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Assessment of Concrete Compressive Strength Prediction Models

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Abstract

Theoretical and phenomenological models for predicting the compressive strength of concrete proposed in literature have been reviewed. The applicability and accuracy of these models in predicting the compressive strength of concrete at 28 days and at different ages were investigated using experimental data reported in the literature. Assessment of the models using visual display and numerical methods have revealed that the Average Paste Thickness (APT) compressive strength model, which accounts for the type of cement, cement degree of hydration, mixture proportions, aggregate proportions, gradations, packing density, and air content, provides the highest predictability at 28 days and at different ages. The assessment revealed that the majority of models give acceptable predictions because strength is mostly affected by water-to-cement ratio (w/c) in comparison to aggregates' properties and gradation.

Keywords: *concrete, compressive strength, models, water-cement ratio, mixture design, packing density*

1. Introduction

Abrams (1919) was one of the pioneers to recognize the significance of concrete compressive strength and proposed to use water-to-cement ratio (w/c) to quantify its value. Since then, it has been shown that there are other parameters that contribute to the compressive strength of concrete, namely cement properties, bonding strength/gradation of aggregates, mixture proportions, chemical admixtures and mineral admixtures (de Larrad, 1999). Cement's chemical and physical properties contribute greatly to the compressive strength of concrete. The composition and proportions of its major chemical phases (C_3S, C_2S, C_3A, C_4AF) affect the cement hydration reactions and products formed, which in turn affect the strength of the cement paste (Mehta and Monteiro, 2006). Physical properties particularly cement fineness and particle size distributions affect concrete strength (Kolias and Georgiou, 2005; Mehta and Monteiro, 2006). Greater cement fineness increases the surface available for hydration, causing greater early compressive strength and more rapid generation of heat (Kolias and Georgiou, 2005). Research has also shown that the particle size distribution of cement is of equal importance.

The aggregate bonding strength is influenced by the size, shape and surface roughness of aggregates. Larger aggregates reduce the specific surface area of the aggregates which leads to a reduction in bond strength (Quiroga, 2003). Crushed and rough aggregates provide a higher aggregate to paste bonding strength in comparison to round and smooth aggregates (Quiroga, 2003). Research has shown that mixtures with aggregate gradation that is uniformly distributed lead to higher packing which results in concrete with higher density, reduced porosity, and higher compressive strength (Quiroga, 2003).

Concrete mixture proportions such as cement content, w/c, aggregate volume fraction and fine-to-coarse aggregate ratio (FA/CA) also influence the compressive strength of concrete (Kolias and Georgiou, 2005; Basheer *et al.*, 2005). An increase in the w/c due to an increase in the water content or due to a reduction in the cement content can dilute the cement paste increasing the volume and thus reducing the compressive strength (Kolias and Georgiou, 2005). The higher the aggregate volume fraction, the longer is the crack path, which results in an increase in the absorbed energy and thus an increase in the compressive strength (Kolias and Georgiou, 2005). An increase in the FA/CA results in a reduction in w/c due to an increased absorption of mix water by sand, which results in an increase in the compressive strength (Basheer *et al.*, 2005).

Chemical and mineral admixtures have been shown to influence the compressive strength of concrete. The use of an air entraining agent results in an increase in porosity and a reduction in strength (Neville, 1981). The addition of mineral admixtures such as fly ash, blast furnace slag and silica fume, have been shown to enhance strength due to their pozzolanic properties (Neville, 1981).

Because compressive strength of concrete is a design requirement, various models have been proposed in the literature to predict it using concrete mixtures (de Larrad, 1999). The majority

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of these models are phenomenological models and follow either Feret's postulation (1892) or Abram's (1919). An overview of the mathematical laws that govern the different models are presented in the next section. This paper provides a quantitative assessment of the models' predictive capabilities using experimental data reported in the literature.

2. Concrete Compressive Strength – A Brief Overview

Compressive strength models are developed from a conceptual postulation of how particles and hydrated cement interact and bond. Accordingly, controlling parameters were identified in the development of these models. This section presents the evolution of these models in brief.

Feret was the first to propose a model for predicting concrete compressive strength. It postulates that strength is a function of the ratio of cement to paste and air (Feret, 1892) as given in Eq. (1).

$$f_c' = A\left(\frac{V_c}{V_c + V_w + V_a}\right)^B \tag{1}$$

Experimental data have shown poor agreement with this ratio (Popovics, 1985). In addition, the coefficients of proportionality need to be determined by trial mixes, which has limited its use (Popovics, 1985). Abrams (1919) proposed a replacement of the ratio of cement to paste and air with water to cement ratio because it is simpler and it gave better correlation with strength data for non-air entrained concrete (Popovics, 1985) as given in Eq. (2).

$$f_c' = \frac{A}{B^{w/c}} \tag{2}$$

However, the model is not complete because different coefficients of proportionality values are needed whenever any factor affecting the strength of concrete changes. The coefficients of proportionality parameters depend on cement type and strength, aggregate gradations and proportions, admixtures, curing conditions, testing conditions, and age of concrete.

Powers postulated that the compressive strength of cement paste is directly related to the gel to space ratio, X, which is the ratio of the gel volume to the volume of capillary porosity (Powers and Brownyard, 1960). The gel space ratio equals the volume of hydration products divided by the sum of the volume of hydration products and capillary porosity. The final form of the model after fitting using Feret's relationship is given in Eq. (3).

$$f_c' = A(X)^B = A\left(\frac{0.66\alpha}{\frac{w+V_a}{c} + 0.32\alpha}\right)^B \tag{3}$$

The effect of air is considered in practice as an extra volume of mixing water (Karni, 1974). Powers was the first to introduce degree of hydration (α) to the strength model. However, his model fails to account for other factors such as aggregates properties, standard cement strength and assumes strength starts developing at zero age. Karni (1974) studied Powers model and then postulated that the volume of the gel should be related to the total volume of the paste (cement + water) minus cement. This was recommended to avoid incorporating the capillary pores fraction which depends on the factors affecting the compressive strength i.e. *w/c* and α, as given in Eq. (4).

$$f_c' = A\frac{\alpha \times 100}{1.53\left(\frac{w+V_a}{c}\right) + \frac{\alpha}{2.06}} - B \tag{4}$$

Power's law requires knowing the value of α. Tango (2000) worked on simplifying Powers law by substituting for α. Function of transformed time was proposed to account for initial hydration and derivative of initial hydration as given in Eq. (5).

$$f_c' = \frac{A}{B^{\left(\frac{w}{c}\right)} E^{\left(\frac{w}{c}\right)t^{-0.5}} D^{t^{-0.5}}} \tag{5}$$

Although the model is simpler to apply, two calibration parameters were added thus limiting the application of the model. In addition, the model fails to account for cement and aggregate properties.

In 1985, Popovics modified Bolomey's model, which is a linear function of w/c, by incorporating the effect of air empirically as given in Eq. (6).

$$f_c' = A\frac{c}{w} + B \times 10^{C.V_a} \tag{6}$$

In 1998, Popovics worked on generalizing Abrams' model in order to include some of the strength-effecting factors (Popovics, 1998). The mathematical model is based on a cement model, in which the hardening of cement is the sum of two hardening processes of first order reaction as given in Eq. (7).

$$f_c' = \frac{A}{B^{w/c}}\sqrt{\frac{S_s}{S_0}} \cdot \frac{100 - C_3Se^{-b_1t} - (100 - C_3S)e^{-b_2t}}{100 - C_3Se^{-90b_1} - (100 - C_3S)e^{-90b_2}} \times 10^{C.V_a} \tag{7}$$

The model also accounts for air content empirically but fails to consider aggregate properties and gradations. In 2008, Popovics proposed a refinement to Abrams' model by incorporating the cement content parameters after noting that if two comparable concretes have the same *w/c*, the one with more cement has a lower strength (Popovics and Ujhelyi, 2008). In comparison to Abrams model, the revised model accounts for air and cement content empirically as given in Eq. (8).

$$f_c' = \frac{A}{B^{\left(\frac{w}{c} + Ec\right)}} \times 10^{C.V_a} \tag{8}$$

Pann *et al*. (2003) worked on Abrams' model and incorporated empirically the cement paste capillary porosity. Pann stated that capillary porosity, P_c, depends on the degree of hydration and developed a statistical model to predict α at 28 days, which is a function of w/c as given in Eq. (9).

$$f_c' = \frac{A}{B^{w/c}} + \frac{C}{D^{P_c}} \quad (9)$$

Pann *et al*. claimed that the proposed model is superior to that of Abram's because it accounts for the capillary porosity term. The model, however, includes many parameters that need re-calibration when other factors not accounted for by the model are changed. These factors include the properties and gradation of both cement and aggregates.

de Larrard (1999) postulated that if a concrete mixture is subjected to compressive forces then the paste located between the coarse aggregate will be subjected to maximum stress. He then stipulated that the distance between the aggregates is the maximum paste thickness, MPT, as given in Eq. (10).

$$MPT = D_{\max}\left(\sqrt[3]{\frac{\phi_{\max}}{\phi}} - 1\right) \quad (10)$$

The volume fraction of the aggregates, ϕ, is based on the mixture proportions. The maximum packing density, $\phi_{\max}$, is computed using the Compressible Packing Model (CPM) (de Larrard, 1999). Knowing the volume fractions of the aggregate particles from the mixture, their densities, specific gravities, method of compaction (packing index of 4.5 for rodding), and their characteristic diameters, the CPM was employed to calculate $\phi_{\max}$ for the aggregates mixture. de Larrard then revised Feret's model and developed a model that also incorporates the effects of aggregates maximum size and packing density on strength. The effect of age on strength was considered empirically as given in Eq. (11).

$$f_c' = KR_{c28}\left[A\log(t/28) + \left(\frac{V_c}{V_c + V_w + V_a}\right)^B\right]MPT^C \quad (11)$$

His model was later revised to also incorporate the effect of pozzolanic admixtures and limestone llers using empirical relationships (de Larrard, 1999). A longer form of the model was also presented which considers the type of aggregates through two additional coefficients, the bond effect coefficient 'p' and the ceiling effect coefficient 'q' that require calibration for any given aggregates. de Larrard's model, which was a signicant improvement to current strength models, did not account for the chemical properties of cement and their degree of hydration. Mechling *et al*. (2009) revised de Larrad model and incorporated the effect of the C_3S content on the strength of hydrated cement paste as given in Eq. (12).

$$f_c' = KR_{c28}(0.2.[C_3S] - 1.65)\left[A\log(t/28) + \left(\frac{V_c}{V_c + V_w + V_a}\right)^B\right]MPT^C \quad (12)$$

Chidiac *et al*. (2013) postulated that particles interact according to excess paste theory, which states that concrete may be considered a mixture of aggregate and cement paste and that the cement paste in excess of the amount needed to fill up the voids between the aggregate particles disperses the particles and lubricates the concrete mixture (Wong and Kwan, 2008). A mathematical model for the average paste thickness, *APT*, was developed and was found to account for concrete mixtures and aggregate gradation through packing density. The proposed model consists of two fundamental models; cement hydration model and the *APT* model. Moreover, the proposed model accounts for the paste to aggregate bond strength, cement properties and strength at 28 days, degree of hydration, the amount of water filled capillary pores, aggregate properties and its gradation, mixture proportions, porosity, age, and air entrainment as given in Eqs. (13) and (14).

$$f_c'(t) = \begin{cases} 0 & \alpha(t) \le \alpha_{cr} \\ KR_{c28}\left(\frac{APT}{D}\right)^A B^{\frac{w+V_a}{c}}(\alpha(t) - \alpha_{cr}) & \alpha(t) > \alpha_{cr} \end{cases} \quad (13)$$

where

$$APT \approx -\frac{1}{2}\left(D_s + \frac{\phi_{ca}D_s^2}{\phi_s D_{ca}} + \frac{\phi D_s^2(1-\phi_{\max})}{\phi_s\phi_{\max}D}\right) + \frac{1}{2}\sqrt{\left(D_s + \frac{\phi_{ca}D_s^2}{\phi_s D_{ca}} + \frac{\phi D_s^2(1-\phi_{\max})}{\phi_s\phi_{\max}D}\right)^2 + \frac{4}{3}\frac{(\phi_{\max}-\phi)}{\phi_{\max}}\frac{D_s^2}{\phi_s}} \quad (14)$$

The degree of cement hydration, α, can be obtained using a cement hydration model such as the one proposed by Schindler and Folliard (2005), which is adopted in this study. The critical degree of hydration, α_{cr}, at which concrete starts developing strength, can be predicted according to Rasmussen *et al*. (2002). The adopted models are given in Eqs. (15) to (20).

$$\alpha_{cr} = k \times w/c \quad (15)$$

$$\alpha = \alpha_u.\exp\left(-\left[\frac{\tau}{t_e}\right]^\beta\right) \quad (16)$$

$$\tau = 66.78.p_{C3A}^{-0.154}p_{C3S}^{-0.401}.Blaine^{-0.804}.P_{SO3}^{-0.758}.\exp(2.187.p_{SLAG} + 9.5.p_{FA}.p_{FA-CaO}) \quad (17)$$

$$\beta = 181.4.p_{C3A}^{0.146}.p_{C3S}^{0.227}.Blaine^{-0.535}.p_{SO3}^{0.558}.\exp(-0.647.p_{SLAG}) \quad (18)$$

$$\alpha_u = \frac{1.031.w/c}{0.194 + w/c} + 0.50.p_{FA} + 0.30.p_{SLAG} \le 1.0 \quad (19)$$

$$t_e = \sum_0^t \exp\left(\frac{E_A}{R}\left(\frac{1}{293} - \frac{1}{T+273}\right)\right).\Delta t \quad (20)$$

Many models that are purely empirical or statistical in nature have also been reported in the literature. Cheng Yeh (2006) modified Abrams law by including the effect of age on strength empirically. Models such Nipatsat and Tangtermsirikul (2000), Hwang *et al*. (2001), and Namyong *et al*. (2004) are examples of statistical models that are only functions of the mixture composition and do not account for cement properties, aggregate properties, their gradations and concrete age.

3. Experimental Data

Over 250 concrete strength values with concrete age ranging from 3 days to 191 days reported from 7 different studies published in the literature are presented in this part and used to evaluate the predictive capabilities of the strength models. The common Ordinary Portland Cement (OPC) was used for all mixes. The corresponding experimental data conform to ASTM C192-02 (2002) for mixing, casting, consolidating, finishing, and curing of the mixtures, and to ASTM C39-05 (2005) for testing the hardened concrete. Each strength value represents an average of three or more measured values. Overview of the experimental programs and composition of the mixtures are summarized in this paper, and the corresponding detail information can be retrieved from the referenced papers. A typical chemical composition of OPC consists of SiO_2, Al_2O_3, Fe_2O_3, CaO, MgO, Na_2O, SO_3, and K_2O with a corresponding oxide weight percentage of 19.7, 4.9, 2.6, 62.1, 2.8, 1.3, 3.2, and 0.6, respectively. The loss of ignition is the remaining percentage. It should be noted that missing information was assumed to be the same as those reported by Chidiac *et al.* (2013). Specifically, the Blaine of cement and weight ratios in terms of total cement content for the C_3A, C_3S and SO_3 compounds are 428 cm^2/g, 0.09, 0.58 and 0.034, respectively.

3.1 Chidiac *et al.* (2013)

The concrete mixtures are composed of OPC CSA GU-type 10, crushed limestone Coarse Aggregate (CA) with 20 mm and 14 mm nominal maximum aggregate sizes, and siliceous sand with a Fineness Modulus (FM) of 2.71. The mixtures w/c range from 0.4 to 0.7, the water content from 175 to 228 kg/m^3, the cement content from 250 to 570 kg/m^3, and the bulk volume of CA per unit volume of concrete from 0.45 to 0.69. The specific gravity of cement is 3.15. The specific gravities and bulk densities for the 20 mm CA are 2.75 and 1636 kg/m^3, for the 14 mm CA are 2.74, and 1576 kg/m^3, and for sand are 2.71 and 1812 kg/m^3, respectively. The characteristic diameters and mean diameters for the 20 mm CA are 14.3 and 12 mm, for the 14 mm CA are 10.4 and 9.1 mm, and for sand are 1.1 and 0.74 mm, respectively. Twenty mixtures are air-entrained and 8 non air-entrained. The concrete compressive strength was tested at 3 days and 28 days. Concrete mixture proportions and strength data are given in Table 1.

3.2 Zain and Abd (2009)

Concrete mixtures consist of OPC ASTM type I, crushed granite CA with nominal maximum aggregate size of 20 mm, and mining sand with a FM of 3.01. The w/c, water content, cement content and bulk volume of CA are 0.50, 200 kg/m^3, 400

Table 1. Concrete Mixture Proportions and Strength Data (Chidiac *et al.*, 2013)

Mixture #	w/c	Size	Cement (kg/m^3)	Coarse aggregate (kg/m^3)	Sand (kg/m^3)	Air (%)	$f'_{c\ 3\ days}$ (MPa)	$f'_{c\ 28\ days}$ (MPa)
1	0.40	14	483	794	851	5.9	32.7±0.8	37.8±2.3
2	0.60	14	322	971	815	5.3	16.3±0.2	23.1±1.2
3	0.40	14	513	971	618	4.8	29.1±2.0	36.4±1.0
4	0.60	14	342	794	939	5.1	18.1±0.7	27.2±1.1
5	0.40	20	460	1134	563	4.6	30.7±0.9	35.7±1.8
6	0.60	20	307	928	898	5.6	18.5±0.4	23.9±2.0
7	0.40	20	493	928	703	4.8	32.1±1.1	38.4±1.4
8	0.60	20	328	1134	641	3.1	16.5±1.3	23.8±2.3
9	0.50	14	386	794	934	5.7	23.3±0.6	30.4±0.8
10	0.70	14	250	794	1100	8.8	11.8±0.2	14.9±0.8
11	0.50	14	410	794	881	5.1	24.5±0.7	32.9±1.0
12	0.70	14	276	794	1029	7.5	10.9±0.3	18.4±0.7
13	0.50	14	386	971	759	5.1	23.2±1.4	30.7±2.6
14	0.70	14	250	971	925	8.7	8.5±0.2	13.6±0.6
15	0.50	14	410	971	706	4.8	25.8±0.2	32.5±0.2
16	0.70	14	276	971	854	5.6	11.6±0.4	18.6±2.0
17	0.50	14	398	883	820	5.0	23.6±1.2	31.9±0.6
18	0.50	14	398	706	994	7.8	20.8±0.3	26.3±1.0
19	0.50	14	398	1059	645	6.7	20.8±0.6	25.5±1.3
20	0.50	14	398	883	820	5.8	22.0±3.7	33.1±2.2
21	0.40	14	540	794	807	2.1	-	42.6±1.5
22	0.60	14	360	971	787	1.0	-	31.0±0.3
23	0.40	14	570	971	574	1.8	31.0±0.6	43.8±3.5
24	0.60	14	380	794	912	1.2	22.5±0.7	29.7±0.3
25	0.40	20	513	1134	542	1.6	31.1±0.7	40.5±2.4
26	0.60	20	342	928	892	1.5	24.4±0.2	31.9±2.3
27	0.40	20	540	928	692	1.4	35.1±0.9	43.3±1.4
28	0.60	20	360	1134	643	1.5	21.5±0.5	27.3±0.7

Table 2. Concrete Mixture Proportions and Strength Data (Zain and Abd, 2009; Kim *et al*., 1998; Gardner, 1990)

Mix #	w/c	Size	Cement (kg/m^3)	Coarse aggregate (kg/m^3)	Sand (kg/m^3)	Air (%)	f'_c (MPa)				
							3d	7d	14d	28d	91d
1	0.35	25	495	940	759	4.3	30.9	39.6	45.5	49.6	-
2	0.55	25	335	959	835	5.4	16.4	25.1	31.7	35.9	-
3	0.55	20	305	1040	830	-	20.0	27.7	31.0	35.2	-
4	0.34	20	409	1040	820	-	29.3	39.1	42.8	46.6	-
5	0.50	19	400	1055	707	6.0	18.0	24.7	26.9	31.0	32.3

kg/m^3, and 0.67, respectively. The specific gravity of cement is 3.15. The specific gravities and assumed bulk densities for the 20 mm CA are 2.62 and 1576 kg/m^3 and for sand are 2.62 and 1812 kg/m^3, respectively. The assumed characteristic diameters and mean diameters for the 20 mm CA are 15.8 and 13.2 mm and for sand are 1.1 and 0.74 mm, respectively. The concrete is 6% air entrained. Concrete was tested for compressive strength at the ages of 3, 7, 14, 28, and 91 days. The mixture proportions and corresponding strength data are given in Table 2.

3.3 Gallo and Popovics (2005)

Concrete mixtures are composed of OPC ASTM Type I, crushed limestone CA with nominal maximum aggregate size of 25 mm, and well graded sand. The w/c ranges from 0.40 to 0.52, the water content from 164 kg/m^3 to 187 kg/m^3, and the cement content from 344 to 423 kg/m^3. The bulk volume of CA per unit volume of concrete was 0.72. The specific gravity of cement is around 3.15. The assumed specific gravities and bulk densities for the 20 mm CA are 2.62 and 1576 kg/m^3 and for sand are 2.62 and 1812 kg/m^3, respectively. The assumed characteristic diameters and mean diameters for the 25 mm CA are 19.7 and 16.5 mm and for sand are 1.1 and 0.74 mm, respectively. Mixes 1 to 4 are 5% air entrained. Mixtures were tested at the age of 28 days only. Concrete mixture proportions and strength data are given in Table 3.

3.4 Namyong, Sangchun, and Hongbum (2004)

ASTM Type I OPC, angular crushed CA with nominal maximum aggregate size of 25 mm, and sea sand and crushed sand with FM ranging from 2.70 to 3.0 were used in the experimental program. The w/c ranges from 0.39 to 0.62, water content from 164 to 196 kg/m^3, the cement content from 289 to 463 kg/m^3, and the bulk volume of CA per unit volume of concrete from 0.55 and 0.72. The specific gravity of cement for all mixes ranges from 3.14 to 3.15. The specific gravities and assumed bulk densities for the 25 mm CA are 2.63 and 1576 kg/m^3, and 2.60 and 1812 kg/m^3 for sand, respectively. The assumed characteristic diameters and mean diameters for the 25 mm CA are 19.7 and 16.5 mm and for sand are 1.1 and 0.74 mm, respectively. All mixes are non air-entrained. The amount of air content was not presented and was therefore reasonably assumed to be around 1.5%. Concrete was tested at the ages of 7 and 28 days. Mixture proportions and strength data are given in Table 4.

Table 3. Concrete Mixture Proportions and Strength Data (Gallo and Popovics, 2005)

Mix #	w/c	Size	Cement (kg/m^3)	Coarse aggregate (kg/m^3)	Sand (kg/m^3)	Air (%)	$f'_{c\ 28\ days}$ (MPa)
1	0.40	25	418	1129	585	3.3	39.2
2	0.44	25	372	1128	636	4.4	31.0
3	0.48	25	344	1136	654	6.3	25.8
4	0.52	25	319	1140	675	5.1	23.8
5	0.40	25	463	1124	578	1.2	45.7
6	0.44	25	423	1129	613	1.5	40.2
7	0.48	25	388	1134	641	1.5	33.8
8	0.52	25	360	1139	663	2.7	34.7

Table 4. Concrete Mixture Proportions and Strength Data (Namyong *et al*., 2004)

Mix #	w/c	Size	Cement (kg/m^3)	Coarse aggregate (kg/m^3)	Sand (kg/m^3)	Air (%)	$f'_{c\ 7\ days}$ (MPa)	$f'_{c\ 28\ days}$ (MPa)
1	0.60	25	244	900	933	-	15.5	21.1
2	0.60	25	254	860	927	-	17.5	24.2
3	0.61	25	253	860	926	-	16.3	23.0
4	0.58	25	236	863	961	-	21.5	26.2
5	0.60	25	256	862	904	-	18.6	24.0
6	0.62	25	255	859	911	-	17.4	22.5
7	0.60	25	260	975	821	-	15.8	22.6
8	0.50	25	252	942	886	-	23.2	31.2
9	0.48	25	298	988	805	-	19.1	26.9
10	0.49	25	276	914	858	-	23.3	30.7
11	0.52	25	263	931	839	-	22.6	28.8
12	0.45	25	294	1000	810	-	20.7	27.6
13	0.45	25	298	940	847	-	18.9	28.5
14	0.49	25	264	902	882	-	24.0	32.0
15	0.49	25	284	887	866	-	23.3	30.5
16	0.50	25	263	913	885	-	23.0	31.6
17	0.50	25	286	874	865	-	21.6	31.7
18	0.45	25	312	894	835	-	22.0	30.2
19	0.45	25	308	939	790	-	22.4	30.2
20	0.47	25	313	981	759	-	20.4	29.7
21	0.50	25	294	955	804	-	16.0	29.8
22	0.47	25	312	962	778	-	19.8	26.7
23	0.47	25	319	962	778	-	18.1	25.3
24	0.48	25	306	924	810	-	20.1	27.8
25	0.48	25	295	902	807	-	22.0	29.1
26	0.48	25	292	924	818	-	22.8	29.2
27	0.48	25	303	922	812	-	21.2	27.6
28	0.46	25	247	905	846	-	22.8	28.9
29	0.47	25	291	878	889	-	21.3	27.3
30	0.47	25	302	862	879	-	21.2	27.6

Table 4. (continued)

Mix #	w/c	Size	Cement (kg/m^3)	Coarse aggregate (kg/m^3)	Sand (kg/m^3)	Air (%)	$f'_{c\ 7\ days}$ (MPa)	$f'_{c\ 28\ days}$ (MPa)
31	0.44	25	300	903	868	-	21.1	29.1
32	0.48	25	305	956	812	-	20.0	26.2
33	0.45	25	299	940	815	-	21.5	28.3
34	0.48	25	300	923	807	-	20.7	27.5
35	0.46	25	306	978	754	-	23.7	31.3
36	0.49	25	297	946	800	-	21.9	28.7
37	0.46	25	304	888	863	-	20.6	28.2
38	0.48	25	284	965	794	-	21.0	27.2
39	0.46	25	309	947	730	-	25.0	30.9
40	0.46	25	326	951	732	-	20.7	30.7
41	0.43	25	325	927	784	-	22.7	30.0
42	0.41	25	348	977	713	-	22.6	30.7
43	0.41	25	346	983	704	-	23.6	31.0
44	0.42	25	336	889	804	-	22.8	29.8
45	0.44	25	329	882	812	-	22.4	30.2
46	0.44	25	324	920	785	-	22.8	30.7
47	0.44	25	314	921	780	-	23.9	30.1
48	0.44	25	316	923	791	-	23.6	30.9
49	0.41	25	317	936	811	-	22.6	33.3
50	0.43	25	327	891	813	-	22.9	30.8
51	0.44	25	327	956	783	-	20.8	29.3
52	0.48	25	314	879	786	-	22.4	30.7
53	0.45	25	319	883	818	-	22.8	29.5
54	0.44	25	306	900	780	-	25.9	34.3
55	0.44	25	312	914	783	-	24.5	32.4
56	0.40	25	336	970	760	-	24.7	29.4
57	0.49	25	318	954	771	-	23.4	30.8
58	0.42	25	322	937	782	-	23.7	31.5
59	0.44	25	293	969	768	-	24.9	30.6

3.5 Kim, Moon, and Eo (1998)

OPC ASTM Type I was used for the experimental program along with angular crushed CA whose nominal maximum aggregate size is 25 mm, and river sand with FM of 2.64. The values for w/c are 0.35 and 0.55, for water content 175 and 185 kg/m^3, for cement content 335 and 495 kg/m^3, and for bulk volume of CA per unit volume of concrete 0.61 and 0.62. The specific gravity of cement is around 3.15. The specific gravities and bulk densities for the 25 mm CA are 2.66 and 1537 kg/m^3 and for sand are 2.65 and 1469 kg/m^3, respectively. The assumed characteristic diameters and mean diameters for the 25 mm CA are 19.7 and 16.5 mm and for sand are 1.1 and 0.74 mm, respectively. The mixtures are air entrained. Concrete was tested at the ages of 3, 7, 14, and 28 days. The mixture proportions and corresponding strength data are given in Table 2.

3.6 Gardner (1990)

CSA Type 10 OPC and crushed stone CA with nominal maximum aggregate size of 20 mm were used. Values for w/c are 0.34 and 0.55, for water content 138 and 169 kg/m^3, and for the cement content 305 and 409 kg/m^3. The specific gravity of cement is around 3.15. The bulk volume of CA per unit volume of concrete is 0.66. The specific gravities and assumed bulk densities for the 20 mm CA are 2.66 and 1576 kg/m^3 and for sand are 2.65 and 1812 kg/m^3, respectively. The assumed characteristic diameters and mean diameters for the 25 mm CA are 15.8 and 13.2 mm and for sand are 1.1 and 0.74 mm, respectively. The cement Blaine is 375 cm^2/g. Concrete was tested for compressive strength at the ages of 3, 7, 14, 28, 56, and 112 days. The mixture proportions and corresponding strength data up to 28 days are given in Table 2. The strength values at 56 and 112 days for w/c of 0.34 are 49.4 and 54.3 MPa and for w/c of 0.55 are 38.9 and 42.6 MPa, respectively.

3.7 Bloem and Walker (1960)

OPC ASTM Type I, hard, sub-angular and quartzite CA with nominal maximum aggregate sizes of 9.5 mm, 19 mm, 38 mm and 63 mm, and siliceous sand with FM of 2.70 formed the main ingredients for the concrete mixtures. The range of w/c is 0.37 to 0.89, 167 to 197 kg/m^3 for water content, 219 to 453 kg/m^3 for cement content, and 0.59 to 0.68 for bulk volume of CA per unit volume of concrete. The specific gravity of cement is around 3.15. The specific gravities for CA and sand are 2.64 and 2.64, respectively. The bulk densities for the 9.5, 19, 38 and 63 mm CA are 1647, 1721, 1742 and 1828 kg/m^3, respectively. The characteristic diameters and mean diameters are assumed to be 7.5 and 6.3 mm for 9.5 mm CA, 15 and 12.6 mm for 19 mm CA, 30 and 25.1 mm for 38 mm CA, and 50 and 41.9 mm for 63.5 mm CA, respectively. The characteristic diameter and mean diameter of sand are assumed to be 0.95 and 0.74, respectively. Mixtures 13 to 24 are air entrained. Concrete was tested at the ages of 7, 28, and 191 days. Concrete mixture proportions and strength data are given in Table 5.

4. Procedure for Calibration and Evaluation

Using experimental data reported in the literature, the strength models were first calibrated using one to two data points for each source (10 calibration points in total), and then evaluated using all the remaining points. For the formulas that require the use of cement hydration model, Schindler and Folliard hydration model (2005) was adopted. Calibration of the compressive strength models was done by minimizing the standard error using least squares analysis. Standard error (σ), which provides a global assessment of the model prediction, is defined by (Montgomery and Runger, 2003):

$$\sigma = \sqrt{\frac{\sum_i (f_{cmodel_i}' - f_{cexp_i}')^2}{n-p}} \tag{21}$$

Where $f'_{c\_model\_i}$ and $f'_{c\_exp\_i}$ are the model and experimental compressive strengths corresponding to mix i respectively, n is the number of test points, and p is the number of model constants. The goodness of fit of the models were evaluated using statistical

Table 5. Concrete Mixture Proportions and Strength Data (Bloem and Walker, 1960)

Mix #	w/c	Size	Cement (kg/m^3)	Coarse aggregate (kg/m^3)	Sand (kg/m^3)	Air (%)	$f'_{c\ 7\ days}$ (MPa)	$f'_{c\ 28\ days}$ (MPa)	$f'_{c\ 191\ days}$ (MPa)
1	0.89	9.5	222	977	945	0.5	10.7	16.0	16.8
2	0.80	19.1	223	1137	795	1.8	12.9	19.2	20.7
3	0.74	38.1	223	1163	795	2.2	13.7	20.1	22.8
4	0.73	63.5	223	1159	808	2.2	13.9	20.1	20.8
5	0.56	9.5	335	985	821	2.3	26.2	34.7	37.0
6	0.51	19.1	336	1143	722	1.5	27.8	35.1	39.6
7	0.50	38.1	334	1163	721	1.5	27.2	33.9	36.4
8	0.48	63.5	335	1161	734	1.4	25.7	32.2	35.8
9	0.44	9.5	445	982	721	2.0	32.7	41.0	45.8
10	0.41	19.1	442	1129	616	1.9	32.3	39.1	44.5
11	0.39	38.1	442	1158	622	1.1	31.5	38.2	42.1
12	0.39	63.5	441	1148	632	1.4	30.0	35.2	41.2
13	0.86	9.5	219	963	879	4.5	10.5	16.3	17.7
14	0.75	19.1	221	1125	768	4.7	14.7	20.8	22.8
15	0.70	38.1	221	1151	771	4.7	15.1	21.4	22.8
16	0.70	63.5	220	1145	789	4.3	14.8	20.4	21.6
17	0.53	9.5	337	990	778	4.5	27.0	35.3	38.3
18	0.48	19.1	337	1146	661	4.8	25.6	32.2	36.1
19	0.46	38.1	337	1174	649	4.6	25.2	32.3	34.9
20	0.45	63.5	338	1170	664	4.6	25.6	30.8	31.4
21	0.41	9.5	451	995	654	4.7	32.5	39.3	44.6
22	0.39	19.1	444	1132	550	5.0	28.3	34.2	38.6
23	0.38	38.1	451	1177	521	4.2	26.4	32.4	36.2
24	0.37	63.5	453	1174	537	4.1	26.5	31.8	34.3

tools namely visual display methods and numerical methods. Visual display methods provide comparisons between model and experiment whereas numerical methods provide measurements that reveal the overall accuracy of the model. Visual methods used include plots such as residual plots which reveal whether the models provide good representation of the experimental data and other plots that demonstrate the trend of models compared to experiment, deviation from exact location, and systematic deviation.

Numerical methods used include determining σ, the correlation coefficient (R^2) of each model, and measures of the systematic deviation of the models from exact location. R^2 measures the degree of correlation between model and experiment (Montgomery and Runger, 2003). Measures of the systematic deviation of the models from exact location were carried out by determining the regression coefficients in the linear regression model. For any model, if the 95% confidence interval for the intercept includes zero and the 95% confidence interval for the slope includes one then this would signify that the model is unbiased with no deviation from exact location.

Since the aggregate type used is different for the different sources from literature, the constant K used in equations 11, 12, and 13, and the constant A used in other models were re-calibrated for each aggregate source. Note that de larrard's shorter model form was used i.e. K calibration constant for aggregates rather than two additional p and q calibration constants for given type of aggregates. In addition, calibration parameters such as B in Eqs. (11) and (12) and in other models were re-calibrated in this study instead of limiting them to certain given values that were based on less experimental data and less variation in properties. Since many models were developed to predict the 28-day strength only, all models were first evaluated at 28 days then extended to all ages using the models accounting for age. It should be noted that in order to allow some models such as Pann and Tango to account for air, the w/c term was replaced with $(w+a)/c$ as recommended by Popovics (1985). This ensures that all the models evaluated account for air, which is an important variable that influences the compressive strength of concrete.

Table 6. Strength Model Calibration Results

Eqn. #	Author	Calibration Parameters
13	Chidiac *at al.* (2013)	$K{*}R_{c28}$ = 142-155, A = -0.1, B = 0.215
12	Mechling *et al.* (2013)	$K{*}R_{c28}$ = 162-180, A = 0.0327, B = 1.58, C = -0.0775
11	de Larrard (1999)	$K{*}R_{c28}$ = 162-180, A = 0.0327, B = 1.58, C = -0.0775
5	Tango (2000)	A = 81-98, B = 4.08, D = 1.24, E = 2.71
9	Pann *et al.* (2003)	A = 109-130, B = 7.08, C = -9.44, $D = 7 \times 10^9$
7	Popovics (1998)	A = 71-108, B = 5.63, C = -2.31, $b1$ = 1.08, $b2$ = 33.1
4	Karni (1974)	A = 0.9-1.1, B = 23.3
8	Popovics (2008)	A = 29-52, B = 2.59, C = -2.44, E = -0.0015
3	Powers (1960)	A = 81-99, B = 1.76
6	Popovics (1985)	A = 13.5-19, B = 1.67, C = 1.61
1	Feret (1892)	A = 232-276, B = 2

through *APT*, gave better predictions of strength in comparison to other models. The effect of *APT*, which accounts for aggregate proportions and gradations through packing density, on strength was studied using experimental data and was shown to have up to 7% affect on the strength values (Chidiac *et al.*, 2013).

Table 9 shows that all models give good results. Looking at the form of the models, one can observe that most models are function of w/c. It is well established in the literature that w/c is

Table 7. Results of Theoretical Values of Variables Used in Strength Models

Source	Mix #	V_w	V_c	V_a	ϕ_s	ϕ_{ca}	ϕ	$\phi_{\max}$	D_s (mm)	D_{ca} (mm)	D (mm)	$D_{\max}$ (mm)	*MPT* (mm)	*APT* (mm)	P_c	α_{cr}	α_u	α_{28}
Chidiac *et al.*	1	0.193	0.153	0.059	0.31	0.29	0.60	0.81	0.74	8.7	6.8	13.2	1.39	0.154	0.020	0.17	0.69	0.65
	2	0.193	0.102	0.053	0.30	0.35	0.65	0.81	0.74	8.7	7.1	13.2	1.01	0.123	0.158	0.26	0.78	0.73
	3	0.205	0.163	0.048	0.23	0.35	0.58	0.81	0.74	8.7	7.4	13.2	1.55	0.208	0.020	0.17	0.69	0.65
	4	0.205	0.108	0.051	0.35	0.29	0.64	0.80	0.74	8.7	6.7	13.2	1.08	0.117	0.158	0.26	0.78	0.73
	5	0.184	0.146	0.046	0.21	0.41	0.62	0.83	0.74	12.5	10.9	19	1.89	0.204	0.020	0.17	0.69	0.65
	6	0.184	0.097	0.056	0.33	0.34	0.66	0.82	0.74	12.5	10.0	19	1.39	0.115	0.158	0.26	0.78	0.73
	7	0.197	0.156	0.048	0.26	0.34	0.60	0.83	0.74	12.5	10.4	19	2.19	0.193	0.020	0.17	0.69	0.65
	8	0.197	0.104	0.031	0.24	0.42	0.66	0.83	0.74	12.5	10.8	19	1.50	0.155	0.158	0.26	0.78	0.73
	9	0.193	0.123	0.057	0.34	0.29	0.63	0.80	0.74	8.7	6.7	13.2	1.12	0.122	0.081	0.22	0.74	0.69
	10	0.175	0.079	0.088	0.39	0.28	0.67	0.80	0.74	8.7	6.5	13.2	0.81	0.085	0.235	0.30	0.81	0.75
	11	0.205	0.130	0.051	0.32	0.29	0.61	0.81	0.74	8.7	6.8	13.2	1.25	0.138	0.081	0.22	0.74	0.69
	12	0.193	0.088	0.075	0.37	0.28	0.65	0.80	0.74	8.7	6.6	13.2	0.94	0.100	0.235	0.30	0.81	0.75
	13	0.193	0.123	0.051	0.28	0.35	0.63	0.82	0.74	8.7	7.2	13.2	1.16	0.145	0.081	0.22	0.74	0.69
	14	0.175	0.079	0.087	0.33	0.34	0.67	0.81	0.74	8.7	7.0	13.2	0.88	0.103	0.235	0.30	0.81	0.75
	15	0.205	0.130	0.048	0.26	0.36	0.62	0.82	0.74	8.7	7.3	13.2	1.29	0.166	0.081	0.22	0.74	0.69
	16	0.193	0.088	0.056	0.31	0.35	0.67	0.81	0.74	8.7	7.0	13.2	0.91	0.111	0.235	0.30	0.81	0.75
	17	0.199	0.126	0.050	0.30	0.32	0.62	0.81	0.74	8.7	7.0	13.2	1.21	0.141	0.081	0.22	0.74	0.69
	18	0.199	0.126	0.078	0.36	0.25	0.61	0.80	0.74	8.7	6.5	13.2	1.27	0.130	0.081	0.22	0.74	0.69
	19	0.199	0.126	0.067	0.23	0.38	0.61	0.81	0.74	8.7	7.4	13.2	1.30	0.180	0.081	0.22	0.74	0.69
	20	0.199	0.126	0.058	0.30	0.32	0.62	0.81	0.74	8.7	7.0	13.2	1.25	0.146	0.081	0.22	0.74	0.69
	21	0.216	0.171	0.021	0.30	0.29	0.59	0.81	0.74	8.7	6.9	13.2	1.47	0.165	0.020	0.17	0.69	0.65
	22	0.216	0.114	0.010	0.29	0.36	0.65	0.81	0.74	8.7	7.1	13.2	1.00	0.124	0.158	0.26	0.78	0.73
	23	0.228	0.181	0.018	0.21	0.36	0.57	0.81	0.74	8.7	7.5	13.2	1.65	0.228	0.020	0.17	0.69	0.65
	24	0.228	0.121	0.012	0.34	0.29	0.64	0.81	0.74	8.7	6.7	13.2	1.09	0.119	0.158	0.26	0.78	0.73
	25	0.205	0.163	0.016	0.20	0.41	0.61	0.83	0.74	12.5	11.0	19	1.96	0.216	0.020	0.17	0.69	0.65
	26	0.205	0.108	0.015	0.33	0.34	0.67	0.82	0.74	12.5	10.0	19	1.33	0.110	0.158	0.26	0.78	0.73
	27	0.216	0.171	0.014	0.26	0.34	0.60	0.83	0.74	12.5	10.4	19	2.21	0.196	0.020	0.17	0.69	0.65
	28	0.216	0.114	0.015	0.24	0.41	0.65	0.83	0.74	12.5	10.8	19	1.59	0.163	0.158	0.26	0.78	0.73
Bloem and Walker	1	0.197	0.070	0.005	0.36	0.37	0.73	0.83	0.74	6.3	5.0	9.525	0.42	0.066	0.348	0.38	0.85	0.79
	2	0.179	0.071	0.018	0.30	0.43	0.73	0.86	0.74	12.6	10.5	19.05	1.02	0.096	0.304	0.35	0.83	0.78
	3	0.165	0.071	0.022	0.30	0.44	0.74	0.87	0.74	25.1	21.1	38.1	2.00	0.098	0.266	0.32	0.82	0.76
	4	0.162	0.071	0.022	0.31	0.44	0.75	0.87	0.74	41.9	35.1	63.5	3.35	0.099	0.254	0.31	0.81	0.76
	5	0.187	0.106	0.023	0.31	0.37	0.68	0.83	0.74	6.3	5.1	9.525	0.64	0.105	0.125	0.24	0.77	0.72
	6	0.172	0.107	0.015	0.27	0.43	0.71	0.86	0.74	12.6	10.7	19.05	1.29	0.125	0.089	0.22	0.75	0.70
	7	0.165	0.106	0.015	0.27	0.44	0.71	0.87	0.74	25.1	21.4	38.1	2.58	0.131	0.078	0.21	0.74	0.69
	8	0.162	0.106	0.014	0.28	0.44	0.72	0.87	0.74	41.9	35.6	63.5	4.31	0.131	0.070	0.21	0.74	0.69
	9	0.194	0.141	0.020	0.27	0.37	0.65	0.84	0.74	6.3	5.2	9.525	0.86	0.145	0.039	0.19	0.71	0.67
	10	0.180	0.140	0.019	0.23	0.43	0.66	0.86	0.74	12.6	10.9	19.05	1.77	0.179	0.024	0.17	0.70	0.65
	11	0.174	0.140	0.011	0.24	0.44	0.67	0.87	0.74	25.1	21.8	38.1	3.44	0.182	0.017	0.17	0.69	0.65
	12	0.172	0.140	0.014	0.24	0.43	0.67	0.88	0.74	41.9	36.2	63.5	5.91	0.187	0.016	0.17	0.69	0.64
	13	0.188	0.069	0.045	0.33	0.36	0.70	0.83	0.74	6.3	5.1	9.525	0.57	0.090	0.334	0.37	0.84	0.79
	14	0.166	0.070	0.047	0.29	0.43	0.72	0.86	0.74	12.6	10.6	19.05	1.17	0.110	0.271	0.32	0.82	0.77
	15	0.155	0.070	0.047	0.29	0.44	0.73	0.87	0.74	25.1	21.2	38.1	2.27	0.111	0.236	0.30	0.81	0.75
	16	0.155	0.070	0.043	0.30	0.43	0.73	0.87	0.74	41.9	35.2	63.5	3.76	0.111	0.239	0.30	0.81	0.76
	18	0.160	0.107	0.048	0.25	0.43	0.68	0.86	0.74	12.6	10.8	19.05	1.52	0.152	0.064	0.20	0.73	0.68
	19	0.156	0.107	0.046	0.25	0.44	0.69	0.87	0.74	25.1	21.7	38.1	3.09	0.163	0.056	0.20	0.73	0.68
	20	0.152	0.107	0.046	0.25	0.44	0.69	0.88	0.74	41.9	36.1	63.5	5.18	0.163	0.047	0.19	0.72	0.67

Table 7. (continued)

Source	Mix #	V_w	V_c	V_a	ϕ_s	ϕ_{ca}	ϕ	ϕ_{max}	D_s (mm)	D_{ca} (mm)	D (mm)	D_{max} (mm)	MPT (mm)	APT (mm)	P_c	α_{cr}	α_u	α_{28}
Bloem and Walker	21	0.185	0.143	0.047	0.25	0.38	0.62	0.84	0.74	6.3	5.3	9.525	0.98	0.171	0.025	0.18	0.70	0.65
	22	0.172	0.141	0.050	0.21	0.43	0.64	0.86	0.74	12.6	11.0	19.05	2.04	0.216	0.014	0.17	0.69	0.64
	23	0.171	0.143	0.042	0.20	0.45	0.64	0.88	0.74	25.1	22.3	38.1	4.15	0.238	0.011	0.16	0.68	0.64
	24	0.167	0.144	0.041	0.20	0.44	0.65	0.89	0.74	41.9	37.0	63.5	7.02	0.239	0.007	0.16	0.68	0.63
Kim *et al.*	1	0.175	0.157	0.043	0.28	0.35	0.63	0.75	0.74	16.5	13.5	25	1.54	0.115	0.002	0.15	0.67	0.62
	2	0.185	0.106	0.054	0.31	0.35	0.66	0.75	0.74	16.5	13.4	25	1.06	0.078	0.120	0.24	0.76	0.71
Gardner	1	0.169	0.097	0.015	0.32	0.40	0.72	0.84	0.74	13.2	10.8	20	1.09	0.093	0.121	0.24	0.76	0.72
	2	0.138	0.130	0.015	0.31	0.40	0.71	0.84	0.74	13.2	10.9	20	1.11	0.096	0.000	0.15	0.65	0.62
Zain & Abd	1	0.200	0.127	0.060	0.25	0.38	0.63	0.85	0.74	13.2	11.1	19	1.94	0.181	0.081	0.22	0.74	0.69
Gallo & Popovics	1	0.167	0.133	0.033	0.23	0.44	0.66	0.85	0.74	16.5	14.4	25	2.14	0.176	0.020	0.17	0.69	0.65
	2	0.164	0.118	0.044	0.24	0.43	0.67	0.85	0.74	16.5	14.2	25	2.02	0.160	0.042	0.19	0.72	0.67
	3	0.165	0.109	0.063	0.24	0.42	0.67	0.85	0.74	16.5	14.2	25	2.10	0.164	0.067	0.21	0.73	0.69
	4	0.166	0.101	0.051	0.25	0.43	0.69	0.85	0.74	16.5	14.1	25	1.88	0.146	0.096	0.22	0.75	0.70
	5	0.185	0.147	0.012	0.22	0.43	0.65	0.85	0.74	16.5	14.4	25	2.26	0.186	0.020	0.17	0.69	0.65
	6	0.186	0.134	0.015	0.23	0.43	0.66	0.85	0.74	16.5	14.3	25	2.14	0.171	0.042	0.19	0.72	0.67
	7	0.186	0.123	0.015	0.24	0.43	0.68	0.85	0.74	16.5	14.2	25	1.99	0.158	0.067	0.21	0.73	0.69
	8	0.187	0.114	0.027	0.25	0.43	0.68	0.85	0.74	16.5	14.2	25	2.00	0.155	0.096	0.22	0.75	0.70
Namyong *et al.*	1	0.174	0.078	0.015	0.37	0.35	0.71	0.84	0.74	16.5	13.0	25	1.43	0.088	0.159	0.26	0.78	0.73
	2	0.184	0.081	0.015	0.36	0.33	0.70	0.84	0.74	16.5	12.9	25	1.62	0.098	0.155	0.26	0.78	0.73
	3	0.183	0.080	0.015	0.37	0.34	0.70	0.84	0.74	16.5	12.9	25	1.60	0.096	0.162	0.26	0.78	0.73
	4	0.173	0.075	0.015	0.38	0.33	0.71	0.84	0.74	16.5	12.8	25	1.44	0.086	0.138	0.25	0.77	0.72
	5	0.190	0.082	0.015	0.36	0.33	0.69	0.84	0.74	16.5	13.0	25	1.74	0.105	0.160	0.26	0.78	0.73
	6	0.190	0.081	0.015	0.36	0.33	0.69	0.84	0.74	16.5	12.9	25	1.71	0.103	0.170	0.26	0.78	0.73
	7	0.184	0.083	0.015	0.32	0.38	0.70	0.85	0.74	16.5	13.4	25	1.70	0.112	0.154	0.26	0.78	0.73
	8	0.164	0.080	0.015	0.35	0.37	0.71	0.85	0.74	16.5	13.2	25	1.48	0.094	0.081	0.22	0.74	0.69
	9	0.176	0.095	0.015	0.31	0.38	0.69	0.85	0.74	16.5	13.5	25	1.80	0.120	0.066	0.21	0.73	0.69
	10	0.178	0.088	0.015	0.33	0.35	0.69	0.85	0.74	16.5	13.2	25	1.78	0.113	0.077	0.21	0.74	0.69
	11	0.178	0.083	0.015	0.33	0.36	0.69	0.85	0.74	16.5	13.3	25	1.74	0.112	0.098	0.23	0.75	0.70
	12	0.165	0.093	0.015	0.31	0.39	0.70	0.85	0.74	16.5	13.5	25	1.68	0.113	0.045	0.19	0.72	0.67
	13	0.164	0.094	0.015	0.33	0.37	0.70	0.85	0.74	16.5	13.3	25	1.66	0.107	0.046	0.19	0.72	0.67
	14	0.169	0.084	0.015	0.35	0.35	0.70	0.85	0.74	16.5	13.1	25	1.64	0.102	0.071	0.21	0.74	0.69
	15	0.181	0.090	0.015	0.34	0.34	0.68	0.85	0.74	16.5	13.1	25	1.87	0.115	0.073	0.21	0.74	0.69
	16	0.171	0.084	0.015	0.35	0.35	0.70	0.85	0.74	16.5	13.1	25	1.63	0.102	0.081	0.22	0.74	0.69
	17	0.181	0.091	0.015	0.34	0.34	0.68	0.85	0.74	16.5	13.1	25	1.86	0.115	0.079	0.21	0.74	0.69
	18	0.179	0.099	0.015	0.33	0.35	0.67	0.85	0.74	16.5	13.2	25	1.99	0.125	0.046	0.19	0.72	0.67
	19	0.180	0.098	0.015	0.31	0.36	0.67	0.85	0.74	16.5	13.4	25	2.02	0.132	0.050	0.19	0.72	0.67
	20	0.183	0.099	0.015	0.30	0.38	0.67	0.85	0.74	16.5	13.6	25	2.04	0.138	0.058	0.20	0.73	0.68
	21	0.175	0.093	0.015	0.32	0.37	0.69	0.85	0.74	16.5	13.4	25	1.79	0.118	0.081	0.22	0.74	0.69
	22	0.183	0.099	0.015	0.30	0.37	0.67	0.85	0.74	16.5	13.5	25	2.02	0.134	0.060	0.20	0.73	0.68
	24	0.184	0.097	0.015	0.32	0.36	0.67	0.85	0.74	16.5	13.3	25	2.00	0.128	0.067	0.21	0.73	0.69
	25	0.183	0.094	0.015	0.32	0.35	0.67	0.85	0.74	16.5	13.3	25	2.01	0.128	0.070	0.21	0.74	0.69
	26	0.183	0.093	0.015	0.32	0.36	0.68	0.85	0.74	16.5	13.3	25	1.96	0.126	0.070	0.21	0.74	0.69
	27	0.184	0.096	0.015	0.32	0.36	0.67	0.85	0.74	16.5	13.3	25	2.00	0.128	0.066	0.21	0.73	0.69
	28	0.175	0.078	0.015	0.34	0.36	0.70	0.85	0.74	16.5	13.2	25	1.68	0.106	0.116	0.24	0.76	0.71
	29	0.173	0.092	0.015	0.35	0.34	0.69	0.85	0.74	16.5	13.0	25	1.76	0.107	0.059	0.20	0.73	0.68
	30	0.179	0.096	0.015	0.35	0.33	0.68	0.84	0.74	16.5	13.0	25	1.89	0.115	0.058	0.20	0.73	0.68
	31	0.168	0.095	0.015	0.34	0.35	0.69	0.85	0.74	16.5	13.1	25	1.75	0.109	0.043	0.19	0.72	0.67
	32	0.182	0.097	0.015	0.32	0.37	0.68	0.85	0.74	16.5	13.4	25	1.93	0.126	0.066	0.21	0.73	0.69
	33	0.172	0.095	0.015	0.32	0.36	0.69	0.85	0.74	16.5	13.4	25	1.85	0.120	0.048	0.19	0.72	0.67
	34	0.183	0.095	0.015	0.32	0.36	0.68	0.85	0.74	16.5	13.4	25	1.98	0.128	0.070	0.21	0.74	0.69
	35	0.184	0.097	0.015	0.29	0.38	0.67	0.85	0.74	16.5	13.6	25	2.09	0.141	0.053	0.20	0.72	0.68
	36	0.185	0.094	0.015	0.31	0.36	0.68	0.85	0.74	16.5	13.4	25	1.99	0.130	0.071	0.21	0.74	0.69
	37	0.175	0.097	0.015	0.34	0.34	0.68	0.85	0.74	16.5	13.1	25	1.85	0.115	0.052	0.20	0.72	0.68

Table 7. (continued)

Source	Mix #	V_w	V_c	V_a	ϕ_s	ϕ_{ca}	ϕ	ϕ_{max}	D_s (mm)	D_{ca} (mm)	D (mm)	D_{max} (mm)	MPT (mm)	APT (mm)	P_c	α_{cr}	α_u	α_{28}
Namy-ong *et al.*	38	0.172	0.090	0.015	0.31	0.38	0.69	0.85	0.74	16.5	13.5	25	1.80	0.119	0.066	0.21	0.73	0.69
	39	0.189	0.098	0.015	0.29	0.37	0.66	0.85	0.74	16.5	13.6	25	2.25	0.152	0.053	0.20	0.72	0.68
	40	0.189	0.103	0.015	0.29	0.37	0.66	0.85	0.74	16.5	13.6	25	2.24	0.151	0.054	0.20	0.73	0.68
	41	0.180	0.103	0.015	0.31	0.36	0.67	0.85	0.74	16.5	13.4	25	2.11	0.137	0.035	0.18	0.71	0.66
	42	0.184	0.111	0.015	0.28	0.38	0.65	0.85	0.74	16.5	13.7	25	2.31	0.159	0.025	0.18	0.70	0.65
	43	0.184	0.110	0.015	0.27	0.38	0.65	0.85	0.74	16.5	13.8	25	2.33	0.162	0.023	0.17	0.70	0.65
	44	0.183	0.107	0.015	0.31	0.34	0.66	0.85	0.74	16.5	13.3	25	2.19	0.139	0.030	0.18	0.71	0.66
	45	0.187	0.104	0.015	0.32	0.34	0.66	0.85	0.74	16.5	13.2	25	2.19	0.138	0.042	0.19	0.72	0.67
	46	0.183	0.103	0.015	0.31	0.36	0.66	0.85	0.74	16.5	13.4	25	2.14	0.139	0.039	0.19	0.71	0.67
	47	0.183	0.100	0.015	0.31	0.36	0.66	0.85	0.74	16.5	13.4	25	2.13	0.139	0.043	0.19	0.72	0.67
	48	0.183	0.100	0.015	0.31	0.36	0.67	0.85	0.74	16.5	13.4	25	2.11	0.137	0.043	0.19	0.72	0.67
	49	0.168	0.101	0.015	0.32	0.36	0.68	0.85	0.74	16.5	13.4	25	1.90	0.123	0.025	0.18	0.70	0.65
	50	0.180	0.104	0.015	0.32	0.34	0.66	0.85	0.74	16.5	13.3	25	2.11	0.133	0.033	0.18	0.71	0.66
	51	0.183	0.104	0.015	0.30	0.36	0.67	0.85	0.74	16.5	13.5	25	2.08	0.137	0.043	0.19	0.72	0.67
	52	0.196	0.100	0.015	0.31	0.34	0.65	0.85	0.74	16.5	13.3	25	2.28	0.145	0.070	0.21	0.74	0.69
	53	0.187	0.101	0.015	0.32	0.34	0.66	0.85	0.74	16.5	13.2	25	2.14	0.135	0.049	0.19	0.72	0.67
	54	0.183	0.097	0.015	0.31	0.35	0.66	0.85	0.74	16.5	13.4	25	2.16	0.139	0.043	0.19	0.72	0.67
	55	0.183	0.099	0.015	0.31	0.35	0.66	0.85	0.74	16.5	13.4	25	2.14	0.138	0.043	0.19	0.72	0.67
	56	0.176	0.107	0.015	0.29	0.37	0.66	0.85	0.74	16.5	13.6	25	2.14	0.144	0.017	0.17	0.69	0.65
	57	0.182	0.101	0.015	0.30	0.37	0.67	0.85	0.74	16.5	13.5	25	2.10	0.140	0.040	0.19	0.71	0.67
	58	0.175	0.102	0.015	0.31	0.36	0.67	0.85	0.74	16.5	13.4	25	2.06	0.135	0.028	0.18	0.70	0.66
	59	0.171	0.093	0.015	0.30	0.38	0.68	0.85	0.74	16.5	13.6	25	1.91	0.128	0.041	0.19	0.72	0.67

Table 8. Results of Models 28-day Compressive Strength Values

Source	$f'_{c\,28}$ (MPa)												
	Mix #	Exp.	Chidiac	Mechling	de Larrard	Tango	Pann	Popovics (1998)	Karni	Popovics (2008)	Powers	Popovics (1985)	Feret
Chidiac *et al.*	1	37.8	37.1	35.7	35.7	36.0	33.7	33.4	33.7	30.8	33.3	35.9	34.8
	2	23.1	25.7	24.5	24.5	24.4	24.4	24.4	23.7	21.0	23.5	24.9	21.0
	3	36.4	37.5	37.3	37.3	37.5	35.8	35.2	35.9	34.0	35.6	37.3	37.2
	4	27.2	26.5	24.9	24.9	25.0	25.1	24.6	24.3	21.8	24.1	25.1	21.5
	5	35.7	37.2	36.3	36.3	37.2	35.3	35.7	35.4	32.0	35.1	37.6	36.7
	6	23.9	25.2	23.3	23.3	23.7	23.5	23.9	22.8	20.1	22.7	24.6	20.2
	7	38.4	37.6	36.1	36.1	37.3	35.5	35.2	35.6	33.1	35.2	37.3	36.9
	8	23.8	28.0	26.4	26.4	27.3	28.0	27.3	27.1	23.9	26.7	27.0	23.9
	9	30.4	31.5	29.2	29.2	29.5	29.5	28.4	28.0	24.7	27.5	29.2	26.4
	10	14.9	16.5	17.2	17.2	15.4	14.0	17.0	14.4	14.1	15.8	18.9	13.1
	11	32.9	32.2	30.0	30.0	30.6	31.0	29.3	29.4	26.4	28.9	29.9	27.7
	12	18.4	18.4	18.7	18.7	17.5	16.4	18.2	16.8	15.8	17.7	19.9	14.7
	13	30.7	31.6	29.8	29.8	30.1	30.4	29.2	28.8	25.5	28.3	29.8	27.2
	14	13.6	16.4	17.2	17.2	15.6	14.1	17.1	14.6	14.2	16.0	19.0	13.2
	15	32.5	32.0	30.3	30.3	30.9	31.4	29.8	29.8	26.9	29.3	30.2	28.1
	16	18.6	20.3	20.4	20.4	19.6	18.7	20.1	19.2	17.5	19.6	21.3	16.4
	17	31.9	32.0	30.0	30.0	30.4	30.8	29.4	29.2	26.0	28.7	29.9	27.6
	18	26.3	29.0	26.7	26.7	27.3	26.7	25.4	25.3	22.3	25.0	27.0	23.9
	19	25.5	29.3	27.9	27.9	28.5	28.3	26.9	26.8	23.7	26.4	28.2	25.3
	20	33.1	30.9	29.0	29.0	29.5	29.6	28.2	28.0	24.9	27.6	29.1	26.4
	21	42.6	41.7	41.8	41.8	41.0	40.6	40.7	40.8	41.1	41.0	41.2	42.8
	22	31.0	31.7	30.3	30.3	30.3	31.9	30.5	30.9	28.1	30.4	29.2	27.4
	23	43.8	40.9	42.2	42.2	41.5	41.3	41.5	41.7	43.8	42.0	41.8	43.7
	24	29.7	31.7	30.0	30.0	30.2	31.7	30.3	30.8	28.7	30.3	29.0	27.3
	25	40.5	41.2	41.6	41.6	41.5	41.3	41.8	41.6	40.7	42.0	42.0	43.7
	26	31.9	31.4	29.0	29.0	29.6	31.0	29.9	30.0	26.7	29.5	28.7	26.6
	27	43.3	42.0	41.8	41.8	41.9	41.9	42.4	42.3	43.0	42.7	42.4	44.5
	28	27.3	30.3	28.7	28.7	29.7	31.1	29.9	30.2	27.4	29.7	28.7	26.7

Table 8. Results of Models 28-day Compressive Strength Values

Source	$f'_{c\,28}$ (MPa)												
	Mix #	Exp.	Chidiac	Mechling	de Larrard	Tango	Pann	Popovics (1998)	Karni	Popovics (2008)	Powers	Popovics (1985)	Feret
Bloem and Walker	1	16.0	17.9	20.5	20.5	18.2	18.7	18.1	19.7	19.3	19.9	18.8	15.5
	2	19.2	18.9	19.8	19.8	19.0	19.6	19.5	19.9	19.4	20.1	19.6	16.2
	3	20.1	20.7	19.9	19.9	20.3	21.3	21.2	21.0	20.1	21.0	20.7	17.4
	4	20.1	21.5	19.5	19.5	20.9	22.1	21.9	21.6	20.5	21.5	21.2	17.9
	5	34.7	30.6	29.9	29.9	28.6	31.9	29.0	29.0	27.9	28.5	27.0	26.1
	6	35.1	33.8	32.1	32.1	32.0	36.1	32.9	32.7	30.6	32.2	30.2	30.6
	7	33.9	34.6	31.3	31.3	32.8	36.9	33.8	33.4	31.0	33.0	31.1	31.7
	8	32.2	35.3	30.9	30.9	33.6	37.7	34.6	34.2	31.5	33.9	31.9	32.8
	9	41.0	37.5	38.2	38.2	36.1	39.4	36.4	36.6	37.2	36.4	34.5	36.6
	10	39.1	38.3	38.4	38.4	37.9	40.4	38.5	38.4	38.3	38.4	37.0	39.6
	11	38.2	40.0	39.0	39.0	39.8	42.5	41.1	40.9	40.6	41.3	39.3	43.0
	12	35.2	39.7	37.2	37.2	39.7	41.9	40.7	40.5	40.0	40.8	39.3	42.7
	13	16.3	13.9	16.6	16.6	14.2	13.8	15.4	14.6	15.7	15.8	16.7	12.2
	14	20.8	16.9	17.7	17.7	16.7	16.8	18.3	16.8	17.3	17.5	18.7	14.2
	15	21.4	18.7	17.9	17.9	18.1	18.5	20.0	18.0	18.1	18.5	20.0	15.4
	16	20.4	19.1	17.5	17.5	18.5	19.0	20.3	18.5	18.5	18.9	20.2	15.7
	18	32.2	30.2	28.5	28.5	29.0	30.9	29.3	27.9	26.3	27.4	28.6	26.7
	19	32.3	30.9	27.8	27.8	29.8	31.6	30.2	28.7	26.9	28.2	29.6	27.7
	20	30.8	31.5	27.3	27.3	30.5	32.0	31.0	29.3	27.3	28.8	30.4	28.6
	21	39.3	34.9	35.4	35.4	34.2	35.2	33.0	33.1	33.1	32.7	33.1	33.7
	22	34.2	34.7	34.4	34.4	35.0	34.9	33.8	33.5	32.9	33.1	34.6	35.0
	23	32.4	35.7	34.4	34.4	36.6	36.6	35.7	35.5	35.1	35.2	36.3	37.4
	24	31.8	36.3	33.8	33.8	37.3	37.0	36.5	36.1	35.6	35.9	37.4	38.6
Kim *et al.*	1	49.6	45.9	44.1	44.1	46.8	46.0	46.7	45.3	58.6	45.3	47.2	46.0
	2	35.9	31.7	27.9	27.9	30.3	31.6	31.3	29.5	36.4	29.1	29.5	24.9
Gardner	1	35.16	34.2	31.0	31.0	32.7	35.4	33.1	33.5	32.9	32.6	30.0	27.9
	2	46.62	47.5	48.6	48.6	47.1	46.4	48.1	47.8	46.8	48.2	48.3	49.5
Zain & Abd	1	31	29.6	27.4	27.4	30.3	31.3	28.6	29.1	30.8	28.3	28.0	25.3
Gallo & Popovics	1	39.2	39.2	39.2	39.2	39.2	36.8	39.2	39.2	39.2	39.2	39.2	39.2
	2	31	35.2	33.9	33.9	34.5	32.9	34.5	33.9	33.2	33.6	34.3	32.4
	3	25.8	29.9	28.3	28.3	29.2	27.8	29.1	28.1	27.7	27.8	29.5	25.9
	4	23.8	29.4	27.8	27.8	28.4	27.8	28.9	27.9	27.5	27.7	28.5	25.0
	5	45.7	42.3	43.5	43.5	42.7	41.5	43.8	44.5	47.0	45.1	42.4	45.0
	6	40.2	39.7	39.5	39.5	39.4	39.4	40.3	40.8	42.0	41.0	38.2	39.5
	7	33.8	37.5	36.5	36.5	36.8	37.5	37.6	38.2	38.5	38.1	35.2	35.6
	8	34.7	33.3	31.8	31.8	32.6	33.0	32.9	33.3	33.3	33.0	31.2	29.9
Namyong *et al.*	1	21.1	25.3	25.4	25.4	24.7	24.0	23.1	26.3	22.7	26.5	22.7	23.5
	2	24.2	24.7	24.8	24.8	24.3	23.6	23.3	25.7	23.5	26.1	22.9	23.1
	3	23	24.7	24.8	24.8	24.3	23.6	22.9	25.8	23.1	26.2	22.6	23.1
	4	26.2	24.7	24.6	24.6	23.8	22.9	24.2	24.7	23.7	25.2	23.7	22.5
	5	24	23.9	24.1	24.1	23.7	22.9	23.0	25.0	23.6	25.4	22.7	22.4
	6	22.5	23.7	24.0	24.0	23.5	22.7	22.6	25.0	23.1	25.4	22.3	22.2
	7	22.6	25.0	25.3	25.3	25.0	24.4	23.3	26.6	23.5	26.8	23.0	23.8
	8	31.2	28.1	27.7	27.7	27.2	25.9	27.5	27.6	26.5	27.8	27.1	26.3
	9	26.9	30.6	30.5	30.5	30.3	29.2	28.6	31.3	28.6	31.3	28.2	30.3
	10	30.7	28.2	27.8	27.8	27.7	26.4	27.8	28.2	27.9	28.3	27.4	26.9
	11	28.8	26.6	26.4	26.4	26.2	25.1	26.4	26.8	26.3	27.1	25.9	25.2
	12	27.6	32.1	32.2	32.2	31.8	29.7	30.3	32.4	29.6	32.3	30.2	32.2
	13	28.5	32.9	32.8	32.8	32.3	30.5	30.2	33.2	29.4	33.1	30.1	32.9
	14	32	28.5	28.1	28.1	27.8	26.3	28.2	28.1	27.6	28.2	27.8	27.1
	15	30.5	28.4	28.1	28.1	28.1	26.7	28.1	28.5	28.4	28.6	27.6	27.4
	16	31.6	28.1	27.7	27.7	27.3	26.1	27.5	27.8	27.0	28.0	27.1	26.5
	17	31.7	28.6	28.2	28.2	28.2	27.1	27.7	28.9	27.9	29.0	27.2	27.6
	18	30.2	31.3	31.2	31.2	31.3	29.2	30.1	31.8	30.8	31.7	30.0	31.5

Table 8. (continued)

Source	$f'_{c\,28}$ (MPa)												
	Mix #	Exp.	Chidiac	Mechling	de Larrard	Tango	Pann	Popovics (1998)	Karni	Popovics (2008)	Powers	Popovics (1985)	Feret
	19	30.2	30.7	30.6	30.6	30.8	28.8	29.9	31.2	30.5	31.2	29.7	30.8
	20	29.7	30.5	30.6	30.6	30.7	29.3	29.2	31.5	30.0	31.5	28.9	30.8
	21	29.8	30.4	30.2	30.2	30.1	29.5	27.5	31.5	27.3	31.5	27.1	29.9
	22	26.7	30.6	30.6	30.6	30.7	29.4	29.0	31.6	29.7	31.5	28.7	30.7
	24	27.8	30.0	29.8	29.8	29.9	28.8	28.5	30.8	29.2	30.8	28.1	29.7
	25	29.1	29.0	28.8	28.8	28.9	27.6	28.3	29.5	28.9	29.6	27.9	28.4
	26	29.2	28.7	28.5	28.5	28.6	27.2	28.3	29.1	28.9	29.2	27.9	28.0
	27	27.6	29.7	29.5	29.5	29.6	28.3	28.6	30.4	29.3	30.4	28.2	29.3
	28	28.9	25.2	25.1	25.1	24.7	23.7	25.4	25.4	25.0	25.8	24.9	23.5
	29	27.3	30.7	30.3	30.3	30.1	28.6	29.1	30.7	28.9	30.7	28.8	29.9
	30	27.6	30.7	30.3	30.3	30.3	28.8	29.2	31.0	29.5	30.9	28.9	30.2
	31	29.1	32.4	32.2	32.2	31.9	29.7	30.4	32.4	30.1	32.4	30.4	32.3
	32	26.2	30.2	30.0	30.0	30.1	28.9	28.6	30.9	29.1	30.9	28.2	29.9
	33	28.3	31.4	31.3	31.3	31.2	29.2	30.0	31.7	29.9	31.7	29.8	31.3
	34	27.5	29.5	29.3	29.3	29.4	28.3	28.3	30.2	28.9	30.2	27.9	29.1
	35	31.3	29.7	29.7	29.7	29.9	28.0	29.6	30.2	30.5	30.2	29.3	29.7
	36	28.7	28.9	28.7	28.7	28.9	27.6	28.2	29.5	28.9	29.6	27.8	28.4
	37	28.2	31.5	31.2	31.2	31.1	29.5	29.7	31.8	29.8	31.8	29.5	31.3
	38	27.2	29.8	29.7	29.7	29.5	28.2	28.6	30.2	28.3	30.3	28.2	29.2
	39	30.9	29.0	29.0	29.0	29.4	27.4	29.6	29.6	31.0	29.6	29.3	29.1
Namyong *et al.*	40	30.7	30.6	30.7	30.7	31.1	29.5	29.5	31.8	30.9	31.8	29.3	31.2
	41	30	32.1	32.3	32.3	32.4	29.7	31.2	32.7	32.3	32.7	31.3	33.0
	42	30.7	32.9	33.7	33.7	33.9	30.5	32.2	34.2	34.3	34.2	32.7	35.2
	43	31	32.7	33.5	33.5	33.7	30.1	32.4	33.9	34.5	33.8	33.0	34.9
	44	29.8	32.6	32.8	32.8	33.0	30.0	31.6	33.3	33.3	33.2	31.9	33.9
	45	30.2	31.4	31.4	31.4	31.7	29.4	30.5	32.1	32.1	32.1	30.5	32.1
	46	30.7	31.5	31.6	31.6	31.8	29.4	30.8	32.2	32.0	32.1	30.8	32.3
	47	30.1	30.7	30.6	30.6	30.9	28.5	30.4	31.1	31.5	31.1	30.3	31.0
	48	30.9	30.8	30.8	30.8	31.0	28.7	30.4	31.3	31.5	31.3	30.3	31.2
	49	33.3	33.5	33.8	33.8	33.6	30.2	32.2	33.8	32.4	33.8	32.7	34.8
	50	30.8	32.3	32.4	32.4	32.5	29.7	31.3	32.8	32.5	32.8	31.6	33.2
	51	29.3	31.9	32.0	32.0	32.1	30.1	30.4	32.8	31.5	32.8	30.3	32.7
	52	30.7	28.6	28.5	28.5	28.9	27.6	28.3	29.5	30.0	29.6	27.9	28.4
	53	29.5	30.6	30.5	30.5	30.8	28.8	29.9	31.2	31.3	31.2	29.8	30.8
	54	34.3	29.9	29.8	29.8	30.1	27.5	30.4	30.0	31.5	30.0	30.3	29.9
	55	32.4	30.4	30.4	30.4	30.6	28.2	30.4	30.8	31.5	30.8	30.3	30.7
	56	29.4	33.4	34.1	34.1	34.1	29.8	33.1	34.0	34.6	33.9	33.9	35.5
	57	30.8	31.1	31.2	31.2	31.4	29.0	30.7	31.7	31.8	31.7	30.7	31.7
	58	31.5	32.6	32.9	32.9	32.9	29.6	31.9	33.0	32.8	32.9	32.3	33.7
	59	30.6	30.6	30.6	30.6	30.6	28.0	30.6	30.6	30.6	30.6	30.6	30.6

5. Analysis Results and Statistical Evaluation of Models

The calibration results for all models are given in Tables 6. Table 7 presents results of all theoretical variables required in the calculation of strength models for all sources of data mixes. Table 8 presents the 28-day compressive strength theoretical values for all models and all mixes.

Residual plots were first employed for model assessment. The residuals were plotted against the compressive strength for models that account for age as shown in Fig. 1. As shown, the residuals are randomly scattered around zero which reveals that the models provide good representation of the experimental data. In addition, the models were assessed by studying the trend, deviation from exact location, and systematic deviation as shown in Figs. 2 and 3. These figures demonstrate that most of the models follow the same trend as the experimental data and also reveal no deviation or systematic deviation.

The results of the evaluation of the models at 28 days in terms of the number of calibration constants (p), σ, and R^2 are given in Table 9. Results show that the model proposed by Chidiac *et al.* (2013), which mathematically accounts for packing density

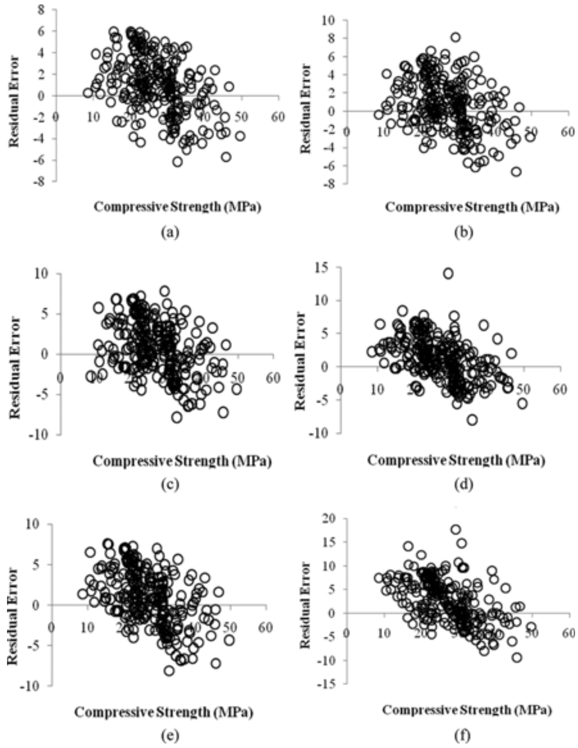

Fig. 1. Residual Error Versus Compressive Strength: (a) Chidiac *et al.* (2013), (b) Tango (2000), (c) Karni (1974), (d) Mechling *et al.* (2009) and de Larrad (1999), (e) Powers (1960), (f) Popovics (1998)

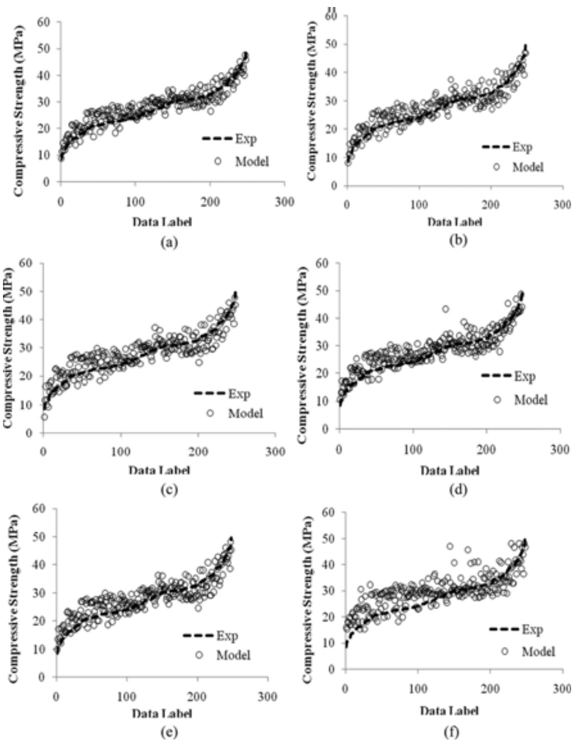

Fig. 2. Data Label Versus Compressive Strength: (a) Chidiac *et al.* (2013), (b) Tango (2000), (c) Karni (1974), (d) Mechling *et al.* (2009) and de Larrad (1999), (e) Powers (1960), (f) Popovics (1998)

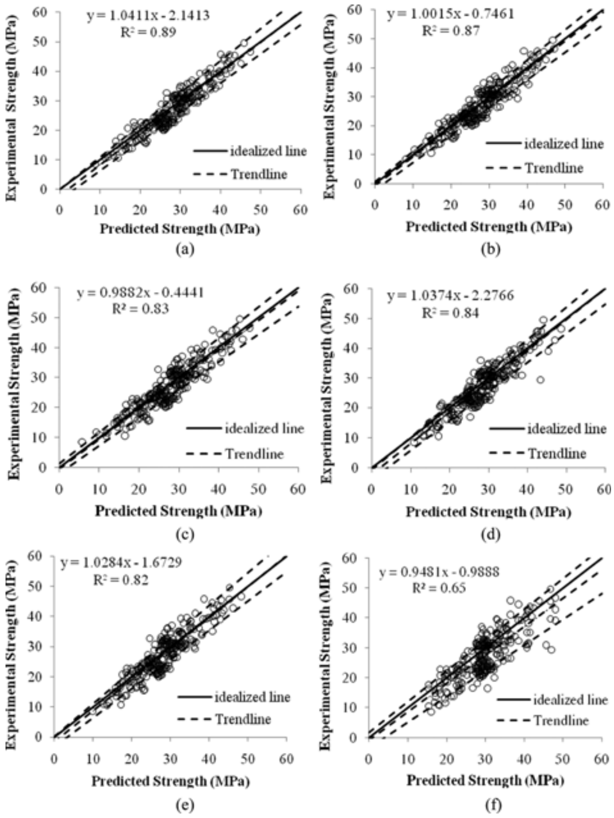

Fig. 3. Predicted Compressive Strength Versus Experimental Compressive Strength: (a) Chidiac *et al.* (2013), (b) Tango (2000), (c) Karni (1974), (d) Mechling *et al.* (2009) and de Larrad (1999), (e) Powers (1960), (f) Popovics (1998)

Table 9. Evaluation of Compressive Strength Models at 28 days

Model	p (#)	σ (MPa)	R^2
Chidiac *et al*. (2013)	3	2.4	0.87
Mechling *et al*. (2009)	4	2.6	0.85
de Larrard (1999)	4	2.6	0.85
Tango (2000)	4	2.6	0.85
Pann *et al*. (2003)	4	2.7	0.85
Popovics (1998)	5	2.5	0.87
Karni (1974)	2	2.8	0.83
Popovics (2008)	4	3.2	0.82
Powers (1960)	2	2.9	0.82
Popovics (1985)	3	2.7	0.84
Feret (1892)	2	3.6	0.80

the most significant factor affecting the compressive strength of concretex (Abrams, 1919; Neville, 1981; Popovics and Ujhelyi, 2008). Fig. 4 demonstrates the effect of w/c on the 28-day strength for non air entrained and for air entrained concrete mixes.

The results display the common trend where an increase in w/c yields a decrease in strength. The results also reveal the degree of significance of w/c on strength in which an increase in w/c from 0.30 to 0.90 resulted in a reduction in strength from 47 to 16 MPa. The scatter in the plot is due to the influence of other

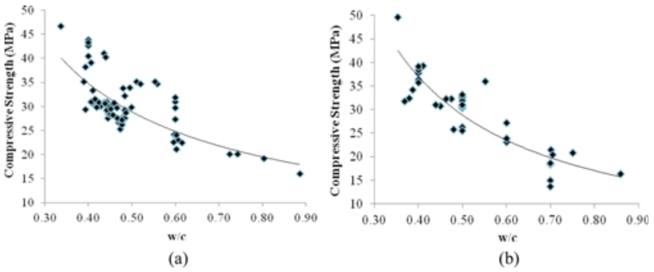

Fig. 4. Compressive Strength vs. w/c: (a) non air entrained concrete, (b) air entrained concrete (range: 3% to 9% air).

Table 10. Evaluation of Compressive Strength Models for All Ages

Model	p (#)	Age (days)	σ (MPa)	R^2
Chidiac *et al*. (2013)	3	3 7 28 191 All	2.4 3.5 2.4 3.0 2.9	0.88 0.85 0.87 0.91 0.89
Tango (2000)	4	3 7 28 191 All	2.6 3.5 2.6 3.6 3.0	0.87 0.82 0.86 0.88 0.87
Karni (1974)	2	3 7 28 191 All	2.7 4.1 2.8 3.8 3.4	0.84 0.78 0.83 0.87 0.83
Mechling *et al*. (2009)	4	3 7 28 191 All	4.4 4.2 2.6 3.1 3.5	0.85 0.81 0.85 0.92 0.84
de Larrard (1999)	4	3 7 28 191 All	4.4 4.2 2.6 3.1 3.5	0.85 0.81 0.85 0.92 0.84
Powers (1960)	2	3 7 28 191 All	2.6 4.3 2.9 3.9 3.5	0.83 0.76 0.82 0.86 0.82
Popovics (1998)	4	3 7 28 191 All	8.8 7.0 2.5 4.6 5.3	0.77 0.87 0.87 0.89 0.65

variables that need be accounted for such as aggregate mixture proportions, sizes, gradation, bonding capability, etc. Table 9 also shows that Feret's model (1892) which is function of the cement to paste ratio, is the least accurate model. This coincides with Popovics' claim that experimental data were shown not to have a very good agreement with this ratio (Popovics, 1985).

Models that account for the age of concrete were also evaluated. The same 9 points were used for calibration and the remaining points were used for evaluation. Results of the evaluation of these models are shown in Table 10. Results reveal that Chidiac *et al.* model (2013), which accounts for age through α, is more accurate than other models. Powers (1960) and Karni (1974)

Table 11. Analysis of Goodness of Fit Using Linear Regression

Model	Age (days)	Intercept	95% Confidence bound	Slope	95% Confidence bound
Chidiac *et al*. (2013)	3 7 28 191 All	-2.2 -1.6 -0.2 1.6 -2.1	[-5.8 1.4] [-4.0 0.8] [-2.4 2.1] [-3.1 6.2] [-3.5 0]	1.08 0.95 1.00 0.97 1.04	[0.93 1.24] [0.96 1.04] [0.92 1.07] [0.83 1.12] [0.99 1.09]
Tango (2000)	3 7 28 191 All	1.6 0.6 1.2 0.9 -0.8	[-1.7 4.9] [-1.8 3.1] [-1.1 3.5] [-4.7 6.5] [-2.2 0.7]	0.91 0.88 0.97 0.99 1.00	[0.77 1.05] [0.78 0.97] [0.89 1.04] [0.82 1.16] [0.95 1.05]
Karni (1974)	3 7 28 191 All	3.0 -0.2 1.5 1.2 -0.4	[-0.4 6.4] [-3.1 2.6] [-1.1 4.0] [-4.5 6.9] [-2.1 1.2]	0.88 0.89 0.95 1.01 0.99	[0.73 1.03] [0.78 1.00] [0.87 1.03] [0.83 1.19] [0.93 1.05]
Mechling *et al*. (2009)	3 7 28 191 All	1.5 0.1 0.2 -4.8 -2.3	[-2.0 5.0] [-2.6 2.7] [-2.2 2.6] [-10.1 0.4] [-4.0 -0.6]	0.83 0.87 1.01 1.14 1.04	[0.70 0.96] [0.77 0.97] [0.93 1.08] [0.98 1.30] [0.98 1.09]
de Larrard (1999)	3 7 28 191 All	1.5 0.1 0.2 -4.8 -2.3	[-2.0 5.0] [-2.6 2.7] [-2.2 2.6] [-10.1 0.4] [-4.0 -0.6]	0.83 0.87 1.01 1.14 1.04	[0.70 0.96] [0.77 0.97] [0.93 1.08] [0.98 1.30] [0.98 1.09]
Powers (1960)	3 7 28 191 All	-0.3 -2.0 1.2 1.4 -1.7	[-4.5 3.8] [-5.2 1.3] [-1.5 3.8] [-4.5 7.3] [-3.4 0.1]	1.01 0.95 0.96 1.00 1.03	[0.84 1.19] [0.83 1.07] [0.88 1.05] [0.82 1.19] [0.97 1.09]
Popovics (1998)	3 7 28 191 All	-1.4 -2.9 0.5 1.0 -1.0	[-6.6 3.7] [-5.2 -0.6] [-1.7 2.7] [-4.3 6.3] [-3.6 1.7]	0.80 0.88 1.00 1.06 0.95	[0.64 0.97] [0.80 0.95] [0.92 1.07] [0.89 1.24] [0.86 1.03]

models account for α, however failure to account for aggregate properties and gradation, standard cement strength, and α_{cr} have negatively affected the accuracy of their models. Mechling *et al.* (2009) and de Larrard (1999) models, which account for packing density, have yielded good accuracy for predicting the strength at 28 days, however the accuracy diminished for other ages because of the empirical relationship between age and strength. The predictive capability of the models that account for age empirically are limited by the type and range of experimental data used and thus might require recalibration for a new set of data. Overall, the predictions of the 11 models tested are found acceptable.

Furthermore, measures of the systematic deviation of the models from exact location were carried out by determining the regression coefficients in the linear regression model. As shown in Table 11, values for the intercept are close to zero and most of the 95% confident intervals include zero. In addition, the values of the slope are close to one and many of the 95% confidence intervals include one. However, for early age prediction, the interval includes one only for models that account for cement hydration. These results signify that the models have very little bias with slight deviation from exact location with Chidiac *et al.* (2013) model being the most accurate.

6. Conclusions

The results of this study can be summarized as follows:

1. An evaluation of published compressive strength models has revealed that the majority of the models predict well the measured values because strength is mostly affected by w/c. Feret's model which is function of cement to paste ratio resulted in the lowest predictability.
2. Review of models proposed in the literature has revealed that Chidiac *et al.* (2013) is the most comprehensive model. Other models did not account for the chemical properties of cement, cement degree of hydration, and/or packing density of aggregates, and some have too many calibration factors that depend on the concrete materials and gradations.
3. Assessment of models using visual display methods and numerical measures has revealed very little bias with slight deviation from exact location with Chidiac *et al*. (2013) model displaying the least scatter.
4. Assessment of models at 28 days has shown that Chidiac *et al*. (2013) model, which mathematically accounts for packing density, aggregate proportions and gradations, resulted in the highest predictability and require only 3 calibration parameters in comparison with Mechling *et al.* (2009) and de Larrard (1999) models which gave high predictability but require 4 calibration parameters.
5. Assessment of concrete strength at any age has shown that Chidiac *et al*. (2013) resulted in the highest predictability. Although Mechling *et al.* (2009) and de Larrard (1999) models account for aggregate packing density and gradations, the effect of age on strength is through an empirical relationship. This led to a drop in their predictability for all ages in comparison with 28 days. Tango (2000) and Karni (1974) models account for the effect of cement degree of hydration on strength thus giving good predictability for all ages but these models fail to account for aggregate properties, standard cement strength and do not include an incubation period for strength development.

It should be noted that due to the absence of mixture properties and characteristics for some experimental data presented in the bibliography, which are input for few models, the author has assumed them to be fixed using realistic values. Therefore, the results of the analysis give tendencies and the accuracy of the models is relative rather than absolute.

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Notations

$A, B, C, D, E, b_1, b_2, k$ = Calibration constants

APT = Average paste thickness

c, w = Quantities of cement and water respectively

D, D_s, and D_{ca} = Mean diameters of the total particles gradation, fine aggregate gradation, and coarse aggregate gradation respectively

D_{max} = Maximum diameter of the total particles gradation

E_A = Activation energy

f'_c = Compressive strength of concrete

$f'_{c\,model\_i}, f'_{c\,exp\_i}$ = The model and experimental compressive strengths corresponding to mix i respectively

K = Paste to aggregate bond strength constant

MPT = Maximum paste thickness

n = Number of test points

p = Number of model constants

P_c = Volume of fraction of capillary porosity

$p_{C3A}, p_{C3S}, p_{SO3}$ = Weight ratios in terms of total cement content

$p_{FA}, p_{SLAG}, p_{FA\text{-}CaO}$ = Weight ratios of mineral admixtures in terms of total cement content

R_{c28} = Standard cement strength at 28 days

S_s, S_o = Specific surface of the cement and that of a typical ASTM Type 1 cement respectively

t, t_e = Age and equivalent age respectively

T = Concrete temperature

V_c, V_w, V_a = Volume fractions of cement, water and air, respectively

$\alpha, \alpha_{cr}, \alpha_u$ = Degree of cement hydration, critical degree of cement hydration and ultimate degree of cement hydration respectively

ϕ, ϕ_s, and ϕ_{ca} = Volume fraction of aggregates, fine aggregates and coarse aggregates respectively

ϕ_{max} = Maximum packing density of aggregate

τ, β = Hydration time parameter and hydration shape parameter respectively

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