved firefly algorithm with harmony search (IFA-HS), which is based on the combined concepts of the Firefly Algorithm (FA) and the Harmony Search (HS) technique, is proposed to solve the design problems of RC retaining walls. The main principle of the hybrid IFA-HS is to integrate the HS operators into the FA, thus increasing the diversity of the population and the FA's ability to escape the local optima.

Furthermore, to improve the computational efficiency of the proposed algorithm for tackling the cost optimization problems of RC retaining walls, the so-called Upper Bound Strategy (UBS) is applied. The UBS is a recently proposed strategy for reducing the total number of structural analyses during the design optimization process. In the UBS, the main strategy is to identify those candidate solutions that have no chance of improving the search during the design optimization process and directly exclude them from the structural analysis stage, thus reducing the total computational effort. The UBS mechanism was first proposed by Kazemzadeh Azad *et al.* (2013) for the design optimization of steel frames. They evaluated the effect of the UBS on the computational efficiency of the BB-BC. Moreover, Kazemzadeh Azad and Hasançebi (2013) applied the UBS to reduce the computational cost of the PSO algorithm for structural design optimization.

The coupled IFA-HS and UBS model is a multi-purpose framework and can be extended to tackle other design optimization problems associated with the civil engineering area such as size and topology optimization of structures, water distribution networks optimization, and calibration of hydrological model parameters which are appealing for its practical applications.

The remainder of this paper is organized as follows. In Section 2, the design procedure of retaining walls is described, and the problem is formulated. In Section 3, the IFA-HS and its implementation details are presented. In Section 4, the statistical optimization results and their comparisons with other reported optimizers for considered test problems are given. Finally, conclusions are drawn in Section 5.

2. Formulation of the Optimum Design of RC Retaining Walls

Often, in the construction of buildings or bridges, it is necessary to retain the earth in a relatively vertical position. Considering a retaining wall shown in Fig. 1, typically three failure modes are considered in the analysis of the retaining structure: overturning, sliding, and bearing capacity.

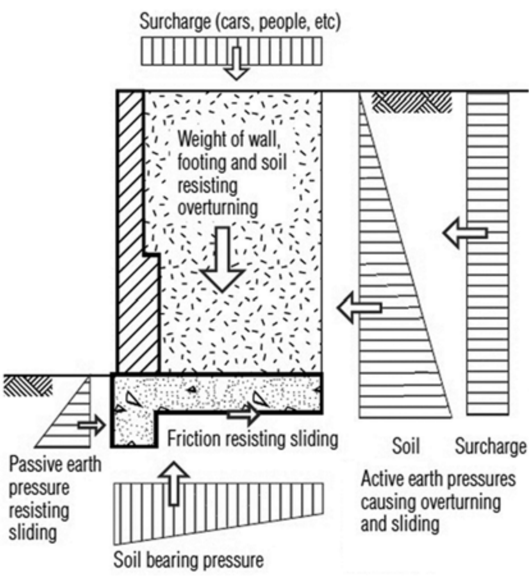

Fig. 1. A Schematic View of a Cantilever Retaining Wall

The overturning moment about the toe of the wall is a balance of the force caused by the active soil pressure of the retained soil weight and the self-weight of the concrete structure, the soil above the base, and the surcharge load. For the sliding mode of failure, only the horizontal component of the active force is considered.

Horizontal resisting forces result from the weight of the wall and soil on the base, surcharge load, friction between the soil and the base of the wall, and passive force owing to soil on the toe and base shear key sections. Under normal conditions, earth pressure at rest is so intense that the wall deflects, relieving itself of this type of pressure, resulting in active pressure.

For this reason, most retaining walls are designed for active pressure due to the retained soil. Moreover, the actual pressure intensity diagram is complex, and it is usual to assume a linear pressure distribution due to active or passive pressure. In this study, both active and passive pressures have been considered in the analysis of retaining wall.

The intensity is assumed to increase with depth as a function of weight of the soil, so that the horizontal pressure of the earth against the wall is often called equivalent fluid pressure. Experience has shown that walls can be safely designed using the approximate forces obtained from mathematical theories such as those of Coulomb and Rankine.

On the other hand, the stem of the wall will bend as a cantilever, so that the tensile face will be toward the backfill. The heel slab of the wall will have net pressure acting downward and will bend as a cantilever, with the tensile face upward. Hence, considering a concrete retaining wall, the four primary concerns relating to the design of these types of walls are as follows (Brooks, 2010):

- An acceptable safety factor with respect to overturning is required.
- The allowable soil bearing pressures should not be exceeded.
- An acceptable safety factor with respect to sliding is required.
- The stresses within the components (i.e., stem and footing) should be within code allowable limits to adequately resist imposed vertical and lateral loads.

These safety factors can be expressed as follows:

Check for overturning:

$$FS_O = \frac{\sum M_R}{\sum M_O} \tag{1}$$

Check for sliding along the base:

$$FS_S = \frac{F_R}{F_d} \tag{2}$$

Check for bearing capacity failure:

$$FS_b = \frac{q_u}{q_{\max}} \tag{3}$$

where F_R = Sum of the horizontal resisting forces
F_d = Sum of the horizontal driving forces
q_u = Ultimate bearing capacity
q_{max} = Maximum bearing pressure
ΣM_O = Sum of the moments of forces that tend to overturn about the toe
ΣM_R = Sum of the moments of forces that tend to resist overturning about the toe

The optimum design cost of a RC cantilever retaining wall is proposed to be determined using the minimum costs of concrete and steel reinforcement. Therefore, objective function can be expressed as follows:

$$Cost = C_1 \times V_{conc} + C_2 \times W_{steel} \tag{4}$$

where V_{conc} and W_{steel} are the volume of concrete (m^3/m) and the weight of steel reinforcement in units of length (kg/m), respectively; C_1 is the cost of the concrete (unit/m^3) and C_2 is the cost of the steel (unit/kg). It is worth mentioning that the weight of steel reinforcement is calculated per unit length (kg/m). Therefore, the length of reinforcement is kept constant for the illustrated examples in the following sections.

As mentioned in Section 1, the optimum design of cantilever retaining walls is formulated as a constraint problem. These constraints may be categorized into four groups, namely, stability, capacity, reinforcement configuration, and geometric limitations. Feasible retaining wall designs should provide minimum factor of safety coefficients for overturning, sliding, and bearing capacity failure modes. These constraints are defined as follows:

$$FS_O \geq 1.5, FS_S \geq 1.5, FS_b \geq 3, \frac{M_u}{\phi_b M_n} \leq 1, \text{and } \frac{V_u}{\phi_v V_n} \leq 1 \tag{5}$$

where FS_o, FS_s, and FS_b are the safety factors against overturning, sliding, and bearing capacity, respectively; M_u and V_u are the design moment and shear strength in the stem, toe, or heel of the retaining wall, respectively; M_n and V_n are the flexural and shear nominal strengths, respectively, and φ is the strength reduction factor (φ_b = 0.9 and φ_v = 0.85).

In this research, shears and moments (demands and nominal capacities) are calculated based on *ACI 318-05* codes (ACI, 2005). The mment capacity of any RC wall section (stem, toe, or heel) should be greater than the design moment of the structure. Similarly, shear capacities of wall sections should be greater than the design shear forces. The flexural strength and shear strength are calculated, respectively, as given (chapters 10 and 11 of ACI 318-05):

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \tag{6}$$

$$V_n = 0.17 bd\sqrt{f_c'} \tag{7}$$

where A_s is the cross-sectional area of steel reinforcement, f_y is the yield strength of steel, d is the distance from compression surface to the centroid of tension steel, a is the depth of stress block, f_c' is the compression strength of concrete, and b is the width of the section.

The design variables for the RC retaining wall are shown in Fig. 2. These variables are categorized into two groups: the

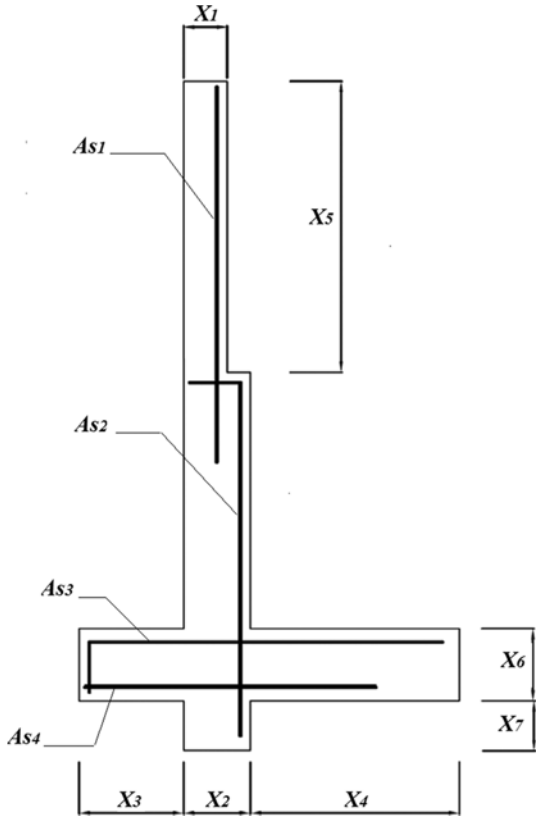

Fig. 2. Design Variables of a RC Retaining Wall

geometric variables that prescribe the dimensions of the wall cross section (X_j, j = 1,…,7), and those that are related to the steel reinforcement (A_{si}, i = 1,2,3,4). In total, there are eleven design variables. Here the possible range of design variables can be determined according to the specific case studies. However, there are also some useful suggestions for quantifying the maximum and minimum allowable limits of the X_j and $A_{si.}$ (Brooks, 2010).

An optimization algorithm initiates the design process by selecting random values for the design variables. Then, the algorithm checks the wall for stability, and if the dimensions satisfy the stability criteria, the algorithm calculates the required reinforcement and checks the strength.

In this procedure, choosing the design parameters that fulfill all of the design requirements and that incur the lowest possible cost is the primary concern. To handle the constraints, a penalty approach is utilized. In this method, the aim of the optimization is redefined by introducing a penalized cost function as given in the following equation:

$$F_{cost} = Cost\left(1 + \sum_{k=1}^{n} \max(0, g_k)\right)^{\psi} \quad (8)$$

where,

$$g_k = \begin{cases} g_1 = \dfrac{1.5 - FS_O}{1.5} \le 0, g_2 = \dfrac{1.5 - FS_S}{1.5} \le 0, g_3 = \dfrac{3 - FS_b}{3} \le 0 \\ g_4 = M_u - \phi_b M_n \le 0, \text{ and } g_5 = V_u - \phi_v V_n \le 0 \end{cases} \quad (9)$$

and ψ is a penalty constant, g_k is the amount of violation of the k^{th} constraint, and n is the total number of constraints (in this paper, $n = 5$).

In the first steps of the search process, ψ is set to 1.5 and ultimately increased to 5. Therefore, each of the design constraint is posed as a penalty on the overall objective function of the design and is nonzero only when violated. In other words, if the design is feasible, the sum of the constraint penalties will be zero (see Eq. (8)). In fact, this approach has the feature of allowing highly infeasible solutions early in the search (smaller values for ψ), while continually increases the penalty imposed (higher values for ψ) to eventually move the final solution to the feasible region.

In the first iterations, if large values of ψ are selected, the solutions tend to narrow the search space to feasible designs that are expensive than the optimal design and reduce the exploration of the solution space. However, within the last iterations, if ψ has a small value, the solutions have an undesirable tendency to converge to infeasible least-cost designs that have a very small penalty. Therefore, setting a large value for ψ for last iterations may help to prevent convergence to infeasible designs.

3. Hybrid IFA and HS

3.1 A Brief Review of FA and HS Algorithms

To begin with, since the proposed IFA-HS involves the FA and HS algorithms, a brief background to the main mechanisms of both methods is provided in the following sections.

3.1.1 Firefly Algorithm

Among phenomenon-mimicking methods, algorithms inspired from the collective behavior of species such as ants, bees, wasps, termites, fishes, and birds are referred to as swarm intelligence-based algorithms. Recently, Yang (2009) proposed the FA as a novel SI algorithm that mimics the natural behavior of fireflies.

Various applications of the FA in various research areas have been reported. Gandomi *et al.* (2011) used a FA-based approach for solving mixed continuous/discrete structural optimization problems. Their study revealed the efficiency of the FA in the field of structural optimization.

Gomez (2011) employed the FA for sizing and optimizing the shape of truss structures with dynamic constraints. Kazemzadeh Azad and Kazemzadeh Azad (2011) developed an improved FA (IFA) algorithm for optimizing the design of planar and spatial truss structures using both sizing design and shape design variables and reported promising results.

As mentioned earlier in this section, the FA is a nature-inspired heuristic search technique based on the natural behavior of fireflies. According to Yang (2009), to develop the FA, the natural flashing characteristics of fireflies have been idealized using the following three rules:

1) All fireflies are unisex, therefore, one firefly will be attracted to the other fireflies regardless of their gender.
2) The attractiveness of each firefly is proportional to its bright-

ness; thus, for any two flashing fireflies, the less bright firefly will move toward the brighter one. The attractiveness is proportional to the brightness, and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.

3) The brightness of a firefly is determined according to the nature of the objective function.

The attractiveness of a firefly is determined by its brightness or light intensity, which is obtained from the objective function of the optimization problem. However, the attractiveness β, which is related to the judgment of the beholder, varies with the distance between two fireflies. The attractiveness β can be defined by the following equation:

$$\beta = \beta_0 \exp(-\gamma r^2) \tag{10}$$

where r is the distance of two fireflies, β_0 is the attractiveness at $r = 0$, and γ is the light absorption coefficient. The distance between two fireflies i and j at x_i and x_j, respectively, is determined as follows:

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \tag{11}$$

where $x_{i,k}$ is the k^{th} parameter of the spatial coordinate x_i of the i^{th} firefly and d is the dimensionality of the search space. In the standard FA, the new position of a firefly i, x_i^{new}, toward a more attractive (i.e., brighter) firefly j is determined by the following equation:

$$x_i^{new} = x_i + \beta_0 \exp(-\gamma r_{ij}^2)(x_j - x_i) + \alpha \varepsilon_i \tag{12}$$

where the second term in Eq. (12) is related to the attraction, and the third term is randomization with the vector of random variables, ε_i, generated by a normal distribution, and α is a scaling parameter. Typically, $\alpha = [0.01L, 0.1L]$ is sufficient for most applications where L is the scale of variables (Yang, 2010).

For the movement stage of the improved version of the FA, the following equation is used (Kazemzadeh Azad and Kazemzadeh Azad, 2011):

$$x_i^{new} = x_j + \beta_0 \exp(\gamma r_{ij}^2)(x_j - x_i) + \alpha \varepsilon_i \tag{13}$$

In the standard FA, the movement of a firefly i toward a brighter firefly j is determined by Eq. (12). Since x_j is brighter than x_i, in Eq. (13), instead of moving firefly i toward j, searching the vicinity of firefly j, which is a more reliable area, is proposed for updating the position of firefly i based on the current position of firefly j. To do this, x_i is replaced by x_j, and Eq. (13) is implemented for the movement stage of the IFA. In Eq. (13), the mean value of the normal distribution for generating ε_i is set to zero, and the standard deviation is taken as the standard deviation of the k^{th} parameter of the spatial coordinate x_i of all fireflies in each generation.

In the IFA, the position of a firefly is updated only if the new position found is better to be than the previous one, to avoid missing the brighter fireflies of the population. Therefore, during optimization, each candidate design will be replaced only with a better design. It is clear that Eq. (13) may generate fireflies outside the bounds of the design variables. To avoid this problem, the parameters of fireflies that are not created within the bounds of the design variables are rounded into the boundary values.

3.1.2 Harmony Search Algorithm

The HS method is another optimization algorithm inspired by the working principles of musical harmony improvisation (Geem *et al*., 2001). Similar to other nature-inspired approaches, the HS is a random search technique. It does not require any prior domain knowledge such as gradient information of the objective function.

However, unlike population-based approaches, it utilizes only a single search memory to evolve. Therefore, the HS method has the distinctive feature of algorithm simplicity (Geem *et al*., 2001; Kim *et al*., 2001). The HS is a metaheuristic search technique without the need for derivative information and with reduced memory requirements. In comparison with other metaheuristic methods, the HS is computationally effective and easy to implement for solving various types of engineering optimization problems (Im *et al*., 2013; Yoo *et al*., 2014). There are four principal steps used in the HS as follows:

Step 1: Initialize a Harmony Memory (HM). The initial HM consists of a certain number of randomly generated solutions for the optimization problem under consideration. For an *n*-dimension problem, a HM with the size HMS can be represented as follows:

$$\mathbf{HM} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_n^1 \\ x_1^2 & x_2^2 & \cdots & x_n^2 \\ \cdots & \cdots & \cdots & \cdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_n^{HMS} \end{bmatrix} \tag{14}$$

where $(x_1^i, x_2^i, \ldots, x_n^i)$, ($i$ = 1, 2, .., HMS) is a candidate solution. HMS is typically set to be between 10 and 100.

Step 2: Improvise a new solution (x'_1, x'_2, x'_n) from the HM. Each component of this solution, x'_j, is obtained based on the Harmony Memory Considering Rate (HMCR). The HMCR is defined as the probability of selecting a component from the HM members, and 1-HMCR is, therefore, the probability of generating it randomly.

If x'_j comes from the HM, it can be further mutated according to the Pitching Adjust Rate (PAR). The PAR determines the probability of a candidate from the HM for mutation.

Step 3: Update the HM. First, the new solution from Step 2 is evaluated. If it yields a better fitness than that of the worst member in the HM, it will replace that one. Otherwise, it is eliminated.

Step 4: Repeat Steps 2 and 3 until a termination criterion (e.g., maximum number of iterations) is met.

The usage of the HM is important because it ensures that good harmonies are considered as elements of new solution vectors. To use this memory effectively, the HS algorithm adopts a parameter HMCR ∈ (0, 1), called the harmony memory considering (or accepting) rate.

If this rate is too low, only a few elite harmonies are selected, and it may converge too slowly. If this rate is extremely high (i.e., near one), the pitches in the harmony memory are mostly used, and other ones are not explored well, so that good solutions are missed. Therefore, typically, the appropriate values for the HMCR may vary from 0.7 to 0.95 (Yang, 2010b).

Note that a low PAR with a narrow bandwidth (*bw*) may slow down the convergence of HS because of the limitation of the exploration of only a small subspace of the whole search space. On the other hand, a very high PAR with a wide *bw* may cause the solution to scatter around some potential optima as in a random search. Furthermore, large PAR values with small *bw* values usually improve the best solutions in final generations.

3.2 IFA-HS Method for Optimizing the Design Cost of RC Walls

3.2.1 Proposed IFA-HS

The hybrid IFA-HS combines the optimization capabilities of the HS and IFA. In the HS, the diversification is controlled by random selection. Random selection explores the global search space more widely and efficiently, and the pitch adjusting operator ensures that the new solution is sufficiently worthy and near existing good solutions.

The intensification in the HS is controlled by memory consideration, guiding the search process toward the search space of good solutions (Geem *et al.*, 2001). Moreover, the use of the HM in HS allows the selection of the best vectors, which may represent different regions in the search space.

On the other hand, the disadvantages of the basic FA are premature convergence and sometimes not obtaining efficacious experiences among solutions in a population. To obtain a high-quality solution, the strategies given in Sections 3.1.1 and 3.1.2 are combined. Since the FA is memory-less, no information is extracted dynamically during the search, whereas the hybrid IFA-HS uses a memory that contains some information extracted online during the search.

In the other words, some history of the search stored in the memory can be used in the generation of the candidate list of solutions and in the selection of the new solution. Using the standard configuration of the IFA, the new harmonies generated are based on the newly generated firefly each iteration after the firefly's position has been updated. The updated harmony vector substitutes the newly generated firefly only if it has better fitness (cost).

This selection scheme is rather greedy and often outperforms the standard HS and FA. The proposed IFA-HS involves two phases of optimization: (*i*) the IFA using a heuristic search technique and (*ii*) the HS algorithm using memory consideration, random selection, and pitch adjustment. The framework of the proposed hybrid algorithm is illustrated in Fig. 3.

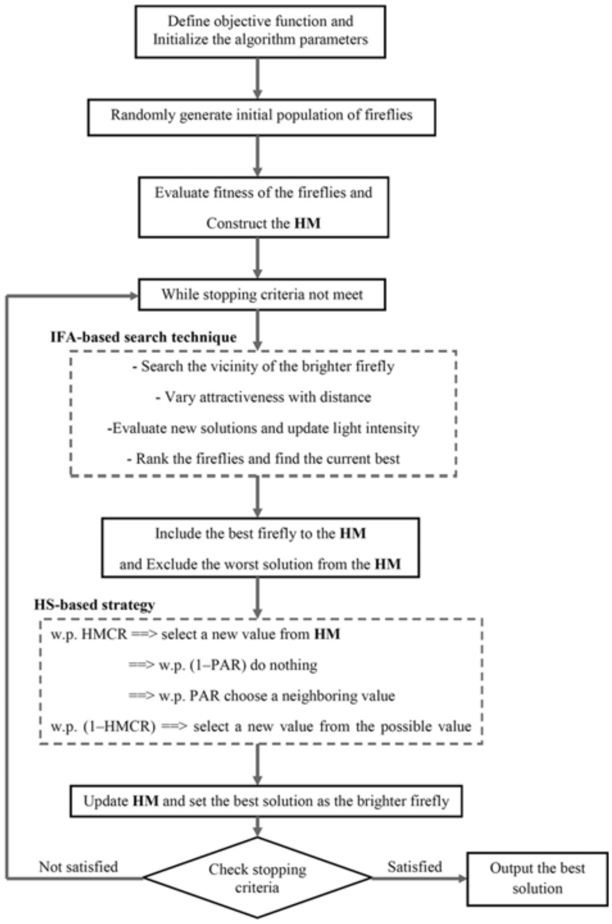

Fig. 3. Flowchart of the Proposed IFA-HS Algorithm

The hybrid IFA-HS has another beneficial feature. It iteratively explores the search space by combining multi-search space regions to visit a single search space region. The IFA-HS iteratively recombines the characteristics of many solutions to make one solution.

The IFA-HS is able to fine-tune obtained solution to which the algorithm converges using neighborhood structures. Throughout the process, recombination is represented by memory consideration, randomness by random consideration, and neighborhood structures by pitch adjustment and variation of firefly attractiveness. Therefore, the IFA-HS has the advantage of combining key components of population-based and local search based methods in a simple optimization model.

3.2.2 Incorporating the UBS Into the IFA-HS (IFA-HS-UBS)

In the UBS, excessive structural analyses are avoided during the optimization by using a simple and efficient mechanism. The main concern is to recognize those candidate solutions that have no chance of improving the search during the optimization. After identifying those non-improving solutions, they are excluded from the structural analysis stage, thus reducing the total amount of computational labor (Kazemzadeh Azad and Hasançebi, 2013).

Considering the standard UBS, the penalized cost of a current solution (i.e., a firefly) can be considered as the upper bound limit for the net cost of a newly generated candidate solution. Accordingly, a new candidate solution with a net cost greater than this limit can be excluded from the structural analysis stage.

Thus, after a new firefly is generated, first, only the net cost of the newly generated firefly is calculated, not the penalized cost. This computation is straightforward and can be accomplished with a trivial amount of computational effort. If this firefly has a net cost smaller than or equal to the penalized cost of the current firefly's best objective function, the structural analysis of the sampled firefly is processed, and its penalized cost is computed.

In the opposite case, however, the upper bound rule is activated, and the firefly is automatically excluded from the structural analysis stage because such a candidate solution is unlikely to improve the current firefly's best vector (Kazemzadeh Azad and Hasançebi, 2013; Kazemzadeh Azad *et al*., 2013). A brief description of the steps in the implementation of the IFA-HS model for the design optimization of RC walls is given as follows:

Step 1: Generate N (N = *popsize*) populations of fireflies randomly in the solution space. Each of the N population represents a possible combination of design variables.

Step 2: Compute the wall cost (see Eq. (4)) for each of the N solutions.

Step 3: Implement the UBS and exclude non-improving fireflies from the structural analysis.

Step 4: Call the RC retaining-wall analyzer and update the input file (the design variables are changed).

Step 5: Perform the analysis of each wall.

Step 6: Check the wall for stability and strength, regardless of whether these dimensions satisfy the criteria, and then compute the penalty function.

Step 7: Calculate the penalized cost (F_{cost}) using the wall net cost and the penalty found in Step 6 (see Eq. (8)).

Step 8: The total cost found in Step 7 is utilized as the fitness value for each of the trial designs.

4. Design Examples and Optimization Results

In this section, two numerical examples are optimized using the proposed hybrid method. The final optimization results of the IFA-HS are compared with the solution obtained from the other standard algorithms, to demonstrate the performance of the proposed approach.

For the proposed hybrid algorithm, a population size of 100 and Harmony Memory Size (HMS) of 70 were used. The HS parameters were set to HMCR = 0.95 and PAR = 0.35 for reported examples.

The maximum number of function evaluations was 10,000. Note that for these parameters, the IFA-HS exhibited good performance in terms of solution quality and required a reasonably small computational overhead.

Sensitivity of optimal responses for user parameters is one of the important issues in the optimum cost design of RC walls. As a result, here a sensitivity analysis is performed for the internal parameters of the IFA-HS algorithm. The algorithm parameters used in this study include HMS, HMCR, PAR, and *popsize* (population size). In order to avoid the possible randomness of the search process due to the use of different initial solutions, the first example is solved 20 times for different parameter configurations. The considered optimizers were implemented in MATLAB.

4.1 Example I

To check the performance of the proposed hybrid method, a retaining wall studied by Saribaş and Erbatur (1996) is considered. The details of this wall and other necessary input parameters are listed in Table 1. Note that all of the values listed in this table are for a unit length of the wall.

Table 1. Input Parameters for the Example *I*

Parameter	Value	Unit
Height of stem	4.5	m
Yield strength of reinforcing steel	400	MPa
Compressive strength of concrete	21	MPa
Surcharge load	30	KPa
The angle of wall friction	15	Degree
Internal friction angle of retained soil	36	Degree
Internal friction angle of base soil	34	Degree
Unit weight of retained soil	17.5	kN/m^3
Unit weight of base soil	18.5	kN/m^3
Unit weight of concrete	23.5	kN/m^3
Cohesion of base soil	100	KPa
Depth of soil in front of wall	0.75	m
Cost of steel	0.40	$/kg
Cost of concrete	40	$/m^3

Table 2. Comparison of Statistical Results for the Example *I* using Different Optimizers "N/A" Stands foR Not Available

Design Variables	IFA-HS-UBS	ACO	IFA	HS
X_1 (m)	0.25	0.25	0.25	0.25
X_2 (m)	0.37	0.25	0.36	0.35
X_3 (m)	1.18	1.14	1.18	1.20
X_4 (m)	1.70	1.38	1.80	1.80
X_5 (m)	2.82	4.50	2.82	2.82
X_6 (m)	0.45	0.40	0.45	0.45
X_7 (m)	0.30	–	0.30	0.30
A_{s1} (cm^2)	28.85	29.50	29.0	30.0
A_{s2} (cm^2)	30.34	29.50	32.0	32.0
A_{s3} (cm^2)	13.98	14.00	15.33	15.31
A_{s4} (cm^2)	13.47	14.00	13.50	13.50
Minimum cost ($/$m$)	186.438	201.185	189.627	189.976
Average cost ($/$m$)	204.667	N/A	207.219	211.434
Maximum cost ($/$m$)	220.892	N/A	229.728	229.286
Standard deviation	3.748	N/A	5.135	4.797
No. of analyses	4,200	N/A	6,700	4,700

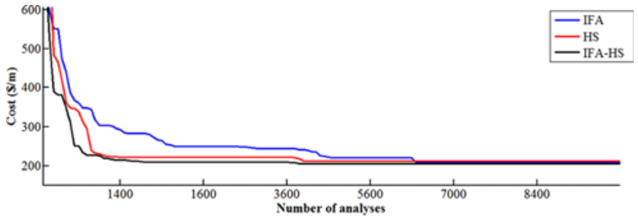

Fig. 4. Convergence History Obtained by the IFA-HS, IFA, and HS (Example *I*)

The obtained results of the cost optimization design for the IFA-HS, HS, IFA, and ACO (Ghazavi and Bazzazian Bonab, 2011) algorithms are summarized in Table 2. As listed in Table 2, the minimum cost for IFA-HS is 186.438 ($/m) (with 4,200 function evaluations), whereas the best costs for the ACO and Sariba° and Erbatur (1996) are 201.185 and 189.546 ($/m), respectively.

Thus, the optimum cost of the proposed method is 7.33% and 1.64% lower than those of ACO and Saribaş and Erbatur (1996), respectively. Since the single IFA-HS and the IFA-HS with UBS (IFA-HS-UBS) use the same formulation for the search procedure, the optimum designs reported for the IFA-HS-UBS are valid for the single IFA-HS as well.

However, the number of structural analyses required to reach the optimum cost will be different as a result of employing the UBS in the former algorithm. Here, the number of structural analyses performed in the IFA-HS-UBS is calculated by counting candidate designs that undergo structural analysis.

For this example, the number of structural analyses performed by the IFA-HS is 7,350. However, when the UBS is employed, it is found that, in fact, 4,200 structural analyses are required in the optimization process. This indicates that the number of saved structural analyses using the IFA-HS-UBS is 3,150 in this example (see Table 2 for the other optimizers).

Figure 4 demonstrates comparisons of convergence rate among considered optimizers. By judging Fig. 4, it can be observed that the proposed hybrid method has the advantage of fast convergence compared with the other optimization methods.

4.1.1 Sensitivity Analyses

The main objective of this section is to evaluate and study the effects of each initial parameter used in the hybrid IFA-HS. One of the most important concerns related to optimization algorithms is to find the most efficient user parameters. Performing sensitivity analyses show the stability of methods and importance level of initial parameters to find the optimal solution against any changes in user parameters.

Especially, when the problem is complex having many local optima, it is crucially important to use the proper and efficient user parameters to boost the convergence speed and efficiency of the algorithms for finding global solution without getting trapped in local optima. In order to further clarify the setting of the initial parameters for the IFA-HS, Example *I* is selected.

In order to tuning the internal parameters for the proposed IFA-HS applied to Example *I*, a sensitive study on two parameters is performed, while fixing other parameters (HMS = 50 and HMCR = 0.9). For various values of *popsize* and *PAR*, this example is launched several times (20 times for each value of *popsize* and *PAR*) and the average design costs are shown in Table 3.

Table 3. Sensitivity Analysis of Optimization Results for Various *PAR* and *popsize* Values for the Example *I*

PAR	*popsize*	Average Cost ($/m)	No. of analyses
0.10	50	220.347	9,850
0.20	70	218.012	8,200
0.25	80	215.032	7,000
0.35	100	205.136	5,050
0.45	150	210.971	5,000
0.55	200	204.721	4,250
0.85	250	205.074	4,180

Table 3 shows that when the values of PAR and *popsize* increase, the optimum cost of RC walls decreases. From Table 3, it can be concluded that, small values of PAR and *popsize* can cause poor performance of the algorithm and considerable increase in computational effort needed to find optimum solution.

On the other hand, a very large *PAR* may cause the solution to scatter around some potential optima as in a random search. As shown in the table, *PAR* = 0.35 and *popsize* = 100 are suitable values for the IFA-HS algorithm.

The result of sensitivity analysis of *HMS* and *HMCR,* while fixing other parameters (*popsize* = 100 and *PAR* = 0.35) are represented in Table 4. As shown in Table 4, the IFA-HS algorithm with a larger *HMS* and *HMCR* performed better for this case study.

The best average cost is obtained when the HMS is set to 60 and 70, and the *HMCR* is equal to 0.93 and 0.95. However, they require different computational effort to reach the same final solution as shown in Table 4. Therefore, considering both the solution quality and efficiency, the HMS of 70 and HMCR of 0.95 were selected as the best internal parameters.

It is worth pointing out that different types of problem need different parameter settings. Therefore, for different problems of interest (different scenarios), user may need to fine tune initial parameters. This can be considered as one of the disadvantages of metaheuristic optimization algorithm.

Table 4. Sensitivity Analysis of Optimization Results for Different Values of *HMS* and *HMCR* for the Example *I*

HMS	HMCR	Cost ($/m)	No. of analyses
30	0.80	224.012	9,100
40	0.85	218.945	7,150
50	0.90	205.136	5,050
60	0.93	204.641	4,850
70	0.95	204.667	4,200

4.2 Example II

For further validation of the developed optimization method, another example is considered, and optimization results are compared with IFA and HS algorithm to demonstrate the efficiency of the IFA-HS and the effectiveness of the UBS mechanism. A wall with a height of 5.5 *m* is considered in this example. Other specifications for the design of this retaining wall are presented in Table 5.

Table 6 shows the statistical optimization results obtained and the number of function evaluations (analyses) required for convergence in the IFA-HS compared with the standard IFA and HS. The IFA-HS found the best feasible solution with the value of 255.470 ($/m) after 4,800 analyses (i.e., function evaluations), whereas the IFA and HS found the best solutions of 264.175 and 273.430 ($/m) after 7,600 and 5,400 analyses, respectively.

The optimum cost of the IFA and HS is 3.29% and 6.57% more expensive than the optimum cost obtained by the proposed IFA-HS. In addition, in terms of other statistical results including average and maximum costs, the proposed hybrid method is superior over other reported optimizers.

Table 5. Input Parameters for the Example *II*

Parameter	Value	Unit
Height of stem	5.5	m
Yield strength of reinforcing steel	400	MPa
Compressive strength of concrete	21	MPa
Surcharge load	25	KPa
The angle of wall friction	10	Degree
Internal friction angle of retained soil	36	Degree
Internal friction angle of base soil	0	Degree
Unit weight of retained soil	17.5	kN/m^3
Unit weight of base soil	18.5	kN/m^3
Unit weight of concrete	23.5	kN/m^3
Cohesion of base soil	120	KPa
Depth of soil in front of wall	0.75	M
Cost of steel	0.40	$/kg
Cost of concrete	40	$\$/m^3$

Table 6. Optimal Design Comparison for the Example *II*

Design Variables	IFA-HS-UBS	IFA	HS
X_1 (*m*)	0.25	0.25	0.25
X_2 (*m*)	0.45	0.45	0.47
X_3 (*m*)	2.00	2.10	2.12
X_4 (*m*)	1.50	1.44	1.85
X_5 (*m*)	3.20	3.20	3.20
X_6 (*m*)	0.50	0.50	0.50
X_7 (*m*)	0.25	0.35	0.25
A_{s1} (cm^2)	38.0	36.00	30.00
A_{s2} (cm^2)	30.0	40.00	40.00
A_{s3} (cm^2)	15.22	15.50	16.00
A_{s4} (cm^2)	16.00	16.00	16.10
Minimum cost ($/m)	255.470	264.175	273.430
Average cost ($/m)	268.825	275.784	282.102
Maximum cost ($/m)	276.011	288.172	286.985
Standard deviation	4.561	6.284	5.848
No. of analyses	4,800	7,600	5,400

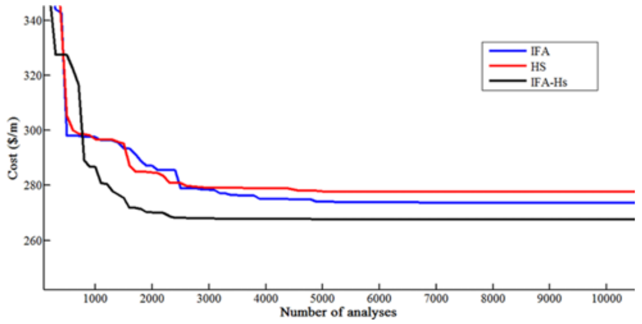

Fig. 5. Convergence History Obtained by the IFA-HS, IFA, and HS for Example *II*

The best obtained solutions are highlighted in bold in Table 6 for the Example *II*. The convergence histories for the average results obtained from the IFA-HS, IFA, and HS are presented in Fig. 5. The cost reduction history given in Fig. 5 demonstrates the effectiveness and efficiency of the proposed hybrid method.

5. Conclusions

In this study, a new hybrid metaheuristic algorithm, the so-called improved firefly algorithm with harmony search scheme (IFA-HS), is proposed to optimize the design cost of Reinforced Concrete (RC) retaining walls based on the combined concepts of the FA and HS schemes. The obtained numerical optimization results indicate that the new hybrid algorithm is an appropriate method for solving RC retaining wall design problems.

The main principle of the hybrid IFA-HS is the integration of the HS operators into the FA, thus increasing the diversity of the population and the ability of the FA to escape the local optima. Practically, harmony memory vectors become a FA population, and then the evolution is accomplished in the form of the usual IFA procedure. Another improvement in the IFA-HS is the addition of pitch adjusting operation in the FA to fine-tune the solution using neighborhood structures. Moreover, to improve the computational efficiency of the proposed hybrid method, the recently developed Upper Bound Strategy (UBS) was incorporated into the IFA-HS (IFA-HS-UBS).

The optimization results reveal that the IFA-HS-UBS found the best solution in a fewer number of function evaluations than the other reported algorithms. This means that the IFA-HS-UBS can be used to find good quality results in a shorter time and clearly indicates that the UBS mechanism has a positive influence on performance of the IFA-HS.

The performance of the IFA-HS was investigated using two case studies, and the optimization results were compared with literature. For the both considered examples, the IFA-HS-UBS surpassed other applied algorithms in terms of statistical results offering minimum cost (i.e., cheaper design) in fewer number of function evaluation (number of analyses).

Therefore, the obtained numerical results demonstrate that the

IFA-HS may be an efficient method for finding the optimum solution for structural optimization problems. Also, a sensitivity analyses was performed for fine tuning internal parameters of the proposed algorithm for each reported examples.

In this study, the design process is limited to obtain the singly reinforcement scenario. However, doubly reinforcement scenario (with or without singly reinforcement scenario) may be considered and a multi-criteria optimization can be applied as a further research.

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