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A Chaotic Imperialist Competitive Algorithm for Optimum Cost Design of Cantilever Retaining Walls

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Abstract

This paper develops a model to obtain the optimum cost of the cantilever retaining walls having different cases of backfill satisfying the stability criteria, according to the height and properties of earth that the wall is required to support. Chaotic Imperialist Competitive Algorithm (CICA) is utilized to find the economical sections as the output after minimizing the cost for sections adhering to provisions of ACI 318-05. The CICA, one of the recently developed meta-heuristic algorithms utilizes imperialism and imperialistic competition process combined with chaos theory as a source of inspiration. Cost design problem of cantilever retaining walls is tested using the new method and the results are compared to those of other algorithms. In addition, a detailed sensitivity analysis for selected design variables, parameters and related safety factors will be presented.

Keywords: *chaotic imperialist competitive algorithm, optimum cost design, cantilever retaining walls, meta-heuristics*

1. Introduction

Retaining walls are structures that are used to retain earth (or any other material) in a position where the ground level changes abruptly. Retaining walls can be grouped into three distinct categories by considering the way in which they resist the lateral pressure exerted by the soil and any surcharge:

- Gravity retaining walls: These walls use their own weight and any captured soil/fill weight to resist the lateral soil pressure.
- Cantilever retaining walls: These walls cantilever vertically from the concrete footing and typically resist overturning by the mass of the soil/material on the heel of the footing.
- Piled retaining walls: These walls use the embedded depth of vertical posts and the strength of the posts to resist lateral soil forces.

Among these three groups the "cantilever wall" is the most common type of retaining wall and when its heights is less than 8m, it is an economical design. The lateral force due to earth pressure is the main force that acts on the retaining wall which has the tendency to bend, slide and overturn it (Pillai and Menon, 2003).

On the other hand, current design of concrete retaining walls is highly dependent on the experience of engineers. The structure is defined on a trial-and-error basis. Tentative design must satisfy the limit states prescribed by concrete codes. This process leads to safe designs, but the cost of the reinforced concrete retaining walls is, consequently, highly dependent upon the experience of the designer (Ghazavi and Salavati, 2011). So, in order to economize the cost of the concrete retaining walls under design constraints, it is advantageous for designer to cast the problem as an optimization problem. Optimum design of retaining walls has been the subject of a number of studies (Dembicki and Chi, 1989; Keskar and Adidam, 1989; Rhomberg and Street, 1981, Saribas and Erbatur, 1996; Ceranic and Fryer, 2001; Basudhar and Lakshman, 2006; Sivakumar and Munwar, 2008; Yepes *et al.*, 2008; Ahmadi and Varaee, 2009; Ghazavi and Bazzazian Bonab, 2011; Kaveh and Shakouri, 2011). In these studies there are two general methods to optimize the cost function, namely, mathematical programming and meta-heuristic methods. The mathematical programming algorithms use gradient information to search the solution space near an initial starting point. In general, gradient-based methods converge faster and can obtain solutions with higher accuracy compared to stochastic approaches in fulfilling the local search task.

As an alternative to the conventional mathematical approaches, the meta-heuristic optimization techniques have been used to obtain global or near-global optimum solutions, due to their capability of exploring and finding promising regions in the search space in an affordable time. Many of these methods are created by the simulation of the natural processes. Genetic Algorithms (Gas), Simulated Annealing (SA), Particle Swarm Optimization (PSO), ant Colony Optimization (ACO), Harmony Search (HS), Charged System Search (CSS) and Imperialist Competitive Algorithm (ICA) are some familiar examples of meta-heuristic

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algorithms (Kaveh and Talatahari, 2010a). Ceranic and Fryer (2001) applied the SA to the minimum cost design of the retaining walls. Also, Yepes *et al.* (2008) implemented a parametric study of optimum earth-retaining walls by the SA. Ahmadi and Varaee (2009) proposed an optimization algorithm based on the PSO for optimum design of retaining walls. Ghazavi and Bazzazian Bonab (2011) applied a methodology to arrive at the optimal design of concrete retaining wall using the ACO algorithm. In addition, Kaveh and Shakouri (2011) used the recently developed improved HS method to cost optimization of the reinforced concrete cantilever soil retaining wall of a given height satisfying some structural and geotechnical design constraints.

In this study, the Chaotic Imperialist Competitive Algorithm (CICA) is used to determine the optimum cost design of cantilever retaining walls. Imperialist Competitive Algorithm (ICA) is a socio-politically motivated optimization algorithm. Initial population of the algorithm is determined randomly. Each individual agent of an empire is called a “country”, and the countries are categorized into “colony” and “imperialist” states that collectively form “empires”. Imperialistic competitions among these empires form the basis of the ICA. During this competition, weak empires collapse and powerful ones take possession of their colonies. Imperialistic competitions direct the search process toward the powerful imperialist or the optimum points. Kaveh and Talatahari improved the ICA by defining two new movement steps and investigated the performance of this algorithm to optimize the design of skeletal structures (Kaveh and Talatahari, 2010b) and engineering optimization problems (Kaveh and Talatahari, 2010c). This algorithm is called Orthogonal Imperi-Alist Competitive Algorithm (OICA). Recently, Talatahari and co-workers (2012a, b) presented the Chaotic Imperialist Competitive Algorithm (CICA). This algorithm is obtained by modifying the movement stage of the OICA. In the CICA, the chaos theory has been used to improve the movement step of the algorithm. CICA not only improves the reliability property but also enhances quality of the results. Here the CICA is used to find the minimum cost for concrete retaining walls and Coulomb lateral earth pressure theory is used to derive the lateral total thrust on the wall.

The rest of this paper is organized as follows. In the next section the review of the CICA is summarized. Then, in Section 3 design procedure is described. Problem formulation is introduced in section 4. The CICA method for optimal design of the concrete retaining walls is provided in Section 5. In Sections 6 and 7, some numerical verification and sensitivity analysis is performed. Conclusions in Section 8 close the paper.

2. Chaotic Imperialist Competitive Algorithm

The ICA simulates the social political process of imperialism and imperialistic competition. The agents of this algorithm or countries are divided into two types; imperialist states (some of the best ones) and colonies (the rest). All the colonies of initial countries are divided among the imperialists based on their power which is inversely proportional to the countries cost. A simple model of assimilation policy is simulated in the algorithm by moving the colonies in each empire toward their relevant imperialist country. As another aspect of the algorithm, the imperialistic competition begins in which any empire that is not able to succeed in this competition and cannot increase its power (or at least prevent losing its power) will be eliminated from the competition. It means that weak empires will loose their power and ultimately they will collapse. The movement model along with competition simulation and also the collapse mechanism will hopefully direct the agent toward the optimum point. The pseudo-code of the algorithm is as follows:

2.1 Step 1: Initialization

The primary locations of the countries are determined randomly in the interval [$x_{\min}$, $x_{\max}$] in which $x_{i,\min}$ and $x_{i,\max}$ are the minimum and the maximum allowable values for the variables.

For each country, the cost identifies its usefulness. The related cost of a country is found by evaluation of the cost function f_{cost} of the corresponding variables considering the related objective function. Total number of initial countries is set to $N_{country}$ and the number of the most powerful countries to form the empires is taken as N_{imp}. The remaining N_{col} of the initial countries will be the colonies each of which belongs to an empire. In this paper, 10 percent of countries belong to empires and the remaining is used as colonies. To form the initial empires, the colonies are divided among imperialists based on their power. To fulfill this aim, the normalized cost of an imperialist is defined as:

$$C_n = f_{\text{cost}}^{(imp,n)} - \max_i (f_{\text{cost}}^{(imp,i)}) \tag{1}$$

where $f_{\text{cost}}^{(imp,n)}$ is the cost of the nth imperialist and C_n is its normalized cost. The initial colonies are divided among empires based on their power or normalized cost, and for the nth empire it will be as follows:

$$NC_j = Round\left(\left|\frac{C_j}{\sum_{i=1}^{N_{imp}} C_i}\right| \cdot N_{col}\right) \tag{2}$$

where NC_j is the initial number of the colonies related to the jth empire which are selected randomly among the colonies. These colonies together with the jth imperialist form the empire number j.

2.2 Step 2: Colonies Movement

In ICA, the assimilation policy, pursued by some of former imperialist states, is modeled by moving all the colonies toward the imperialist. According to this movement, a colony moves toward the imperialist by a random value that is uniformly distributed between 0 and $b \times d$:

$$\{x\}_{new} = \{x\}_{old} + U(0, \beta \times d) \times \{V_1\} \tag{3}$$

where b is a control parameter and d is the distance between colony and imperialist. $\{V_1\}$ is a vector which its start point is the previous location of the colony and its direction is toward the imperialist locations. The length of this vector is set to unity.

In the original ICA, to increase the searching around the imperialist, a random amount of deviation, q, is added to the direction of movement. q is a random number with uniform distribution.

In order to improve the ICA performance, the Orthogonal Imperialist Competitive Algorithm (OICA) was developed (Kaveh and Talatahari, 2010b). This algorithm not only uses different random values, but also utilizes the orthogonal colony-imperialistic contacting line instead of q for deviating the colony as follows:

$$\{x\}_{new} = \{x\}_{old} + \beta \times d \times \{rand\} \otimes \{V_1\} + U(-1,+1) \times \tan(\theta) \times d \times \{V_2\},\ \{V_1\} \cdot \{V_2\} = 0,\ \|\{V_2\}\| = 1 \quad (4)$$

where $\{V_2\}$ is perpendicular to $\{V_1\}$. Since this vector must be crossed the point obtained from the two first terms, we use a random value by using $U(-1,+1)$ for the third term of the Eq. (4) which changes its value in addition to its direction by using negative values.

Finally using chaotic maps, Chaotic Imperialist Competitive Algorithm (CICA) is obtained (Talatahari *et al.*, 2012a, b). In CICA The vector $\{rand\}$ and the parameter $U(-1,+1)$ of Eq. (4) are modified by the selected chaotic maps and the assimilation (moving the colonies of an empire toward the imperialist) equation is modified by:

$$\{x\}_{new} = \{x\}_{old} + \beta \times d \times \{cm_i\} \otimes \{V_1\} + cm \times \tan(\theta) \times d \times \{V_2\},\ \{V_1\} \cdot \{V_2\} = 0,\ \|\{V_2\}\| = 1 \quad (5)$$

where $\{cm_i\}$ and cm are a chaotic vector and variable based on the selected map, respectively. Here Sinusoidal map (May, 1976) is utilized as the selected chaotic map. This map is presented by:

$$x_{k+1} = a x_k^2 \sin(\pi x_k) \quad (6)$$

And for $a = 2.3$ and $x_0 = 0.7$, it has the following simplified form:

$$x_{k+1} = \sin(\pi x_k) \quad (7)$$

2.3 Step 3: Imperialist Updating

If the new position of the colony is better than that of its relevant imperialist (considering the cost function), the imperialist and the colony change their positions and the new location with lower cost becomes the imperialist. Then the other colonies move toward this new position.

2.4 Step 4: Imperialistic Competition

In this step, all empires try to take the possession of colonies of other empires and control them. The imperialistic competition is modeled by just picking some (usually one) of the weakest colonies of the weakest empires and making a competition among all empires to possess these (this) colonies. Based on their total power, in this competition, each of the empires will have a likelihood of taking possession of the mentioned colonies.

Total power of an empire is affected by the power of imperialist country and the colonies of an empire as:

$$TC_j = f_{\cos t}^{(imp,j)} + \xi \cdot \frac{\sum_{i=1}^{NC_j} f_{\cos t}^{(col,i)}}{NC_j} \quad (8)$$

where TC_n is the total cost of the jth empire and x is a positive number which is considered to be less than 1. Also, the normalized total cost is defined as:

$$NTC_j = TC_j - \max_i(TC_i) \quad (9)$$

where NTC_j is the normalized total cost of the jth empire. Finally, the possession probability of each empire is evaluated by:

$$P_j = \left| \frac{NTC_j}{\sum_{i=1}^{N_{imp}} NTC_i} \right| \quad (10)$$

2.5 Step 5: Implementation.

When an empire loses all its colonies, it is assumed to be collapsed. In this model implementation, where the powerless empires collapse in the imperialistic competition, the corresponding colonies will be divided among the other empires.

2.6 Step 6: Terminating Criterion Control

Moving colonies toward imperialists are continued and imperialistic competition and implementations are performed during the search process. When the number of iterations reaches a predefined value or the amount of improvement in the best result reaches a pre-defined value, the searching process is stopped.

3. Overview of Retaining Walls Design Procedure

Retaining walls are designed to withstand lateral earth and water pressures and for a service life based on consideration of the potential long-term effects of material deterioration on each of the material components comprising the wall. Permanent retaining walls should be designed for a minimum service life of 50 years. Temporary retaining walls should be designed for a minimum service life of 5 years.

The cantilever wall is the most common type of retaining walls. This type of wall is constructed of reinforced concrete. They can be used in both cut and fill applications. They have relatively narrow base widths. They can be supported by both shallow and deep foundations. The position of the wall stem relative to the footing can be varied to accommodate right-of-way constraints. They are most economical at low to medium wall heights. The cantilever wall generally consists of a ‘vertical stem’, and a base slab. The base slab is made up of a ‘heel’ and a ‘toe’ slabs. All three components behave like one-way cantilever slabs: the stem acts as a vertical cantilever under the lateral earth pressure; the

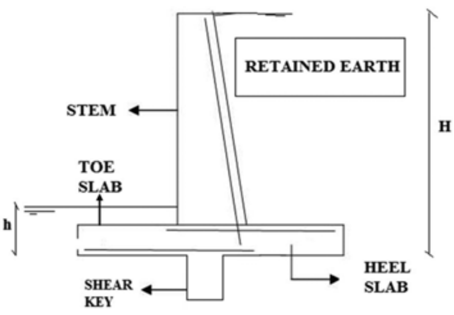

Fig. 1. Cantilever Retaining Wall's Detailing

heel and toe slab acts as a horizontal cantilever under the action of the resulting soil pressure. The reinforcement detailing is given in Fig. 1. The weight of the earth retained helps in maintaining the stability of the wall (Jena and Ramanujam, 2010). The stem (CD) will bend as cantilever, so that tensile face will be towards the backfill. The heel slab will have net pressure acting downwards, and will bend as a cantilever, having tensile face upwards. The pressure distribution will be as shown in Fig. 2. The critical section will be at point D, where cracks may occur if it is not reinforced properly at the upper face. The net pressure on toe slab will act upwards; hence it must be reinforced at the bottom face. The thickness of stem, hell and toe slab must be sufficient to withstand compressive bending stresses.

Considering a concrete retaining wall, the four primary concerns relating to the design of these walls are (Brooks, 2010):

1. That it has an acceptable factor of safety with respect to overturning.
2. That the allowable soil bearing pressures are not exceeded.
3. That it has an acceptable factor of safety with respect to sliding.
4. That the stresses within the components (stem and footing) are within code allowable limits to adequately resist imposed vertical and lateral loads.

This means that there are three different modes of instabilities, namely sliding, overturning and bearing capacity which should be checked. Considering these modes of instabilities, the design usually follows this order:

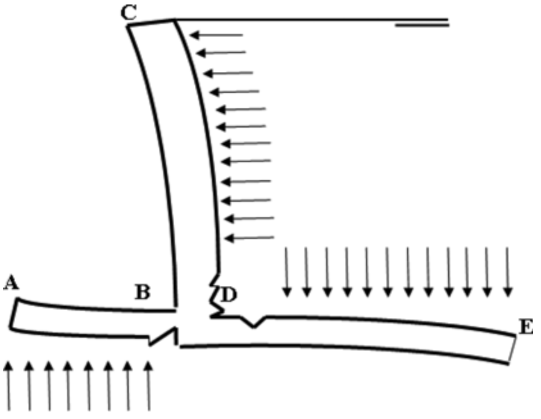

Fig. 2. The Pressure Distribution

1. Establish all design criteria based upon the applicable building codes.
2. Compute all applied loads, soil pressures, seismic, wind, axial, surcharges, impact, or any others.
3. Design the stem. This is usually an iterative procedure.
4. Compute overturning moments; calculate about the front (toe) edge of the footing.
5. Compute resisting moments based upon an assumed footing width, and again calculated about the toe.
6. Based upon 4 and 5 calculate the eccentricity of the total vertical loads. Is it within or outside the middle-third of the footing width?
7. Calculate the soil pressure at the toe and heel.
8. Select reinforcing. Design footing for shears and moments.
9. Check sliding. A key or adjusting the footing depth may be required.
10. Check and review.

4. Problem Formulation

The optimal design of a concrete cantilever retaining wall is proposed to be determined by the minimum of the costs of concrete, steel reinforcement and formworking. The objective function can be expressed as follows:

$$\text{Minimize: } f_{\cos t} = C_1 \times V_{conc} + C_2 \times W_{steel} + C_3 \times A_{fw} \tag{11}$$

$$\text{subject to: } FS_o \geq 1.5$$
$$FS_S \geq 1.5$$
$$FS_b \geq 2 \tag{12}$$
$$\frac{M_u}{\phi_b M_n} \leq 1$$
$$\frac{V_u}{\phi_b V_n} \leq 1$$

where V_{conc}, W_{steel} and A_{fw} are the volume of concrete (m^3/m), the weight of reinforcement steel in the unit of length (kg/m) and the area of the formwork (m^2/m), respectively; C_1 is the cost of the concrete (€/m^3); C_2 is the cost of steel (€/kg); C_3 is the total cost of formworking (€/m); and here, we use 58, 6, 25, for C_1, C_2 and C_3, respectively. FS_o, FS_s and FS_b are the factors of safety against overturning, sliding and bearing capacity, respectively; f is the strength reduction factor. Here, shears and moments (V, M) are calculated based on *ACI 318-05* codes (2005). The design variables have been shown in Fig. 3.

5. The CICA Method for Optimal Design of the Concrete Retaining Walls

The CICA algorithm initiates the design process by selecting random values for the design variables. Then the algorithm checks the wall for stability and if the dimensions satisfy stability criteria, the algorithm calculate the required reinforcement and checks the strength. In this procedure choosing design parameters that

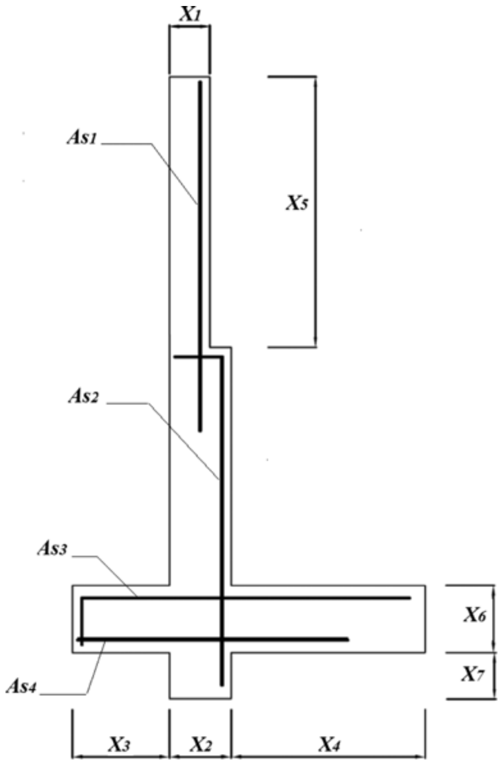

Fig. 3. The Design Variables (Concrete Cantilever Retaining Wall)

fulfill all design requirements and have the lowest possible cost is concerned, *i.e.,* the main objective is to comply with basic standards but also to achieve good economic results. In order to handle the constraints, a penalty approach is utilized. In this method, the aim of the optimization is redefined by introducing the penalized cost function as:

$$F_{\cos t} = (1+\varepsilon_1 \nu)^{\varepsilon_2} \times f_{\cos t},\quad \nu = \sum_{j=1}^{n} \max(0, g_j) \qquad (13)$$

where n represents the number of evaluated constraints for each individual design; v denotes the sum of the violations of the design and g_j presents the jth design constraint. The constant e_1 and e_2 are selected considering the exploration and the exploitation rate of the algorithm. Here, e_1 is set to unity; e_2 is selected in a way that it decreases the penalties and reduces the variables. Thus, in the first steps of the search process, e_2 is set to 1.5 and ultimately increased to 3 (Kaveh and Talatahari, 2010d).

From the structural design point of view, the CICA determines the appropriate value for each group of variables so that with these set of values the response of the retaining wall is within the limitations imposed by the design condition when it has the minimum cost. The chaotically movement of colonies towards their relevant imperialist states along with competition among empires and also the collapse mechanism will hopefully cause all the countries to converge to a state in which there exist just one empire in the world and all the other countries are colonies of that empire. In this ideal new world, colonies will have the same position and power as the imperialist.

6. Verification

In this Section, a numerical example is optimized with the proposed method. The final result of the CICA is compared to the solution of the OICA and the standard ICA method to demonstrate the efficiency of the present approach. For the proposed algorithm, a population of 30 countries consisting of 3 empires and 27 colonies are used.

For the considered example, the geometry condition of the wall is as H = 6 m and h = 0.5 m. The backfill has shear strength parameters of $c_1 = 0$ kg/m^2, $f_1 = 30°$ (internal friction angle) and $g_1 = 18$ kN/m^3 (density). The wall is founded on a soil with $c_2 = 40$ kg/m^2, $f_2 = 20°$ and $g_2 = 19$ kN/m^3. Also the material density for the wall is 24 kN/m^3 (concrete wall). In this example, the angle of the wall friction (d) is 20° and the inclination of ground surface behind the wall to horizontal is zero. Surcharge load (w_s) is 10 kN/m^2. The 28 days concrete cylinder strength (f_c') is 21 MPa, Rebar yield stress (f_y) is 300 MPa, and the allowable soil pressure is taken as $q_u = 250$ kN/m^2. The clear concrete cover is 50 mm. The codes have been prepared in MATLAB™ 7 (MathWorks, Natick, MA, USA), and all the runs for the problem have been implemented on a Pentium IV PC with 2.8-GHz six cores and 2-GB RAM.

The results obtained by the CICA, OICA and ICA are summarized in Table 1. As shown in this table, the result for the CICA algorithm is 1337.42 (€/m), which is less than the result of the OICA and standard ICA algorithms. In addition, the average weight of 30 different runs for the CICA algorithm is 2.22% and 6.71% less than the average results of the OICA and ICA algorithms, respectively. As another investigation and for testing the degree of the consistency of the algorithms, the standard deviation of the results is paid attention in which it is the smallest one for the results of the CICA in 30 independent runs. From Table 1,

Table 1. Optimal Cost Design Obtained by the ICA-based Methods

Design variables (m)	ICA	OICA	CICA
X_1	0.300	0.329	0.300
X_2	0.650	0.650	0.650
X_3	1.200	1.200	1.200
X_4	2.550	2.555	2.488
X_5	3.027	3.166	3.027
X_6	0.500	0.500	0.476
X_7	0.250	0.250	0.250
As_1(cm^2)	10.00	10.00	10.00
As_2(cm^2)	25.79	25.79	25.79
As_3(cm^2)	24.48	24.55	25.00
As_4(cm^2)	11.27	11.27	11.97
FS_o	4.08	4.09	3.98
FS_s	2.89	2.90	2.86
FS_b	2.00	2.00	2.00
Best cost (€/m)	1339.56	1338.17	1337.42
Average cost (€/m)	1445.55	1379.15	1348.52
SD (€/m)	88.96	41.26	26.31

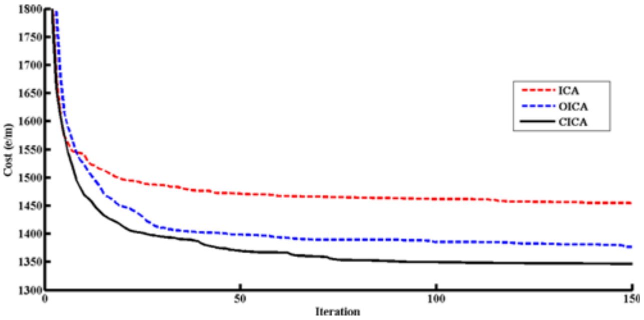

Fig. 4. Convergence History Comparison between the Three Algorithms (Average of 30 Different Runs)

the safety factors of bearing capacity and sliding (contrary to overturning safety factors) are the active constraints. The Convergence history comparison for this example is shown in Fig. 4. Clearly, the figure shows a good performance of the CICA.

7. Sensitivity Analysis

Sensitivity analysis is mathematically defined as a process of computing the derivatives of the solution to governing equations with respect to the parameters defining the equations (Elishakoff and Ohsaki, 2010). Any optimum design problem involves a design vector and a set of problem parameters. In many cases, we would be interested in knowing the sensitivities or derivatives of the optimum design (design variables and objective function) with respect to the problem parameters because for a designer this is very useful to know which data values are more influential on the design and how changes in prices influence the total cost. Sensitivity of optimal responses to these parameters is one of the important issues in the optimum design of retaining walls. As a result, in this section a sensitivity analysis is performed for the optimum cost design of concrete cantilever retaining wall parameters. The design parameters used in this study include a wide range of parameters related to loading, geometry, soil properties and etc.

In this study, results concerned with sensitivity of optimum solutions with respect to height, the base friction coefficient, the type of fill as regards its angle of internal friction and safety factor for sliding and compressive strength of concrete are presented. The basic parameters considered for sensitivity analysis are given in Table 2.

Table 2. Input Parameters for Sensitivity Analysis

Input parameters for sensitivity analysis	Value	Unit
γ_1	22	kN/m^3
γ_2	19.4	kN/m^3
c_1	0	kN/m^2
c_2	50	kN/m^2
d	2/3 (ϕ_1)	degree
w_s	10	kN/m^2
h	1	m
q_u	300	kN/m^2
f_y	300	MPa

7.1 Influence of the Base Friction Conditions on the Optimum Results

The coefficient of friction is usually determined by designers, and different tests have shown that actual friction coefficients are closer to 0.70. This parameter is used to calculate the friction resistance which is equal to the total vertical weight multiplied by the coefficient of friction between the base of the footing and the soil. The sum of all the horizontal forces tends to slide the wall away from the fill. The tendency to resist this is achieved by the friction at the base.

Figure 5 illustrates the cost variation for different base friction conditions. The internal friction angle of the base soil can vary from 20° to 35° with an increment of 5°. All four curves in Fig. 5 have a good parabolic variation in terms of the total height of the wall and a quadratic function can describe the relation between the height of the wall and the average cost as:

$$F_{cost} = 95.48H^2 - 696.3H + 1926 \tag{14}$$

The related regression coefficient is $R^2 = 0.9890$ in this condition.

7.2 Influence of the Internal Friction Angle of Backfill Soil on the Optimum Results

The internal friction angle of backfill soil is the most important value for determining the lateral pressure and the bearing capacity of the granular (non-cohesive) soil. It is a measurement of the shearing resistance of the soil because of intergranular friction, obtained from several laboratory tests such as direct shear test.

Figure 6 shows cost variation against internal friction angle of backfill soil (f_1). The internal friction angle of the backfill soil can vary 34° to 40° with 2° increment. The results show that when the height of the walls increases, influence of the internal friction angle of backfill soil on the optimum results becomes very important. For example, for a wall with H = 7 m height, choosing $f_1 = 40°$, causes cost reduction of 13.50% in comparison with the cost of the wall when $f_1 = 34°$. Also, Fig. 6 explains why it is beneficial to use more compacted soil offering greater internal friction angles.

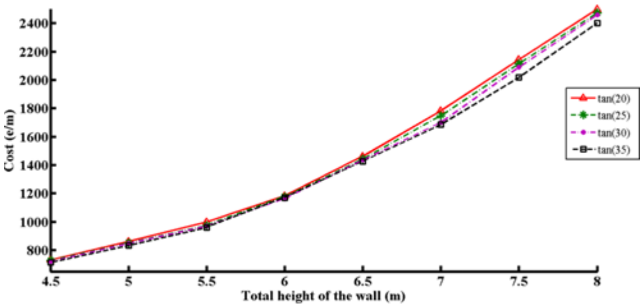

Fig. 5. Cost Variation for Different Base Friction Conditions

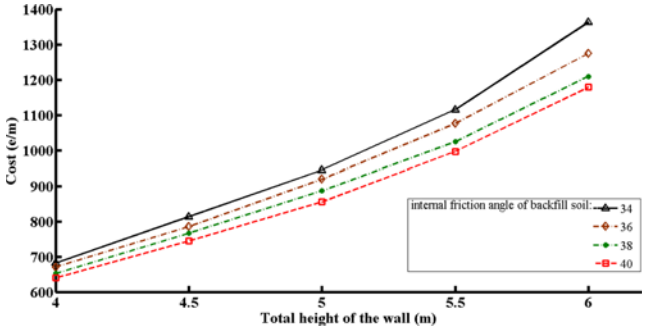

Fig. 6. Cost Variation Against Internal Friction Angle of Backfill Soil

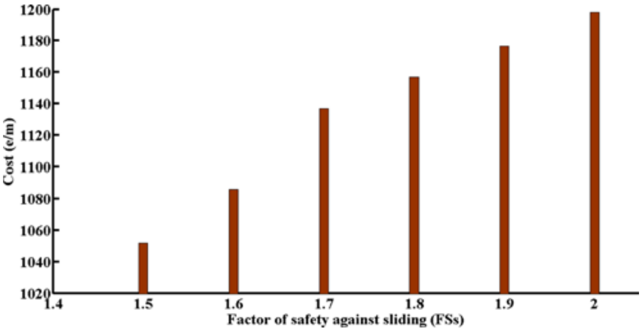

Fig. 7. Cost Variation Against Different FS_s

7.3 Influence of the Safety Factor of Sliding on the Optimum Results

The factor of safety for sliding of the wall is defined as the resisting forces divided by the driving force, as:

$$FS_s = \frac{\begin{array}{c}\text{Sliding resistance force}\\ \text{+Allowable passive resualtant force}\end{array}}{\text{Active earth pressure resultant force}} \quad (15)$$

If the wall is found to be unsafe against sliding, shear key below the base is provided. Such a key develops passive pressure which resists completely the sliding tendency of the wall. The customary minimum safety factor against sliding is 1.5, with some agencies requiring more. In the determination of FS_s, the effect of passive lateral earth pressure resistance in front of a wall footing or a wall footing key shall only be considered when competent soil or rock exists which will not be removed or eroded during the structure life. Not more than 50 percent of the available passive lateral earth pressure shall be considered in determining the FS_s.

In Fig. 7, the cost variation against the safety factor of sliding is depicted. In this case, the height of the wall is constant and is equal to 5.5 m; the internal friction angle of the backfill soil is equal to 35° and base friction coefficient is equal to 0.24. It is interesting to emphasis that a small coefficient as $FS_s = 1.5$ causes an average decrease in cost of 12.5% compared to a bigger one such as $FS_s = 2$.

7.4 Influence of the Strength of used Concrete on the Optimum Results

Table 3 displays the minimum cost and the reinforcement requirement for different compressive strengths of concrete starting from 20 MPa to 40 MPa. It was observed that with the use of higher grade of concrete, the minimum cost and the total reinforcement (per unit length) may increase. Also it can be concluded that as the value of the compressive strength of concrete increases, the optimum steel in toe and heel adjust accordingly.

8. Conclusions

Determining optimum cost design and sensitivity analysis of cantilever retaining walls is presented in detail, using the CICA. The CICA simulates the social political process of imperialism and imperialistic competition. Two simple models containing the assimilation policy simulated by moving the colonies in each empire and the imperialistic competition simulated by eliminating the week empire from the competition form the base of CICA. This algorithm is modified in the movement step by using chaotic maps. To fulfill this aim, the orthogonal imperialist competitive algorithm, as an improved ICA with two movement steps, and the sinusoidal chaotic map are utilized. The results from the considered numerical example show the ability of the new algorithm to find optimal results and it has been demonstrated that the presented algorithm is able to find rapidly the minimum cost, justified geometry and specifications for cantilever retaining walls.

A sensitivity analysis is performed for the optimum cost design of concrete cantilever retaining wall parameters using the developed algorithm in which the base friction coefficient, the type of fill as regards its angle of internal friction and safety factor for sliding and compressive strength of concrete are concerned. Different base friction conditions have a small effect on optimum results while the influence of the internal friction angle of backfill soil on the optimum results is very important especially when the height of the walls increases. The results related to the influence

Table 3. Results for Different Grades of Concrete

Compressive strength of concrete (f_c')	20 MPa	25 MPa	30 MPa	35 MPa	40 MPa
Optimum steel at stem (mm^2)	2646.01	2634.89	2619.24	2661.09	2611.28
Optimum steel in heel (mm^2)	624.66	637.69	734.23	704.92	752.86
Optimum steel in toe (mm^2)	579.74	589.42	639.07	626.28	648.85
Total reinforcement per unit length (kg/m)	68.16	70.58	72.75	73.04	73.45
Minimum cost(€/m)	877.17	895.49	908.62	927.77	940.02

of the safety factors of sliding show that as expected, a large safety factor causes a costly wall compared to a small one. Finally the investigations show that with the use of higher grade of concrete, the minimum cost and the total reinforcement (per unit length) may increase.

Notations

A_{fw} = Area of the formwork (m^2/m)
c = Shear strength parameter (kN/m^2)
C_1 = Cost of the concrete (€/m^3)
C_2 = Cost of steel (€/kg)
C_3 = Cost of formworking (€/m)
cm = Chaotic value
C_n = Normalized cost
d = Distance between colony and imperialist
f_c' = Concrete cylinder strength (MPa)
f_{cost} = Cost function
FS_b = Safety factor against bearing capacity
FS_o = Safety factor against overturning
FS_s = Safety factors against sliding
f_y = Rebar yield stress (MPa)
g_j = *j*th design constraint
NC_j = Initial number of the colonies related to the *j*th empire
$N_{country}$ = Total number of initial countries
N_{imp} = Number of the most powerful countries
NTC_j = Normalized total cost of the jth empire
q_u = Allowable soil pressure (kN/m^2)
TC_n = Total cost of the jth empire
V_{conc} = Volume of concrete (m^3/m)
W_{steel} = Weight of reinforcement steel in the unit of length (kg/m)
$x_{i,max}$ = Maximum allowable values for the variables
$x_{i,min}$ = Minimum allowable values for the variables
$\{cm_i\}$ = Chaotic vector
u = Sum of the violations for the design constraints
f = Internal friction angle (degree)
g = Density (kN/m^3)
d = Angle of wall friction (degree)

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