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Effect of Frame Irregularity on Accuracy of Modal Equivalent Nonlinear Static Seismic Analysis

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Abstract

This study investigates the effect of frame set-back irregularity in height on accuracy of Modal Pushover Analysis (MPA) for predicting target displacement, story drifts, and base shear. In order to have a thorough investigation of frame irregularity, 21 irregularity layouts are considered for a 5-story steel moment resisting frame. Each irregular frame is designed to represent low and high values of response reduction factor (R). For each frame model, nonlinear dynamic analyses for 14 ground motions (7 pairs) and nonlinear static pushover analysis up to the MPA-predicted target displacement are performed using computer program IDARC-2D with fiber beam - column elements. The results of nonlinear static analyses are compared with the results for nonlinear dynamic analyses and FEMA440 modified coefficient method. Correlations of predicted measures with respected to results from nonlinear dynamic analyses are computed for all models and also each model. For Irregular frames, MPA is inconservative for estimating median of dynamic responses for selected ground motions though error quantity depends on models R value and kind of response. MPA estimated results have good correlation with respect to FEMA in displacement and drift results for all models under all ground motions are considered.

Keywords: *equivalent nonlinear static analysis, irregular frames, modal pushover analysis, seismic modeling, steel moment resisting frames*

1. Introduction

Estimating seismic demands at low performance levels, such as life safety and collapse prevention, requires explicit consideration of inelastic behavior of the structure. While nonlinear response history analysis is the most rigorous procedure to compute seismic demands, current civil engineering practice prefers to use the Non-linear Static Procedure (NSP) or pushover analysis (Chopra and Goel, 2002). In performance assessment and design verification of building structures, approximate Nonlinear Static Procedures (NSPs) are becoming commonplace in engineering practice to estimate seismic demands. For seismic evaluation and design of building structures, simplified design-oriented modeling procedures using static analyses are more practical than nonlinear dynamic modeling procedures. Such equivalent static procedures are supposed to estimate building seismic displacement and force demands with practical accuracy. In fact, some seismic codes have begun to include them to aid in performance assessment of structural systems e.g., Eurocode 8 (2001), Japanese Design Code (2001), (Kalkan and Kunnath, 2006). Nonlinear Static Procedures are often employed because they offer better accuracy and simplicity when compared with linear static and nonlinear dynamic procedures, respectively (Elnashai, 2001). NSP is widely used in recent years for practical evaluation of seismic demands and for structural design. One of well-established static procedures is the equivalent nonlinear static procedure is Modal Pushover Analysis. Recently, Modal Pushover Analysis (MPA), was introduced by Chopra and Goel (2002) based on the assumptions that the response of a structure is controlled by a single mode and the shape of that mode remains constant with time, MPA can lead to good estimates of the seismic demands of a building (Lignos and Gantes, 2005).

The seismic response of vertically irregular building frames, which is the subject of many research papers, started getting attention in the late 1970s. A large number of papers have focused on plan irregularity resulting in torsion in structural systems. Vertical irregularities are characterized by vertical discontinuities in the distribution of mass, stiffness and strength. Very few research studies have been carried out to evaluate the effects of discontinuities in each one of these quantities independently (Karavasilis *et al*., 2008), and majority of the studies have focused on the elastic response (Kreger and Sozen, 1989; Mahin *et al*., 1976). In setback structures there is a sudden change in the vertical distribution of mass, stiffness, and in some cases, strength. A setback structure is thought of being made up of two parts: A base (the lower part having many bays), and a tower (the upper part with fewer bays) (Soni and Mistry, 2006).

The Uniform Building Code started to distinguish vertically

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irregular structures from regular ones based on certain limits on the ratio of strength, stiffness, mass, setbacks or offsets of one story with respect to an adjacent story. These limits are based in part on analytical e.g., Humar and Wright (1977), Esteva (1992), Valmundsson and Nau (1997) and experimental studies such as Moehle (1984), Wood (1992). Most previous investigations collectively pinpoint significantly altered drift and ductility demands in the vicinity of structural irregularities. Recent parametric studies on Two-Dimensional (2D) generic frames by Al-Ali and Krawinkler (1998) and code-compliant 2D Special-Moment-Resisting Frames (SMRFs) by Das and Nau (2003) Provide more insight into the influences of variation of vertical irregularity along the height on seismic performance of buildings when subjected to different types of ground motions (Devesh P. Soni and Bharat B. Mistry, 2006).

Previous investigations of MPA's accuracy and efficiency dealt with regular frames. Very recent works by Chintanapakdee and Chopra (2003) address the effects on floor displacements, story drifts and plastic hinge rotations of 'vertically' irregular frames. This study represents a further attempt to evaluate the effect of contributed modes on accuracy of MPA for target displacement, story drift and base shear for the case of frames with setback irregularity, by comparing results to those obtained with Nonlinear Time History Analysis (NL-THA).

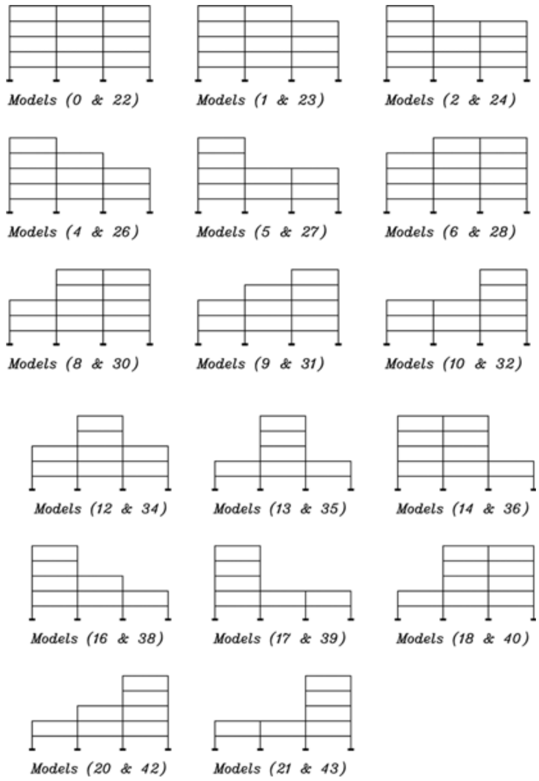

Fig. 1. Steel Moment Resisting Frames

2. Description of Modeling and Ground Motions

Models in this study are the irregular moment resisting frame from Mazzolani and Piluso (1996) research that they was loaded by UBC-97 and designed as Intermediate Moment Resisting Frame (IMRF) using AISC-LRFD99 code. Each irregular frame is designed to represent low and high values of response reduction factor (R, numerical coefficient representative of the inherent over strength and global ductility capacity of lateral Force-resisting systems presented in UBC-97). Therefore there are 44 models and here they are called in two classes, class "I", the models from number 0 to 21 that are designed by low value of R and class "II", the models from number 22 to 43 that are designed by high value of R. These frames (Fig. 1) are having three spans at 10 meter and five stories. The height of first story is 3.4 meter and others have identical height of 3.6 meter. In these frames all beams and columns are assumed to bent at their strong axis and connect to each other by welding. For time history analyses are selected fourteen (7 couples) natural records referred to as W1 to W14 in Table 1. These accelerograms belong to strong ground motions that had been recorded in soil

Table 1. Ground Motions used in This Study

W	Earthquake	Identifier	Magnitude	Dist.“Km”	PGA“g”	Scaled PGA“g”
1	Chi-Chi, Taiwan	CHY101W	Ms = 7.6	11.14	0.353	0.596
2	Chi-Chi, Taiwan	CHY101N	Ms = 7.6	11.14	0.44	0.596
3	Imperial Valley	E11230	Ms = 6.9	12.6	0.38	1.154
4	Imperial Valley	E11140	Ms = 6.9	12.6	0.364	1.154
5	Loma Prieta	G03000	Ms = 7.1	14.4	0.555	0.813
6	Loma Prieta	G03090	Ms = 7.1	14.4	0.367	0.813
7	Northridge	CNP106	Ms = 6.7	15.8	0.356	0.562
8	Northridge	CNP196	Ms = 6.7	15.8	0.42	0.562
9	Superstitn	ICC000	Ms = 6.6	13.9	0.358	0.750
10	Superstitn	ICC090	Ms = 6.6	13.9	0.258	0.750
11	Northridge	LOS000	Ms = 6.7	13	0.41	0.664
12	Northridge	LOS270	Ms = 6.7	13	0.482	0.664
13	Loma Prieta	G02000	Ms = 7.1	12.7	0.367	0.705
14	Loma Prieta	G02090	Ms = 7.1	12.7	0.322	0.705

type C in USCGS category and having same specialty such as magnitude and distance. They are all having similar magnitude between 6.6 to 7.6 and similar distance to the fault. The calculated inelastic dynamic response is sensitive to the characteristic of the input motions, thus the selection of representative acceleration time-histories is important. The ground motions were selected from the Pacific Earthquake Engineering Research (PEER) center strong ground motion database (available at http://peer.berkeley.edu).

In order to apply the outline procedure for evaluation of NSP responses with median response of NL-THA, scaling of the records is required. The selected ground motions are scaled using procedure suggested by FEMA356 while design spectrum is as recommended in the Iranian seismic code 2800 for site type C. According to this procedure for each earthquake record (containing two horizontal components of a ground motion record), the square root of the sum of the squares (SRSS) of the 5%-damped Site-specific spectrum of the scaled horizontal component shall be constructed. The damped spectra and their SRSS combination of the 2nd earthquake ground motion (Imperial Valley) are shown in Fig. 2.

The data sets shall be scaled such that the average value of the

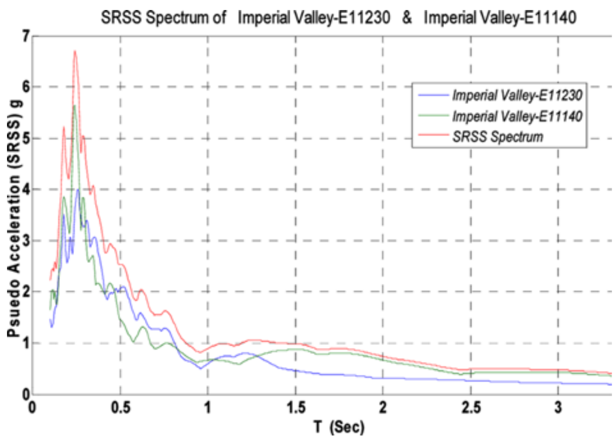

Fig. 2 Second Earthquake Record's Damped Spectra and SRSS Combination

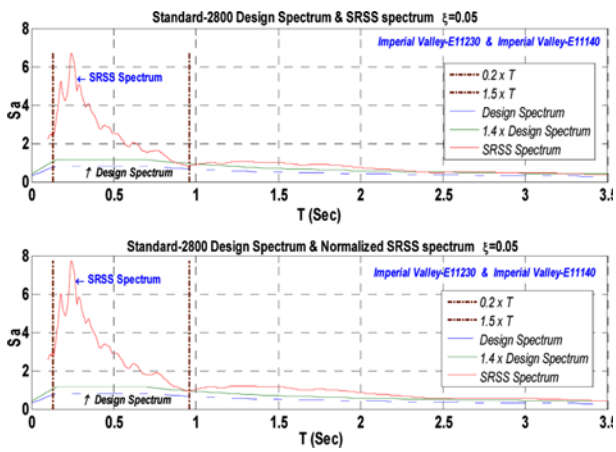

Fig. 3 Scaling Procedure for Second Earthquake Ground Motions

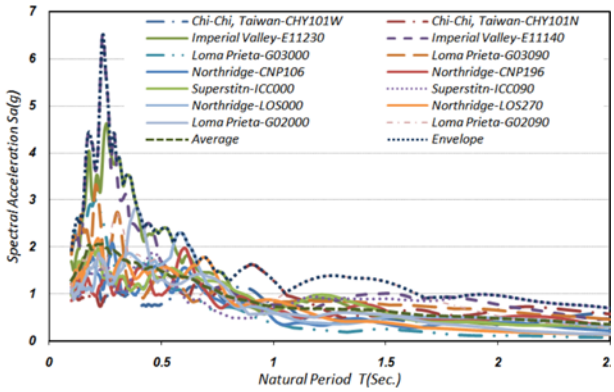

Fig. 4. Scaled Acceleration Spectra

SRSS spectra does not fall below 1.4 times the 5% damped spectrum for the design earthquake for periods between 0.2 T seconds and 1.5 T seconds, where the T is the fundamental period of the building. Fig. 3 shows the procedure for the 2nd earthquake ground motions. All scaled acceleration response spectra's for 5% damping are shown in Fig. 4. The range of scale factors is between 1.465 and 3.035. The computed scaled Peak Ground Acceleration (PGA) for each ground motion is shown in Table 1.

3.Nonlinear Analyses

The inelastic analyses have been performed using static and dynamic structural analysis program IDARC-2D which is developed for nonlinear analysis of reinforced concrete and steel structures under static and dynamic loading. This program use fiber approach for inelastic analysis.

3.1 Nonlinear static Procedures

Two major nonlinear static procedures are used in this study; Modal pushover analysis method and modified coefficient method of FEMA440. They are explained in continuous, concisely.

3.1.1 Modified Coefficient Method of FEMA440

One of the well-established nonlinear static procedures is the equivalent nonlinear static procedure summarized in FEMA356 based on nonlinear static pushover analysis using the target displacement predicted by the Coefficient Method (CM). CM utilizes a displacement modification procedure in which several empirically derived factors are used to modify the response of a single-degree-of freedom model of the structure assuming that it remains elastic. FEMA440 has suggested some recommendations for improving the performance of CM leading to a Modified Coefficient Method (MCM).

FEMA440 MCM suggests that the maximum demands (displacements and forces) for a nonlinear time history analysis can be estimated from a nonlinear static analysis where roof displacement is the same as maximum roof displacement estimated by the nonlinear time history analysis. The structure layout, boundary conditions, and nonlinearities are the same in both analyses. The lateral loading pattern for the nonlinear static analysis is limited to recommendation in Chapter 3 of FEMA356. In order to make the nonlinear static analysis independent from the nonlinear time history analysis, FEMA440 MCM estimates the target roof displacement (δ_t) using the following formula:

$$\delta_t = C_0 C_1 C_2 S_a \frac{T_e^2}{4\pi^2} g \quad \& \quad R < R_{\max} \tag{1}$$

Where the modification factor C_0 relates the spectral displacement of an equivalent SDOF system to the roof displacement of the MDOF building, the modification C_1 relates expected maximum inelastic displacements to displacements calculated for linear elastic response, the modification factor C_2 represents the effect of pinched hysteretic degradation on displacement response and is recommended to be 1.0, T_e is the effective fundamental period, S_a is the response spectrum acceleration at T_e and damping ratio of the building, g is the acceleration of gravity, R is the ratio of elastic strength demand to calculated yield strength coefficient at the target displacement δ_t, and R_{max} is the recommended maximum R value for limiting plastic P-Δ instability.

3.1.2 Modal Pushover Analysis

The MPA procedure is extended to estimate the seismic demands for inelastic systems: First, a pushover analysis determines the peak response r_{no} of the inelastic MDF system to individual modal terms, $\mathbf{p}_{\mathrm{eff},n}(t) = \mathbf{s}_{ng}(t)$, in the modal expansion of the effective earthquake forces, $\mathbf{p}_{\mathrm{eff},n}(t) = \mathbf{m}\boldsymbol{l}\ddot{u}_g(t)$. The base shear-roof displacement ($V_{bn} - u_m$) curve is developed from a pushover analysis for modal force distribution $\mathbf{s}_n^*$ that is the only force distribution that produces displacements proportional to the nth vibration mode. This pushover curve is idealized as bilinear and converted to the force-deformation relation for the nth-"mode inelastic SDF system. The peak deformation of this SDF system is used to determine the roof displacement, at which the seismic response, r_{no}, is determined by pushover analysis. Second, the total demand, r_o, is determined by combining the r_{no} (n = 1, 2,) according to an appropriate modal combination rule (Anil K. Chopra and Rakesh K. Goel, 2001).

3.2 Nonlinear Dynamic Procedure

The nonlinear dynamic analysis is used in order to employ the average or median value of each response as the acceptability value and as a bench mark for the accuracy of NSP responses.

4. Results

For each of selected 44 frames, nonlinear static analyses (MPA) and one nonlinear time history analysis are performed for each of 14 selected scaled ground motions (7 pairs) and nonlinear static pushover analysis up to the MPA predicted target displacement, base shear and story drift for all models. More than 3000 nonlinear time histories and static pushover analyses are performed. The results of modal pushover analysis with considering different modes are compared with the results of nonlinear dynamic analyses to evaluate the accuracy and conservatism of MPA.

4.1 Errors and Correlation Definition

For evaluation of NSP purposed by MPA, one should determine the errors in resulted responses with regard to the same one which obtained from NL-THA in that ground motion. So determination of average, median and maximum error for this method is important. In this study for purpose of error calculation relative error is used as the accuracy indicator for this method of analysis. Relative error has illustrated in Eq. (1) in percent (1).

$$error_i(\%) = \frac{-Q_i^{NL-THA} - Q_i^{MPA}}{Q^{NL-THA}} \times 100 \tag{2}$$

In these equations Q_i^{NL-THA} is the nonlinear time history response (such as target displacement, base shear, drift and etc.) for the i[th] ground motion and Q_i^{MPa} is the analogous response, resulted from nonlinear static procedure for i[th] ground motion. For evaluation of outcome results we need a parameter to verify the dependency or independency of the results. For this purpose statistics represents a non dimensional parameter which it is known as correlation factor. For a band including n-couple results, Eq. (3) may be used to calculate this factor.

$$\rho = \frac{\sum_{i=1}^{n}(Q_i^{NL-THA} - \overline{Q}^{NL-THA})(Q_i^{MPA} - \overline{Q}^{MPA})}{\sqrt{\sum_{i=1}^{n}(Q_i^{NL-THA} - \overline{Q}^{NL-THA})^2} \times \sqrt{\sum_{i=1}^{n}(Q_i^{MPA} - \overline{Q}^{MPA})^2}} \tag{3}$$

In the recent equation, Q_i^{NL-THA} and Q_i^{MPA} are define as in Eqs. (1) and (2) and $\overline{Q}_i^{NL-THA}$ is used as the average of nonlinear time history results and $\overline{Q}_i^{MPA}$ defines the average of nonlinear static results. It is clear that if $\rho = \pm 1$ the complete linear correlation is approved.

To illustrate the accuracy and the correlation rate of this approximate method for predicting of the responses, NL-THA responses resulted from nonlinear dynamic analyses are plotted versus MPA responses obtained from nonlinear static analyses to each of the 14 ground motions in all models which will be presented in relative following text.

4.2 Review of Displacement Results

By scatter plotting target roof displacements estimated by MPA versus maximum roof displacement estimated by the nonlinear time history analyses, as shown in Fig. 5, the accuracy and conservatism of MPA for estimating roof displacement can be presented. The correlation factor using 616 nonlinear time history results and 616 results obtained from more than 1800 nonlinear static analyses (using three or more modes) is equal to 0.8094 showing reasonable correlation between MPA estimated roof displacements and actual maximum roof displacements.

As mentioned before for investigation of the accuracy of this response, one can use relative error. Relative errors for this

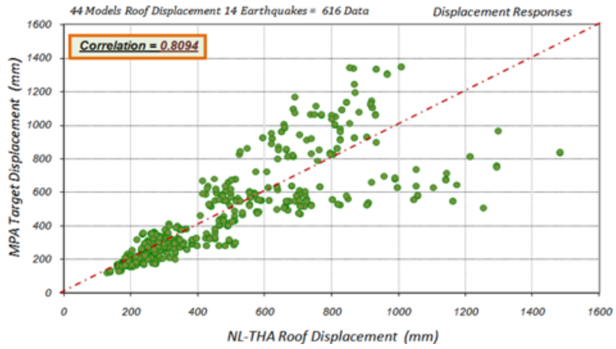

Fig. 5. Scatter Plot of Estimated Frame Target Displacements

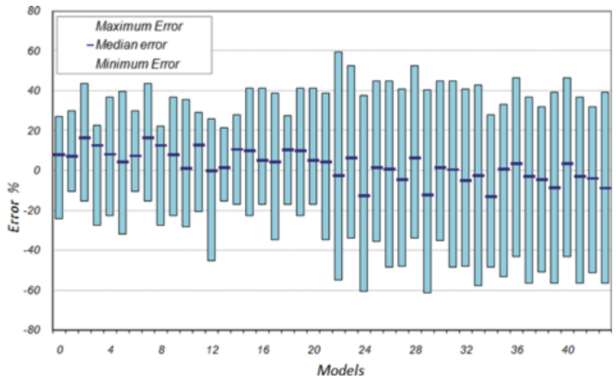

Fig. 6. Relative Errors of Estimated Frame Target Displacements

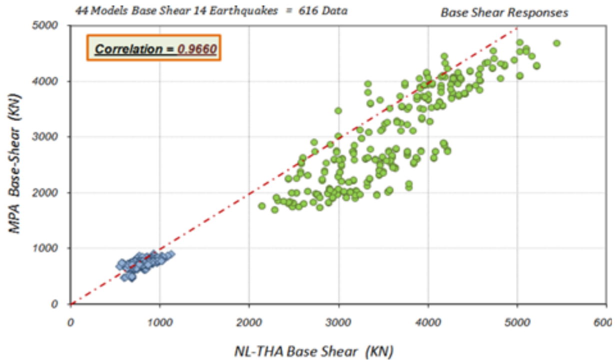

Fig. 7. Scatter Plot of Estimated Frame Base Shears

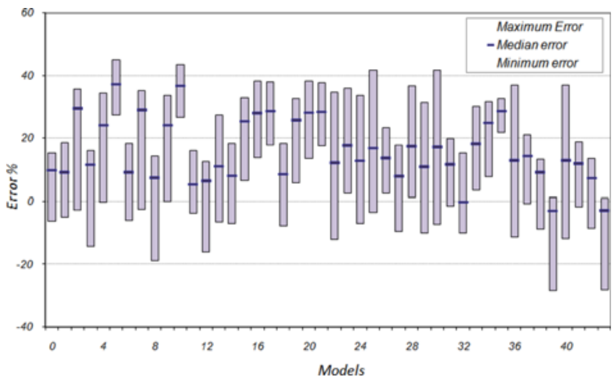

Fig. 8. Relative Errors of Estimated Frame Base Shears

response are shown in Fig. 6 in percent. In this figure. each bar shows the relative error interval for all ground motions and it is existed for each model separately. Therefore, the maximum of relative errors occurred in all ground motion for a certain model, is appeared at the top or bottom of the bar and the median value of relative error is shown by the dash. In Fig. 6, it is observed that error domain intermediate maximum error and minimum error in class “I” models, are between -30 to +40 percent whenever, in class “II” models are between -60 to +50 percent. Tolerance of median errors in all models is between -15 to +15 percent.

4.3 Review of Base Shear Results

In order to investigate the accuracy of base shear results of MPA method, the correlation ratio of base shear for all analyses that including 616 nonlinear time history results versus 616 results obtained from more than 1800 nonlinear static analyses (determined by MPA considering at least three modes) is calculated which is equal to 0.9660 (Fig. 7). The correlation value which is obtained shows the high accuracy of the MPA for prediction of peak inelastic displacement of buildings in a certain earthquake. By the way the graph shows the scatter rate of this response.

For observation of the deviation of base shear response, the occurred relative errors in estimation of this response are shown in Fig. 8 in percent. In this figure. each bar shows the relative error interval for all ground motions and it is illustrated for each model separately. Thus, the maximum of relative error occurred in all ground motion for a certain model, is appeared at the top or bottom of the bar which is shown by the dash. Moreover, it is observed in this figure. that domain of error is between -25 to +45 percent.

4.4 Review of Story Drift Results

One of the other seismic responses which can be affected by irregularity in buildings is story drift. It should be noted that by moving from global responses such as target displacement toward local response like story drift leads to less accuracy in prediction of responses and this limitation is in the nature of pushover analysis as be shown in research by Valmundson and Nau (1997). To finding the ability of this method (NSP) for prediction of story drift, the values of NSP for this response is plotted versus resulted of NL-THA in Fig. 9.

Figure 9 is consisted of 3080 Cartesian points and it includes NSP and NL-THA drift for each story of 44 frame models in all of 14 ground motion. The correlation factor of this response is 0.8150 and as expected it shows good accuracy for prediction of this response. For studying of the accuracy of story drift response, the relative errors of drift estimation are shown in Fig. 10 in percent. In addition, it observed in Fig. 10 that the error domain are different for class “I” and class “II” models as in class “I” models are between 0 to +45 percent while in class “II” models are between -60 to +50 percent. Also median errors are between 0 to +40 percent.

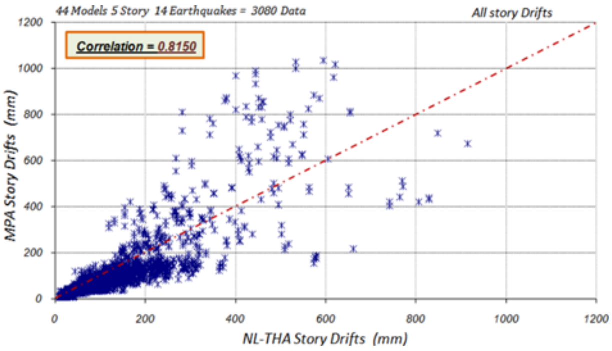

Fig. 9. Scatter Plot of Estimated Frame Drifts

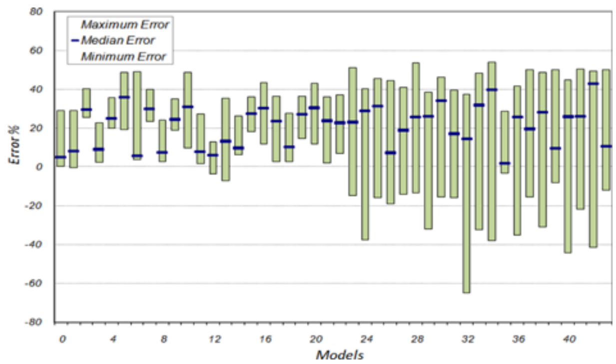

Fig. 10. Relative Errors of Estimated Frame Drifts

5. Comparison and Evaluation of the Analysis Results of MPA and MCM-FEMA440

The earthquake induced demands for the models are determined by many analyses; MPA considering at least three modes, pushover analysis using the two force distributions in FEMA-440, and NL-THA responses which were induced in all 14 ground motions. The force distributions of FEMA are uniform and triangle which maximum results of analysis are considered. The pushover analyses are implemented for a target roof displacement, by using each of these force distributions. Here, used the results of FEMA analysis from Momtahen and Banan research (2008).

5.1 Comparison of Median Results

Figure 11 presents; MPA median target displacement are more accurate than FEMA results in both models classes however for class “I” models error is less in comparison with class “II” models. The Median base shear demands are presented Fig. 12. The MPA results are as well as FEMA results in all models wherever in class “I” models for some models, the FEMA results more accurate than MPA ones.

The median story drift demands in 5th, 4th and 3d floor are shown in Figs. 13, 14 and 15, respectively. The MPA drift results trace closely NL-THA results in all models and less error than FEMA ones in class “I” models and each floor. However, for class “II” models in 4th floor FEMA has conservative results.

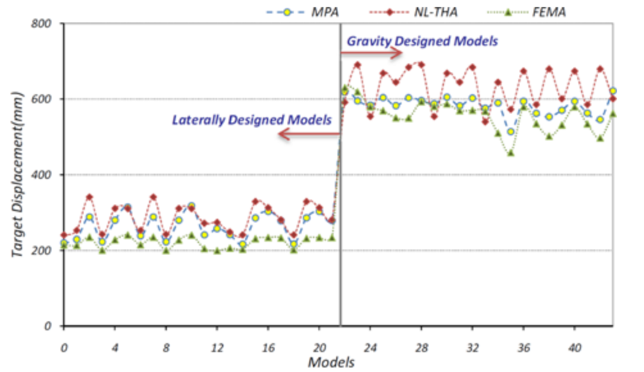

Fig. 11. Comparison of MPA and FEMA Median Target Displacement with NL-THA Results

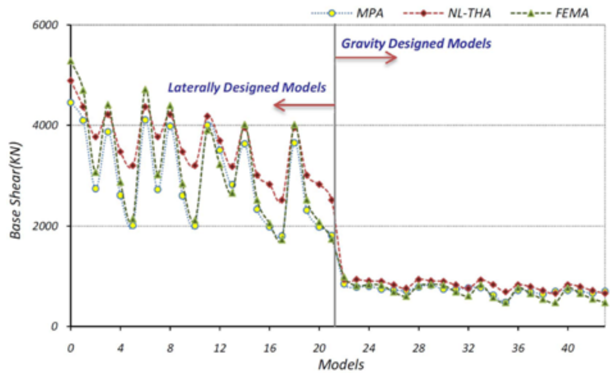

Fig. 12. Comparison of MPA and FEMA Median Base Shear with NL-THA Results

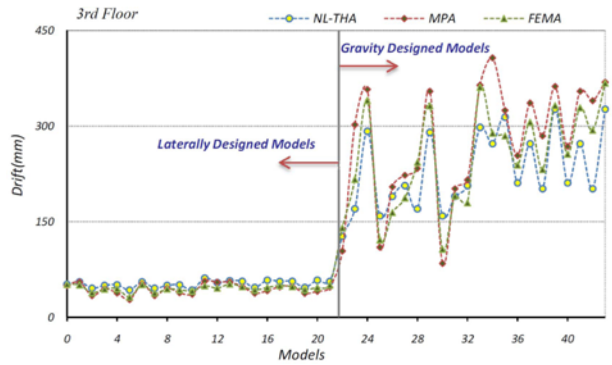

Fig. 13. Comparison of MPA and FEMA Median Drift in 3rd Floor with NL-THA Results

5.2 Comparision of Correlation Factor

The correlation factor of MPA and FEMA results with NL-THA results are compared with in Fig. 16. In this Fig., considerabl difference are observed in displacement results of class “I” models that 0.91 is for MPA and 0.88 is for FEMA.

5.3 Comparision of Story Responses

The MPA story results in some models are compared with FEMA story results in Figs. 17 and 18.

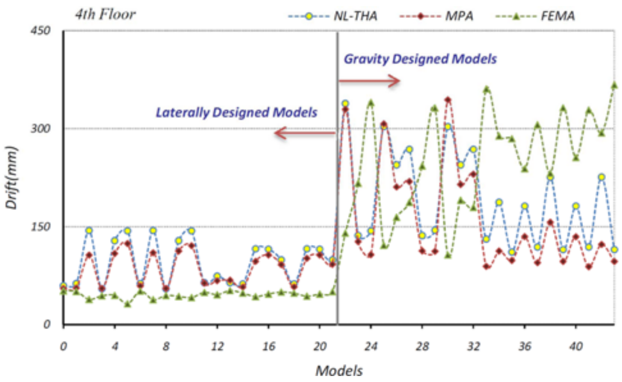

Fig. 14. Comparison of MPA and FEMA Median Drift in 4th Floor with NL-THA Results

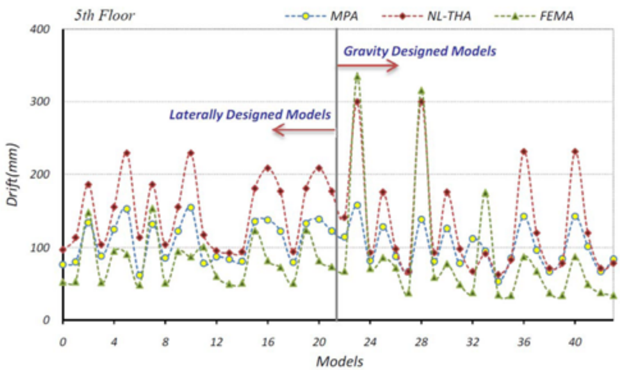

Fig. 15.Comparison of MPA and FEMA Median Drift in 5th Floor with NL-THA Results

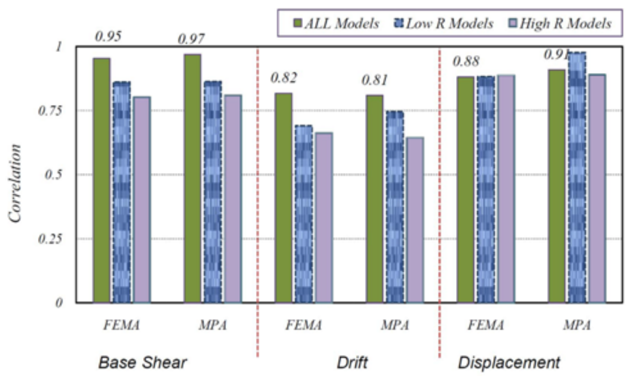

Fig. 16. Comparison of Correlation Factor on MPA and FEMA Analysis

5.4 Comparision of Errors

The error comparisons of the results that are obtained from the MPA and FEMA procedures are given in Figs. 19 and 20. As seen in these Fig, the median and maximum error in MPA responses is almost lower than FEMA in each response and two models class however, for median error considerable difference are seen in drift responses, furthermore, for maximum errors, substantial difference are seen in displacement responses which the MPA has a lower error.

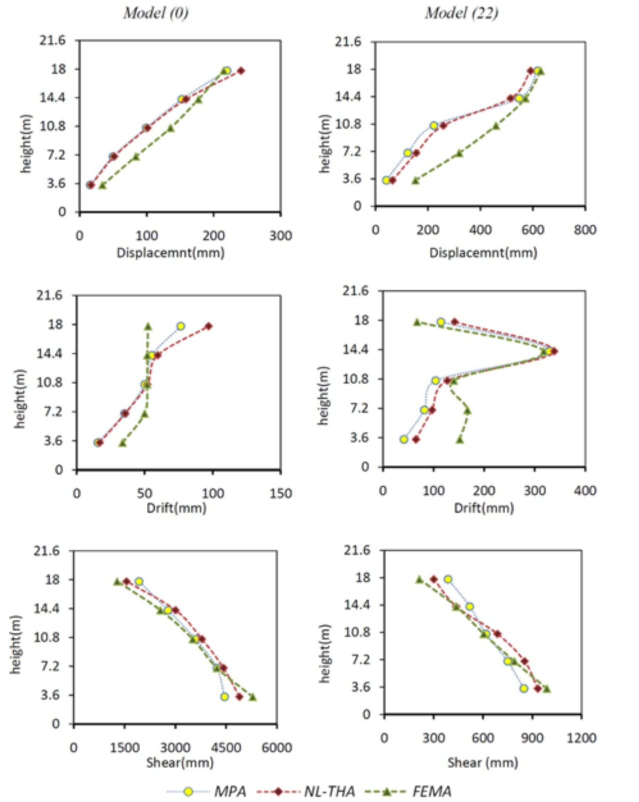

Fig. 17. Comparison of MPA and FEMA Median Responses for models 0 and 22 with NL-THA Results

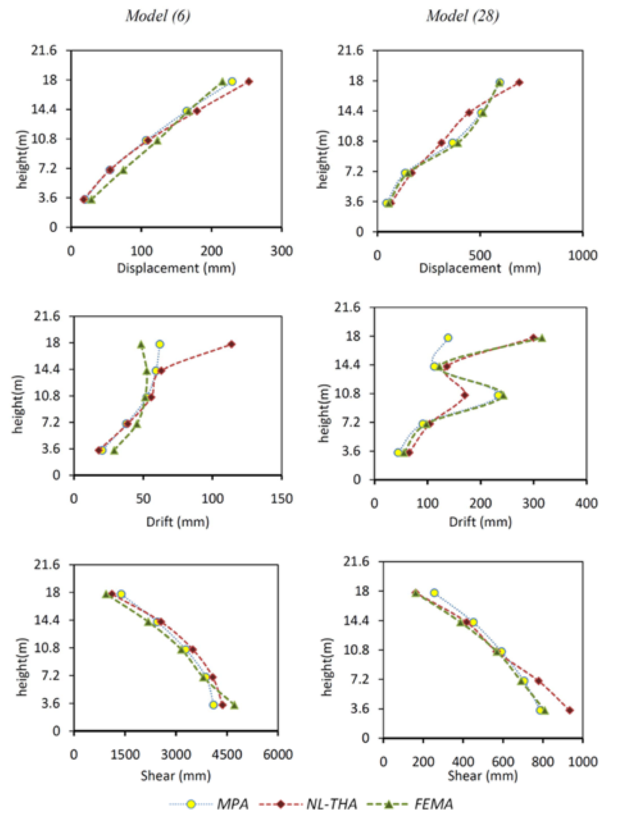

Fig. 18. Comparison of MPA and FEMA Median Responses for Models 6 and 28 with NL-THA Results

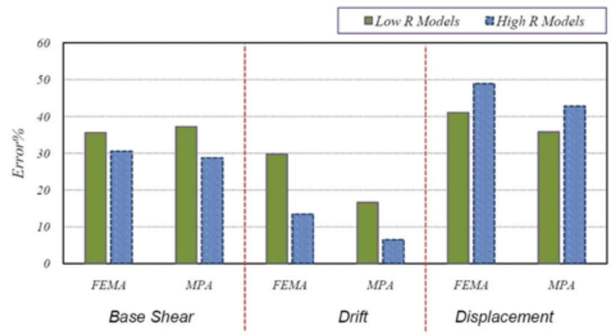

Fig. 19. Comparison of MPA and FEMA Results Median Error

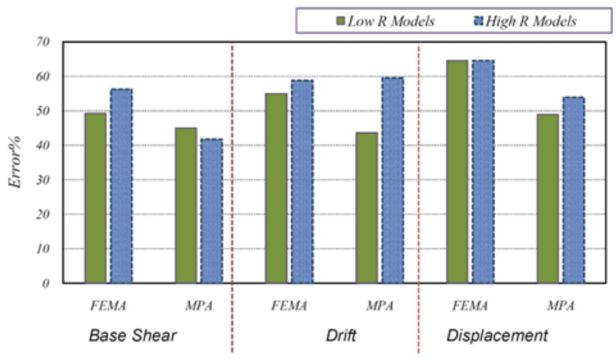

Fig. 20. Comparison of MPA and FEMA Results Maximum Error

6. Conclusions

Summary of a detailed investigation on performance of MPA analysis for equivalent nonlinear static analysis for estimating frame maximum roof displacement, base shear, and median story drifts is presented for steel moment resisting frames with irregularities in elevation. Results of nonlinear dynamic analyses of 44 irregular frames subjected to a family of 14 ground motions are compared with MPA and FEMA440 MCM results. The following trends are concluded:

1. For Irregular frames, MPA is inconservative for estimating median of dynamic responses for selected ground motions. The maximum median error is about 22%, 37%, and 42% for estimating frame roof displacement, base shear, and story drifts, respectively.
2. MPA is conservative for estimating roof displacement in models with a high-R values.
3. The absolute value of error between estimated MPA response and computed response from nonlinear dynamic analyses is increased with increasing of the R value for frame roof displacment and drift and it is reduced for frame base shear.
4. MPA estimated results have a good correlation with computed results of nonlinear dynamic analysis with correlation values of 0.81, 0.96 and 0.91 for frame roof displacement, base shear, and story drifts, respectively.
5. MPA results are compared with FEMA440 MCM. It was concluded that MPA results have better correlation with NL-THA results and less error than FEMA results for story displacements, drifts, and shears for all considered models.

Approximation of error by using MPA is partially due to occurance of incomplete failure mechanisms. Further research is required for quantification of irregularity and developing modifications to MPA for improving its performance for irregular structures.

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