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Structural Damage Detection Using Incomplete Modal Data and Incomplete Static Response

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Abstract

This paper presents novel approaches to structural damage detection and estimation using incomplete modal data and incomplete static response of a damaged structure. The proposed methods use modal data or static displacement to formulate objective functions. Damage location and severity in structural elements are determined using optimization of the objective functions by the simulated annealing algorithm. The presented methods are applied to a simply supported beam and a three-story plane frame with and without noise in modal data and containing several damages. Moreover, the performance of the proposed methods has been verified through using experimental modal data of a mass-stiffness system. The results indicate that the proposed methods perform quite well using different objective functions in spite of the incomplete data.

Keywords: *damage detection, incomplete modal data, incomplete static response, optimization, simulated annealing algorithm*

1. Introduction

Structural damage detection has attracted much attention in recent decades in order to assess the reliability of structural systems during their service life. Most of damage detection methods are based on the changes of dynamic characteristics and static responses. Many researchers studied modal parameters of structure including natural frequencies and mode shapes that are very sensitive to structural properties like stiffness (Law *et al*., 2001; Kim *et al*., 2003; Ghodrati Amiri *et al*., 2011). On the other hand, static responses are more locally sensitive to damage than natural frequencies (Jenkis *et al*., 1997) and the equipments of static testing, and precise static displacements of structures could be obtained rapidly and economically (He and Hwang, 2007). However, there are two main drawbacks in the static damage identification methods: (1) Static testing provides less information as compared to dynamic testing; (2) The effect of damages on static responses for damage detection may be cryptic due to limited load paths (He and Hwang, 2007).

In both dynamic and static damage detection methods, the incompleteness of the test data is a great obstacle. Therefore, some researchers have inspected damage detection using incomplete modal data and incomplete static responses. Hajela and Soeiro (1990) presented a damage detection algorithm based on static displacements, mode shapes and frequencies. To solve an unconstrained optimization problem, an iterative non-linear programming method was developed. Also, this algorithm can be useful when the measured data is incomplete. Numerically and experimentally verification of the proposed algorithm is done using a planner truss and clamped beam, respectively. Wang *et al*. (2001) proposed a damage identification that employs the structural static deformation and the first several natural frequencies. To locate damage in the structure, the Damage Signature Matching (DSM) technique is improved through a proper definition of Measured Damage Signatures (MDS) and Predicted Damage Signatures (PDS). In their study, after they obtained the possible damage location, an iterative estimation scheme for solving non-linear optimization programming problems based on the quadratic programming technique, was proposed to predict the damage extent. A main effort of the presented approach is that it can be directly applied in the cases of incomplete measured data. Chen *et al*. (2005) developed a structural damage detection method by using a limited test static displacement based on grey system theory. The grey relation coefficient of displacement curvature is defined and used to locate damage in a structure. Duan *et al*. (2007) extended the damage locating vector method to the case of ambient vibration with incomplete measured Degree of Freedoms (DOFs). Rahai

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et al. (2007) presented a global algorithm for detecting and estimating damage in structures based on the parameter estimation method, in which elemental damage equations, which partially relate the measured mode shape of the damaged structure to the change of structural parameter, are developed using incomplete measured mode shapes. Recently, Li *et al*. (2008) developed the Cross-Model Cross-Mode (CMCM) method for damage detection that is capable of identifying the damage to individual members of offshore jacket platforms, when limited, spatially incomplete modal data is available.

Furthermore, some researchers used model updating for damage detection of structures with incomplete modal data. Yuen *et al*. (2006) studied a Bayesian structural model updating methodology which can treat incomplete modal data. Ching *et al*. (2006) developed a gibbs sampler approach for linear Bayesian structural model updating, in which the goal is to detect and quantify any damage using incomplete modal data obtained from small-amplitude vibrations measured before and after a severe loading event. Carvalho *et al*. (2007) presented a direct method for model updating with incomplete modal data. The proposed method uses an algorithmic way without requiring any model reduction or modal expansion techniques. Huajun *et al*. (2008) extended the CMCM method to simultaneously update the mass, damping and stiffness matrices of a finite element model when only few spatially incomplete, complex-valued modes are available. The results reveal that applying the CMCM method, along with an iterative Guyan reduction scheme can yield good damage detection in general. Also, Chen (2008) presented an approach for detecting local damage in large scale frame structures by utilizing regularization methods with incomplete noisy data. A system of linear basic equations for determining the damage indicators has been developed by directly adopting the measured incomplete modal data.

In this study, new methods are introduced to detect and estimate damage in structures using modal data and static displacements of a damaged structure. The damage detection is carried out through applying simulated annealing method to minimize objective function. Simulated annealing is a probabilistic method proposed by Kirkpatrick *et al*. (1983) and Cerny (1985) to find the global minimum of a cost function that may possess several local minima. It works by emulating the physical process whereby a solid is slowly cooled so that when eventually its structure is “frozen” this happens at a minimum energy configuration (Bertsimas *et al*., 1993). The simulated annealing method has been widely used in different fields of engineering as a robust and promising method (Levin *et al*., 1998; Wong *et al*., 2002; Chang *et al*., 2011; Chen and Chen, 2009; Puoza *et al*., 2011). The presented method for damage identification has been applied to two numerical examples, namely a simply supported beam and a three-story plane frame containing several damages. In addition, the experimental data from the vibration test of a mass-stiffness system are used in the present approach. The obtained results show that the proposed methods perform quite well in spite of the incomplete modal data and static displacements.

2. Problem Formulation

In this section, three methods are proposed for structural damage detection and estimation. The first method uses the incomplete mode shapes and frequencies to formulate a dynamic residue vector as an objective function. The second method uses the difference between the measured displacements and the corresponding computed displacements. Finally, in the third method, an objective function is formulated by using static residue force vector. Then, the simulated annealing optimization algorithm for minimizing the objective functions is presented.

2.1 First Proposed Method

The modal characteristics of a structure without damage are described by the following equations:

$$([\mathbf{K}^{ud}]-\lambda_i[\mathbf{M}])\{\Phi_i\} = 0 \quad i = 1, 2, \ldots, n \tag{1}$$

where, $\mathbf{K}^{ud}$ and $\mathbf{M}$ are undamaged stiffness and mass matrices, respectively; λ_i is the square of the natural frequency corresponding to the mode shape $\mathbf{\Phi}_i$; and n is the total number of obtained mode shapes.

One of the simplest techniques to determine damage-induced alteration stiffness is the degradation in Young's modulus of an element as follows:

$$E_j^d = E_j^{ud}(1-d_j) \tag{2}$$

where E_j^d and E_j^{ud} are the damaged and undamaged Young's modulus of the jth element in the finite element model, respectively; and d_j indicates the damage severity at the jth element in the finite element model whose values are between 0 for an element without damage and 1 for a ruptured element.

Moreover, it is assumed that no change would occur after damage in the mass matrix, which seems to be reasonable in most real problems.

Thus, as it was mentioned above, the eigenvalue equations for a damaged structure became:

$$([\mathbf{K}^d]-\lambda_i^d[\mathbf{M}])\{\Phi_i^d\} = 0 \quad i = 1, 2, \ldots, n \tag{3}$$

where $\mathbf{K}^d$ is the damaged stiffness matrix; λ_i^d and $\mathbf{\Phi}_i^d$ are the square of the ith natural frequency and the ith mode shape of the damaged structure, respectively.

As the number of sensors used to measure modal data is normally limited and usually is less than the number of DOFs in the finite element model, either the model reduction method should be used to match with incomplete measured mode shapes or the measured mode shapes must be expanded to the dimension of the analytical mode shapes. Because of no convergence in the proposed optimization method using the modal expansion, the first option has been adopted using the Guyan (1965) static reduction method. This method is employed to condense the mass and stiffness matrices. In this method, the mass and stiffness matrices, and the displacement and acceleration vectors in Eq. (1) are partitioned into a set of master and slave DOFs:

$$\begin{bmatrix}\mathbf{M}_{mm} & \mathbf{M}_{ms}\\ \mathbf{M}_{sm} & \mathbf{M}_{ss}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{x}}_m\\ \ddot{\mathbf{x}}_s\end{Bmatrix}+\begin{bmatrix}\mathbf{K}_{mm} & \mathbf{K}_{ms}\\ \mathbf{K}_{sm} & \mathbf{K}_{ss}\end{bmatrix}\begin{Bmatrix}\mathbf{x}_m\\ \mathbf{x}_s\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix} \tag{4}$$

In which, $\mathbf{x}$ and $\ddot{\mathbf{x}}$ are the displacement and acceleration vectors, respectively and the subscript m and s are the master and slave coordinates, respectively. To eliminate the slave DOFs, the inertia terms for the second set of equations are neglected that leads to:

$$\begin{Bmatrix}\mathbf{x}_m\\ \mathbf{x}_s\end{Bmatrix}=[\mathbf{T}]\{\mathbf{x}_m\} \tag{5}$$

where,

$$[\mathbf{T}]=\begin{bmatrix}\mathbf{I}\\ -\mathbf{K}_{ss}^{-1}\mathbf{K}_{sm}\end{bmatrix} \tag{6}$$

Substitution of Eq. (5) into Eq. (4), followed by permultiplication by $[\mathbf{T}]^{\mathrm{T}}$ and using incomplete mode shapes, yields the reduced eigenproblem as:

$$([\mathbf{K}_r^d]-\lambda_{i,r}^d[\mathbf{M}_r])\{\Phi_{i,r}^d\}=0 \qquad r=1,2,\ldots,n \tag{7}$$

where,

$$[\mathbf{K}_r^d]=[\mathbf{T}]^T[\mathbf{K}^d][\mathbf{T}] \tag{8}$$

$$[\mathbf{M}_r]=[\mathbf{T}]^T[\mathbf{M}][\mathbf{T}] \tag{9}$$

in which, $\mathbf{K}_r^d$ and $\mathbf{M}_r$ are the reduced stiffness and mass matrices of damaged structure, respectively; $\lambda_{i,r}^d$ and $\mathbf{\Phi}_{i,r}^d$ are the square of the ith natural frequency and the ith incomplete mode shape of the damaged structure in the reduced state, respectively.

Applying the incomplete mode shapes and natural frequencies of damaged structure to Eq. (7) leads to form the inverse problem of determining the damage severity parameter. The definition of a local damage severity parameter d in the finite element model allows estimating damage quantity and location together, since damage identification is then carried out at the element level. The problem can be formulated as optimization problem of objective function, while using some transforms as a direct inversion to obtain solution is impossible most of the time.

Localizing and quantifying damage is often considered as a difficult and complex problem, requiring a sophisticated optimization procedure. In typical optimization problem, there may be lots of locally optimal layouts; therefore, a downhill-proceeding algorithm in which a steady declining value of objective function is created in iterations, may be stuck into a locally optimal point instead of providing global optimal solution. For that reason, global search algorithms like the simulated annealing method is adopted by the authors in order to characterize damage. Simulated annealing is a popular technique due to its ability to 'escape' from local minima; it is able to move to areas of less desirable solutions than that which it currently explores, so that it can eventually locate the global optimum (Smith, 2006). A particular attraction of simulated annealing is the existence of the proof of Geman and Geman (1984) that guarantees convergence to the global minimum provided that the annealing rate is sufficiently slow.

The simulated annealing method attempts to find the best solution to a given problem by minimizing an objective function. In any optimization process, existence of objective function is an indispensable part of problem.

The general statement for the objective function is:

$$F=f(d_1,d_2,\ldots,d_{N_e}) \tag{10}$$

In the process of substituting the incomplete measured modal parameters of the damaged structure in Eq. (10), a dynamic residue vector can be defined over each measured mode as follows:

$$[\mathbf{R}_i(d)]=([\mathbf{K}_r^d]-\lambda_{i,r}^m[\mathbf{M}_r])\{\Phi_{i,r}^m\} \qquad i=1,2,\ldots,m \tag{11}$$

where $\lambda_{i,r}^m$ and $\mathbf{\Phi}_{i,r}^m$ are the square of the ith natural frequency and the ith incomplete mode shape from measurements, respectively; and m is the number of available mode shape for damage detection.

Then, if structural damages are determined correctly, the residue vector would be next to 0 in Eq. (11). Therefore, the problem of damage detection can be formulated as an optimization problem. So, the first objective function can be formulated as follows:

$$f_1(d)=\sum_{i=1}^{m}\|[\mathbf{R}_i(d)]\|^2 \tag{12}$$

$$0\le d_1\le 1\ ,\ 0\le d_2\le 1\ ,\ \ldots,\ 0\le d_{N_e}\le 1$$

where $\|\ \|$ represents the Euclidean length of $\mathbf{R}_i(d)$.

2.2 Two Last Proposed Methods

The static equilibrium equation of a structure in a displacement based finite element frame work can be expressed as follows:

$$[\mathbf{K}^{ud}]\{\mathbf{x}\}=\{\mathbf{F}\} \tag{13}$$

where $\mathbf{F}$ and $\mathbf{x}$ are the force and displacement vectors; respectively. From Eq. (13), the static equilibrium equation of a damaged structure can be obtained as:

$$[\mathbf{K}^d]\{\mathbf{x}^d\}=\{\mathbf{F}\} \tag{14}$$

where superscript d is noted as the damage state. In fact, not all displacements in $\mathbf{x}^d$ can be measured. Therefore, Eq. (14) is partitioned into the master and slave coordinates as below:

$$\begin{bmatrix}\mathbf{K}_{mm}^d & \mathbf{K}_{ms}^d\\ \mathbf{K}_{sm}^d & \mathbf{K}_{ss}^d\end{bmatrix}\begin{Bmatrix}\mathbf{x}_m^d\\ \mathbf{x}_s^d\end{Bmatrix}=\begin{Bmatrix}\mathbf{F}_m\\ \mathbf{F}_s\end{Bmatrix} \tag{15}$$

in which, the subscripts m and s are the master and slave coordinates, respectively. The vector of slaved displacements $\mathbf{x}_s^d$ is condensed out, following static condensation and Eq. (15) reduces to the following:

$$[\mathbf{K}_r^d]\{\mathbf{x}_m^d\}=\{\mathbf{F}_r\} \tag{16}$$

where,

$$[\mathbf{K}_r^d] = ([\mathbf{K}_{mm}^d] - [\mathbf{K}_{ms}^d][\mathbf{K}_{ss}^d]^{-1}[\mathbf{K}_{sm}^d])\{\mathbf{x}_m^d\} \tag{17}$$

$$\{\mathbf{F}_r\} = \{\mathbf{F}_m\} - [\mathbf{K}_{ms}^d][\mathbf{K}_{ss}^d]^{-1}\{\mathbf{F}_m\} \tag{18}$$

in which, $\mathbf{K}_r^d$ and $\mathbf{F}_r$ are the condensed stiffness matrix and the condensed load vector of damaged structure; respectively.

From Eq. (16), the measured displacement of damaged structure can be obtained as:

$$\{\mathbf{x}_m^d\} = [\mathbf{K}_r^d]^{-1}\{\mathbf{F}_r\} \tag{19}$$

The second objective function is defined in terms of output errors between computed and measured displacements as follow:

$$f_2(d) = \sum_{i=1}^{p}(\beta(\mathbf{x}_{m,i}^d - \mathbf{x}_{t,i}^d)^2) \tag{20}$$

where $\mathbf{x}_{m,i}^d$, $\mathbf{x}_{t,i}^d$ are the measured and theoretically computed displacement of the ith point of a damaged structure, respectively. β is a weighing factor and p is the number of a considered displacement point.

Also, the third objective function, f_3 formulates as a static residue force vector as follow:

$$f_3(d) = \|\gamma(\mathbf{F}_r - \mathbf{K}_r^d\mathbf{x}_m^d)\|^2 \tag{21}$$

where γ is a weighing factor. The weighting factors are introduced to produce a more appropriate value of the objective functions.

2.3 Optimization by using Simulated Annealing Algorithm

Simulated annealing is the simulation of annealing of a physical many particle system to find the global optimum solutions of a large combinatorial optimization problem (Davis, 1987). It uses a temperature parameter that controls the search. At each step the temperature is slowly “cooled” or lowered and a new point is generated using an annealing function. At each step, the new points distance from the current point is proportional to the temperature. If the energy (cost) of this new point is lower than that of the old point, the new point is accepted. If the new energy is higher, the point is accepted probabilistically, with probability dependent on a “temperature” parameter. This unintuitive step sometime helps identify a new search region in hope of finding a better minimum.

In this paper, fast annealing function that takes random steps with size proportional to temperature and generates a point based on the current point and the current temperature is used. Also, the Boltzmann acceptance probability is used which is based on the chances of obtaining a new state E_{k+1}, relative to a previous state E_k:

$$\mathrm{Pro}_{(k+1)} = \frac{\exp(-E_{k+1}/T)}{\exp(-E_{k+1}/T) + \exp(-E_k/T)} = \frac{1}{1+\exp(\Delta E/T)} \tag{22}$$

$$\Delta E = E_{k+1} - E_k \tag{23}$$

where ΔE represents the energy difference between the present and previous values of the target function, and T is temperature.

The proposed algorithm uses the fast Cauchy annealing schedule for temperature, T decreasing linearly in annealing-time k:

$$T = \frac{T_0}{k} \tag{24}$$

where T_0 is initial temperature.

Also, Reannealing is performed after a certain number of points (Reanneal Interval) are accepted. Reannealing raises the temperature in each dimension, depending on sensitivity information and the search is resumed with the new temperature values (Matlab User Manual, 2008).

The Simulated annealing optimization algorithm stops when any of following situations occur (Matlab User Manual, 2008):

- The number of iteration or evaluation of objective function reaches the max value.
- Alteration in the improvement of objective function is less than the function tolerance.
- The time algorithm runs reaches the max value.

Figure 1 shows the flowchart of the proposed method for estimation and localization of the damage via simulated annealing method.

After terminating the simulated annealing algorithm, another minimization function is utilized. For this purpose, we used the

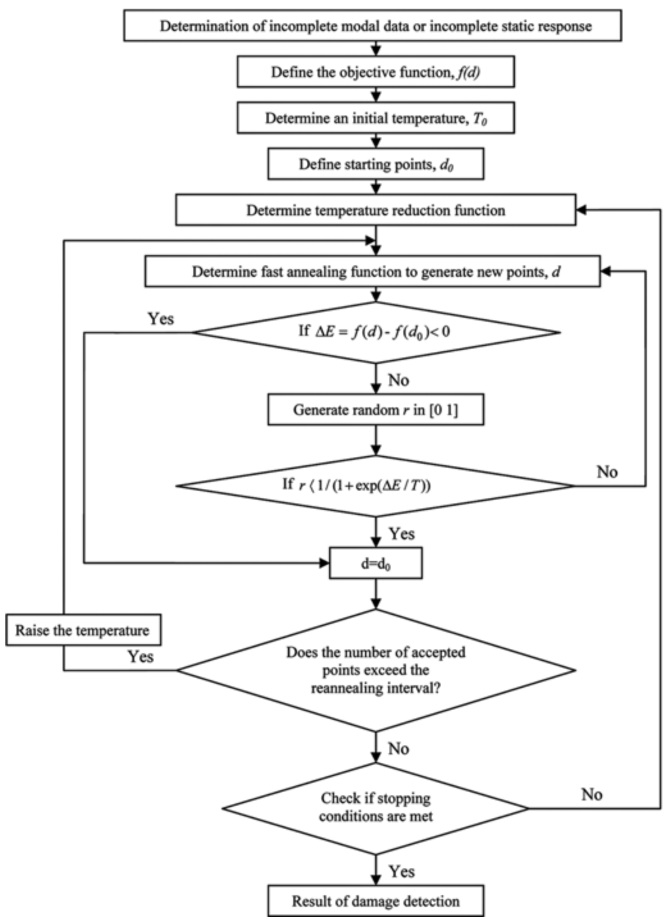

Fig. 1. Flowchart of the Damage Detection Method using Simulated Annealing Method

Optimization Toolbox function *fmincon* in MATLAB to perform constrained minimization. This routinely implement Sequential Quadratic Programming (SQP) to minimize the nonlinear cost function was subjected to linear and nonlinear equality and inequality constraints. SQP converts a nonlinear minimization to a linear minimization using a Hessian matrix of cost function and gradient of nonlinear constraints.

Table 2. Input Parameters for the Simulated Annealing Algorithm

Maximum function evaluations	18000-200000
Annealing function	Fast annealing
Temperature update function	Linear temperature update
Reannealing interval	600-2000
Initial temperature	25-80

3. Numerical Examples

In this section, the efficiency and effectiveness of the proposed methods is evaluated through some numerically simulated damage identification tests using incomplete modal data, which may be noisy or noise-free. A simply supported beam and three-story plane frame are chosen with two different scenarios of damage for each of them for the purpose.

3.1 Simply Supported Beam

A Simply supported beam as illustrated in Fig. 2 with a finite-element model consisting of 10 beam elements and 11 nodes is considered. To formulate the last two objective functions, two vertical point loads have been used. For the considered concrete beam, the material properties include Young's modulus of E=25 GPa, mass density of ρ=2500 kg/m^3. The cross-sectional area and the moment of inertia of the beam are A=0.12 m^2 and I=0.0016 m^4, respectively.

In this example, two damage scenarios are represented as the elements with reduction in Young's modulus. The damage severity in each element is given by the reduction factor listed in Table 1. In this case, only 9 translational DOFs are selected as measured DOFs.

Damage in the simply supported beam can be determined by using the proposed method. The simulated annealing method input parameters adopted for the following analyses are summarized in Table 2.

Considering the low values of the applied loads on the structure, so values of the formed objective functions will be small. Therefore, it has been tried to prevent diverging of the proposed algorithm by defining two weighting factors β and γ that are large enough and magnifying the objective function. So, the selected values of the weighing factors β and γ are 1030 and 1020, respectively. The selection of these parameters is based on trial and error and the selected values of the weighing factors depend on the values of applied loads in the structure.

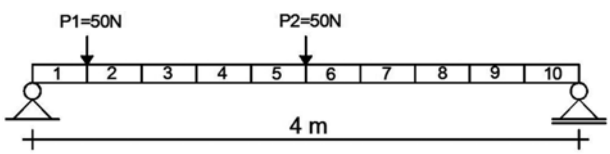

Fig. 2 The Simply Supported Beam with the Finite Element Model

Table 1. Damage Scenarios for Simply Supported Beam

Scenario 1		Scenario 2	
Element 1	45%	Element 2	35%
Element 6	50%	Element 4	25%
Element 9	20%	Element 8	50%

To be more suited with the real dynamic cases, another examination has been performed in which the natural frequencies with 5% noise are utilized to damage identification considering the same patterns mentioned before. To perform this, some random noise has been added to the theoretically calculated natural frequencies. The contaminated frequency with noise can be obtained from the frequency without noise using the following equation:

$$\omega_i^m = \omega_i(1 + \eta\ rand\ [-1\ \ 1]) \tag{25}$$

where ω_i^m and ω_i are the frequencies of ith mode contaminated with noise and without noise, respectively. η is the noise level (e.g., 0.05 relates to a 5% noise level) and *rand* is a random number in the range [-1 1].

Figure 3 shows the results of damage identification in the simply supported beam for two damage scenarios using the first objective function. As mentioned above, the first objective function is based on incomplete first three mode shapes and frequencies which may be contaminated by 5% noise in natural frequencies or noise free. The results illustrate that the proposed method is a robust and effective method in detecting and quantifying various damage scenarios in spite of incomplete modal data and measurement inaccuracies. Although in the case of noisy data some

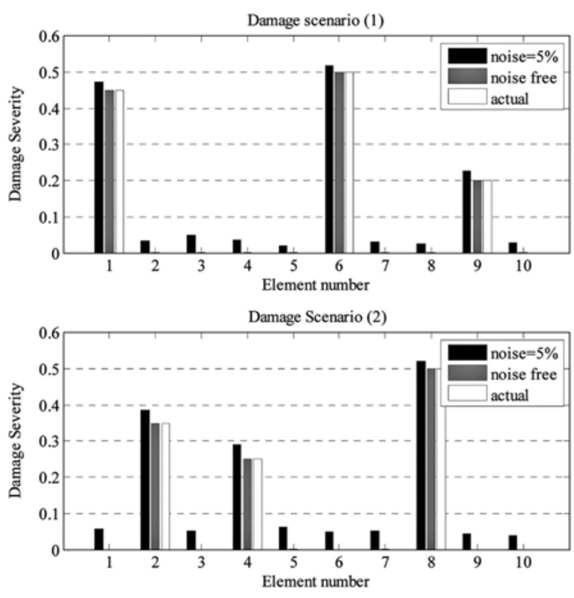

Fig. 3. The Obtained Results of the Simply Supported Beam for the First Objective Function

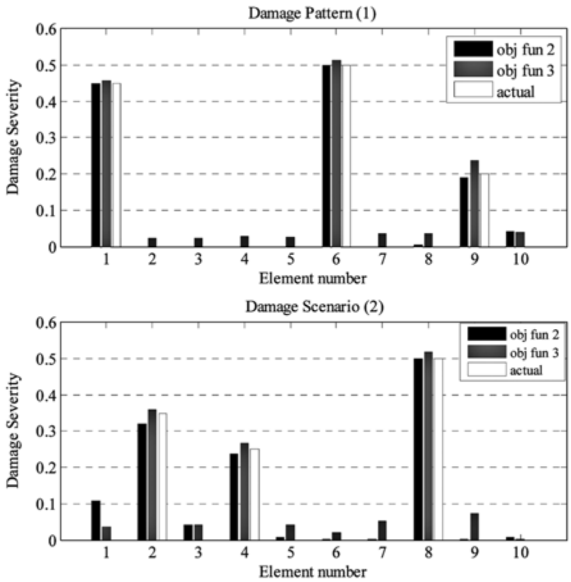

Fig. 4. The Obtained Results of the Simply Supported Beam for the Last Two Objective Functions

undamaged elements are detected as damaged by mistake which value of damage is very low.

The obtained results of damage detection and quantification using the objective functions that are based on incomplete static displacements of structure, are shown in Fig. 4. The results show that the proposed method is robust and promising in localizing and quantifying of different damage scenarios.

3.2 Three Story Plane Frame

A three-story plane steel frame as illustrated in Fig. 5 with finite-element model consists of nine elements (six columns and three beams) and six free nodes are considered. To formulate the last two objective functions, uniformly distributed load of 50 kN/m at the beam elements has been used. For the considered steel frame, the material properties of the steel include Young's modulus of E=200 GPa, mass density of ρ=7850 kg/m^3. The mass per unit length, moment of inertia, and cross-sectional area

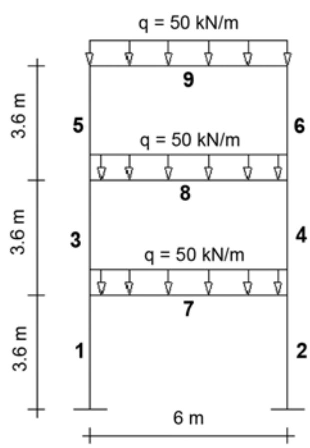

Fig. 5. The Three-story Plane Frame with the Finite Element Model

of the columns are: m=117.75 kg/m, I=3.3×10^{-4} m^4 and A=1.5×10^{-2} m^2, respectively; for the beams are: m=119.71 kg/m, I=3.69×10^{-4} m^4 and A=1.52×10^{-2} m^2. Also, the damage severity in each element is given by the reduction factor listed in Table 3.

In this case, only 6 translational DOFs are selected as measured DOFs in the process of damage detection and quantification. As

Table 3. Damage Scenarios for the Three-story Plane Frame

Scenario 1		Scenario 2	
Element 2	20%	Element 1	30%
Element 3	25%	Element 6	30%
Element 8	30%	Element 7	20%

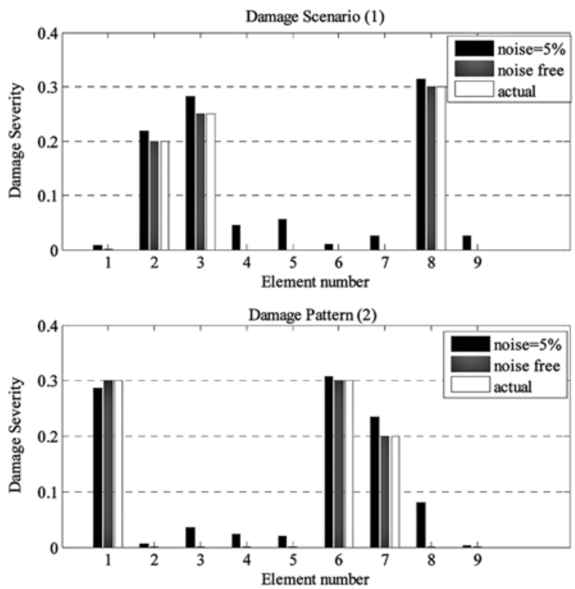

Fig. 6. The Obtained Results of the Three-story Plane Frame for the First Objective Function

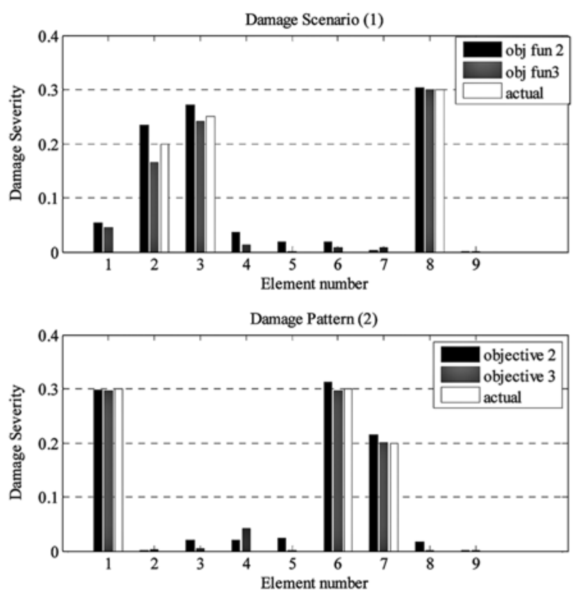

Fig. 7. The Obtained Results of the Three-story Plane Frame for the Last Two Objective Functions

it mentioned above to formulate the last two objective functions, the selected values of the weighting factors β and γ are 10^{30} and 10^{20}, respectively.

Using modal parameters includes the first three incomplete mode shapes and natural frequencies of the damaged frame, the proposed method was applied to detect and quantify the damage in the considered frame. Figs. 6 and 7 show the identified damaged elements using the first and the last two objective functions. It can be seen that the damage severity and locations can be obtained, for two different scenarios considered.

4. Experimental Validation Study

To validate the suggested approach by real data, the first two experimental mode shapes and frequencies of an 8 DOFs spring-mass system are used. The 8 DOFs spring-mass system which tested by Duffey *et al*. (2001) is formed with eight translating masses connected by springs which is shown in Figs. 8 and 9. The undamaged configuration of the system is the state for which all springs are identical and have a linear spring constant. The nominal values of the system parameters are as follows: m_1=559.3 grams (This mass is located at the end and is greater than others because the hardware requires to attach the shaker), m_2 through m_8=419.4 grams and spring constants are 56.7 kN/m. Damage is simulated by replacing an original spring with another spring which has a spring constant 14% less than that of original in spring 5 (Duffey *et al*., 2001).

Based on the measured data, two first frequencies and mode shapes which are measured only in the last 5 DOFs, were utilized for damage detection and quantification. It means that, after condensation of mass and stiffness matrices in last 5 DOFs, only the measured mode shapes in these DOFs are used. The condensed mass and stiffness matrix of the damaged system can be calculated as follows:

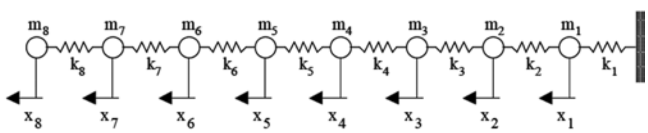

Fig. 8. Schematic of Analytical 8 DOFs System (Duffey *et al*., 2001)

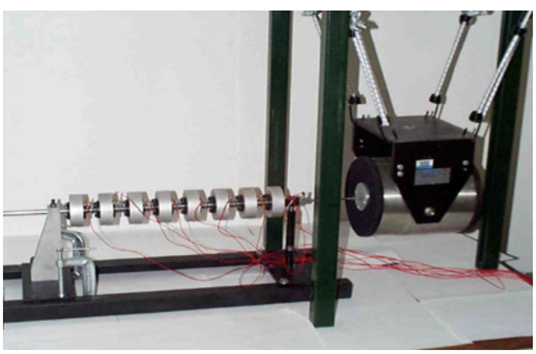

Fig. 9. Experimental 8 DOFs System with Excitation Shaker Attached (Duffey *et al*., 2001)

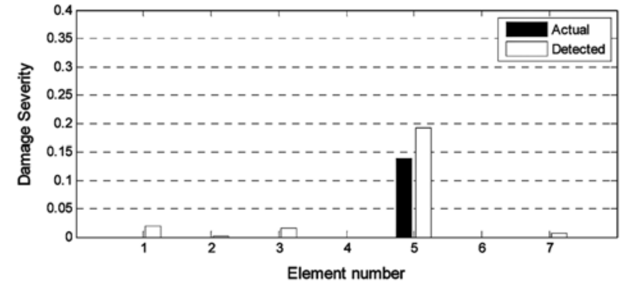

Fig. 10. The obtained Result for the Experimental 8 DOFs System

$$M_r = \begin{bmatrix} 1.8175 & 0 & 0 & 0 & 0 \\ 0 & 0.4194 & 0 & 0 & 0 \\ 0 & 0 & 0.4194 & 0 & 0 \\ 0 & 0 & 0 & 0.4194 & 0 \\ 0 & 0 & & 0 & 0.4194 \end{bmatrix} kg \tag{26}$$

$$K_r^d = \begin{bmatrix} 56.7 & -56.7 & 0 & 0 & 0 \\ -56.7 & 105.462 & -48.762 & 0 & 0 \\ 0 & -48.762 & 105.462 & -56.7 & 0 \\ 0 & 0 & -56.7 & 113.4 & -56.7 \\ 0 & 0 & 0 & -56.7 & 56.7 \end{bmatrix} \frac{kN}{m} \tag{27}$$

Figure 10 shows the capability of the proposed method for detection and quantification of the damage in the experimental 8 DOFs system, using incomplete mode shapes. The obtained results indicated that the proposed method can be characterized as a robust and viable method for damage detection and quantification of actual structures.

5. Conclusions

In this paper, new structural damage detection methods were proposed for identification of structural damage by using the simulated annealing algorithm. Several objective functions using condensed mass and stiffness matrices or the only stiffness matrix, are formulated. Localizing and quantifying the damage severity is done via an optimization problem of the objective functions, applying the simulated annealing method.

In numerical examples, the proposed method is applied to a simply supported beam and a three-story plane frame. Also, an experimental validation using a mass-stiffness system has been done. The obtained results indicated that the proposed method is a promising procedure to damage identification in spite of incomplete noisy modal data and incomplete static response.

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