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Effect of Stress Level on the Bearing Capacity Factor, N_γ, by the ZEL Method

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Abstract

It has been known that soil shear strength parameters are stress level dependent. On the other hand, foundation size has a significant effect on the level of imposed stress on subsoil elements. In this study, the Zero Extension Lines (ZEL) method, which has wide applications in determination of bearing capacity and load-displacement behavior of foundations and retaining walls, is employed to consider the stress level dependent nature of soil shear strength parameters to predict the actual bearing capacity of foundations. The ZEL equations which are capable of considering variations in soil shear strength parameters have been employed to consider their dependency on stress level and solved numerically by a computer code developed for this study. The presented approach has been compared with experimental data showing reasonable predictions when the effect of stress level is taken into account. It is then utilized to develop some design charts showing modified values of N_γ, as a function of foundation size and soil properties based on Bolton (1986) equation for stress level effect in cases of smooth and rough base foundations. The charts represent the decreasing tendency in N_γ with an increase in foundation size and it shows the decreasing tendency in the reduction rate when the foundation size increases.

Keywords: *ZEL, bearing capacity, foundation, stress level, plasticity, numerical solution*

1. Introduction

Determination of the bearing capacity and prediction of load-displacement behavior of foundations have been a major concern in foundation design. Several classical methods have been developed to predict the bearing capacity of foundations in the last century. Prandtl (1921), Hill (1926), Fellenius (1926), Terzaghi (1943), Taylor (1948), Meyerhof (1963), and Vesić (1973) can be mentioned here for their works on determination of the bearing capacity of soil. The famous triple-*N* formula, originally from Terzaghi (1943) can be considered as the most valuable advantage of their works presented as follow:

$$q_{ult} = cN_c + qN_q + 0.5\gamma BN_\gamma \tag{1}$$

where, q_{ult} is the ultimate bearing capacity, c is the cohesion, q is the surcharge pressure, B is the foundation width, γ is the soil density and the N coefficients, which are functions of soil friction angle, are the bearing capacity factors.

A number of full-scale load tests had been performed to verify the bearing capacity formula or to predict the bearing capacity of foundations in different soil types (Ismael, 1985; Fellenius, 1994; Briaud and Gibbens, 1999). Investigations show that for foundations on relatively dense soils, the measured value of the bearing capacity is often higher than the values obtained from the theoretical equation based on a critical state friction angle; meanwhile, using a peak friction angle results in higher bearing capacities than measured values (Clark, 1998; Eslami, *et al.*, 2004). Also, the third bearing capacity factor, N_γ, increases illogically with friction angle which leads to very high bearing capacity values for large foundations. It seems that such shortcomings are results of neglecting the stress level dependent behavior of the soil mass beneath the foundation. Eslami *et al.* (2004) presented a review of delusions in the bearing capacity of shallow foundations and inconsistency between test results and theoretical approaches and concluded that the bearing capacity obtained from theoretical methods are often over-conservative for small footings but irrationally high for large foundations.

Effects of a number of important parameters have been investigated to adjust experimental data with theoretical analysis results. Among them, depth, scale and base roughness of foundations have been found to be important factors on the intensity of a possible limited load on foundations. Ismael (1985) performed several tests on square footings of 0.5 m and 1.0 m in width with various depth to width ratios ranging from 0.5

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through 4.0. The results indicated that the influence of footing depth is very small. However, based on these results, Fellenus and Altaee (1994) showed that the width of foundation plays a very important role in the bearing capacity. It can be related to soil behavior under different states of imposed stress. Soil shear strength parameters have been believed to be dependent on the level of experienced stress, also known as stress level (Holtz and Kovacs, 1981; Bolton, 1986; Clark, 1998; Maeda and Miura, 1999; Kumar *et al*., 2007; Budhu, 2007). This dependency may be considered as a reason of differences between experimental and theoretical approaches in determination of the bearing capacity of foundations. Since the first two factors in the bearing capacity formula, i.e., N_c and N_q have been well determined by closed form solutions of a plasticity problem in soil mechanics, the major problem in the bearing capacity is often the third factor, i.e., N_γ. The bearing capacity of shallow foundations over cohesionless soils is highly dependent on the third bearing capacity factor, N_γ. Theoretically, the third factor, varies significantly with an increase in soil friction angle (Bowles, 1996; Michalowski, 1997; Eslami *et al*., 2004). There are some evidences, from using small footings up to about 1m, that the BN_γ term does not increase the bearing capacity without a bound.

Hansen and Odgaard (1960) performed circular footing tests on sand at various relative densities. Cerato (2005) has recompiled their data, showing that the bearing capacity factor, N_γ, decreases by an increase in foundation size, but the scale effect becomes less marked with a decrease in the relative density of the soils.

Experimental Data from De Beer (1965) revealed a decrease in N_γ with an increase in foundation size. Many centrifugal tests showed similar results (Kimura *et al.*, 1985; Bolton and Lau, 1989; Clark, 1998; Zhu *et al.*, 2001). There are also many recent small and large scale footing load tests indicating that the bearing capacity factor, N_γ, is a function of foundation size (Cerato, 2005; Cerato and Lutenegger, 2007; Kumar and Khatri, 2008; Yamamoto *et al.*, 2009). According to these results, N_γ, has a decreasing tendency with foundation size, for small size foundations and approach a constant value for reasonably large foundations.

For very large values of B, both Vesi (1969) and De Beer (1965) suggest that the limiting value of q_{ult} approaches that of a deep foundation. Thus, a reduction factor as a function of foundation width is often applied to limit the value of N_γ for large foundations (De Beer, 1965; Bowles, 1996; Clark, 1998; Eslami *et al*., 2004; Cerato, 2005).

Since the normal stress is variable from point to point along a possible sliding surface underneath a footing, it is necessary to consider the intrinsic curve in order to calculate the ultimate bearing capacity. The choice of friction angle beneath a footing may be important in explaining scale effects. Meyerhof (1950) assumed the mean value of the normal stress, $\sigma_{g,M}$, along the slip surface to be equal to about 1/10 of the ultimate bearing capacity and calculated the ultimate bearing capacity with a formula in which he introduced the angle ϕ, corresponding to the secant connecting the origin to the point $\sigma_{g,M}$ as illustrated in Fig. 1.

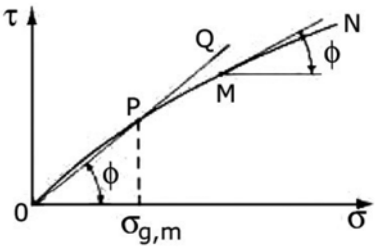

Fig. 1. Meyerhof's Method of Dealing with the Mohr-Coulomb Strength Envelope in Calculating the Bearing Capacity of a Shallow Foundation

There are also a number of researches indicating that base roughness has an increasing effect on N_γ of foundations (Bolton and Lau, 1993; Bowles, 1996; Michalowski, 1997; Kumar and Kouzer, 2007). According to Bolton and Lau (1993), in rough base foundations, a rigid entrapped wedge (or cone in axi-symmetric problems) is stabilized beneath the foundation. This rigid wedge or cone is assumed to be inclined at an angle equal to $\pi/4+\phi/2$ and soil friction angle is assumed to be fully mobilized on its inclined interfaces. It forms a larger plastic zone and thus, larger bearing capacity factor, N_γ, for rough base foundations in comparison with smooth base foundations. This effect can roughly increase the values of N_γ, by a factor of 3 for plane strain problems and 5 for axi-symmetric problems.

Since there are a few theoretical works with practical applications known in the literature, this study focuses on the effect of stress level on the bearing capacity of foundations through a numerical study which is expected to have some practical advantages. The results are also compared with existing experimental data. This study investigates the effect of stress level on the bearing capacity of shallow foundations over dense frictional soils with relatively high internal friction angles. The main objective of this work is to study the effect of stress level on the third bearing capacity factor, N_γ, for different size shallow foundations with low surcharge pressure on cohesionless soils. Since the method of Zero Extension Lines, with many applications in some plasticity problems in soil mechanics, is capable of considering the variations of soil shear strength parameters, the idea of employing these equations for considering the variations in these parameters due to change in stress level, has been utilized in this study. Based on the ZEL method, a computer code has been developed for this research to investigate the effect of stress level on N_γ, and to provide modified values of N_γ for practical uses.

2. Influence of Stress Level on the Shear Strength and Bearing Capacity of Soil

2.1 Stress Level Effect on Soil Shear Strength

Variations of maximum achieved friction angle in the standard shear tests with normal or confining pressure had been reported by different researchers. It is widely recognized that the peak friction angle of soils decreases with stress level (Meyerhof, 1950; De Beer, 1965; Holtz and Kovacs, 1981; Bolton, 1986) and the Mohr failure envelope is curved rather than being a straight line. Few reports indicate that the critical state friction angle might also depend on stress level (Clark, 1998); but, this requires further investigations as there are more evidences that it is indeed a constant parameter and therefore it is taken so in this study.

Lee and Seed (1967) presented data from dense Ottawa Sand and dense Sacramento River Sand which shows a decrease in friction angle with an increase in confining pressure. Two experimental direct shear tests on Monterey 0/30 Sand and Danish Normal Sand were conducted by Gan *et al.* (1988) showing a decreasing tendency in friction angle with an increase in applied normal stress.

There are also relationships between normal or confining pressure and soil angle of dilation from laboratory tests. Bolton (1986) proposed to correlate maximum friction angle to soil relative density, D_r, and applied effective stress, σ, as the following simplified forms:

$$\phi_{max} = \phi_{c,s.} + 0.8\,\nu_{max} \tag{2a}$$

$$\phi_{max} = \phi_{c,s.} + 5I_R \quad \text{(in plane-strain condition)} \tag{2b}$$

$$\phi_{max} = \phi_{c,s.} + 3I_R \quad \text{(in triaxial condition)} \tag{2c}$$

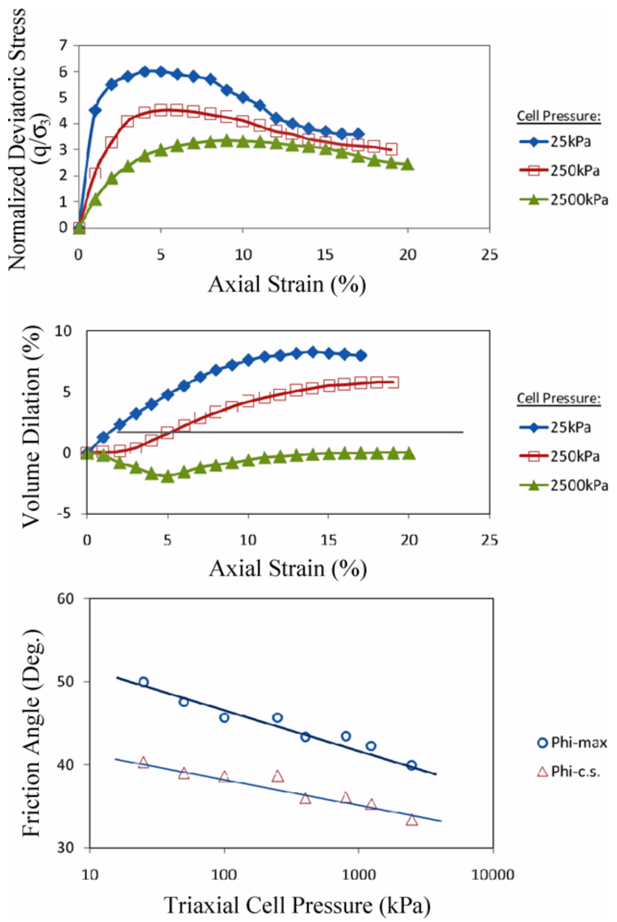

Fig. 2. Test Results of Silica Sand in Different Stress Levels and Variation of Peak and Critical State Friction Angle as a Function of Stress Level (Data from Clark, 1998)

$$I_R = D_r(Q - \ln(\sigma)) - R \tag{2d}$$

where, ϕ_{max} is the maximum mobilized friction angle, $\phi_{c.s.}$ is the critical state friction angle, ν_{max} is the maximum dilation angle, I_R is the dilatancy index, D_r is the soil relative density (in decimals), σ is the effective stress (in kPa), and Q and R are constants. Bolton (1986) recommended to use Q=10 and R=1. Kumar *et al.* (2007) performed a number of shear tests on Bangalore sand and utilized these equations to express stress level dependency of tested specimens.

Clark (1998) performed a series of triaxial tests on a dense silica sand with a density index of 88%, a mean grain size (d_{50}) equal to 0.2 mm, the coefficient of uniformity, C_u=1.69 and a mass density of 15 kN/m^3 at different confining pressures. Triaxial test results on this sand, performed by Clark (1998), are reproduced and illustrated in Fig. 2. Their stress-strain curves are shown in Fig. 2(a) and 2(b). In Fig. 2(c), variations of measured soil friction angle with confining pressure are presented on a semi logarithmic diagram, showing a linear decreasing tendency.

Based on these results, the following simple equation was suggested to correlate the maximum friction angle to the level of confining pressure (Clark, 1998):

$$\phi = A(\sigma)^M \tag{3}$$

where, ϕ is the maximum mobilized friction angle as a function of σ, σ is the confining pressure which is equal to σ_3, in triaxial test, A is a factor (that can be considered as the peak friction angle at unit confining pressure, i.e., at σ=1.0 kPa) and M is an exponent which are determined experimentally. Using this equation, there is no need to determine the relative density in the laboratory and the coefficients can be determined simply by standard shear tests on soil samples. The coefficients were determined by Clark (1998) for the tested dense silica sand soil in his work.

2.2 Stress Level Effect on Soil Bearing Capacity

As it was stated before, several researchers studied the effect of foundation size on the bearing capacity. This effect is mainly related to stress level effect. In general, in larger foundations, higher stress levels in the soil causes lower soil friction angles to be mobilized, resulting in a decrease in the bearing capacity factor, N_γ. Most of the research works have been focused on small scale footing load tests because of limitations in testing large foundations.

Clark (1998) carried out centrifuge tests on the silica sand. The centrifuge test facilitates modeling large foundations in the laboratory. The model footing 43.7 mm in diameter was tested at accelerations of 1, 10, 40 and, 100 and 160 g. The corresponding diameters of the prototypes would be 0.044 m, 0.437 m, 1.75 m, 4.37 m and 6.99 m. Data from Clark (1998) based on a stress level concept show the effect of foundation size on the bearing capacity. The main reason on this dependency, regarding the stress level concept, is that the larger the foundation size the higher the stresses induced in the soil mass and therefore, the lower the maximum mobilized friction angle at failure. Test results of Clark (1998) have shown the following representation of N_γ as a function of foundation diameter:

$$N_\gamma = 325D^{-0.32} \tag{4}$$

where, D is the diameter of the foundation. The equation shows a decreasing tendency in the value of N_γ, by increasing the foundation size.

Clark (1998) also stated that in engineering practice it is very important to carefully select the soil shear strength parameter, ϕ. The error in calculation of the bearing capacity caused by improper selection of this parameter may be very large. The peak friction angle should be used for determining the ultimate bearing capacity, but N_γ should take into account the effect of footing size (a measure of stress level) on ϕ_{max} (Clark, 1998).

Cerato (2005) and Cerato and Lutenegger (2006), presented the results of a wide study on the effect of foundation size on the bearing capacity. They tested model and prototype foundations ranging from 25.4 mm to 914.4 mm on two compacted sands at three different relative densities (Cerato, 2005; Cerato, and Lutenegger, 2006; Cerato and Lutenegger, 2007). They concluded that the bearing capacity factor, N_γ, is absolutely dependent on the foundation size; in small foundations with low stresses it is very high due to higher mobilized maximum friction angle (Cerato, and Lutenegger, 2006). They also stated that the Mohr-Coulomb failure envelope is a curve rather than a straight line indicating, soil friction angle is influenced by stress level. Fig. 3 shows the results of calculation of the bearing capacity factor, N_γ, for experimental tests on different sizes of foundation, recompiled for this study. Tests and numerical studies were performed on Brown Mortar Sand with friction angles ranging between 48° and 40° in different densities in a 59.9 mm direct shear test apparatus. Considering the results of these tests, the effect of foundation size on the bearing capacity factor, N_γ, is evident for small footings, while for larger foundations; this effect is less considerable (Cerato, 2005).

Kumar and Khatri (2008) studied the effect of foundation size on the bearing capacity factor, N_γ, on the basis of a numerical study by incorporation of soil friction angle as a function of

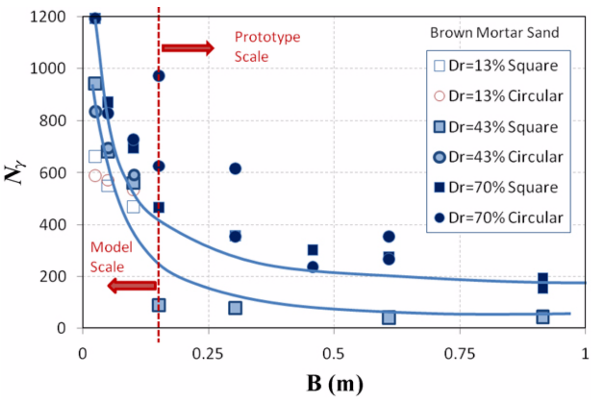

Fig. 3. Variation of Bearing Capacity Factor, N_γ, with the Size of Foundation Resting on Brown Mortar Sand with Peak Friction Angle of 48° (Data from Cerato, 2005)

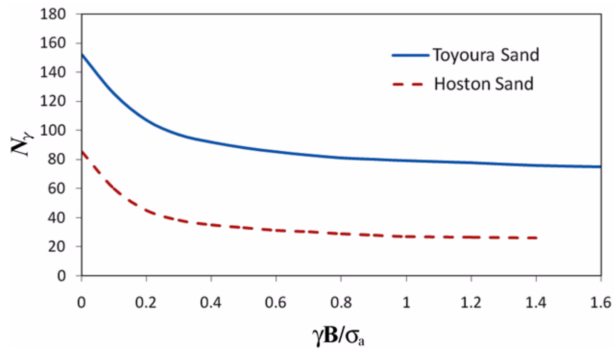

Fig. 4. Variation of the Bearing Capacity Factor, N_γ, with Normalized Foundation width for two Different Sands (Data from Kumar and Khatri, 2008)

mean principal stress. The effect is investigated for smooth base strip foundations on granular medium without any surcharge pressure. A numerical lower bound limit analysis by the means of a finite element programming has been used in their research. Fig. 4, illustrates the reproduced graphs, showing the variation of the bearing capacity factor, N_γ, with foundation width, normalized to the atmospheric pressure and soil density for two different sands, i.e., Toyora sand and Hoston Sand for which the friction angle varied with confining pressure.

Recently, Yamamoto *et al.* (2009) have investigated the effect of foundation size on the bearing capacity factor, N_γ, for natural sands at a wide range of different densities. Their study was established on a MIT-S1 Sand model with a finite element programming. They stated that the variation of peak friction angle raises questions on the applicability of conventional bearing capacity theories, which are based on a constant friction angle with depth. It is obvious that soil friction angle varies with depth as a function of stress level. The assessment of the bearing capacity factor, N_γ, is reported to decrease with foundation size.

As a conclusion, the dependency of soil friction angle to stress level suggests a decrease in the bearing capacity factor, N_γ, with an increase in the size of the foundation. Although there are several experimental works that support this observed fact, a few numerical studies are known to the authors that show this effect.

3. Plasticity Problem in Soil Mechanics and the ZEL Method

Different methods have been used for investigating the bearing capacity of foundations in soil mechanics (Davis and Selvadurai, 2002). These include the limit analysis method using the bound theorems of theory of plasticity, the stress characteristics method and the Zero Extension Line method (ZEL). The ZEL method has been used in this study.

The idea of using the Zero Extension Lines for obtaining the strains and deformations in the soil mass; and thereby predicting the load-deflection behavior of structures in contact with soil was introduced by Roscoe (1970). James and Bransby (1971) used them for prediction of strains and deformations behind a model retaining wall. Habibagahi and Ghahramani (1979) and Ghahramani and Clemence (1980) calculated the soil pressures by considering the force-equilibrium of soil elements between the zero extension lines. The methodology for finding the load-deflection behavior of foundation and retaining walls on the basis of the ZEL theory was presented by Jahanandish (1988) and Jahanandish *et. al.* (1989). The general form of these lines were considered in this methodology and the variations of soil strength parameters c and ϕ, with the induced shear strain due to the deflection of the structure were also taken into account. In 1997, Anvar and Ghahramani used the matrix method for derivation of differential equilibrium-yield equations along the stress characteristics in plane strain condition, and transferred them along the Zero Extension Lines. This approach was important since it allowed integrating the equations along the zero extension directions. In 2003, Jahanandish considered the more general case of axial symmetry, and obtained the equilibrium-yield equations along the Zero Extension Lines by direct transformation, independent from the stress characteristics. Details of these derivations can be found in Anvar and Ghahramani (1997) and Jahanandish (2003). Only the final form of these equations will be presented here.

The zero extension directions are defined by:

$$\frac{dz}{dx} = \tan(\psi \pm \xi) \tag{5}$$

where $\xi=\pi/4-\nu/2$, the angle between the ZEL directions and the major principal stress direction, the minus sign (−) stands for the one direction and the plus sign (+) for the other which are shown in Fig. 5(a), on a Mohr's circle of strain. Fig. 5(b) shows the ZEL directions and stress characteristics for an arbitrary soil element.

Based on Anvar and Ghahramani's (1997) derivation, the equilibrium-yield equations along the ZELs for the plane strain problem are as follows:

Along the – Zero Extension Lines:

$$\begin{aligned} &dS-2(S\tan\phi+c)\left(\bar{\alpha}d\psi+\bar{\zeta}\frac{\partial\psi}{\partial\varepsilon^{+}}d\varepsilon^{-}\right) \\ &\quad = X\bar{\beta}(\tan\phi dz+\bar{\alpha}dx)-Z\bar{\beta}(\tan\phi dz-\bar{\alpha}dz)+(S-c\tan\phi) \\ &\left(\tan\phi d\phi-\frac{1}{\cos\phi}\frac{\partial\phi}{\partial\varepsilon^{+}}d\varepsilon^{-}\right)+\left(\tan\phi dc-\frac{1}{\cos\phi}\frac{\partial c}{\partial\varepsilon^{+}}d\varepsilon^{-}\right) \end{aligned} \tag{6a}$$

Along the + Zero Extension Lines:

$$\begin{aligned} &dS+2(S\tan\phi+c)\left(\bar{\alpha}d\psi+\bar{\zeta}\frac{\partial\psi}{\partial\varepsilon^{-}}d\varepsilon^{+}\right) \\ &= -X\bar{\beta}(\tan\phi dz-\bar{\alpha}dx)+Z\bar{\beta}(\tan\phi dz+\bar{\alpha}dz)+(S-c\tan\phi) \\ &\left(\tan\phi d\phi-\frac{1}{\cos\phi}\frac{\partial\phi}{\partial\varepsilon^{-}}d\varepsilon^{+}\right)+\left(\tan\phi dc-\frac{1}{\cos\phi}\frac{\partial c}{\partial\varepsilon^{-}}d\varepsilon^{+}\right) \end{aligned} \tag{6b}$$

where, S, is the mean stress, i.e., $(\sigma_1+\sigma_3)/2$, X and Z are the body and/or inertial forces along x, z directions and $d\varepsilon^{+}$ and $d\varepsilon^{-}$ are the length of the differential elements along the ZEL directions (measures of distance along the Zero Extension Lines), and $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\zeta}$ are used to provide a simple form of the equations. The values of, $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\zeta}$ are given by:

$$\bar{\alpha}=\frac{1-\sin\phi\sin\nu}{\cos\phi\cos\nu};\quad \bar{\beta}=\frac{\cos\nu}{\cos\phi};\quad \bar{\zeta}=\frac{\sin\phi-\sin\nu}{\cos\phi\cos\nu} \tag{7}$$

Based on Jahanandish (2003), the final form of equilibrium-yield equations along the ZEL directions for the more general case of an axi-symmetric condition is:

Along the – zero extension lines:

$$\begin{aligned} &dS+\frac{\partial T}{\partial\varepsilon^{+}}d\varepsilon^{-}+\frac{2T}{\cos\nu}\left(d\psi-\sin\nu\frac{d\psi}{d\varepsilon^{+}}d\varepsilon^{-}\right) \\ &= [f_x\cos(\psi-\xi)+f_z\sin(\psi-\xi)]d\varepsilon^{-} \end{aligned} \tag{8a}$$

Along the + zero extension lines:

$$\begin{aligned} &dS+\frac{\partial T}{\partial\varepsilon^{+}}d\varepsilon^{+}-\frac{2T}{\cos\nu}\left(d\psi-\sin\nu\frac{d\psi}{d\varepsilon^{-}}d\varepsilon^{+}\right) \\ &= [f_x\cos(\psi+\xi)+f_z\sin(\psi+\xi)]d\varepsilon^{+} \end{aligned} \tag{8b}$$

where, T is the radius of Mohr circle for stress defined as $T=S\sin\phi+c\cos\phi$, and f_x and f_z are expressed by:

$$f_x = X-\frac{nT}{x}(1+\cos 2\psi);\quad f_z = Z-\frac{nT}{x}\sin 2\psi \tag{9}$$

where, n is an integer defining the problem type: it is equal to 1 for the axi-symmetric problems and 0 for the plane strain cases.

As aforementioned, these equations are more general so that those for plane strain can be simply deduced from them by setting n=0 and $T=S\sin\phi+c\cos\phi$. Note that x is the measure of the radial distance for the axi-symmetric problem. It is also

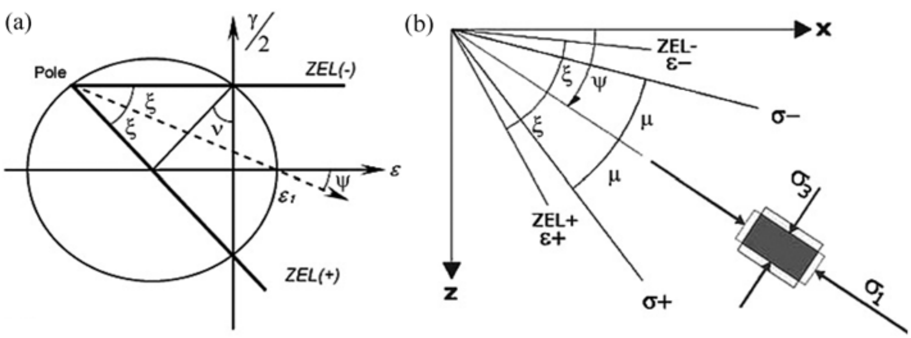

Fig. 5. Zero Extension Lines: a) Directions of Zero Axial Strains on the Mohr Circle of Strains, b) Directions of Stress Characteristics (σ^{-}, σ^{+}) and Strain Characteristics (ε^{-}, ε^{+})

remarkable that for an associative flow rule assumption, it is sufficient to substitute the dilation angle, n, with friction angle, ϕ, in all equations containing these terms which is a particular case of the general ZEL method.

It should be also mentioned that the variation in soil strength parameters c and ϕ has also been considered in these equations. Variation in c and ϕ can be due to nonhomogeneity of the soil mass. It can be also due to the difference in shear stain at different points. This later relation has already been used in obtaining the load–deflection behavior of structures in contact with soil (Jahanandish, 1988; Jahanandish *et al.*, 1989; Anvar and Ghahramani, 1997; Jahanandish, 2003; Jahanandish and Eslami Haghighat, 2003; Jahanandish and Eslami Haghighat, 2004).

In relatively homogeneous soils, the strength parameters c and ϕ at different points may be different due to the difference in the stress level. As mentioned before, the influence of this phenomenon on the bearing capacity cannot be ignored. In this study, the capability of the ZEL method in considering the variation of strength parameters has been used for this purpose and the equations have been solved numerically. For each point, calculations are made for finding the location and stress from the information at the preceding points, as is usual in the method of characteristics. The obtained stress is then used to adjust the friction angle of that point using Bolton's relation given in Eq. (2). The calculations are then repeated until there is no significant change in the location, stress level and hence the friction angle of that point. Note that the friction angle of the point in succeeding iterations is different from that of the preceding points due to difference in the stress level, and this has been observed in the above equations. The calculations are then made for the next point in the same way. This procedure has been implemented in a developed computer code that is further described in the next section.

4. Outline of the Study and Numerical Investigations

The effect of stress level on the bearing capacity of foundations is studied in this work. A computer code in MATLAB 7 is developed to take this effect into account by the ZEL method. As stated before, a plastic mass at failure is assumed to be formed beneath the foundation when the limit load is applied. An associated flow rule is adopted for such condition when seeking for this limit load, so that the results can be comparable with those obtained by the other researchers. The procedure by which the equations are solved numerically is described first. Next, the code is verified with published results and existing values for the bearing capacity factors and later extended to the stress level-dependent case.

4.1 Procedure of Numerical Solution

The developed code in MATLAB comprises of some calculation blocks, subroutines and functions to solve the equations. A summary of the procedure implemented in the code is presented here:

4.1.1 Block 1: Input Data

In the first block, the input parameters are defined containing soil geotechnical properties necessary for the equations and geometry of the problem. These parameters include soil density, shear strength parameters, relative density, foundation size (width) and failure mechanism (for smooth or rough base foundations) and surcharge pressure. There are also some controlling parameters, i.e. number of iterations and convergence criterion (precision of the result) and mesh size (number of divisions).

4.1.2 Block 2: Calculations

In this block, the calculation procedure starts from a boundary on which, all necessary data are prescribed. For the certain case of the bearing capacity problem, this boundary is the ground surface over which a surcharge pressure may or may not exists. It is however necessary to define a very small value of surcharge pressure even in the absence of any actual surcharge load. It is necessary to avoid a trivial solution for the partial differential equations of the ZEL method. The ZEL equations are first rewritten in a finite difference form as presented in Appendix A. Then, a triple-point procedure is used to calculate the unknown variables at the third point, as shown in Fig. 6. Thus, once the stresses are known at the boundaries, stresses within the field can be computed along the ZEL directions. For solving the equations by the finite difference forms of the equations, if the stress states and coordinates of Points A and B are given, one can seek for the unknowns in Point C according to Fig. 6.

It is worth mentioning that in the ZEL equations some coupling terms, i.e. partial derivation of functions c, ϕ, S and ψ in negative direction appear in positive direction and vice versa. The only approximation to be made is to use along the positive direction, AC, the values of negative coupling term along BC instead of AF and along the negative direction, BC, the values of positive coupling term along AC instead of BE. This approximation is acceptable because the coupling terms are ψ dependent and do not change rapidly. Thus, the following approximations are used as suggested in the literature (Anvar and Ghahramani, 1997; Jahanandish, 2003):

$$\left[\frac{\partial h}{\partial \varepsilon^-}\right]_{AF} \approx \left[\frac{\partial h}{\partial \varepsilon^-}\right]_{BC} \quad \text{and} \quad \left[\frac{\partial h}{\partial \varepsilon^+}\right]_{BE} \approx \left[\frac{\partial h}{\partial \varepsilon^+}\right]_{AC} \tag{10}$$

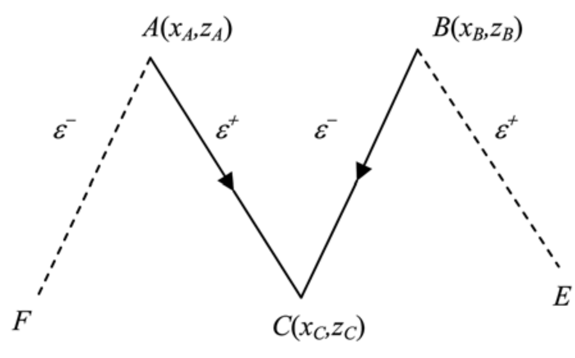

Fig. 6. Directions of the ZEL and Integration Paths for Numerical Solution Procedure

where, h stands for any of the coupling terms in the ZEL equations, e.g. ψ, ϕ, c or T.

There is a subroutine that performs the numerical solutions for every point within the domain. Once the initial stresses are computed for an arbitrary point in the domain, an iteration procedure is performed to compute the corresponding mobilized friction angle in the case of stress level-dependent soil friction angle, as a function of stress level, i.e., confining pressure, σ_3. In this procedure, the stresses are first computed and then corresponding mobilized maximum friction angles are obtained. Since the stresses depend on values of $\phi_{\max}$, the procedure is repeated for convergence. There is also another function within which ϕ_{max} is computed. In this function, dependency of soil friction angle to stress level is defined based on Eq. (2). Later, the computation for the next point in the domain is performed to compute all unknown values in the field. Once the values of soil friction angle have been computed, based on an associative flow rule, the value of dilation angle at each point is set equal to mobilized friction angle. As stated before, computations consist of a calculation procedure between three points, i.e., points A and B (at which the required data exist) and point C (in which the four unknowns should be found). Calculations start from points A and B to find, first the coordinates of point C (Eq. (A1), Appendix A) and then to find S and ψ (Eqs. (A2) and (A3), Appendix A). An iterative procedure is also performed for convergence. A flowchart is also presented in Fig. 7 to show the numerical procedure utilized in the developed code.

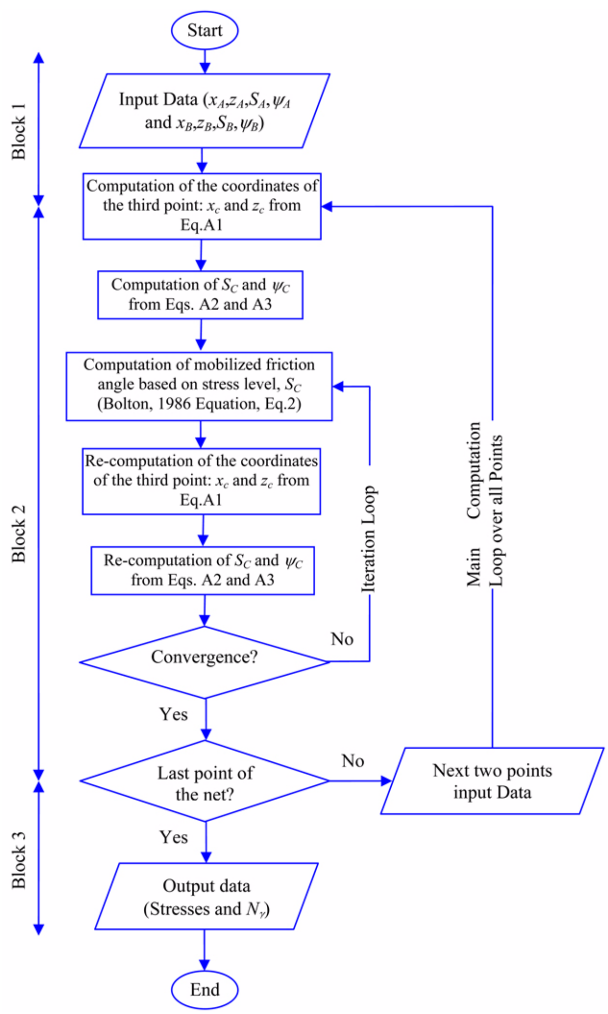

Fig. 7 Flowchart of the Developed Code and Procedure of Numerical Solution

4.1.3 Block 3: Output

In this block, output data is presented. Output contains all unknowns within the field, i.e., S, ψ, x and z of any point in the domain, mobilized maximum friction angle of corresponding points, ultimate bearing pressure and also some optional graphical outputs.

4.2 Boundary Condition

For smooth base foundations, a frictionless interface boundary between the foundation and the soil was assumed. For rough base foundations, there are some assumptions to consider the mobilization of interface friction angle between the base and the soil. As stated before and based on Bolton and Lau (1993) following Meyerhof (1951), stabilization of a rigid wedge (or cone) beneath the rough base foundations has been assumed to be the boundary condition adopted in this research.

4.3 Model Verification without Stress Level Consideration

There are several published works in the literature on determination of the bearing capacity factors based on numerical studies. It has been realized that the bearing capacity factors, N_c and N_q can be obtained in a closed-form solution for a plane strain plasticity problem with associative flow rule assumption as follows (Bowles, 1996; Budhu, 2007)

$$N_q = e^{\pi \tan\phi}\tan^2\left(\frac{\pi}{4}+\frac{\phi}{2}\right) \quad (11a)$$

$$N_c = (N_q - 1)/\tan\phi \quad (11b)$$

The code was first validated for N_c and N_q values. A standard surcharge pressure boundary condition was assumed in the right side of the foundation. Table 1 shows a comparison between the results of the closed-form solution and results obtained from analysis based on the ZEL method with associative flow rule assumption. It is obvious that the results show insignificant

Table 1. Comparison Between Closed-form Solution and ZEL Results for N_c and N_q

ϕ (Deg.)	N_c (Closed-Form)	N_c (ZEL)	N_q (Closed-Form)	N_q (ZEL)
0	5.1	5.1	1.0	1.0
10	8.3	8.3	2.5	2.5
20	14.8	14.8	6.4	6.4
30	30.1	29.9	18.4	18.3
40	75.3	75.0	64.1	63.8

difference. It is remarkable that the closed-from solution can be obtained by the ZEL method when the number of nodes approaches infinity, i.e., in a very fine ZEL net.

There have been several attempts to calculate the bearing capacity factor, N_γ. The N_γ value has the widest suggested range of values of any of the bearing-capacity N factors. A literature review reveals that for ϕ=40º, 38 < N_γ< 192 (Bowles, 1996).

An important difference between different methods is to consider the superposition of two different contributors in the bearing capacity; i.e. weight and surcharge pressure. Terzaghi (1943) assumed that the components of the bearing capacity equation can be safely superposed. Davis and Booker (1971) performed rigorous checks on the superposition assumption for the plane strain case and found that it was indeed conservative. Based on a study on the effect of the surcharge pressure on value of N_γ, Bolton and Lau (1993) concluded that if the ratio of surcharge pressure, q, to γB (resulting in a dimensionless factor, Ω) is 1.0, the effect of surcharge pressure leads to less than 20% error in calculation of the bearing capacity factor, N_γ. This error seems to be acceptable for practical problems in soil mechanics.

The code was verified for the values of the bearing capacity factor, N_γ, in comparison with different methods. The methods consist of different equations suggested by well-known authors i.e., Terzaghi (1943), Meyerhof (1963), Hansen (1970), Vesić (1973), Bolton and Lau (1993) and the results obtained from finite element method, regarding its wide application, by the aid of PLAXIS 8 code. In the last method, an associative flow rule was adapted to PLAXIS 8 model to be consistent with the results of the developed code. In PLAXIS 8, a Mohr-Coulomb soil model was considered for the analyses with yield parameters equal to those assumed in the ZEL analyzed cases with an associative flow rule assumption (i.e., ν=ϕ). Values of the ultimate bearing pressure for this study have been computed in a plastic analysis for 10% settlement of the foundation width, which is suggested by Briaud and Jeanjean (1994) at which plastic deformations could be considered as the main contributors. It is also adopted by Cerato (2005) and Cerato and Lutenegger (2007) because of its simplicity consistency for different size foundations and, may be actually close to the average soil strain at failure when it undergoes significant plastic strains. The soil modulus of elasticity was chosen equal to 10 MPa and Poisson's ratio equal to 0.3 which is common for most of frictional soils in practice. It has to be mentioned that these elasticity coefficients are not so important for ultimate bearing pressure comparison, which is obtained from a plastic analysis. A *Plastic Analysis Type* was chosen in the analyzed cases by PLAXIS 8 and the following *Analysis Phases* were performed in every case:

- Phase 0: *Initial Stress Computation*, in which the initial stresses due to soil density are computed within the field.
- Phase 1: *Plastic Analysis*, in which a gradually increasing load is applied to the foundation to find the ultimate load corresponding to the ultimate bearing capacity.

Soil model parameters, i.e. Mohr-Coulomb failure criterion, and foundation size were set similar to the cases analyzed with the ZEL method. To provide a reasonably rigid foundation in PLAXIS 8, a plate element of 1.0 m thickness, bending stiffness EI=1.67E7 kN/m and axial stiffness, EA=5E7 kNm^2/m was used in the analyses. Therefore, the effect of foundation flexibility was

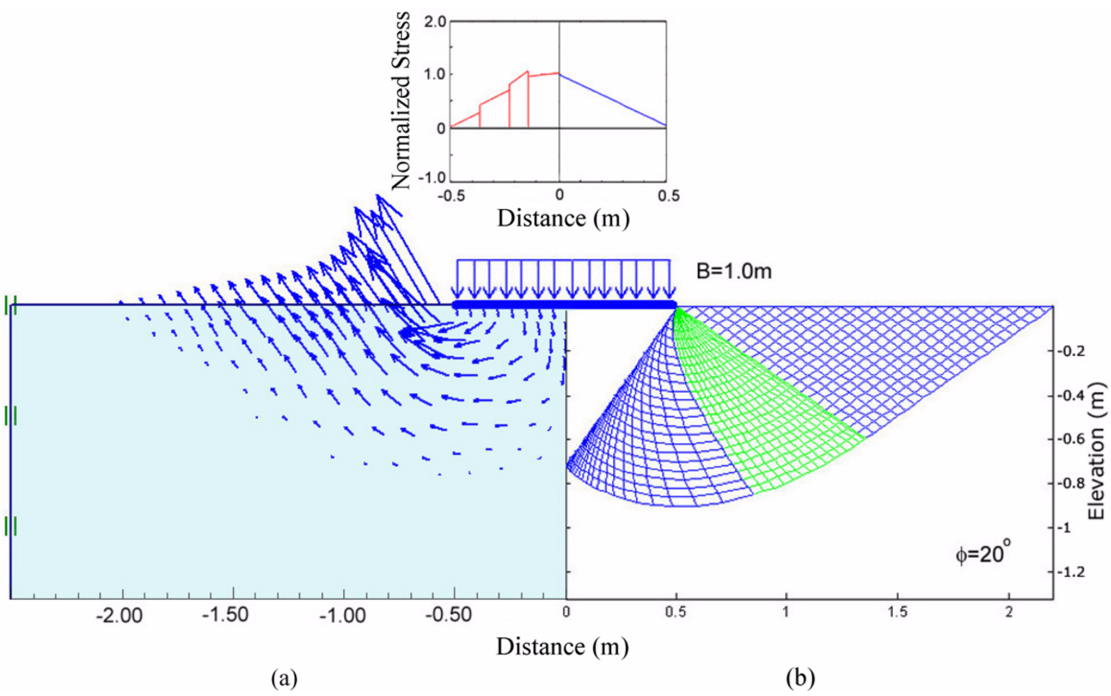

Fig. 8. Comparison of the ZEL and Finite Element Methods for a Plane Strain Problem Rough Mechanism (ϕ=20º): (a) PLAXIS Total Increments Vectors at Failure along with Normalized Stress Distribution, (b) ZEL Net Coinciding on Yielded Planes along with Normalized Stress Distribution

insignificant. It is consistent with the assumption of a rigid foundation over a plastic soil at failure in other researches in which the value of N_γ has been computed (Terzaghi, 1943; Meyerhof, 1963; Hansen, 1970; Vesić, 1973; Bolton and Lau, 1993), and hence, they can be compared.

Fig. 8 shows the comparison between the results of PLAXIS 8 and present study for the case of a 1.0 m width strip foundation over a frictional soil with rough base (ϕ=20°). Fig. 8(a) shows the analyzed case in PLAXIS: the total increment vectors of displacements at failure and normalized stress distribution beneath the foundation. In Fig. 8(b), the ZEL net (which coincides with yielded planes) for the similar case and normalized stress distribution below the foundation is depicted. Variations of stress beneath the foundation seem to be similar in both methods; i.e., the maximum value is achieved at center while it reduces to zero at the edge of the foundation. The results obtained from the ZEL method in this case are in reasonable agreement with results which are obtained from another numerical code, i.e. PLAXIS, when the stress level effect is not considered. But, further study shows some differences between the results obtained from the developed code based on the ZEL method and PLAXIS when soil friction angle exceeds 40°.

A summary of comparison between all methods is illustrated in Fig. 9. It is clear that the results of this study located in the range of different values suggested by different researchers and thus, seem to be acceptable. Values computed by Bolton and Lau (1993) for rough base foundations have the best consistency with

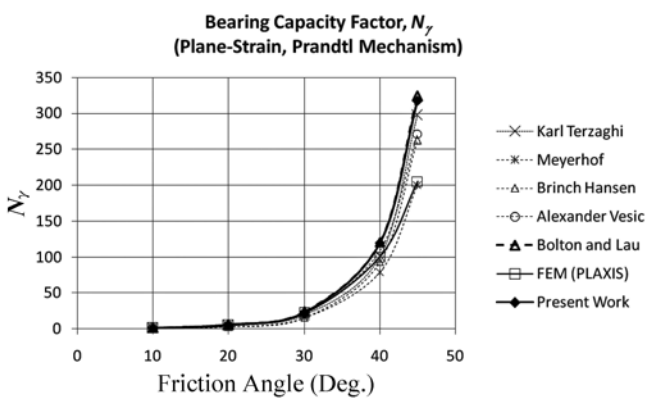

Fig. 9 The Bearing Capacity Factor, N_γ from Different Methods for Rough Base Strip Foundations

the ZEL method among the others. The possible reasons can be related to similarities in the methods and similarities in the boundary condition for a rough-base foundation. Similar values to that of Terzaghi (1943), Hansen (1970) or Vesić (1973) for ϕ=45° are resulted by decreasing the friction angle roughly by 1.0 degree. Also, by decreasing the friction angle roughly by 2.0 degrees similar results to those obtained from PLAXIS or suggested value of Meyerhof (1963) can be obtained, indicating that the computed values are in range. One of the reasons of differences in suggested values of N_γ, in spite of dissimilarities between the methods and assumptions, can be related to the fact that computed values of N_γ at high friction angles are large and therefore, the differences are significant.

4.4 Model Convergence

A thorough study was performed to validate the model convergence. The convergence was checked for an increase in both number of divisions (number of nodes to discretize the boundaries of the problem) and number of iterations (number of try-and-error rounds to achieve minimum error in the numerical calculations at each node) for convergence. Fig. 10 shows the model convergence for the number of divisions and the number of trial rounds for a certain problem of a rough-base foundation resting over a soil (ϕ=40°, γ=20 kN/m^3). It is obvious that the maximum error associated with the solution in determination of the foundation pressure is 20% when the number of divisions is equal to 16.

4.5. Model Verification Considering Stress Level Effect

In the previous parts, the code was verified with conventional methods, including closed form solutions of plasticity equations and other existing results. However, there is a major difference between the developed code and a number of the traditional finite element-based codes. The developed code, based on a stress level dependent ZEL method can take the effects of stress level dependent soil friction angle and its variations into account, which seems to be an important ability to predict actual bearing capacity of different size foundations over a certain soil type. As a matter of fact, when the effect of stress level on soil friction angle and its variation is not considered, a unique prediction of the ultimate bearing capacity factor, N_γ, for foundations of different sizes will be obtained. This is not always an accurate prediction as stated before since the value of N_γ has been found

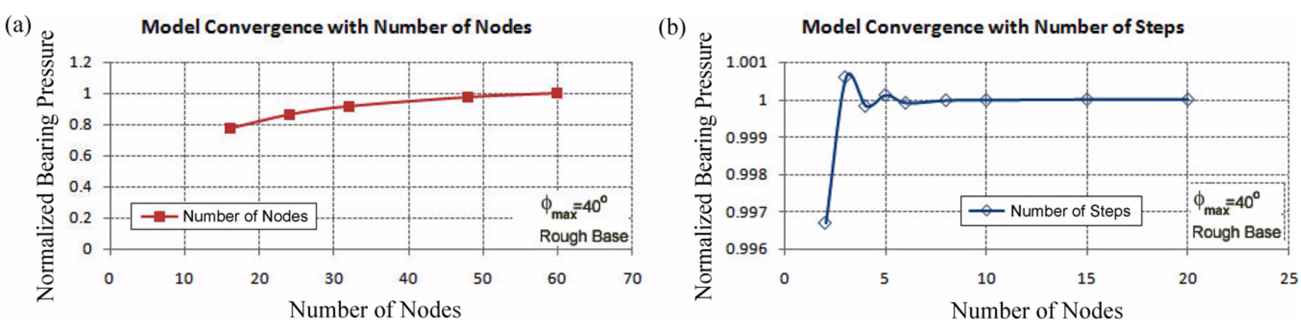

Fig. 10. Model Convergence by Increasing: (a) Number of Nodes (Divisions) and (b) Number of Trial Rounds at 24 Divisions

Table 2. Comparison of the Results for the Bearing Capacity Factor N_γ

Diameter (m)	N_γ		Error (%)
	Clark (1998) Experiments	This Study (Considering Stress Level)	
0.5	406	440	8.4
1.0	325	335	3.1
5.0	194	185	4.6
10.0	156	160	2.6

to be a function of foundation size. Therefore, application of the ZEL method in the developed code can be considered as a useful tool to take the stress level effect into account and hence, it is expected to avoid small and over-conservative values of the bearing capacity factor, N_γ, for small foundations or very high values for larger ones where may be unsafe.

In this part, the numerical model and computer code is validated for experimental results of Clark (1998) which were previously described. The results of N_γ calculated from Eq. (4), suggested by Clark (1998) for foundation load tests, are presented in Table 2.

As it was stated before, the variations of soil maximum friction angle with stress level are employed in this study to predict the bearing capacity factor, N_γ. The foundation was assumed to be over a cohesionless soil with a deep (rough base) mechanism. The stress level based ZEL method through the developed code with sufficient iteration rounds to obtain precise results provided the bearing capacity factor, N_γ for different size foundations. The results are presented in Table 2 along with the results obtained from Clark (1998) experiments. The predictions made by the ZEL method with stress level considerations, seem to be in good agreement with experimental results of Clark (1998). Thus, it can be concluded that the ZEL method and developed code work properly in a stress level dependent condition.

Since the distribution of stress level in different locations below a foundation is very complex, the use of conventional methods, i.e. bearing capacity formula, cannot provide a precise estimation of the bearing capacity of foundations. However, a stress level based ZEL method can reasonably provide a good estimation of the bearing capacity, considering the corresponding shear strength of soil in different points below the foundation as a function of stress level. It is also clear that the use of critical state friction angle in computation of N_γ is over-conservative while using peak friction angle is unsafe.

5. Computation of the Bearing Capacity Factor, N_γ, Based on Stress Level and Discussion of the Results

5.1 Computation of the Stress Level Dependent N_γ

Considering the validity of the code, the values of the bearing capacity factor, N_γ, are computed for different size foundations at different conditions. These results include both smooth base and rough base failure mechanisms. The procedure is described in the following.

5.1.1 Soil Shear Strength Parameters as a Function of Stress Level

The stress level dependency of soil shear strength can be considered as a nonlinear Mohr-Coulomb failure envelope as it was stated before. This dependency can be expressed as a general relationship between soil internal friction angle and confining pressure in the laboratory. Without violating the generality of the problem, a simple assumption for this variation is utilized similar to Eq. (2) recommended by Bolton (1986) and Kumar *et al.* (2007). Employing this equation requires the values of critical state friction angle and relative density of soil to be determined from laboratory tests results.

5.1.2 Model Parameters and Assumptions

Propagation of generated stresses in the soil mass due to the boundary conditions of the problem and soil weight, provide different stress levels in the medium. Mobilized friction angle of the soil mass is supposed to obey stress level dependency, i.e., maximum friction angle, ϕ_{max}, of soil elements are functions of critical state friction angle, $\phi_{c.s.}$, relative density, D_r, and stress level. The variation of the bearing capacity factor, N_γ, is computed for different values of $\phi_{c.s.}$, D_r and foundation size. Since the value of soil density is required for actual stress level at different locations in the soil mass, it is supposed to be equal to 20 kN/m^3 which seems to be valid for most of soils. The values are obtained for both rough base (after Prandtl, 1921) and smooth base (after Hill, 1926) failure mechanisms. The lower most value of ϕ_{max} is limited to $\phi_{c.s.}$ in calculations. Relative densities were considered as 50% and 75% which seem to be reasonable in practice.

5.2 Results of the Numerical Modeling and Discussion

The bearing capacity factor, N_γ, is computed for different cases of soil friction angle with different relative densities and for different foundation sizes. The ZEL net for the case of a *rough base* strip foundation on a soil mass with $\phi_{c.s.}$=30^o, B=1.0 m and D_r=50% is illustrated in Fig. 11(a). The distribution of the maximum mobilized soil friction angle, ϕ_{max}, in the soil mass is shown in Fig. 11(b) which is resulted from different stress levels at different points. As it was expected, the mobilized maximum friction angles are achieved in zones of the lowest stresses, i.e. in passive Rankine zone at the right side. Fig. 11(c) represents the normalized stress distribution beneath the foundation. Fig. 12 shows similar results for B=10 m. In this case, the decrease of the friction angle due to an increase in stress level distribution in the soil mass is evident in comparison to the previous case of smaller foundation.

In Fig. 13, similar results for a *smooth base* strip foundation with B=1.0 m over the same soil are presented. Fig. 14, depicts similar results for larger foundation (B=10 m) on the similar soil with similar properties. Again, larger foundation generally reduces

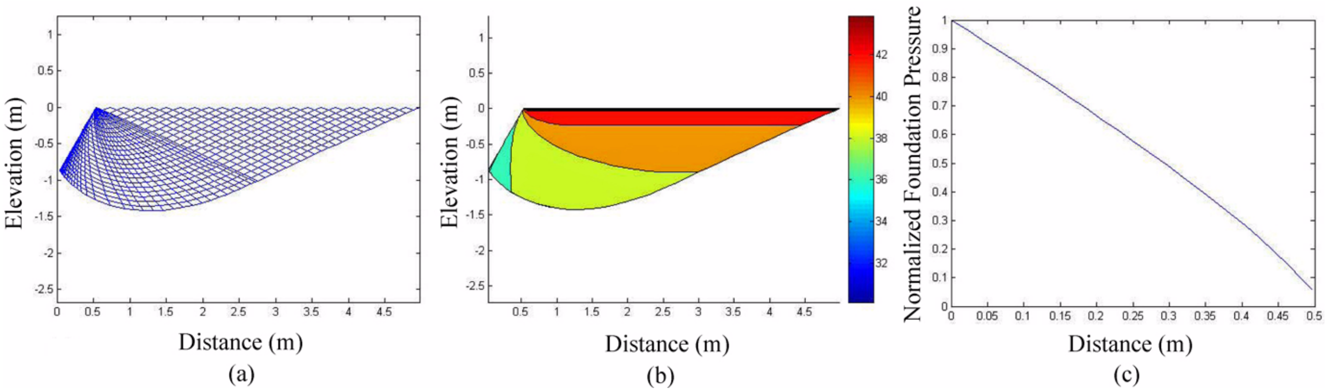

Fig. 11. Strip Foundation with Rough Base ($\phi_{c.s.}$=30°, B=1.0 m and D_r=50%): (a) ZEL Net, (b) Variation of Soil Friction Angle, (c) Normalized stress Distribution Beneath the Foundation

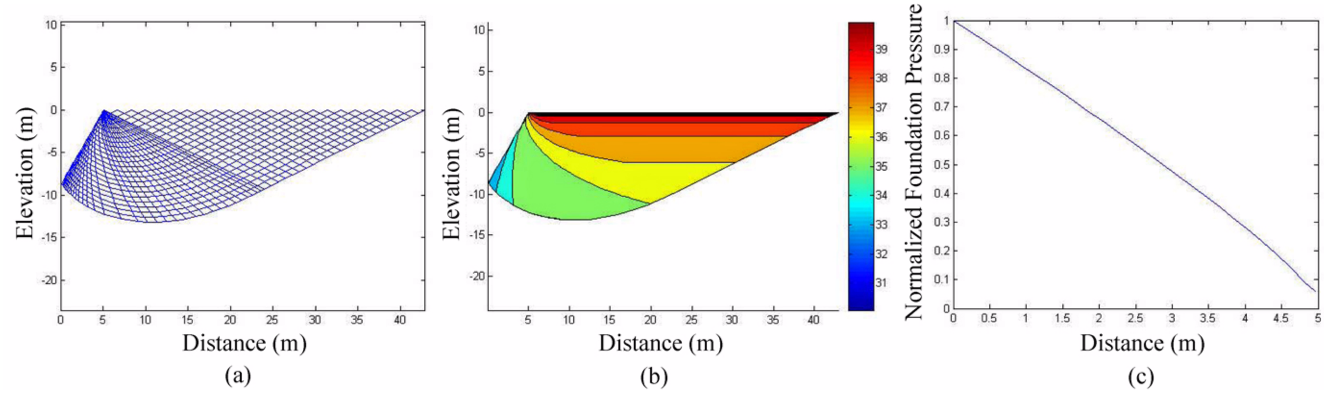

Fig. 12. Strip Foundation with Rough Base ($\phi_{c.s.}$=30°, B=10.0 m and D_r=50%): (a) ZEL net, (b) Variation of Soil Friction Angle, (c) Normalized stress Distribution Beneath the Foundation

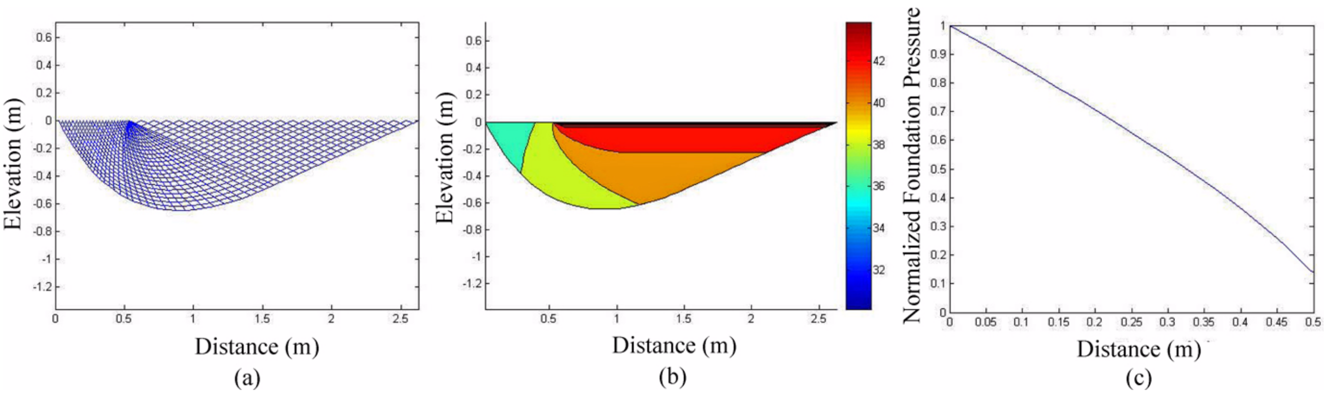

Fig. 13. Strip Foundation with Smooth Base ($\phi_{c.s.}$=30°, B=1.0 m and D_r=50%): (a) ZEL Net, (b) Variation of soil Friction Angle, (c) Normalized stress Distribution Beneath the Foundation

the maximum mobilized friction angle, $\phi_{\max}$, in a major portion of the soil mass. It is worth mentioning to note that for the soil with given properties, according to Eq. (2), this soil has a ϕ_{peak}= 45° at unit confining pressure.

In general, according to the numerical study, it can be concluded that the larger the foundation the higher the imposed stress levels at any arbitrary point in the soil mass below the foundation. It results in lower values of mobilized friction angle and as a consequence, lower values of N_γ. The results obtained from numerical analyses, coincide with experimental observations of many researchers as stated earlier.

Figs. 15 through 18 show the variation of the bearing capacity factor, N_γ, versus foundation size, with critical state soil friction angle and relative density for both rough base and smooth base failure mechanisms. These charts have been provided by a similar scheme described before and in each graph, critical state friction angle, $\phi_{c.s.}$, and soil relative density, D_r, were assumed to be constant. It is also assumed that the soil obeys Bolton's (1986)

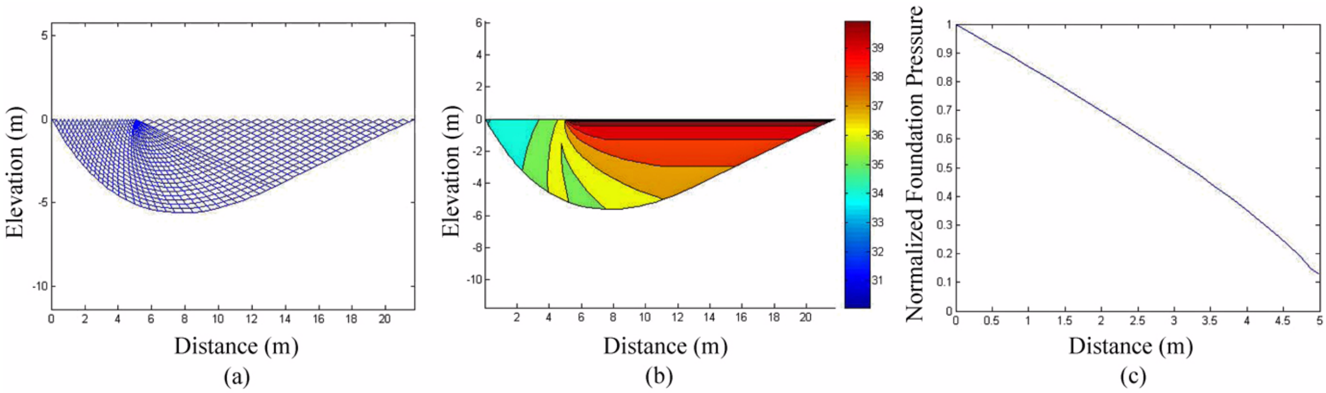

Fig. 14. Strip Foundation with Smooth Base ($\phi_{c.s.}$=30°, B=10.0 m and D_r=50%): (a) ZEL Net, (b) Variation of Soil Friction Angle, c) Normalized stress Distribution Beneath the Foundation

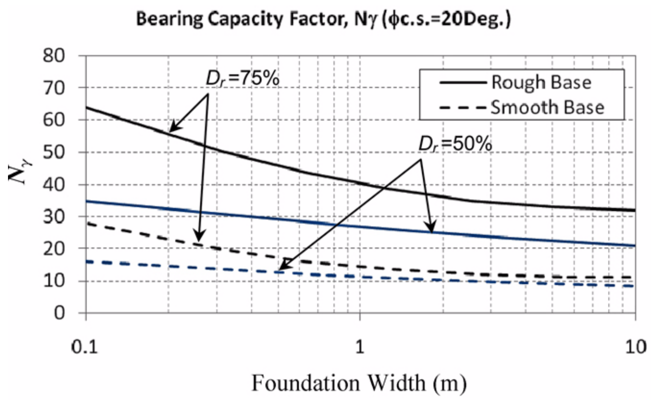

Fig. 15. Variation of the Bearing Capacity Factor, N_γ, with Foundation Size ($\phi_{c.s.}$=20°)

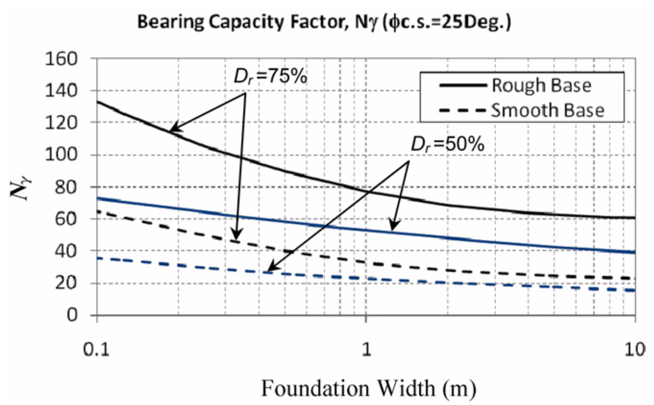

Fig. 16. Variation of the Bearing Capacity Factor, N_γ, with Foundation Size ($\phi_{c.s.}$=25°)

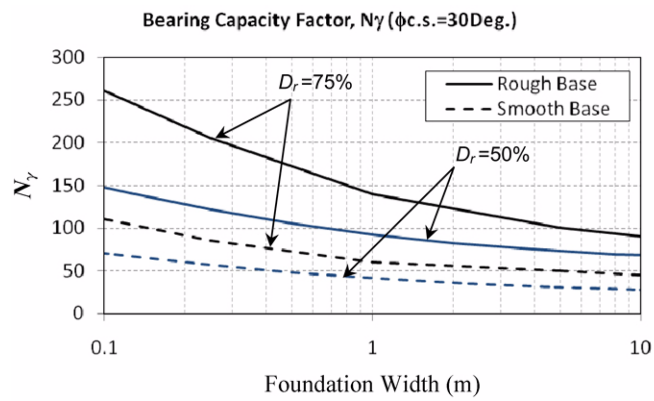

Fig. 17. Variation of the Bearing Capacity Factor, N_γ, with Foundation Size ($\phi_{c.s.}$=30°)

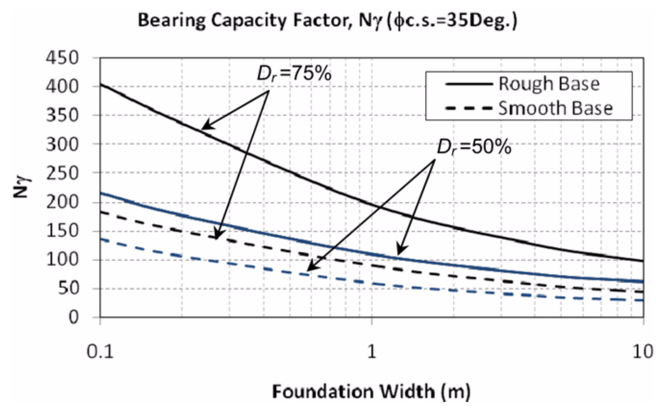

Fig. 18. Variation of the Bearing Capacity Factor, N_γ, with Foundation Size ($\phi_{c.s.}$=35°)

equation which is true for many frictional soils. Changes in N_γ are resulted from variations in mobilized maximum friction angle, ϕ_{max}, due to an increase or decrease in foundation size and in fact, different pattern of stress level distribution beneath the foundation. The decreasing rate in N_γ can be observed but it decreases by increasing the foundation size and for reasonably large foundations, N_γ approaches a constant value corresponding to the critical state soil friction angle. It is also evident that the bearing capacity factor, N_γ, is different for a certain case, i.e., similar foundation size and soil properties, but with different failure mechanisms, foundations with rough base show higher values of N_g. Therefore, the effect of foundation roughness can be considered as another important factor in the bearing capacity of foundations as well as foundation size.

6. Conclusions

There have been several attempts to relate the shear strength parameters of soils to the level of applied stress. Unlimited increase of the bearing capacity factor, N_γ, has been a major concern resulted in very high and illogical values of the ultimate bearing capacity of foundations. Considering the dependency of soil shear strength parameter, i.e. maximum internal friction angle, ϕ_{max}, to the stress level provides a basis for more realistic estimation of the bearing capacity of foundations through a stress level-based numerical model. Stress level beneath the foundations, is mainly affected by the size of the foundation, and the larger the foundation the higher the produced stresses in the soil mass resulted from the foundation load.

The ZEL method with considering the dependency of soil maximum friction angle, $\phi_{\max}$, to the confining pressure was used to predict the bearing capacity factor, N_γ, as a function of foundation size. Thus, the simultaneous effect of stress level and foundation size on the bearing capacity of foundations has been studied.

The stress level-based ZEL method, was first employed in the developed computer code and then verified for a number of existing numerical and experimental results. The verification also included the comparison between the code and similar computer software, i.e. PLAXIS, which is common in geotechnical engineering analyses for cases without considering the effect of stress level. These comparisons showed that the results are located in the common range presented by the researchers. For high values of soil friction angles, e.g., ϕ>40º the results have the best consistency with those suggested by Bolton and Lau (1993) due to similarities in the assumptions and boundary conditions. Further comparison with existing experimental results of Clark (1998), on small to large foundations, showed that the prediction of the bearing capacity considering the stress level effect by the ZEL method with stress level considerations provides reasonable results. It is also noticeable that assuming a constant soil friction angle, as arbitrarily assumed in many conventional methods, provides inaccurate results. The computer code was later employed for prediction of the bearing capacity factor, N_γ, for small to large foundations. The model was applied to both rough base and smooth base foundations.

The results show that the use of stress level based ZEL method can reasonably provide practical values of the bearing capacity factor, N_γ, as a function of foundation size. A number of design charts have been presented to provide practical values of the bearing capacity factor, N_γ, assuming Bolton (1986) equation to take the stress level effect. These charts require the critical state friction angle and soil relative density to be determined from laboratory tests. Values of the bearing capacity factor, N_γ, can then be obtained as a function of foundation size for soils obeying Bolton (1986) equation. As it was expected from experimental observations of many researchers, it is also evident from these charts, obtained by theoretical approach, that stress level and foundation size, as well as base roughness have considerable effect on the bearing capacity of shallow foundations. The value of N_γ, decreases with an increase in foundation size but the reduction rate decreases when foundation size increases. By using these charts, it is possible to find the corresponding value of N_γ, based on the size of the foundation and soil geotechnical parameters.

Acknowledgements

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Notations

A	: Constant parameter
B, D	: Foundation width and diameter
c	: Soil cohesion strength intercept
D_r	: Soil relative density
f_x, f_z	: Defined in Eq. (9)
h	: An arbitrary function
I_r	: Soil dilatancy index
M	: Exponent
N_i	: Bearing capacity factors consists of N_c, N_q and N_γ
n	: An integer equal to 0 for plane strain problems and 1 for axi-symmetric problems
Q, R	: Constants in finding I_r
q	: Surcharge pressure
q_{ult}	: Ultimate bearing capacity
S	: Mean stress, $(\sigma_1+\sigma_3)/2$
T	: Radius of Mohr's circle of stresses equal to $S\sin\phi + c\cos\phi$
X, Z	: Horizontal and vertical body and/or inertial force
x, z	: Measures of distance for horizontal and vertical directions
$\bar{\alpha}, \bar{\beta}, \bar{\zeta}$	: Parameters using in compacted forms of the ZEL equations, defined in Eq. (7)
ε^-, ε^+	: Measures of distance for minus and plus strain characteristics (ZEL) directions
ϕ	: Soil friction angle (in general)
$\phi_{c.s.}$	: Critical state friction angle
$\phi_{\max}$	: Maximum mobilized friction angle
γ	: Soil density
μ	: Angle between the stress characteristics lines and major principal stress direction
$\nu_{\max}$	: Maximum soil angle of dilation
ν	: Soil angle of dilation
σ	: Stress (in general)
σ^-, σ^+	: Measures of distance for minus and plus stress characteristics directions
σ_1, σ_3	: Major and minor principal stresses
σ_x, σ_z	: Normal stress acting in x and z directions
σ_θ	: Circumferential stress in an axi-symmetric problem
τ_{xz}	: Shear stress

ψ : Inclination of the major principal strain and stress with respect to horizontal direction

ξ : Angle between the Zero Extension Lines and major principal strain direction

References

Anvar, S. A. and Ghahramani, A. (1997). "Equilibrium equations on zero extension lines and their application to soil engineering." *Iranian Journal of Science and Technology (IJST)*, Shiraz University Press, Transaction B, Vol. 21, No. 1, pp. 11-34.

Bolton, M. D. (1986). "The strength and dilatancy of sands." *Géotechnique*, Vol. 36, No. 1, pp. 65-78.

Bolton, M. D. and Lau, C. K. (1989). "Scale effect in the bearing capacity of granular soils." *Proc. 12th Int. Conf. Soil Mech. Found. Eng.*, Rio De Janeiro, Brazil, Vol. 2, pp. 895-898.

Bolton, M. D. and Lau, C. K. (1993). "Vertical bearing capacity factors for circular and strip footings on mohr-coulomb soil." *Can. Geotech. J.*, Vol. 30, pp. 1024-1033.

Bowles, J. E. (1996). *Foundation analysis and design.* 5th Ed., McGraw-Hill Companies, Inc.

Briaud, J.-L. and Jeanjean, P. (1994). "Load settlement curve method for spread footings on sand." *Journal of Geotechnical and Geoenvironmental Engineering,* ASCE, Vol. 133, No. 8, pp. 905-920.

Briaud, J.-L. and Gibbens, R. M. (1999). "Behavior of five large spread footings in sand." *Journal of Geotechnical and Geoenviron- mental Engineering,* ASCE, Vol. 125, No. 9, pp. 787-796.

Budhu, M. (2007). *Soil mechanics and foundations*, 2nd Ed., John Wiley and Sons, Inc.

Cerato, A. B. (2005). *Scale Effect of shallow foundation bearing capacity on granular material*, PhD Dissertation, University of Massachusetts Amherst, U.S.A.

Cerato, A. B. and Lutenegger, A. J. (2006) "Bearing capacity of square and circular footings on a finite layer of granular soil underlain by a rigid base." *Journal of Geotechnical and Geoenvironmental Engineering,* ASCE, Vol. 132, No. 11, Nov. 2006, pp.1496-1501.

Cerato, A. B. and Lutenegger, A. J. (2007). "Scale effect of shallow foundation bearing capacity on granular material." *Journal of Geotechnical and Geoenvironmental Engineering,* ASCE, Vol. 133, No. 10, pp. 1192-1202.

Clark, J. I. (1998). "The settlement and bearing capacity of very large foundations on strong soils: 1996 R.M. Hardy keynote address." *Can. Geotech. J.*, Vol. 35, pp. 131-145.

Davis, E. H. and Booker, J. R. (1971). "The bearing capacity of strip footings from the standpoint of plasticity theory." *Proc. 1st Australian-New Zealand Conference in Geomechanics*, Melbourne, Australia, pp. 276-282.

Davis, R. O. and Selvadurai, A. P. S. (2002). *Plasticity and geomechanics,* Cambridge University Press.

De Beer, E. E. (1965). "Bearing capacity and settlement of shallow foundations on sand." *Proc. of the Bearing Capacity and Settlement of Foundations Symposium*, Duke University, Durham, N. C., pp.15-34.

Eslami, A., Fellenius, B. H., and Veiskarami, M. (2004). "Problems on determination of shallow foundations bearing pressure by analytical approach." *Proc. 1st National Congress of Civil Engineering (NCCE, 2004)*, Sharif University of Technology, Tehran, Iran.

Fellenius, B. H. (1999). "Bearing capacity of footings and piles- A delusion?" *DFI Annual Meeting*, October 14-16, 1999, Dearborn, Michigan, U.S.A., pp.1-17.

Fellenius, B. H. and Altaee, A. (1994). "Stress and settlement of footings in sand." *Proc. Conference on Vertical and Horizontal Deformations for Foundations and Embankments*, ASCE, Geotechnical Special Publication, GSP, No. 40, College Station, TX, Vol. 2, pp. 1760-1773.

Gan, J. K. M., Fredlund, D. G., and Rahardjo, H. (1988). "Determination of the shear strength parameters of an unsaturated soil using the direct shear test." *Can. Geotech. J.,* Vol. 35, pp. 500-510.

Ghahramani, A. and Clemence, S. P. (1980). "Zero extension line theory of dynamic passive pressure." *Journal of the Geotechnical Engineering Division,* ASCE, Vol. 106, No. 6, pp. 631-644.

Habibagahi, K. and Ghahramani, A. (1979). "Zero extension line theory of earth pressure." *Journal of the Geotechnical Engineering Division,* ASCE, Vol. 105, No. GT7, pp. 881-896.

Hansen, B. and Odgaard, D. (1960). "Bearing capacity tests on circular plates on sand." *The Danish Geotechnical Institute*: Bulletin No. 8.

Holtz, R. D. and Kovacs, W. D. (1981). *An introduction to geotechnical engineering*, Prentice-Hall Inc., p. 733.

Ismael, N. F. (1985). "Allowable pressure from loading tests on kuwaiti soils." *Can. Geotech. J.*, Vol. 22, pp. 151-157.

Jahanandish, M. (1988). *Zero extension line net and its application in soil mechanics*, M.Sc. Thesis, Shiraz University, Shiraz, Iran.

Jahanandish, M. (2003). "Development of a zero extension line method for axially symmetric problems in soil mechanics." *Scientia Iranica Journal*, Sharif University of Technology Press., Vol. 10, No. 2, pp.1-8.

Jahanandish, M. and Eslami Haghighat, A. (2003). "Load-deformation behavior of strip footings on soils by the zero extension line method." *Proc. 6th Int. Conf. Civil Eng. (ICCE), May 5-7,* Isafahan University of Technology, Isfahan, Iran, Vol. 7, pp. 389-396.

Jahanandish, M. and Eslami Haghighat, A. (2004). "Analysis of boundary value problems in soil plasticity assuming non-coaxiality." *Iranian Journal of Science and Technology (IJST),* Shiraz University Press., Transaction B, Vol. 28, No. B5, pp. 583-594.

Jahanandish, M., Behpoor, L. and Ghahramani, A. (1989). "Load-displacement characteristics of retaining walls." *Proc. 12th Int'l Conf. Soil Mech. Fnd. Engng.,* Rio de Janeiro, Brazil, Vol. 1, pp. 243-246.

James, R. G. and Bransby, P. L. (1971). "A velocity field for some passive earth pressure problems." *Géotechnique,* Vol. 21, No. 1, pp. 61-83.

Kimura, T., Kusakabe, O., and Saitoh, K. (1985). "Geotechnical model tests of bearing capacity problems in a centrifuge." *Géotechnique*, Vol. 35, No. 1, pp. 33-45.

Kumar, J. and Khatri, V. N. (2008). "Effect of footing width on bearing capacity factor N_g." *Journal of Geotechnical and Geoenvironmental Engineering,* ASCE, Vol. 134, No. 9, pp. 1299-1310.

Kumar, J. and Kouzer, K. M. (2007). "Effect of footing roughness on the bearing capacity factor N_g." *Journal of Geotechnical and Geoenvironmental Engineering,* ASCE, Vol. 133, No. 5, pp. 502-511.

Kumar, J., Raju, K. V. S. B., and Kumar, A. (2007). "Relationships between rate of dilation, peak and critical state friction angles." *Indian Geotechnical Journal*, Vol. 37, No. 1, pp. 53-63.

Lee, K. L. and Seed, H. B. (1967). "Drained strength characteristics of sands." *Journal of the Soil Mechanics and Foundations Division,* ASCE. Vol. 93, No. SM6, pp. 117-141.

Maeda, K. and Miura, K. (1999). "Confining stress dependency of mechanical properties of sands." *Soils and Foundations*, Vol. 39, No. 1, pp. 53-67.

Meyerhof, G. G. (1950). *The bearing capacity of sand*, PhD Thesis,

University of London, London, England.
Meyerhof, G. G. (1951). "The ultimate bearing capacity of foundations." *Géotechnique*, Vol. 2, No. 4, pp. 301-332.
Michalowski, R.L. (1997). "An estimate of the influence of soil weight on bearing capacity using limit analysis." *Soils and Foundations*, Japanese Geotechnical Society, Vol. 37, No. 4, pp. 57-64.
Roscoe, K. H. (1970). "The influence of strains in soil mechanics." *Tenth Rankine Lecture, Géotechnique*, Vol. 20, No. 2, pp. 129-170.
Vesić, A. S. (1973). "Analysis of ultimate loads of shallow foundations." *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 99, No.1, pp. 45-73.
Yamamoto, N., Randolph, M. F., and Einav, I. (2009). "Numerical study of the effect of foundation size for a wide range of sands." *Journal of Geotechnical and Geoenvironmental Engineering,* ASCE, Vol. 135, No. 1, pp. 37-45.
Zhu, F., Clark, J. I., and Phillips, R. (2001). "Scale effect of strip and circular footings resting on dense sand." *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol. 127, No. 7, pp. 613-621.

Appendix. Finite Difference Forms of the Equations

As stated before, there are four equations and four unknowns at each point, e.g., point *C*, which should be calculated based on existing data from previous two points, namely, *A* and *B*. The finite difference forms of these equations are as follows:

$$\begin{cases} \text{For} + \text{tive ZEL:} \dfrac{(z_C - z_A)}{(x_C - x_A)} = \tan(\psi + \xi) \\ \text{For} + \text{tive ZEL:} \dfrac{(z_C - z_B)}{(x_C - x_B)} = \tan(\psi - \xi) \end{cases} \tag{12}$$

Jahanandish (2003) Equations:

$$\begin{cases} \text{Along the plus (+) ZEL:} \\ (S_C - S_A) + \dfrac{(T_C - T_B)}{\Delta_{BC}}\Delta_{AC} + \dfrac{2T}{\cos\nu}\left((\psi_C - \psi_A) - \sin\nu \dfrac{(\psi_C - \psi_B)}{\Delta_{BC}}\Delta_{AC}\right) \\ \qquad = [f_r \cos(\psi + \xi) + f_z \sin(\psi + \xi)]\Delta_{AC} \\ \text{Along the minus (-) ZEL:} \\ (S_C - S_B) + \dfrac{(T_C - T_A)}{\Delta_{AC}}\Delta_{BC} - \dfrac{2T}{\cos\nu}\left((\psi_C - \psi_B) - \sin\nu \dfrac{(\psi_C - \psi_A)}{\Delta_{AC}}\Delta_{BC}\right) \\ \qquad = [f_r \cos(\psi - \xi) + f_z \sin(\psi + \xi)]\Delta_{BC} \end{cases} \tag{13}$$

$$\begin{cases} \Delta_{AC} = \sqrt{(x_A - x_C)^2 + (z_A + z_C)^2} = d\varepsilon^+ \\ \Delta_{BC} = \sqrt{(x_B - x_C)^2 + (z_B + z_C)^2} = d\varepsilon^- \end{cases} \tag{14}$$