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Analysis of Time Headway Distribution on Suburban Arterial

Jinhwan Jang*

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Abstract

Vehicular time headway is of fundamental importance in traffic engineering. For any traffic simulation to effectively address traffic problems, accurate vehicle generation is essential. Previous researches on this subject have focused solely on the stochastic modeling of uninterrupted facilities. Yet little research has been conducted on interrupted facilities such as suburban arterials. This paper proposes theoretical headway models for a suburban arterial. Using a traffic detector based on laser sensors, a large amount of accurate headway data were obtained in Korea. To analyze the data according to different states of traffic flow, the headways were categorized into five flow groups. Subsequent runs tests for each headway group rejected the randomness assumption for the lowest flow (5-9 veh/min). Therefore, theoretical modeling was performed only for the four remaining flows (10-14, 15-19, 20-24, and 25-29 veh/min). The Johnson SB model achieved the best fit for a flow of 10-14 veh/min, whereas the Johnson SU model provided the best fits for the three other flows. The Log-Logistic and Log-normal models were also accepted for high flows. In addition to analyses on statistics of the collected headway, the characteristics of the fitted model revealed that the headway has somewhat different features, compared to those of uninterrupted facilities. This paper provides a better understanding of headway for interrupted facilities and constitutes a starting point for developing a descriptively accurate simulation model for signalized arterials.

Keywords: *headway distribution, suburban arterial, Johnson SB model, Johnson SU model, Log-Logistic model, Log-normal model*

1. Introduction

The distribution of time headway has been studied for the last few decades. Time headway−the elapsed time between successive vehicles in a single lane of traffic−affects safety, Level of Service (LOS), and capacity analysis. From the perspective of safety, minimum time headway must be obeyed in case the leading vehicle suddenly stops. In the LOS point of view, one index is the percentage of vehicles in platoon at headways less than 5 s (May *et al.*, 1990). The time headway distribution determines the opportunity for merging and crossing at an intersection or ramp. The capacity of a freeway and the saturation flow rate of an intersection are reciprocals of the minimum time headway. Furthermore, accurate modeling of time headway distribution is fundamentally important for vehicle generation in traffic simulation models, many of which have been developed to solve different traffic problems for interrupted or uninterrupted facilities. A key component determining the performance of such models is the generation of vehicle arrival times as input to the simulations. Hence, traffic simulation researchers have devoted considerable effort to developing theoretical models that adequately describe actual headway distributions.

Many theoretical models that describe time headway distributions have been derived. Hoogendoorn and Botma (1997) and Hoogendoorn and Bovy (1998) proposed a new parameter estimation technique for time headway models based on Fourier-series analysis. They estimated parameters for vehicle-type-specific headway distributions with a Pearson-III-based Generalized Queuing Model (GQM) using time headway data from a two-lane rural highway (Hoogendoorn and Botma, 1997; Hoogendoorn and Bovy, 1998). Luttinen described the principal statistical properties of a Cowan M3 distribution model of time headway, and evaluated the accuracy of moments and least-squares estimation methods with headway data form Finnish two-lane roads (Luttinen *et al*., 1999). Al-Ghamdi, after analyzing substantial time headway data, found that the gamma distribution provided good fits for unsignalized urban arterials, whereas the negative exponential, shifted exponential, or Erlang distribution provided good fits for freeways. He also defined the boundaries of traffic flow states from parameter analyses of the distributions (Al-Ghamdi, 2001). Pueboobpaphan examined time headway distributions of probe vehicles with AVI tags on freeways in Houston (Pueboobpaphan and Park, 2003). Arasan performed an in-depth study on time headway distributions of mixed traffic unaffected by nearby intersections and dominated by smaller vehicles such as motorized two-wheelers (Arasan and Koshy, 2003). Zhang *et al.* conducted comprehensive research on models of time headway distribution using headway data observed on regular and HOV lanes of freeways in the Seattle area (Zhang *et al*., 2007). Ha *et al*. modeled time headway distributions under different contexts, such as the

*Research Specialist, Advanced Transport Research Division, Korea Institute of Construction Technology, Goyang 411-712, Korea (E-mail: jhjang@kict.re.kr)

type of lane, traffic flow, period of day, and change of cross-profile using the composite time headway distribution, GQM, and Log-normal Model on loop event data from the A6 motorway in the south of Paris (Ha *et al*., 2010).

As described above, most of the previous studies on time headway distribution have been conducted only for uninterrupted facilities. However, for traffic analyses using simulation programs to be effective for interrupted facilities, it is essential that vehicle generation features representing interrupted facilities (e.g., traffic flow interruption due to traffic signals) are accurate. Hence, this research examines the characteristics of time headway for a signalized arterial and proposes statistical distribution models from a large amount of accurate time headway data. The Baseline Data Source (BDS) for detector evaluation in Korea was exploited to collect time headway data. To analyze the headway distributions for different traffic flow states, the collected headways were divided into five flow groups. As a result of exploring numerous theoretical distribution models using Stat::Fit, a widely used statistical program, models fitted to each flow are derived. However, for the flow range of 5-9 veh/min, a statistical test to verify the randomness of headway data rejected the Independent and Identical Distribution (IID) assumption. Therefore, statistical models for this flow range could not be obtained.

2. Data Collection

The Baseline Data Source (BDS), which was devised to generate baseline data for evaluation on traffic detectors in Korea, was used to collect time headway data. It allowed the acquisition of predominantly accurate headway data at 100 Hz (1/100 s). The BDS, shown in Fig. 1, employs an adequate algorithm to detect a passing vehicle and calculate its speed and occupancy time. Consequently, a separate procedure for dealing with erroneous data or outliers in the gathered data was not recommended.

The algorithm is initiated when a vehicle crosses lasers 2, 4. If no pulse signals are detected by sensors 1, 3 for 5 min after sensors 2, 4 are switched on, or if any pulse signals are detected by sensors 1, 3 before sensors 2, 4 are activated, the algorithm considers the detected vehicle as straddling or performing a lane changing maneuver; these are preferably not considered for a headway analysis. If all of the detection signals are properly on, the algorithm matches the two pulses generated from sensors 2, 4 and sensors 1, 3 to calculate speed 1 from the rising edge and speed 2 from the falling edge, as illustrated in Fig. 2. It then compares the two speeds; if the difference exceeds a predefined threshold for the given conditions (e.g., a difference of 5 km/h at more than 30 km/h), the data are also regarded as indicating a lane changing maneuver; the data are flagged as “1” and later verified using images from the CCTV installed with the BDS. Next, the occupancy time defined as the time from when the former sensors are on to when the latter ones are off, is calculated. Lastly, calculation of vehicle length is performed with the well-known method using speed and occupancy time (Coifman and Dhoorjaty, 2004). If the length is not within plausible values, the data are also flagged as “1” for later video verification (Jang and Byun, 2011).

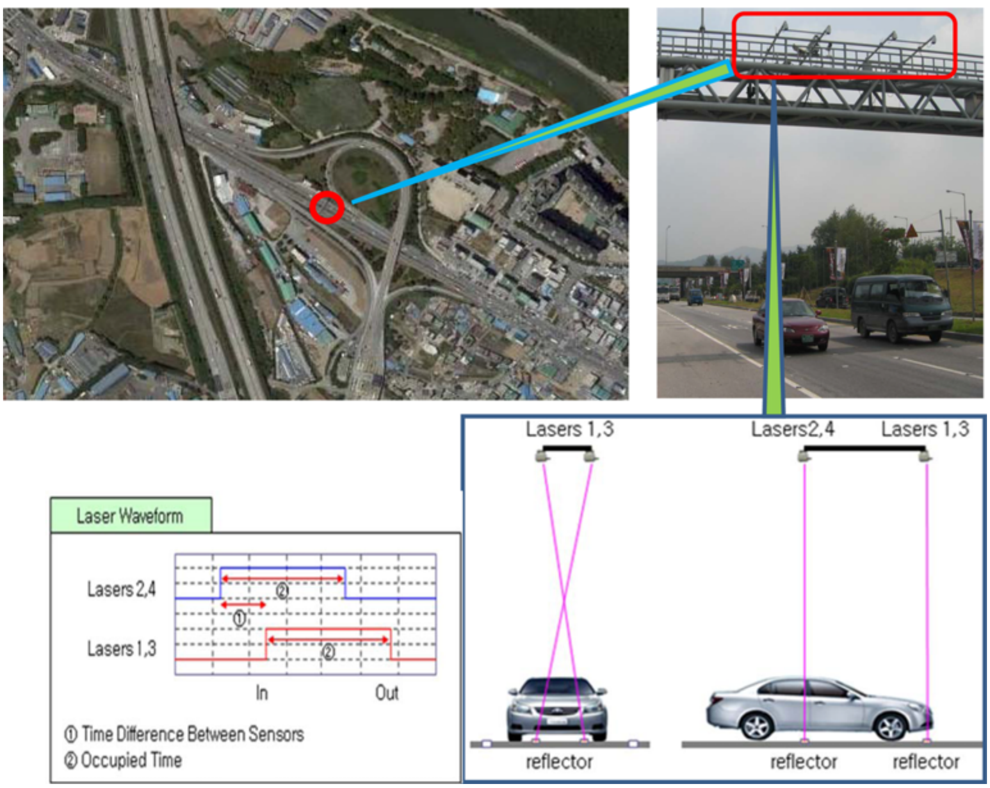

Fig. 1. Study Site and Data Collection Device

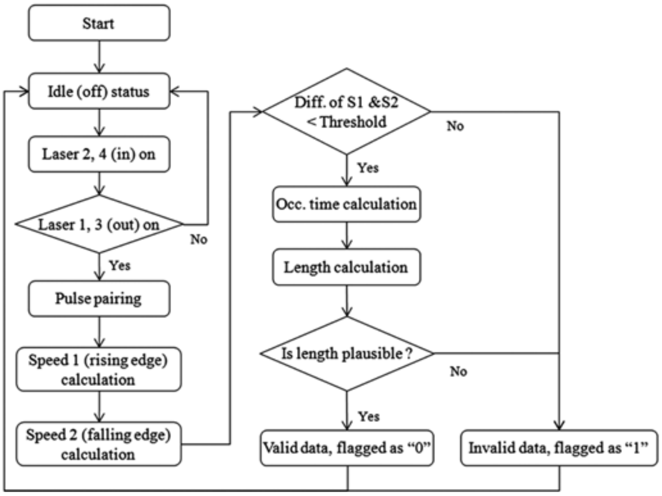

Fig. 2. Data Calculation Process of the Baseline Data Source

After 3 h of data verification against volume data from video images, no errors were detected in the BDS data. Differences were revealed only when straddling or lane-changing maneuvers occurred, conditions which should preferably be removed from time headway analysis. Assisted by the BDS, time headway data were gathered for three consecutive days including peak and non-peak hours on a bi-directional suburban arterial near Seoul, Korea in May, 2009.

The study site is located in the middle of the on- and off-ramp of the freeway, between uncoordinated signalized intersections with green ratios (g/c) of 0.7. The up- and downstream intersections are separated by approximately 1.3 km. The roadway has two lanes in each direction, a level terrain, and an 80 km/h speed limit. All of these characteristics are typical of suburban arterials in Korea (MLTM, 2001). As depicted in Fig. 1, the site is led by the off-ramp and followed by the on-ramp. Hence, the influence of uninterrupted flow from the off-ramp cannot occur as it does at downstream intersections. The main factor impacting the flow was the interruption by the signal placed upstream of the BDS in each direction.

To examine the time headway distributions for different traffic flow states, the data were collected at 5 veh/min increments from 5 veh/min (equivalent to 300 v/h) to 29 veh/min (equivalent to 1,740 v/h). As shown in Table 1, the descriptive statistics of the collected data reasonably followed the general characteristics of time headway; the mode is always less than the median, which in turn is always less than the mean. However, these differences diminish with increasing traffic flow (May *et al*., 1990).

All of the procedures to fit the data to certain distribution models assume that the observed data are Independent and Identically Distributed (IID), that is, each data point is independent of all other data points and all points are drawn from identical distributions. To verify the IID assumption for the collected data, autocorrelation and runs tests were conducted. The autocorrelation, which varies between -1 (negative) and 1 (positive), was analyzed to examine the dependence of the collected data. The analysis revealed low correlations between the data points at all flows, as shown in Table 2. On the other hand, the runs tests to verify the randomness of the data, by investigating the occurrence of uninterrupted sequences of numbers with the same attribute, rejected the randomness assumption for the 5-9 veh/min flow, as shown in Table 2 (i.e., p-value lower than the predefined significance level of 0.05). Therefore, the fitting procedures were performed only for the remaining four flows.

Table 1. Descriptive Statistics of the Collected Headways

Statistics	5-9 veh/min	10-14 veh/min	15-19 veh/min	20-24 veh/min	25-29 veh/min
Sample size	6201	8000	8000	5744	2709
Minimum (s)	0.4	0.4	0.4	0.4	0.4
Maximum (s)	45.2	36.0	23.4	17.6	10.4
Mean (s)	6.6	4.2	3.2	2.6	2.2
Median (s)	4.1	3.0	2.4	2.1	1.9
Mode (s)	1.6	1.7	1.6	1.4	1.5
Std. deviation (s)	6.4	3.7	2.4	1.8	1.2
Coeff. of Var.(CV)	0.97	0.88	0.75	0.69	0.55
Skewness	2.0	2.6	2.4	2.6	2.0
Kurtosis	4.7	9.5	8.9	10.6	5.9

3. Data Analysis

After collecting the data, they were supposed to be analyzed to determine the appropriate distribution model that reasonably describes the observations. After investigating a wide range of distribution models, some were fitted to each flow, as shown in Table 3. Unlike the well-known models for time headway from uninterrupted facilities, such as the shifted negative exponential, Erlang, Gamma, or Pearson Type III distributions, less familiar models including the Johnson SB, Johnson SU, and Log-Logistic were derived. This suggests that the characteristics of time headway for the signalized arterial are different from those of uninterrupted facilities such as freeways, two-lane rural highways, or unsignalized arterials.

The Johnson SB distribution, fitted to the 10-14 veh/min flow, is similar to the Beta distribution. Like the Log-normal and Johnson SU distributions, it is a transformation of the Normal distribution. The Johnson SU distribution, which provided the best fits for flows from 15-19 to 25-29 veh/min, is an unbounded continuous distribution. Similar to the Log-normal and Johnson

Table 2. Results of Independence Tests using Autocorrelation and Runs Tests

Independence tests		5-9 veh/min	10-14veh/min	15-19 veh/min	20-24 veh/min	25-29 veh/min
Autocorrelation	Max. positive	0.06	0.06	0.05	0.04	0.06
	Max. negative	-0.05	-0.06	-0.04	-0.04	-0.07
Runs test ($\alpha = 0.05$)	P-value	8E-11	0.14	0.17	0.31	0.10

Table 3. Equations and Parameters of the Fitted Distribution Models

Distribution	Probability density function	Description of the parameters
Johnson SB	$f(x) = \frac{\delta}{\sqrt{2\pi}y(1-y)\lambda}\exp\left[-\frac{1}{2}\left\{\gamma+\delta\ln\left(\frac{y}{1-y}\right)\right\}^2\right]$	y = (x-ξ)/λ, ξ = location parameter λ = scale parameter > 0 γ = skewness parameter δ = shape parameter > 0
Johnson SU	$f(x) = \frac{\delta}{\lambda\sqrt{2\pi}\sqrt{y^2+1}}\exp\left[-\frac{1}{2}\{\gamma+\delta\ln(y+\sqrt{y^2+1})\}^2\right]$	y = (x-ξ)/λ, ξ = location parameter λ = scale parameter > 0 γ = skewness parameter δ = shape parameter > 0
Log-Logistic	$f(x) = \left\{\alpha\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}\right\}/\beta\left\{1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right\}^2$	α = shape parameter > 0 β = scal parameter > 0 γ = location parameter
Log-normal	$f(x) = \frac{1}{(x-\min)\sqrt{2\pi\sigma^2}}\exp\left[-\frac{\{\ln(x-\min)-\mu\}^2}{2\sigma^2}\right]$	μ = mean of the included Normal σ = std. of the included Normal min = minimum value of x

SB distributions, it can be used to describe most naturally occurring unimodal data sets (Johnson, 1994). The Log-Logistic distribution, which provided the second-best fit to flows of 20-24 and 25-29 veh/min, is a continuous distribution bounded on the lower side. Analogous to the Gamma distribution, it contains three distinct regions. For $\alpha = 1$, it is similar to the Exponential distribution, starting at the minimum x (or γ) and decreasing monotonically thereafter. For $\alpha < 1$, it tends to infinity at γ and decreases monotonically with increasing x. For $\alpha > 1$, it has zero probability at γ, peaks at a value subject to both α and β, and decreases monotonically thereafter (Johnson *et al.*, 1994). The Log-normal distribution, found to be one of the most appropriate models to describe time headway from uninterrupted facilities (Mei and Bullen, 1981), was also accepted for a flow of 25-29 veh/min with the lowest p-value (see Table 4).

Parameter estimation by conventionaL Maximum Likelihood Estimation (MLE) and a goodness-of-fit test for the proposed models by the Kolmogorov-Smirnov (KS) statistic at 5% significance level were conducted as shown in Table 4. For single distribution models such as those proposed in this study, the MLE technique is generally known to be the best unbiased estimator (Zhang *et al*., 2007). Among the common methods for testing goodness-of-fit are the χ^2, KS, and Anderson Darling (AD) tests, the KS test which fits a cumulative distribution to observations point by point is regarded as the most conservative test. That is, it is least likely to falsely reject a correct fit. Many researchers use it for this reason (Mei and Bullen, 1981).

Table 4. Goodness of Fit Tests and Estimated Parameters for the Fitted Distribution Models

Flow (veh/min)	Fitted distribution at sig. = 0.05	Estimated parameters	*K-S* test (p-value)
10-14	Johnson SB	ξ=0.67, λ=104.57, γ=3.71, δ=0.98	0.10
15-19	Johnson SU	ξ=0.78, λ=0.52, γ=-2.18, δ=1.15	0.34
20-24	Johnson SU	ξ=0.66, λ=0.46, γ=-2.45, δ=1.31	0.11
	Log-Logistic	ξ=2.50, β=1.70, γ=0.41,	0.10
25-29	Johnson SU	ξ=0.61, λ=0.49, γ=-2.49, δ=1.49	0.75
	Log-Logistic	α=2.72, β=1.46, γ=0.41	0.70
	Log-normal	μ=0.53, σ=0.56, min=0.20	0.09

Histograms of the observed time headway and fitted distributions for each flow are represented in Fig. 3. With the exception of 20-24 veh/min, distributions tend to be fitted to the observed data with higher p-values as the flow increases. This presumably indicates that the time headway distribution of the signalized arterial transfers from deterministic state to stochastic (or random) state that occurs in uninterrupted facilities. Plots the residuals, depicted in Fig. 4, shows a decreasing pattern with increasing p-values for the K-S tests. Interestingly, all of the residuals have a tendency to show high residuals at a low flow, which diminish as the flow increases. This suggests that the theoretical headway distributions tend to provide poorer fits for car-following states with a shorter headway than for freely moving states with a longer headway.

It can be valuable to investigate the relationships between the variables to further understand the characteristics of time headway for the signalized arterial. The mean time headway, which is the reciprocal of the flow, is highly correlated with the standard deviation of the time headway (Fig. 5a). This is also the case for the flow and Coefficient of Variation (CV) (Fig. 5b). In each

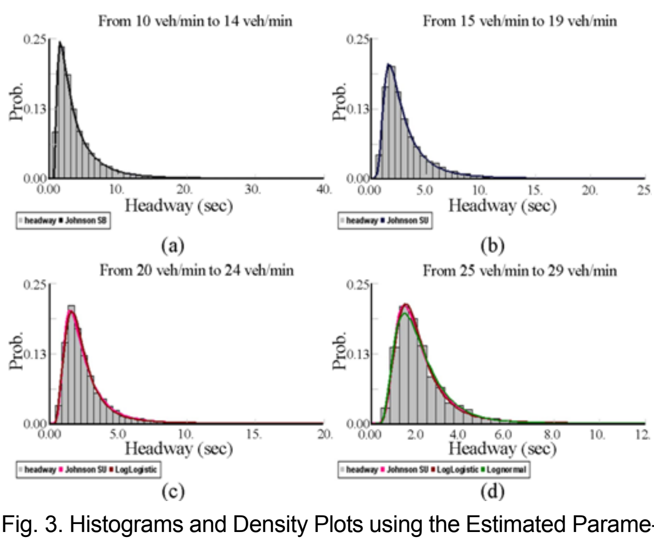

Fig. 3. Histograms and Density Plots using the Estimated Parameters: (a) 10-14 veh/min, (b) 15-19 veh/min, (c) 20-24 veh/min, (d) 25-29 veh/min

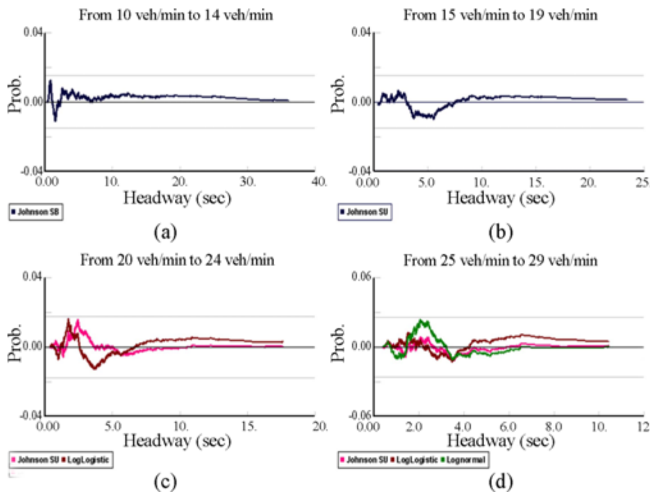

Fig. 4. Residuals of the Statistical Distributions: (a) 10-14 veh/min, (b) 15-19 veh/min, (c) 20-24 veh/min, (d) 25-29 veh/min

case, the coefficients of determination are around 0.99. The standard deviation of the time headway can be directly obtained from the observed flow. Therefore, the relationship between the mean and standard deviation of the headway could be highly regarded. Unlike previous studies, which showed a convex or concave relationship for uninterrupted facilities (Luttinen, 1992), the CV and flow was linearly correlated in this study. Notably, the slope (1.183) of the fitted line for the relationship between the mean and standard deviation was steeper than that for uninterrupted facilities (0.453-0.777) (Al-Ghamdi, 2001). This indicates that as traffic flow increases, the variance of the time headway for the signalized arterial decreases rapidly compared to that for uninterrupted facilities. This is likely to result from the interrupting influence of the signal.

As a result of this research, three notable conclusions were drawn. First, no stochastic distribution model could be accepted for very low flow owing to the rejection of the randomness assumption by runs test. Second, well-known headway models such as the shifted negative exponential, the Erlang, the Gamma, or the Pearson Type III distributions were not fitted. On the other hand, the less familiar Johnson SB and SU models were found to be appropriate. This presumably represents the fact that the headway distribution for the signalized arterial, an interrupted facility, differs from those for freeways or unsignalized arterials

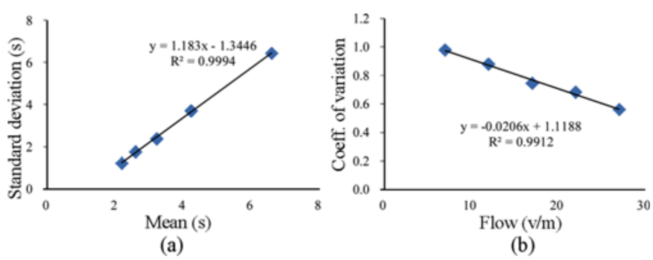

Fig. 5. Relationships of the Headway Statistics: (a) Standard Deviation versus Mean, (b) Coefficient of Variation (CV) versus Traffic Flow

(i.e., uninterrupted facilities). The Johnson SU model, which fitted to most flows, is known as one of the most flexible distribution functions in accommodating asymmetry and excess kurtosis (Choi and Min, 2008). In fact, the kurtosis of the headway in this study, which ranges from 4.7 to 10.6 (see Table 1) is much higher than those for uninterrupted facilities, which ranges from 0.94 to 7.56 (Jang *et al.*, 2011). For interrupted facilities, traffic frequently moves in clusters because of the effects from traffic signals. The resultant platoon probably causes high kurtosis values even at low flows. According to May (May, 1990), headways for uninterrupted facilities generally come to fit the Normal distribution with a kurtosis of three as the flow increases. However, the kurtosis of the headway in this study, even at low flows, is higher than three. This phenomenon explains why the Johnson SU model fitted to the headway. Lastly, the relationships between the standard deviation and mean, as well as the CV and flow, were also different from those of earlier studies performed for uninterrupted facilities. These aspects can be attributed to the aforementioned characteristics of interrupted facilities.

This paper can provide better understanding of time headway for suburban arterials. It can also be used to develop a simulation for signalized arterials that properly describes the vehicle arrival times of interrupted arterials. Unexpected outcomes might be derived by analyzing traffic phenomena on signalized arterials with a simulation program that generates vehicles using a headway distribution model addressing only uninterrupted facilities. Moreover, analyses on the statistics of time headway showed relationships that were different from those for uninterrupted facilities. With increasing traffic flow, the standard deviation of headway in this study declines rapidly compared to that in interrupted facilities. Therefore, applying equations derived from uninterrupted facilities to estimate the standard deviation of headway for the arterial examined in this study can also produce inappropriate estimates, which could lead to unwanted consequences.

4. Conclusions

This paper investigated some intriguing characteristics of time headway gathered on a signalized suburban arterial with the BDS used for detector evaluation in Korea, and proposed several stochastic distributions to fit the collected headway data for different traffic flow levels. The features of time headway for an interrupted facility that differ from those of uninterrupted facilities are as follows. The fact that the randomness test for very low flows (5-9 veh/min) was rejected indicates that the fitting procedure for this flow is meaningless. This is inconceivable for uninterrupted facilities. Furthermore, the models fitted for other flows differ from those for uninterrupted facilities. The standard deviation and mean of the time headway, as well as the CV and flow, are highly correlated with steeper slopes compared to those from uninterrupted facilities. These phenomena were presumably induced by the influence of traffic signals that frequently interrupt moving vehicles and create platoons even in low-traffic conditions. Therefore, the sharp decrease in the standard deviation is reasonable.

As a result of exploring many statistical distributions, the Johnson SB distribution for a flow of 10-14 veh/min, and the Johnson SU distribution for flows of 15-19, 20-24, and 25-29 veh/min were considered as the most appropriate models. The Log-Logistic was also found to potentially explain the headway distribution for flows of 20-24 and 25-29 veh/min. The Log-normal distribution, despite having the lowest p-value, was also accepted for a flow of 25-29 veh/min. Parameters for each model were estimated from MLE, and goodness-of-fit was assessed by the KS test. Most of the derived models other than the Log-normal distribution appear to be unfamiliar even to those substantially knowledgeable on time headway distributions. This suggests that when analyzing the performance characteristics of the signalized arterial using simulation software that generates vehicles with distribution models only appropriate for uninterrupted facilities, undesirable conclusions may be drawn. Therefore, this paper emphasizes the importance of research on headway distributions of interrupted facilities. It can also contribute to pioneering research toward the development of a descriptively accurate simulation for signalized suburban arterials. It is also noted that the spatial transferability of the proposed distribution models should be verified with time headway data from other suburban arterials.

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