KSCE Journal of Civil Engineering (2016) 20(5):1966-1976

DOI 10.1007/s12205-015-0135-4

Structural Engineering

pISSN 1226-7988, eISSN 1976-3808
www.springer.com/12205

Shape Optimization of Arch Dams using Sensitivity Analysis

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Received March 16, 2015/Accepted July 9, 2015/Published Online October 23, 2015

Abstract

The purpose of this paper is to determine the optimal shape of double curvature concrete arch dams by using the sensitivity of design objective to design variables. Design sensitivities quantify the relationship between the design variables such as thickness of crown cantilever, left and right abutment thickness, crown cantilever upstream profile, radius of curvature of water and air faces on one hand and the objective function such as the volume of the concrete arch dam on the other hand. For this aim, first a methodology was developed to provide interface between a finite element method and an optimization algorithm for shape optimization of double curvature concrete arch dams. The constraint conditions are the geometric shape, stress and stability against sliding. Second, the shape optimization of an arch dam was analyzed by using the developed system for sensitivity analysis. The results show that all of the design variables contribute significant and none can be ignored in design. Moreover the crown cantilever upstream profile and Radius of curvature of water face left denoted by USP_7 and R_{LUS16} respectively, show less important role as compared to other design variables.
Keywords: *Arch dam, Shape optimization, Sensitivity Analysis, Latin hypercube sampling (LHS), SIMLAB*

1. Introduction

Structural shape sensitivity analysis, that is, the calculation of quantitative information on how the response of a structure is affected with respect to changes in its parameters that define its shape, plays a key role in shape optimization (Kwak, 1994). Sensitivity analysis can be used to study how the uncertainty in the output of a mathematical model or system (numerical or otherwise) can be apportioned to different sources of uncertainty in its inputs (Saltelli *et al*., 2008). For a successful optimization the requirements are a good finite element model, adequate sensitivities, proper choice of objective function, design variables, constraints and a suitable method of solution of the nonlinear mathematical problem (MotaSoares *et al*., 1992).

Several studies are found in the recent literature that focused on shape sensitivity analysis are Used to decrease optimization processing time (Zhang *et al*., 2009; Akbari *et al*., 2010; Takalloozadeh *et al*., 2014; Duan *et al.*, 2014).

The focus of this paper is on the development and application of optimization and design Sensitivity analysis for determination of the best shape of arch dams. The first section recalls the optimization procedure at regular state. In Section 2, we apply this method to a geometrically arch dam. Section 3 describes the algorithms used to solve the non-linear equilibrium equation at limit point and the optimization problem. Section 4 is devoted to numerical results. Four examples which demonstrate the accuracy and efficiency of the proposed method are shown.

The purpose of this paper is to determine the optimal shape of double curvature concrete arch dams and also sensitivity analysis of design variables in shape optimization of concrete arch dams. In the optimization process, the objective function is the volume of the concrete arch dam, processing the design variables are the geometric shape parameters of the double curvature of arch dam, and the constraint conditions are the geometric shape, the stress and the stability against sliding. Coefficients sensitivity analysis is performed based on the 10000 trails by utilizing Latin Hypercube Sampling. Uniform distributions have been assigned for simulations. Correlation analysis is used to determine relationships between design variables and objective function.

The program basically consists of three parts that are described in the paper: The first section recalls the optimization procedure of an arch dam at regular. In Section 2, sensitivity analysis is performed and section 4 analyzes the results.

2. Mathematical Equation of Arch Dam Design

2.1 Preliminary Design

The main geometric parameters of arch dams are as shown in Table 1.

2.2 The Geometric Model of an Arch Dam

The optimization design of an arch dam involves determining

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Table 1. The Main Geometric Parameters

K	Arch number
EL	Elevation
T_C	The thickness of crown cantilever
T_{AL}	Left Abutment thickness
T_{AR}	Right Abutment thickness
USP	Crown cantilever upstream profile
DSP	Crown cantilever downstream profile
R_{LUS}	Radius of curvature of water face left
R_{RUS}	Radius of curvature of water face right
R_{LDS}	Radius of curvature of air face left
R_{RDS}	Radius of curvature of air face left
x_{eL}	Left abetment curve
x_{eR}	Right abetment curve

the geometric model which should be established firstly. The shape of crown cantilever, radii of curvatures of arches, and incidence angles of arches to the topography and abutment thickening can be considered as the geometrical parameters of major importance for proper development and definition of dam wall geometry (Grigorov *et al.*, 2009). These geometrical parameters can be defined in Table 2.

In Fig. 1, the shape of crown cantilever and layers at control elevations are shown.

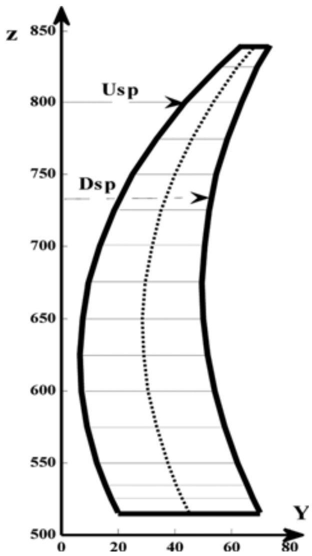

Fig. 1. Cross Section of Crown Cantilever

In Fig. 2, the radius of curvature R is specified at interpolation points by the R_1, R_7, R_{11}, R_{16} as design variables. R is interpolated at each level with the following cubic spline.

In Fig. 3, some related details and geometric parameters are used to define a general horizontal parabolic arch are shown.

Table 2. The Geometric Model of Design Variables

Equation		Quantification of equation		Figure
Crown cantilever shape (Quadratic functions)	$USP(z) = a_0 + a_1 z + a_2 z^2$	**USP:** U/S Crown cantilever upstream profile (U/S)	**Z:** (Vertical coordinates) **a_0, a_1, a_2, b_0, b_1, b_2** (Coefficients)	1
	$DSP(z) = b_0 + b_1 z + b_2 z^2$	**DSP:** Crown cantilever down-stream profile (D/S)		
Thickness of arch dam (Third degree polynomials)	$t_c(z) = c_0 + c_1 z + c_2 z^2 + c_3 z^3$	**$t_c(z)$:** Crown thickness	Z:(Vertical coordinates) **c_0, c_1, c_2, c_3 d_0, d_1, d_2, d_3 e_0, e_1, e_2, e_3** (Coefficients)	-
	$t_{AL}(z) = d_0 + d_1 z + d_2 z^2 + d_3 z^3$	**$t_{AL}(z)$:** Left abutment thickness		
	$t_{AR}(z) = e_0 + e_1 z + e_2 z^2 + e_3 z^3$	**$t_{AR}(z)$:** Right abutment thickness		
Radius of curvature (Cubic spline)	$R_i(x) = a_i + b_i (z - z_i) + c_i (z - z_i)^2 + d_i (z - z_i)^3$ $(i = 0, 1, \ldots, n - 1)$	**R:** Radius of curvature (**R_{LUS}**, water face left) (**R_{RUS}**, water face right) (**R_{LDS}**, air face left) (**R_{RDS}**, air face right)	**Z:** (Vertical coordinates) **a_i, b_i, c_i, d_i** (Coefficients) $i = 0, 1, \cdots, n-1$	2
Horizontal Arches (Parabolic conic functions)	$y = USP + \frac{(x-x_0)^2}{2R_{US}}$	**Y:** Vertical plane	**X (x_L, x_R):** Left and right abutment curve And **$X_0 = 0$**	3
	$y = DSP + \frac{(x-x_0)^2}{2R_{US}}$			
Horizontal section (Parabolic conic functions)	$x_{edl} < x < x_{el}$ $T_{aL}(x) = T_C + \frac{(x-x_{edl})^2(T_{AL}-T_C)}{(x_{el}-x_{edl})^2}$ $x < x_{edl}$ $T_{aL} = T_C$	**TaL(x):** variable thickness in right half of horizontal section	**X_{edR}:** length of segment with constant thickness in and left bank **X_{eL}:** Upstream Left abetment curve **T_C:** The thickness of crown cantilever	4
	$x_{edR} < x < x_{eR}$ $T_{aR}(x) = T_C + \frac{(x-x_{edR})^2(T_{AR}-T_C)}{(x_{el}-x_{edl})^2}$ $x < x_{edR}$ $T_{aR} = T_C$	**$T_{aR}(x)$:** variable thickness in right half of horizontal section	**X_{edR}:** Length of segment with constant thickness in and right bank **X_{eR}:** Upstream right abetment curve **T_C:** The thickness of crown cantilever	

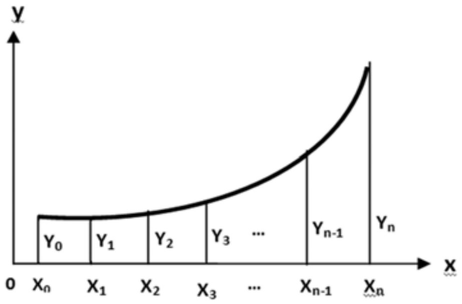

Fig. 2. Cubic Spline

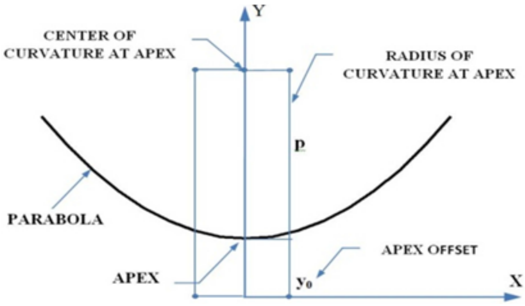

Fig. 3. Definition of Parabola

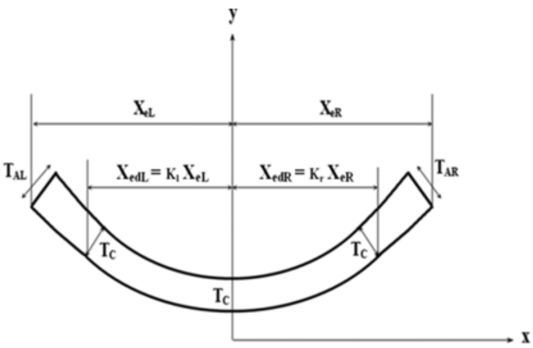

Fig. 4. Horizontal Arch of Dam Body

In Fig. 4. Coefficientskr and k_l determine the portion of arch length with constant thickness in right and left bank. In this paper, k_r and k_l are equal to 2/3.

The number of horizontal arch layers is selected as 16 from the base to the crest elevation. In Table 1, the basic input parameters for definition of crown cantilever and horizontal arches are included. The x coordinate of the vertical middle line can be calculated from the x coordinate of the upstream (x_{eu}) and downstream (x_{id}) surfaces, which can be written as:

$$x_c = \frac{x_{eu} + x_{id}}{2} \tag{1}$$

2.2.1 Programming and Implementation of 3D Model and Loadings

According Eq. (1), a MATLAB finite element analysis program is written for geometrical design of arch dams. The program was developed in APDL programming language of ANSYS code. In the finite element modeling, the geometry of the arch dam is considered as doubled curvature. 1580 eight node elements in the foundation and 180 twenty node elements in the dam body are used. Each node has three degrees of freedom comprised of: translations in the nodal X, Y and Z directions. Two layers of elements were set along the thickness of the dam. The finite element model of the dam is developed which includes the foundation too. As it is shown in Fig. 4, the length and width of the foundation along the global X and Y axes are taken to be 1650 m. For the 3D arch dam analysis, the mass concrete and rock were assumed homogeneous with linear elastic material behaviour. The modulus of elasticity of mass concrete and foundation rock were taken as 28 GPa and 9 GPa respectively. The Poisson's ratio of mass concrete and rock were taken as 0.18 and 0.25, respectively. The mass density of the concrete was chosen as 2400 kg/m^3 and no gravity load was applied on the foundation rock. Concrete and rock were assumed to be homogeneous and isotropic materials. Because foundation is assumed as mass less, only the effects of foundation flexibility are considered in the analysis. For the boundary conditions in the finite element model of the dam, all the degrees of freedom are considered fixed at the outside surfaces of the foundation.

Figure 5. illustrates the dam, foundation and reservoir finite element model. The static usual cases include the effects of silt, tail water pressures and temperature (either summer or winter). While, for optimization purposes, it is convenient to exclude all these effects and merely consider the effects of dam body dead weight and upstream hydrostatic pressures. Two basic loading cases have been considered for the optimization procedure in this research, for the following:

1. SU1 (the first static usual load combination) or self-weight
2. SUN1 (the first static unusual load combination) or self-

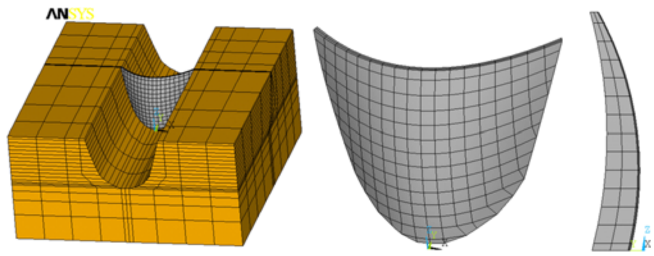

Fig. 5. Three-dimensional Shape of a Typical Arch Dam with Foundation

Table 3. Load Combinations used for Presenting the Analyses Results

Comb.	Single load parts		Factor of Safety		Load combination
ID	Dead weight	Normal water pressure	Tension	Compression	
SU1	√	√	2	3	Static usual
SUN1	√	-	1.5	2	Static unusual

weight and upstream hydrostatic pressures.

Two different load combinations are used to present the analysis results. These load combinations are listed in Table 3.

As the self-weight considered by staged construction method in which dead load is applied in several stages (Pourbakhshian *et al.,* 2015).

3. Optimization Method for Arch Dam Shape

Shape optimization is to minimize the consumed concrete volume while enhancing safety criteria. The shape optimization problem is to find the design variables X while minimizing the objective function $F(X)$ under the constraint functions $h_j(X)$ and $g_j(X)$ that can be stated mathematically as:

$$\begin{aligned} &\text{Find } X = [X_1\ X_2 \dots X_n]^T \\ &a_i \le X \le b_i (i = 1, 2, \dots, n) \\ &\text{To minimize } F(X) \\ &\text{Subject to:} \\ &h_j(X) = 0\ (j = 1, 2, \dots, p) \\ &g_k(X) \le 0\ (k = 1, 2, \dots, m) \end{aligned} \quad (2)$$

The subscripts p, m and n denote the number of equality constraints, behavioural constraints and design variables respectively where a_i and b_i are the allowable lower and upper limits of the design variables which are introduced to deal with various requirements.

3.1 Design Variables

Shape optimization can be improved by increasing the number of design variables but it raises the cost of calculations. According to the geometrical model of arch dams described before in the paper, 31design variables are selected which will be used in the process of optimization as shown in Table 4.

Crown cantilever design variables are shown in Fig. 6.

3.2 Objective Function

The purpose of optimization is to choose proper geometric shape of arch dam to make the project cost minimal on the premise of meeting the needs of strength and stability. Generally, the cost of arch dam is mainly dependent upon the volume of dam body concrete. So the objective function is the dam body volume.

3.3 Constraint Functions

In shape optimization of concrete arch dams, the following three types of constraint sets should be satisfied, as required by

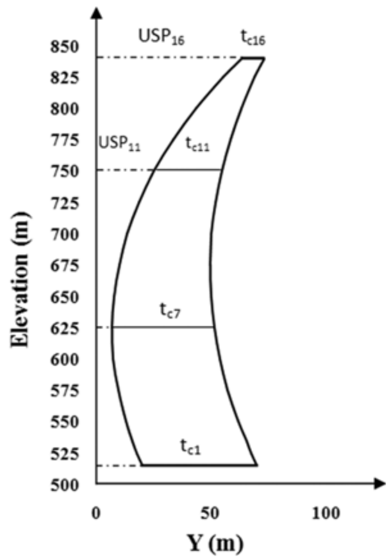

Fig. 6. Crown Cantilever Design Variables

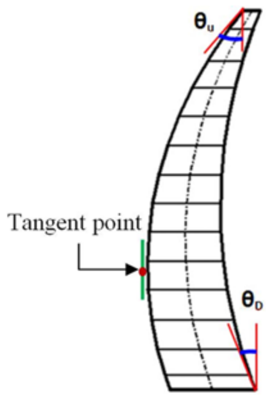

Fig. 7. Slope of Overhang in Upstream

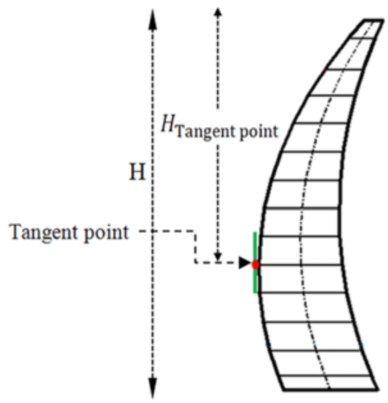

Fig. 8. Location of Tangent Point and Downstream of Arch Dam

Table 4. Design Variables Types

Thickness			Radius				USP
t_{C1}	t_{AL1}	t_{AR1}	R_{LUS1}	R_{RUS1}	R_{LDS1}	R_{RDS1}	USP_1
t_{C7}	t_{AL7}	t_{AR7}	R_{LUS7}	R_{RUS7}	R_{LDS7}	R_{RDS7}	USP_7
t_{C11}	t_{AL11}	t_{AR11}	R_{LUS11}	R_{RUS11}	R_{LDS11}	R_{RDS11}	USP_{16}
t_{C16}	t_{AL16}	t_{AR16}	R_{LUS16}	R_{RUS16}	R_{LDS16}	R_{RDS16}	

Table 5. Constraints in Arch Dam Shape Optimization

Geometrical constraints			Figure
Thickness of horizontal arch (i = 0, 1, …, n)	$T_{c_{i+1}} < T_{c_i} \rightarrow \frac{T_{c_i}}{T_{c_{i+1}}} - 1 \le 0$	Tc decreases from base to crest	6
	$T_{c_i} < T_{AL_i} \rightarrow \frac{T_{c_i}}{T_{AL_i}} - 1 \le 0$ $T_{c_i} < T_{AR_i} \rightarrow \frac{T_{c_i}}{T_{AR_i}} - 1 \le 0$	The crown cantilever thickness (T_c) is lower than abutment thickness (T_A)	
Slope of overhang	$\theta^U_{\max} < \theta^U_{alw}$	To facilitate construction, the maximum slope of overhang at the U/S and D/S faces should be controlled. θ^U_{max} And θ^D_{max} are the allowable maximal overhang slope of the upstream and downstream surfaces, and θ^U_{alw} and θ^D_{alw} are the allowable maximal overhang slope, respectively.	7
	$\theta^D_{\max} < \theta^U_{alw}$		
Crown cantilever profile	$USP_{i+1} < USP_i \rightarrow \frac{USP_{i+1}}{USP_i} - 1 \le 0$ (i = 1, 2, …, n $_{\text{tangent point}}$) $USP_i < USP_{i+1} \rightarrow \frac{USP_i}{USP_{i+1}} - 1 \le 0$ (i = n $_{\text{tangent point}}$, …, n)	Below tangent points, the angles of tangents are negative and above it are positive. The plotting steps should be increased to avoid gap of curves of the upper and lower parts. i = n $_{\text{tangent point}}$ → tangent points	8
	$USP_{i+1} < DSP_i \rightarrow \frac{DSP_{i+1}}{DSP_i} - 1 \le D$ $USP_i < DSP_{i+1} \rightarrow \frac{DSP_i}{DSP_{i+1}} - 1 \le D$	Crown cantilever downstream profile	
The Location tangent points	$H_{\text{tangant point}} < H_{\text{tangant point}_{\max}}$ $\rightarrow \frac{H_{\text{tangant point}}}{H_{\text{tangant point}_{\max}}} - 1 \le 0$	The maximum distance between crest to tangent point = $H_{\text{tangant point}_{\max}}$ = 0.6 H	
Radius of curvature	$R_{LDS_i} < R_{LUS_i} \rightarrow \frac{R_{LDS_i}}{R_{LUS_i}} - 1 \le 0$ $R_{RDS_i} < R_{RUS_i} \rightarrow \frac{R_{RDS_i}}{R_{RUS_i}} - 1 \le 0$	The most important geometric constrains are those that prevent from intersection of upstream and downstream face as (i = 0, 1, …, n)	
	$\frac{R_{LUS_i}}{R_{LUS_{i+1}}} - 1 \le 0, \frac{R_{RUS_i}}{R_{RUS_{i+1}}} - 1 \le 0$ $\frac{R_{LUS_i}}{R_{LDS_{i+1}}} - 1 \le 0, \frac{R_{RDS_i}}{R_{RDS_{i+1}}} - 1 \le 0$	The Variation of radius at crown cantilever along with height of dam should be satisfactory to some kind of nonlinear variation rule	

Stress constraints and Stability constraints			
Stress constraints	Stress distribution in the structure	$\frac{f}{f_c} \le \frac{s}{sf}$ $\frac{f}{f_c} - \frac{S}{sf} \le 0$ (i = 0, 1, …, n_e)	the behavior constraints are defined to prevent from failure of each element (i) of arch dam under specified safety factor (*sf*). For this purpose, the failure criterion of concrete of Willam and Warnke due to multiaxial stress state is employed. Where *f* is a function of the principal stress state f_c $\sigma_1 \le \sigma_2 \le \sigma_3$ and *S* is failure surface expressed in terms of principal stresses, uniaxial compressive strength of concrete (f_c), uniaxial tensile strength of concrete (f_t) and biaxial compressive strength of concrete (f_{cb}).
Stability constraints	Central angle of the arch	$1 - \frac{\phi_U}{90} \le 0, \frac{\phi_U}{110} - 1 \le 0$ $1 - \frac{\phi_D}{90} \le 0, \frac{\phi_D}{110} - 1 \le 0$	ϕ is the sum of central angles at the right and the left arc hand. $90 \le \phi_U \le 110,\ 90 \le \phi_D \le 110$
	Overturning	$USP_1 < Y_{bar} \rightarrow \frac{USP_1}{Y_{bar}} - 1 \le 0$	to verify the overall overturning stability of crown cantilever arch dam monoliths

the demands of design and construction, are shown in Table 5.

1. Geometrical constraints
2. Stress constraints
3. Stability constraints

4. Optimization Algorithm

The Simultaneous Perturbation Stochastic Approximation (SPSA) has recently attracted considerable attention in areas such as statistical parameter estimation, feedback control, simulation-based optimization, signal and image processing and experimental design. However, the SPSA has not been tested yet for structural optimization and it is the first study that is employed for this purpose. The promising feature of the SPSA optimization algorithm is that it requires only two structural analyses in each cycle of optimization process, regardless of the dimension of optimization problem. This attribute can drastically reduce the computational cost of optimization, particularly in problems with large number of variables. The following step-by-step summary shows the process of SPSA in arch dam optimization:

- **Step 1**: *Initialization and coefficient selection.* Set counter index $k = 0$. Pick initial guess and non-negative coefficients a, c, A, α, and γ in the SPSA gain sequences $a_k = a/(A + k + 1)^{\alpha}$ and $\mathbf{c}_k = c/(k + 1)^{\gamma}$. The choice of gain sequences (a_k and c_k) is critical to the performance of SPSA. Spall provides some guidance on picking these coefficients in a practically effective manner.
- **Step 2**: *Generation of the simultaneous perturbation vector.* Generate by Monte Carlo an n_v dimensional random perturbation vector Δ_k, where each of the n_v components of Δ_k is independently generated from a zero mean probability distribution satisfying some conditions. A simple and theoretically valid choice for each component of Δ_k is to use a Bernoulli ±1 distribution with probability of 1/2 for each ±1 outcome. Note that uniform and normal random variables are not allowed for the element of Δ_k by the SPSA regularity conditions.
- **Step 3**: *Fitness function evaluations.* Obtain two measurements of the fitness function f(0) based on the simultaneous perturbation around the current design vector $\hat{\mathbf{x}}_k$: $f(\hat{\mathbf{x}}_k + \mathbf{c}_k\Delta_k)$ and $f(\hat{\mathbf{x}}_k - \mathbf{c}_k\Delta_k)$ with the $\mathbf{c}_k$ and Δ_k from steps 1 and 2.
- **Step 4**: *Gradient approximation.* Generate the simultaneous perturbation approximation to the unknown accurate gradient $G(\hat{\mathbf{x}}_k)$:

$$G_k(\hat{x}_k) \approx \hat{G}_k(\hat{x}_k) = \frac{f(\hat{x}_k + c_k\Delta_k) - f(\hat{x}_k - c_k\Delta_k)}{2c_k}\begin{bmatrix}\Delta_{k1}^{-1} \\ \Delta_{k2}^{-1} \\ \vdots \\ \Delta_{knv}^{-1}\end{bmatrix} \quad (3)$$

Where Δ_{ki} is the ith component of Δ_k vector.

- **Step 5**: *Updating* $\hat{\mathbf{x}}$ *estimate.* Use the standard stochastic approximation (SA) to update $\hat{\mathbf{x}}_k$ to a new value $\hat{\mathbf{x}}_{k+1}$:

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_k - a_k\hat{\mathbf{G}}_k(\hat{\mathbf{x}}_k) \quad (4)$$

- **Step 6**: *Iteration or termination.* Return to step 2 with $k + 1$ replacing k. Terminate the algorithm if the maximum number of iterations (MNI) has been reached (Seyedpoor *et al.*, 2011). The flow chart of the SPSA algorithm for the arch dam optimization problem is as shown in Fig. 9.

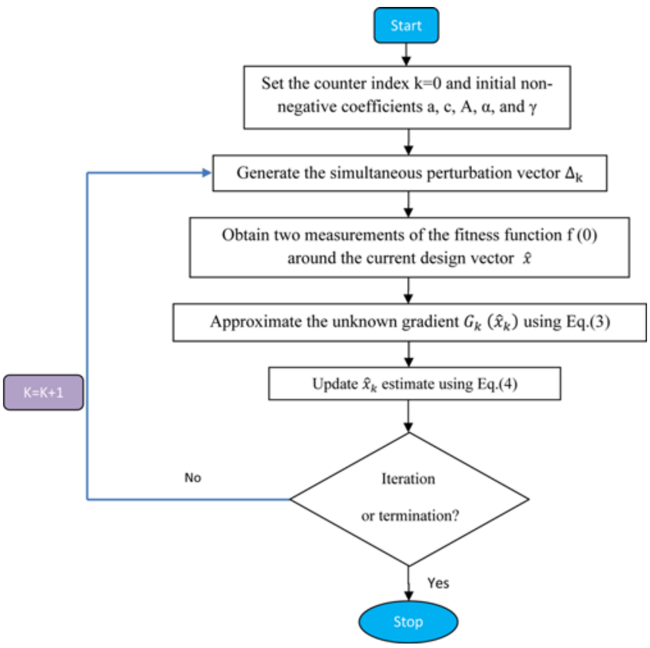

Fig. 9. The Flowchart of SPSA Algoritm

5. Sensitivity Analysis

Sensitivity analysis, in particular, measures the change in the outputs due to changes in the inputs, the latter generated by sampling from their values distributional range. SA is therefore important to identify the key factors affecting the output (Foscarini *et al.*, 2010).

In this section, the shape optimization of an arch dam is analyzed by Design Sensitivity Analysis (DSA).

The model procedure is as follows:

1. Generation of a Sample of Design Variables by SIMLAB Software
2. By applying generation data, the basis for sensitivity analysis is derived by MATLAB software.

5.1 Generation of Design Variables

SIMLAB provides a free development framework for Sensitivity and Uncertainty Analysis (SimLab ver.3.2.6, 2009).

To use SIMLAB the user performs the following operations.

5.1.1 Select a Range and a Distribution (probability distribution functions, pdfs) for Each Input Factor

These selections will be used in the next step for the generation of as ample from the input factors. In this paper, uniform (Rectangular) distribution is used.

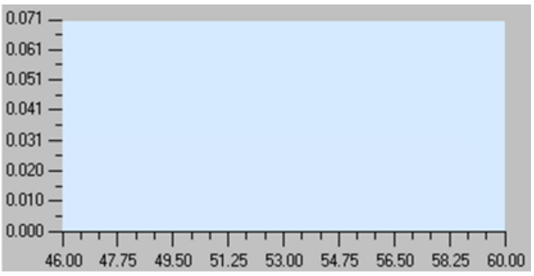

Fig. 10. PDF of an Uniform Distribution, with a = 46.00 and b = 60.00 for T_{C_1}

If it is only possible to define a minimum and maximum value for a factor, a uniform distribution is appropriate. For a uniform distribution all values between the minimum and maximum values are equally likely to be sampled. It is the simplest of all continuous distributions. Parameters are the upper and lower bounds: a and b. The uniform PDF is given as (Ekström, 2005):

$$f(x) = \begin{cases} 0, & x \notin (a,b) \\ \dfrac{1}{b-a}, & x \in (a,b) \end{cases} \tag{5}$$

Figure 10, shows the PDF of a uniform distribution with a minimum value a = 46.00 and a maximum value b = 60.00.

5.1.2 Generate a Sample of Elements from the Distribution of the Inputs Previously Specified

The result of this step is a sequence of sample elements. Among different sampling strategies, LHS is the most appropriate one for the sensitivity analysis of simulation models (McKay *et al.*, 1976). The LHS is a particular case of stratified sampling which is meant to achieve a better coverage of the sample space of the selected input parameters (Saltelli *et al.*, 2000). In this study, LHS is utilized for generating input sample for sensitivity simulations.

The first step in the sample generation phase is to select range sand distributions (probability distribution functions, pdfs) for the input factors. The second step is to select a sampling method from among the available ones. The third step is the actual generation of the sample from the input pdfs. 1000 different samples have been generated with the help of Simlab. For the generation LHS was used.

5.2 By Applying Generation Data, the Basis for Sensitivity Analysis was Derived by MATLAB Software

Correlation coefficients, which are the special cases of regression-based analysis, are typically useful methods for sensitivity analysis (Saltelli *et al.*, 2000). The Pearson correlation coefficient, for instance, can be used to characterize the degree of linear relationship between the output values and sampled values of individual inputs.

A quantitative estimate of linear correlation can be determined

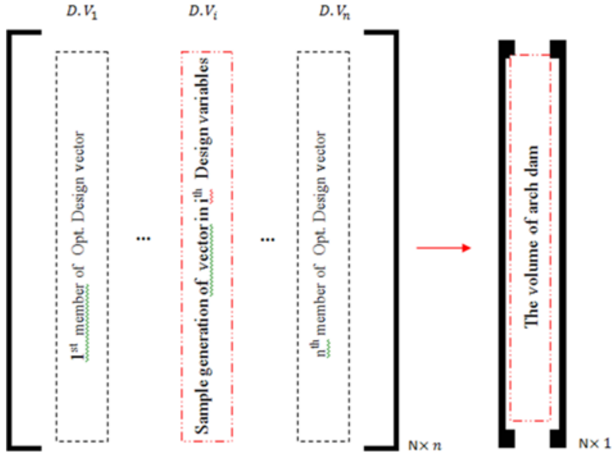

Fig. 11. S.A. Assessment Flowcharts in Arch Dam

by calculating a simple correlation coefficient on the parameter values of input and output. Pearson's product moment correlation coefficient is denoted by r and is defined as

$$r = \frac{\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)(Y_j-\bar{Y})}{\left[\sum_{j=1}^{n}(X_{ij}-\bar{X}_i)^2\sum_{j=1}^{n}(Y_{ij}-\bar{Y}_i)^2\right]^{\frac{1}{2}}} \tag{6}$$

For the correlation between X_i and Y (Conover, 1980). The larger the absolute value of r the stronger the degree of linear relationship between the input and output values. A negative value of r indicates the output is inversely related to the input. A linear regression on the data can be used to determine the correlation coefficient from the square root of the coefficient of determination, R^2 (Hamby, 1994).

In this process, correlation coefficients are obtained for 31 design variables.

$$\text{Opt. Design vector} = \begin{bmatrix} D\cdot V_1 \\ D\cdot V_2 \\ \vdots \\ DV_n \end{bmatrix}_{1\times n} \tag{7}$$

n = Number of design variables
N = Number of samples

$$r_n = \frac{\sum_{j=1}^{N}[(D.V_{nj}-\overline{D.V_{nj}})(V_j-\bar{V})]}{\left[\sum_{j=1}^{N}(D.V_{nj}-\overline{D.V_{nj}})^2\sum_{j=1}^{N}(V_j-\bar{V})^2\right]^{\frac{1}{2}}}$$

V= the volume of arch dam for

$$\rightarrow S.A = r = \text{Correlation coefficients} \rightarrow S.A._{index} = \begin{bmatrix} S\cdot A_1 \\ S\cdot A_2 \\ \vdots \\ S.A_n \end{bmatrix}$$

6. Result

6.1 Result of Optimization of Arch Dam

The optimization process of arch dam according to the

Table 6. Design Variables at the Initial and Optimum Designs

Design Variable Number	Design Variable (Input parameter)		Uniform distribution		Initial values	Optimum values
			Minimum	Maximum		
1	T_{C1}	The thickness of crown cantilever in level 1	46	60	50	54.03
2	T_{C7}	The thickness of crown cantilever in level 7	35	55	44.52	35.00
3	T_{C11}	The thickness of crown cantilever in level 11	25	45	33.22	25.00
4	T_{C16}	The thickness of crown cantilever in level 16	8	11	10	8.00
5	T_{AL1}	Left Abutment thickness in level 1	48	65	50	62.61
6	T_{AL7}	Left Abutment thickness in level 7	55	70	60.1	61.59
7	T_{AL11}	Left Abutment thickness in level 11	40	60	48.87	40.66
8	T_{AL16}	Left Abutment thickness in level 16	8	11	10	10.52
9	T_{AR1}	Right Abutment thickness in level 1	48	65	50	64.35
10	T_{AR7}	Right Abutment thickness in level 7	55	70	60.1	69.71
11	T_{AR11}	Right Abutment thickness in level 11	40	60	48.87	41.05
12	T_{AR16}	Right Abutment thickness in level 16	8	11	10	9.00
13	USP_1	Crown cantilever upstream profile in level 1	15	30	20	25.89
14	USP_7	Crown cantilever upstream profile in level 7	5	8	6.83	7.66
15	USP_{16}	Crown cantilever upstream profile in level 16	50	75	63.12	61.93
16	R_{LUS1}	Radius of curvature of water face left in level 1	90	110	100	102.05
17	R_{LUS7}	Radius of curvature of water face left in level 7	105	125	115.07	106.36
18	R_{LUS11}	Radius of curvature of water face left in level 11	130	155	144.56	130.00
19	R_{LUS16}	Radius of curvature of water face left in level 16	190	210	200	210.00
20	R_{RUS1}	Radius of curvature of water face right in level 1	90	110	100	94.98
21	R_{RUS7}	Radius of curvature of water face right in level 7	105	125	115.07	105.00
22	R_{RUS11}	Radius of curvature of water face right in level 11	130	155	144.56	130.89
23	R_{RUS16}	Radius of curvature of water face right in level 16	190	210	200	197.81
24	R_{LDS1}	Radius of curvature of air face left right in level 1	40	60	50.83	54.54
25	R_{LDS7}	Radius of curvature of air face left right in level 7	50	70	60.81	70.00
26	R_{LDS11}	Radius of curvature of air face left right in level 11	85	105	95.32	105.00
27	R_{LDS16}	Radius of curvature of air face left right in level 16	180	210	192.03	207.10
28	R_{RDS1}	Radius of curvature of air face left in level 1	40	60	50.83	44.92
29	R_{RDS7}	Radius of curvature of air face left in level 7	50	70	61.48	69.33
30	R_{RDS11}	Radius of curvature of air face left in level 11	85	105	94.98	105.00
31	R_{RDS16}	Radius of curvature of air face left in level 16	180	210	192.03	195.99

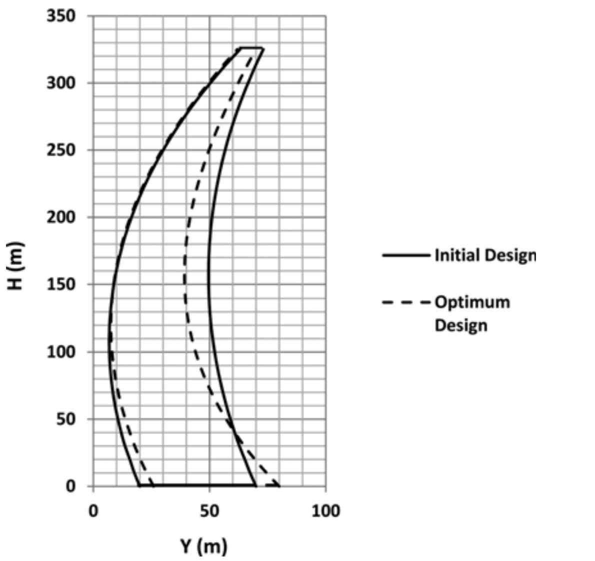

Fig. 12. The Comparison of Shape Crown Cantilever in Initial and Optimum Design

Table 7. Boldness Coefficients for Initial and Optimum Design

Lombardi Boldness Coefficient $\left(\lambda = \frac{s^2}{h \times V}\right)$				
Design	Height (m)	Volume (Mm^3)	Mid surface area (m^2)	λ
Initial	325	4.92485	137314	11.78
Optimum	325	3.93796	137980	14.88

above methodology converged after 1000 iterations. As it can be seen, the volume of the dam body defined by the present optimization is 1057550 m^3 less than the initial volume, i.e. 21% less.

The difference between the initial and optimum design shapes can be seen in Fig. 12. It is observed that the optimal design is thinner than that initial design.

The principal stress for two load cases as shown in Fig.13 and Table 8.

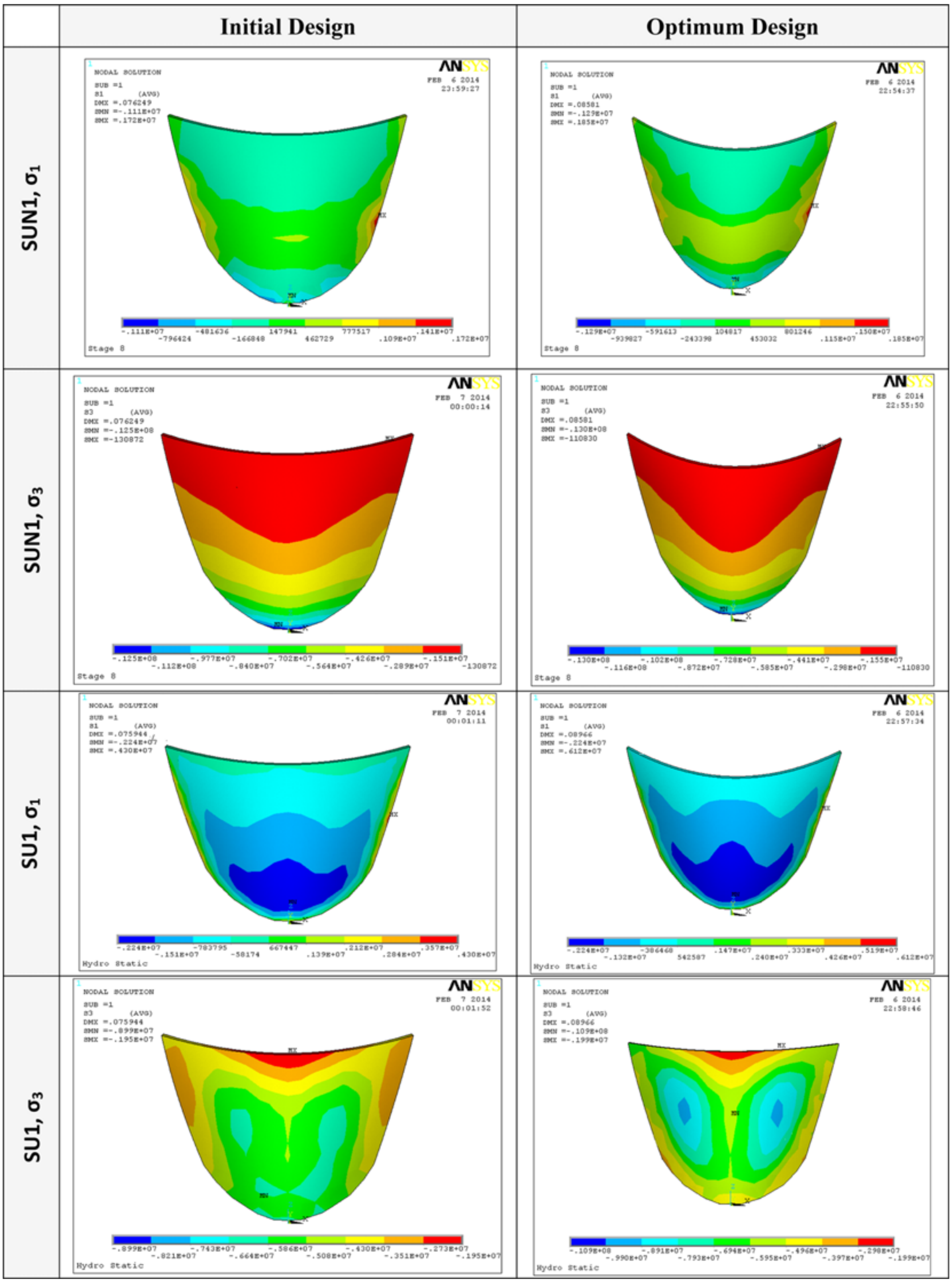

Fig. 13. Principal Stress σ_1 and σ_3 in Initial and Optimum Design Shape Usual Load Combination (SU) and Unusual Load Combination (SUN)

Table 8. Summary of the Results of the Models

		US		DS	
		Maximum tension	Maximum compression	Maximum tension	Maximum compression
Initial design	SUN1	1.72	-12.5	0.77	-4.26
	SU1	4.3	-7.43	2.12	-8.99
Optimum design	SUN1	1.85	-13	1.5	-4.4
	SU1	6.12	-9.9	3.3	-10.9

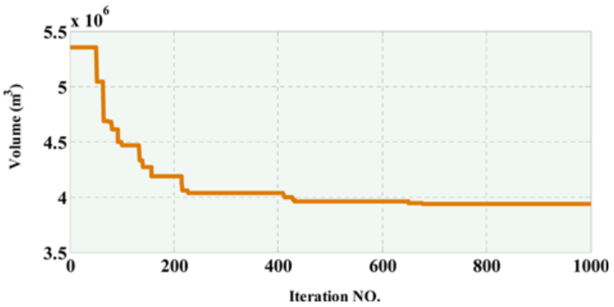

Fig. 14. Convergence Rate of the Dam Body Volume

The maximum tensile stresses are obtained for the U/S face in two cases (SUN1 and SU1).

For the SUN1 case, the maximum compression stress is observed in the U/S face and for the SUN1 case it is obtained in the D/S case.

The values of stresses for the optimum design are bigger than the initial design.

Convergence rate of the objective function in the optimization process is shown in Fig. 14.

After performing the optimization process, the dam volume has decreased by 21% in comparison with the initial design.

6.2 Result of Sensitivity Analysis of Arch Dam

After performing the sensitivity analysis, the results are shown based on correlation coefficients in Table 9 and Fig. 15.

The S.A positive value means that when the design variables value increase, the objective function (volume of arch dam)value increases, and the SA negative value means that when the design variables value increase, the objective function (volume of arch dam)value decreases All of the design variables are important. Moreover USP_7 and R_{LUS16} are of less important role compared to the other design variables.

7. Conclusions

In this study, a methodology to determine the optimal shape of double curvature concrete arch dams and the sensitivity analysis of design variables in shape optimization of concrete arch dams are presented.

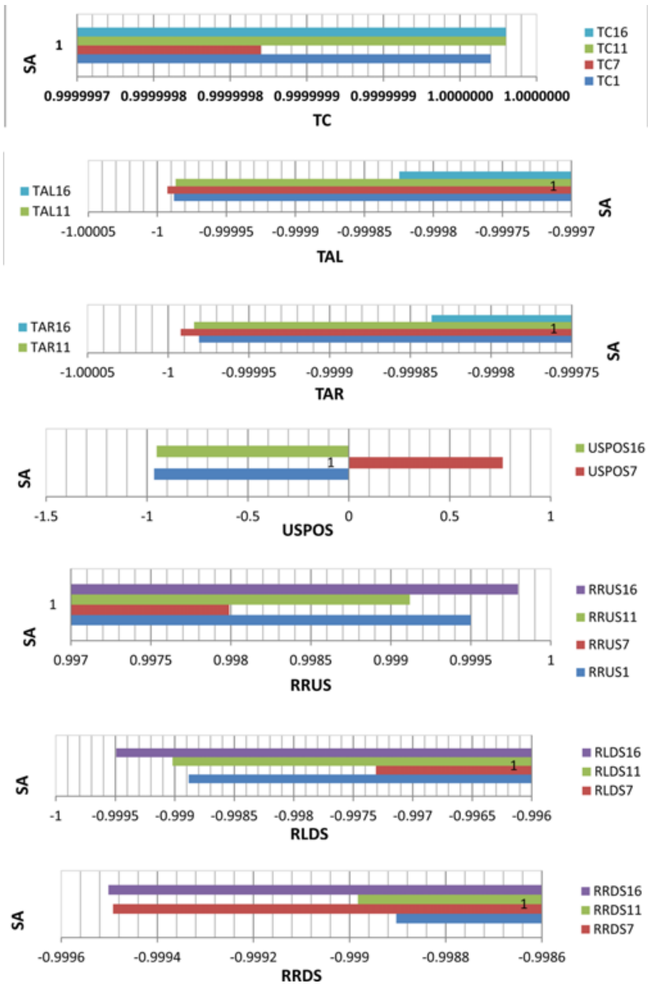

Fig.15. Sensitivity Indices for the Arch Dam Volume

Coefficients of sensitivity analysis are performed based on the 10000 trials by utilizing Latin Hypercube Sampling. Uniform distributions have been assigned for simulations. Correlation analysis is used to determine relationships between design variables and objective function. Results show that all of design variables are important. Moreover USP_7 and R_{LUS16} are of less important role compared to the rest of the design variables.

Table 9. Sensitivity Analysis of the 31 Input Factors

Design Variable	**k** (Arch number)			
	1	7	11	16
$\mathbf{T_C}$ (The thickness of crown cantilever)	0.99999997	0.99999982	0.99999998	0.99999998
$\mathbf{T_{AL}}$ (Left Abutment thickness)	-0.99998788	-0.99999265	-0.99998666	-0.99982468
$\mathbf{T_{AR}}$ (Right Abutment thickness)	-0.99998085	-0.99999221	-0.99998392	-0.99983676
$\mathbf{U_{SP}}$ (Crown cantilever upstream profile)	-0.96462346	0.76253081	-	-0.95171189
$\mathbf{R_{LUS}}$ (Radius of curvature of water face left)	0.99959102	0.99830301	0.99913396	0.56704113
$\mathbf{R_{RUS}}$ (Radius of curvature of water face right)	0.99949929	0.99798867	0.99911864	0.99979497
$\mathbf{R_{LDS}}$ (Radius of curvature of air face left)	-0.99888324	-0.99730991	-0.99901822	-0.99949206
$\mathbf{R_{RDS}}$ (Radius of curvature of air face left)	-0.99890307	-0.99949206	-0.99898286	-0.99950174

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