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Reevaluation of the Factors Influencing the Consolidation of Ground by Incorporating Prefabricated Vertical Drains

N. K. Lee* and S. G. Chung**

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Abstract

In the consolidation of ground by incorporating prefabricated vertical drains, there are a number of rigorous and approximate solutions that can be utilized. These solutions are affected by many factors. Due to a high level of complexity, design and back analysis are generally inefficient in predicting the consolidation behaviour. In this study, the existing solutions for the combined vertical and radial consolidation and the recommended values of each parameter were reviewed. A parametric study was then performed to examine the appropriateness of the solutions and the effects of each factor on the total degree of consolidation U_{vh} to the combined consolidation. Results show that the effect of vertical drainage on U_{vh} is sufficiently insignificant, and the peak influence occurs at U_{vh} of less than 60%. All the solutions for the radial consolidation, except for Hansbo's solution, gave rise to the almost identical results. Hansbo's solution affected well resistance only. It was noted, however, that the significance of well resistance depends on soil permeability and the discharge capacity of drains which is compared to the effects of spacing and smear.

Keywords: *prefabricated vertical drains, consolidation, theories, parametric study*

1. Introduction

In general, there is no definite method that can account for the precise prediction of the consolidation behaviour of a ground improved by prefabricated vertical drain technology, even though a number of consolidation theories or solutions have been proposed (Rendulic, 1935; Carillo, 1942; Barron, 1948; Yoshikuni and Nakanodo, 1974; Hansbo, 1981; Onoue, 1988; Zeng and Xie, 1989; Lo, 1991; and others). The proposed solutions are sophisticated due to a number of factors or parameters affecting the vertical and radial drainages like spacing, smear effects, and well resistance of drains, as well as the installed length and drainage condition. In addition, the differences in the results for each solution are not generally known. The prediction significantly relies on the estimation of consolidation parameters. Generally, such complications make it difficult to predict the consolidation behaviour at the design stage, as well as in the back-analysis. As a typical example of unsatisfactory predictions is the report that at the reclamation works in the Nakdong River estuary in Busan, Korea, the settlement and consolidation time were underestimated by 120-200% and 200-600%, respectively (Chung 1999). Owing to such inaccuracy, it is a usual practice for the predictions that are made at the design stage to be corrected based on the data obtained from observational procedures in the field (Tan *et al.,* 1991; Tan, 1995; Asaoka, 1978; Magnan *et al.,* 1983; Chung *et al.,* 1998, 2009) and from other back-analyses (Lo, 1991; Bergado *et al.*, 1992; Stark *et al.,* 1999).

The purpose of this study is to evaluate how different will the proposed rigorous and approximate consolidation solutions come out in the actual ranges of each parameter, following the selection of a prefabricated vertical drain technology that can be used as a soil improvement technique. To examine the above facts, the existing solutions used extensively for vertical and radial consolidations and the recommended values of each parameter were listed. Using the ranged values of the parameters, a parametric study was attempted to investigate the appropriateness of the solutions and the effects of each factor or parameter on the combined consolidation. Furthermore, discussions and recommendations are presented as practical applications.

2. Consolidation Theories

2.1 Vertical Consolidation

In general, Terzaghi's infinitesimal strain theory for one-dimensional consolidation is exclusively used in practice. This theory is based on the assumptions that the soil is saturated and homogeneous, obeys Darcy's law, and has a linear stress-strain relationship. The average degree of consolidation is:

$$U_v = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \exp(-M^2 T_v) \tag{1}$$

where $M = (\pi/2)(2m+1)$; $T_v = c_v t/H_d^2$; c_v = the vertical coefficient

*Member, Assistant Manager, Development Project Team, Busan Metropolitan Corporation, Busan 614-707, Korea (E-mail: nklee@bmc.busan.kr)
**Member, Professor, Dept. of Civil Engineering, Dong-A University, Busan 604-714, Korea (Corresponding Author, E-mail: sgchung@dau.ac.kr)

of consolidation; and H_d = the vertical drainage distance. Among the closed form approximations of Eq. (1), the following equation provides almost the same results as the above for $0 \le U_v \le 0.997$, or $0 \le T_v \le 6$ (Sivaram and Swamee 1977). This is known as

$$U_v = \frac{(4T_v/\pi)^{0.5}}{[1+(4T_v/\pi)^{2.8}]^{0.179}} \tag{2}$$

2.2 Radial (or Horizontal) Consolidation

A large number of consolidation solutions for radial consolidation have been proposed based on Barron's theory (1944) and the rigorous and approximate solutions are listed in Table 1. All the solutions can be expressed as a general form:

$$U_h = 1-\exp\left(\frac{-8T_h}{F}\right) \tag{3a}$$

Table 1. Theoretical Solutions for Radial Drainage

References		Suggested theoretical solution for radial consolidation
Barron (1944, 1948)	Rigorous (S+WR)	$\overline{U}_h = 1-\exp\left(\frac{-8T_r}{F_{(n+s)}}\right)$ $F_{(n+s)} = \frac{n^2}{n^2-1}\ln\left(\frac{n}{s}\right) - \frac{3n^2-s^2}{4n^2} + \frac{k_h}{k_s}\times\frac{n^2-s^2}{n^2}\ln(s)$ $\approx \ln\left(\frac{n}{s}\right) - \frac{3}{4} + \frac{k_h}{k_s}\ln(s)$ $\overline{U}_h = \frac{1}{2H}\int_0^{2H}\left(1-\exp\left(\frac{-8T_r}{F_{(n+s)}}\times f(z)dz\right)\right)$ $f(z) = \frac{\exp[b(z-2H)]+\exp(-bz)}{1+\exp(-2bH)}$, $b = \left[\frac{2k_h}{k_w}\times\frac{(n^2-s^2)}{R^2\cdot F_{(n+s)}}\right]^{1/2}$, $R = \frac{d_e}{2}$
Scott (1963)	Approximate (S)	$\overline{U}_h = 1-\exp\left(\frac{-8T_r}{F_{(n+s)}}\right)$ $F_{(n+s)} = \frac{n^2}{n^2-1}\ln(n) - \frac{3n^2-1}{4n^2} + \frac{k_h}{r_w\cdot K}\times\frac{n^2-1}{n^2}$ $= \frac{n^2}{n^2-1}\ln(n) - \frac{3n^2-1}{4n^2} + \frac{(s-1)\cdot k_h}{k_s}\times\frac{n^2-1}{n^2}$
Yoshikuni and Nakanodo (1974)	Rigorous (WR)	$\overline{U}_h = 1-\exp\left(\frac{-8T_r}{F_{(n)}+0.8\times L^*}\right) = 1-\exp\left(\frac{-8T_r}{F_{(n)}+2.6G}\right)$ $F_{(n)} = \frac{n^2}{n^2-1}\ln(n) - \frac{3n^2-1}{4n^2} \approx \ln(n) - \frac{3}{4}$, $L^* = \frac{32}{\pi^2}\times\frac{k_h}{k_w}\left(\frac{l}{d_w}\right)^2 = \frac{32}{\pi^2}\times G$
Hansbo (1981)	Approximate (S+WR)	$\overline{U}_h = 1-\exp\left(\frac{-8T_r}{F}\right)$ $F = \frac{n^2}{n^2-1}\left(\ln\frac{n}{s} + \frac{k_h}{k_s}\ln(s) - \frac{3}{4}\right) + \frac{s^2}{n^2-1}\left(1-\frac{s^2}{4n^2}\right)$ $+ \frac{k_h}{k_s}\frac{1}{n^2-1}\left(\frac{s^4-1}{4n^2} - s^2 + 1\right) + \pi z(2l-z)\frac{k_h}{q_w}\left(1-\frac{1}{n^2}\right)$ $\approx \ln\left(\frac{n}{s}\right) + \frac{k_h}{k_s}\ln(n) - \frac{3}{4} + \pi z(2l-z)\frac{k_h}{q_w}\left(1-\frac{1}{n^2}\right)$
Rixner *et al.* (1986a and b)	Approximate (S+WR)	$\overline{U}_h = 1-\exp\left(\frac{-8T_r}{F}\right)$ $F = \frac{n^2}{n^2-1}\left(\ln\frac{n}{s} + \frac{k_h}{k_s}\ln(s) - \frac{3}{4}\right) + \frac{s^2}{n^2-1}\left(1-\frac{s^2}{4n^2}\right)$ $+ \frac{k_h}{k_s}\frac{1}{n^2-1}\left(\frac{s^4-1}{4n^2} - s^2 + 1\right) + \frac{2}{3}\pi\frac{k_h\cdot l^2}{q_w}\left(1-\frac{1}{n^2}\right)$

Table 1. Theoretical Solutions for Radial Drainage (Continued)

References		Suggested theoretical solution for radial consolidation
Onoue (1988)	Rigorous (S+WR)	$\overline{U}_h = 1-\exp\left(\frac{-8T_r}{F_{n'}+0.8L^*}\right)$ $F_{n'} = \frac{(n')^2}{(n')^2-1}\times\ln(n') - \frac{3(n')^2-1}{4(n')^2}$, $n' = n\cdot s^{\eta-1}$, $\eta = k_h/k_s$, $L^* = \frac{32}{\pi^2}\times\frac{k_h}{k_w}\left(\frac{l}{d_w}\right)^2 = \frac{32}{\pi^2}\times G$
Xie (1987) Zeng and Xie (1989)	Rigorous (S+WR)	$\overline{U}_h = 1-\sum_{m=0}^{\infty}\frac{2}{M^2}\exp\left(\frac{-8T_r}{F_{(n+s)}+D}\right) = 1-\exp\left(\frac{-8T_r}{F_{(n+s)}+\pi G}\right)$ $F_{(n+s)} = \ln\left(\frac{n}{s}\right)+\frac{k_h}{k_s}\ln(s)-\frac{3}{4}$, $G = \frac{k_h}{k_w}\left(\frac{l}{d_w}\right)^2$
Lo (1991)	Rigorous (S+WR)	$\overline{U}_h = 1-\exp\left(\frac{-8T_r}{F_{(n+s)}+2.5G}\right)$ $F_{(n+s)} = \frac{n^2}{n^2-1}\left(\ln\frac{n}{s}+\frac{k_h}{k_s}\ln(s)-\frac{3}{4}\right)+\frac{s^2}{n^2-1}\left(1-\frac{s^2}{4n^2}\right)$ $+\frac{k_h}{k_s}\frac{1}{n^2-1}\left(\frac{s^4-1}{4n^2}-s^2+1\right)$

Note: S and WR mean the smear effect and well resistance, respectively.

$$F = F_n + F_s + F_r \tag{3b}$$

where $T_h = c_h t/d_e^2$; d_e = the equivalent diameter of unit cylindrical cell; and F_n, F_s and F_r are the factors for drain spacing, smear effect and well resistance, respectively.

2.3 Vertical and Horizontal Consolidations

Carillo (1942) proposed a solution for the combined vertical and radial drainages based on the assumption that the time for consolidation is identical for both vertical and horizontal flows.

$$U_{vh} = 1-(1-U_v)(1-U_h) \tag{4a}$$
$$= 1-(1-U_v)\cdot\exp(-\lambda T_v) \tag{4b}$$

$$\lambda = \frac{8\,T_h}{F\,T_v} = \frac{8\,c_h/d_e^2}{F c_v/H_d^2} \tag{4c}$$

3. Review on the Used Parameters

The solutions for radial drainage (or consolidation) are comprised of a number of parameters. The calculation results will significantly vary depending on the input values. Some of the main factors and their values proposed by many researchers are provided as follows.

3.1 Equivalent Diameter of PVD

Table 2 shows the equivalent diameters (d_w) proposed for the band-shaped PVD. To quantitatively compare with each other, the dimensions a=10 cm and b=0.4 cm are considered. It appears that the d_w values range between 2.26~6.62 cm in accordance with the proposed methods. Ideally, it is proper to adopt Hansbo's method (1979, 1981) wherein the d_w value is the diameter of an annulus having the equivalent circumference of a PVD. This method gives rise to the largest value (6.62 cm) among the listed values, which is exclusively used in practice. Holtz *et al.* (1991) reported that Hansbo's method is reasonable. However, the other methods present reduced values compared with the above value, probably due to the effectiveness of the size of PVD. Among the methods, the value obtained from the one proposed by Atkinson and Eldred (1981) is approximately the average value, which may be recommended for practical applications.

3.2 Smear Zone

In general, two shapes of mandrel, rhombic and rectangular shapes (120×60 mm), are used for the drain installation in Korea. Remolding will take place inside a volume equal to the volume displaced by the mandrel (Torstensson, 1973). The extent of the zone of the distortion is a function of the stiffness, sensitivity and macrofabric characteristics of the subsoil. Barron (1948) and Hansbo (1979) assumed that the disturbed zone is an annulus having an equivalent diameter d_s and the permeability k_s.

Table 3 shows the extent of the smear zone as proposed by the researchers. In the case of displacement-type circular drains, it is generally assumed as $d_s/d_w \approx 2$ (Holtz and Holm, 1972; Akagi, 1976; Bergado *et al.*, 1992). Recent investigations on a laboratory

Table 2. Suggested Equivalent Diameters of PVD

References	Suggested equation	Equivalent dia.of drain (PVD 100×4 mm)
Hansbo (1979, 1981, 1987)	$d_w = \frac{2\times(a+b)}{\pi}$	6.62 cm
Atkinson and Eldred (1981); Jansen and den Hoedt (1983); and Rixner *et al.* (1986a, b)	$d_w = \frac{a+b}{2}$	5.20
Fellenius and Castonguay (1985)	$d_w = \left(\frac{4ab}{\pi}\right)^{0.5}$	2.26
Pradhan *et al.* (1993)	$d_w = d_e - 2\sqrt{(\bar{s}^2)} + b$ $\bar{s}^2 = \frac{1}{4}d_e^2 + \frac{1}{12}a^2 - \frac{2a}{\pi^2}d_e$	4.41 ($d_e = 1.13\times1.5 = 1.695$ m)
Long and Covo (1994)	$d_w = 0.5a + 0.7b$	5.28

Table 3. d_s/d_w and k_h/k_s for Smear Zone

References	Smear zone (d_s)	d_s/d_w		k_h/k_s	Remarks
		Rhombic	Rectangular		
Akagi (1976)	$d_s = 2\times d_m$	2.60	3.68	-	-
Bergado *et al.*(1991)	$d_s = 2\times d_m$	2.60	3.68	$k_s = k_v$	Back-analysis
Bergado *et al*. (1992)	$d_s = 2.5\times d_w$			10	Back-analysis
Bergado *et al*. (1993)	$d_s = 2\times d_m$	2.60	3.68	10	Back-analysis
Bo *et al*. (2003)	$d_s = (4\sim7)\times d_w$			2-10 (normally 2)	Field and lab. tests
Eriksson *et al*. (2000)	$d_s = 2\times d_m$	2.60	3.68	6	Back-analysis
Hansbo (1981)	$d_s = 1.5\times d_m$	1.95	2.76	3	Recommended
Hansbo *et al*.(1981)	$d_s = 2\times d_w$			2	Back-analysis
Hansbo (1994)	$d_s = 2\times d_w$	2.60	3.68	$k_s = k_v$	Recommended
Hansbo (1997)	$d_s = 2\times d_m$	2.60	3.68	2	Back-analysis
Hird & Moseley (2000)	$d_s = 1.6\times d_w$			3	Lab. test
Holtz and Holm (1973)	$d_s = 2\times d_m$	2.60	3.68	-	Recommended
Indraratna and Redana (1998)	$d_s = (4\sim5)\times d_w$			$k_s = k_v$	Lab. test
Jamiolkowski *et al*. (1983)	$d_s = (2.5\sim3)\times d_m$	3.26~3.91	4.60~5.52	-	Recommended
Sathananthan and Indraratna (2006)	$d_s = 2.5\times d_m$	3.26	4.60	1.34 (1.09~1.64)	Lab. test

Note: d_w and d_m are the equivalent diameters of PVD and mandrel.
$d_m = \sqrt{4A_m/\pi}$ ($d_m = 6.77$ cm for the rhombic type of 120×60 mm; $d_m = 9.57$ cm for the rectangular type of 120×60 mm)
A_m = area of the used mandrel.

scale indicate that $d_s/d_w = 1.6$ (Onoue *et al.*, 1991; Hird and Moseley, 2000). However, a recent back-analysis indicates that the cross-sectional area of the smear zone is 2^2 times the cross-sectional area of the mandrel (Eriksson *et al.*, 2000). Overall, it appears that the d_s/d_w values range from 2 to 7 and the d_s/d_m values from 2 to 3 (2.5 average), where d_m is the equivalent diameter of the mandrel. It also shows that the d_s/d_w values converted from the proposed d_s/d_m values are in the range of 1.95-3.91 and 2.76-5.52 for the rhombic and rectangular shapes, respectively. For this, the following values were assumed: d_w= 5.2 cm and d_m are 6.77 cm and 9.57 cm for the rhombic and rectangular-shaped mandrels (120×60 mm), respectively. As shown above, it should be noted that the converted d_s/d_w value is estimated depending on the shape and size of the mandrel, as well as the estimation of d_w. For example, the average value $d_s/d_m = 2.5$ is equivalent to the d_s/d_w values of 3.26 and 4.60 for both types. According to the result proposed by Eriksson *et al.* (2000), the d_s/d_w values equal 2.60 and 3.68 for both types. Since the proposed ratios have been estimated under various laboratory and field conditions and by assuming the d_w value, it is not apt to simply compare the d_s/d_m values with d_s/d_w.

Table 3 also shows that the ratio k_h/k_s was proposed between 1.34~10, which was obtained from laboratory and field measurements. This value is also determined depending on the soil

Table 4. Discharge Capacity of PVD

References	Discharge Capacity (q_w)	Remarks
Bo (2004)	27~405 m^3/yr	Back-analysis
den Hoedt (1981)	95	Lab. test ($\Delta\delta$=40 mm/day, l=30 m)
Hansbo (1981)	20	Lab. test
Holtz *et al*. (1991)	100~150	Lab. test (σ_c=300~500 kPa)
Indraratana (2002)	100	Back-analysis
Jamiolkowski *et al*. (1983)	10~15	Lab. test (σ_c=300~500 kPa)
Kamon and Suwa (2006)	48~125	Lab. test (σ_c=300 kPa, max. folding strain of 50%)
Koda *et al*. (1989)	100	Lab. test (using the consolidated samples)
Koerner (1994)	50~150	Back-analysis
Dutch recommendation (Kremer *et al*., 1982)	160	Lab. test (σ_c=100 kPa, l=0.6)
Mesri and Lo (1991)	1~90	Back-analysis
Oostveen (1986)	150	Back-analysis
Rixner *et al*. (1986a and b)	500~800 (100~300 for higher lateral pressure)	Lab. test

characteristics, the used mandrel types, and the installation techniques. Bo *et al.* (2003) assumed that $k_h/k_s=c_h/c_v$. The ratio k_h/k_s of 2 that is normally used in practice is probably acceptable as the lower bound.

3.3 Well Resistance

As can be seen in Table 1, the term of well resistance in the proposed solutions includes the discharge capacity q_w and the permeability k_w of the drain. Table 4 shows the q_w values estimated under various testing or measured conditions using different kinds of PVD, ranging between 1~800 m^3/yr. It is interesting to note that a recent study of Kamon *et al.* (1992) and Kamon and Suwa (2006) was based on a triaxial test (the folding radius of 3cm, the maximum folding strain of 50%, and the all-rounded pressure of 300 kPa) and a soil box test (with more severe conditions than that of the triaxial test), using various kinds of sleeve-typed PVDs. It indicates that the q_w values obtained from the soil box are between 48~125 m^3/yr and are (1/5~1/10) times compared to those from the triaxial cell.

4. Parametric Study

The appropriateness of the theoretical solutions for the combined consolidation and the effects of the factors included in the solutions were examined using the recommended values of each parameter. The factors for horizontal drainage were first analyzed followed by the effect of vertical drainage and their weightiness.

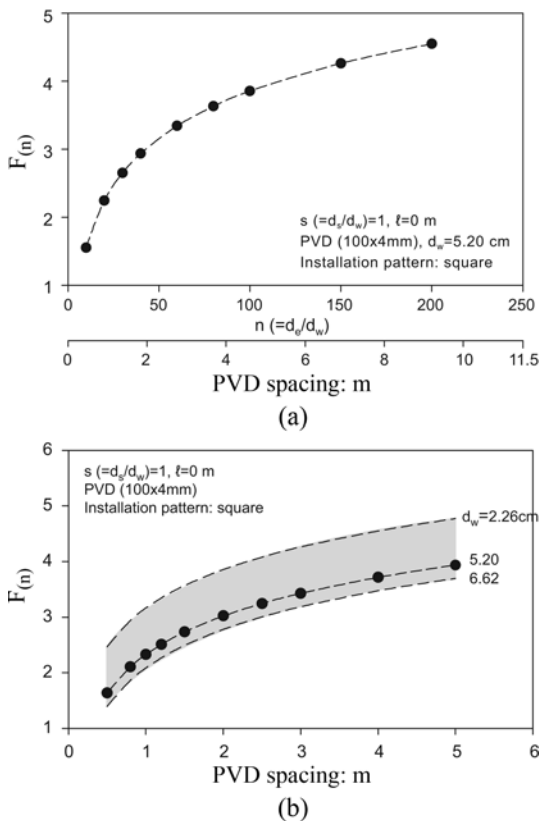

Fig. 1. Effects of Spacing Obtained from Various Solutions and the d_s/d_w Values: (a) Solutions, (b) Effects of d_w

4.1 Effects of Spacing

To isolate the spacing effects (F_n), $s=1$ and $l=0$ in all of the solutions (Table 1). Fig. 1(a) shows the F_n values obtained from various solutions in the range of $n=10$~200 which are equivalent to the spacing range of 0.46-9.20 m. It appears that all the solutions, including the approximate solutions, provide approximately the same value at a given n value, and the F_n values tend to increase rapidly up to n of about 50 (2.3 m spacing). The rate of the increase decreases after. This means that any of the simplified solutions can be used for the spacing effect, that is, the simplified approximate solution proposed by Barron (1948) or Hansbo (1979).

Fig. 1(b) shows the variation of F_n depending on the proposed d_w and the spacing. It appears that the difference in F_n values between the upper and lower bounds of d_w is 1.08, and the F_n values vary from 1.64 to 3.97 between the spacing of 0.5~5.0 m when $d_w=5.2$ cm.

4.2 Effects of Smear Zone

Fig. 2(a) shows the variation of F_s among the solutions where $F_s=F_{(n+s)}-F_n$ when $l=0$. It appears that when $d_s/d_w=3.26$ and $k_h/k_s=2$~10, all the results are almost of the same values. This means that any of the simplified forms can also be used for the smear effect.

Fig. 2(b) shows the variation of F_s with different k_h/k_s and d_s/d_w values, for which the simplified approximate solution proposed by Barron (1948) or Hansbo (1979) was adopted. The F_s values increase linearly with an increasing k_h/k_s and d_s/d_w. For the $d_s/d_w=3.26$, the F_s values vary from 1.18 to 10.63 when $k_h/k_s=2$~10. It is interesting to note that with a small value of k_h/k_s (for example, 2.0), the effect of d_s/d_w is relatively insignificant in the given range (for example, $F_s=0.47$~1.94 for $k_h/k_s=2$ and $d_s/d_w=1.6$~7.0).

4.3 Effects of Well Resistance

Fig. 3(a) shows the comparative result of F_r among the solutions when $k_h=1\times10^{-9}$ m/sec and $q_w=100$ m^3/yr (for the condition $F_{(n+s)}=0$). It appears that the F_r values are resulted to be very close among all the solutions except Hansbo's solution (1979), regardless of the installation depth of PVD. This means that the approximate solution proposed by Hansbo (1979) is inappropriate for well resistance, and any of the other simplified solutions can be used.

Fig. 3(b) shows the effects of the length of PVD and discharge capacity on the F_r value, for which the solution proposed by Zeng and Xie (1989) was used. Note that the length of PVD equals a half of l for the double drainages. It appears that the F_r value dramatically increases with the PVD installation depth when $q_w=10$ m^3/year, while it is insignificant irrespective of the installation depth when $q_w\geq100$ m^3/year ($F_r<0.4$). This is similar to Holtz *et al.* (1991) conclusion. This result is different from Hansbo's study (1994) where the effect of F_r is insignificant when $l\leq20$ m and $q_w=100$ m^3/year. Even for $q_w=50$ m^3/year, which is a lower bound of the values at the soil box test performed by Kamon and Suwa (2006), it also indicates that the effect is not significant for a short PVD length ($F_r<0.8$).

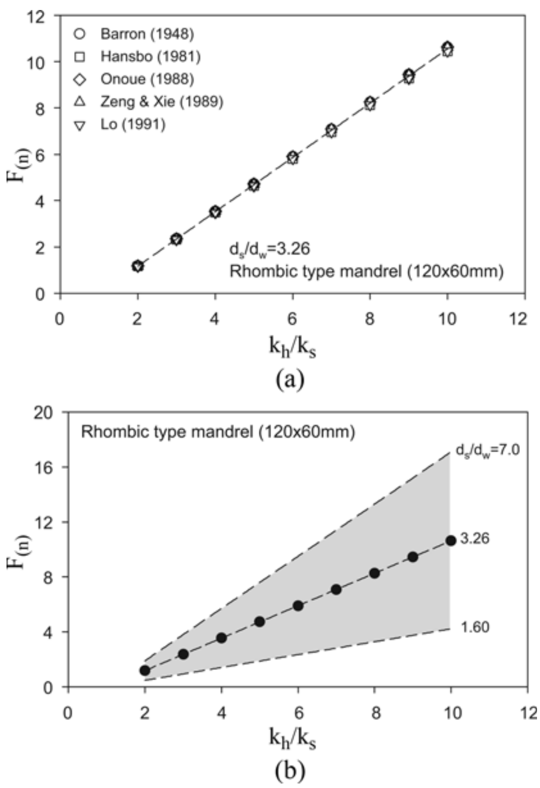

Fig. 2. Effect of k_h/k_s on F_s Values: (a) Solutions, (b) Effects of k_h/k_s and d_s/d_w

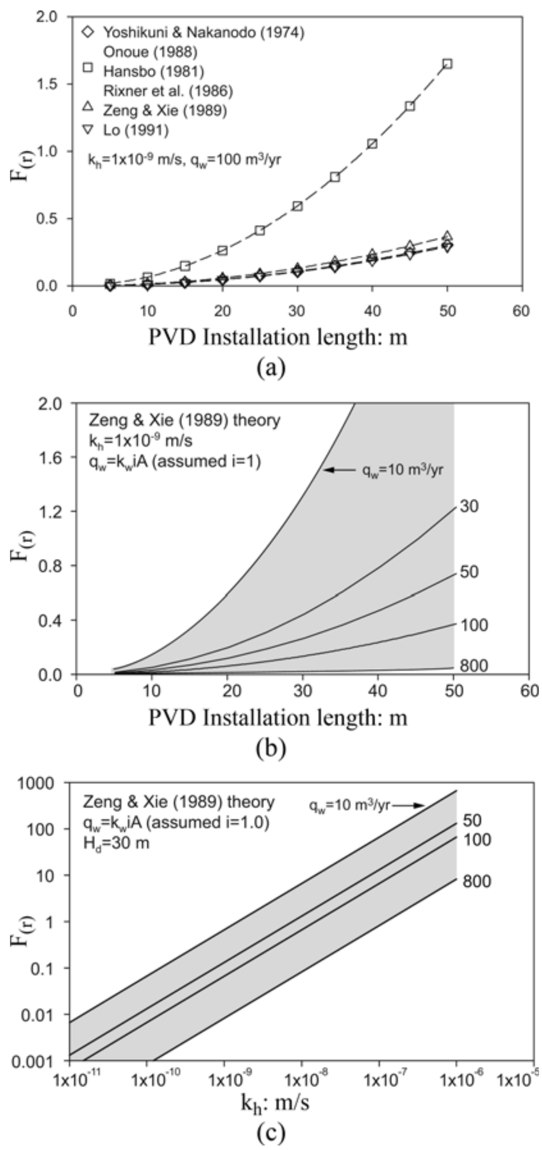

Fig. 3. Effects of l and k_h/k_w on F_r Values: (a) Solutions, (b) Effects of l and q_w, and (c) Effect of k_h and q_w.

Fig. 3(c) shows the effect of k_h with q_w, i.e. k_h/k_w on F_r. For this, Darcy's equation is applied: $q_w=k_w iA=k_w A$, where $i=1.0$ and A is the cross-sectional area of drain. It appears that although q_w is relatively large, F_r dramatically increases with increasing k_h. In clay deposits, the term k_h/k_w dominantly affects F_r due to very small values of k_h, rather than $(l/d_w)^2$ [Xeng and Xie (1989): $F_r=\pi\cdot(k_h/k_w)(l/d_w)^2$].

The above conclusions are almost identical to a number of study results that show that unless the drains are very long and under high lateral stresses, the well resistance does not significantly affect the design from the theoretical point of view. However, due to such factors as the deterioration of the drain filter, the reduced area, and the folding of the drains in practice, studies recommended that the well resistance should be taken into account when designing (Holtz *et al.*, 1991; Bo *et al.*, 2003). Nevertheless, no matter how much of the discharge capacity is taken into account, no method is suggested at present.

4.4 Effects of Vertical Consolidation

To investigate the effect of vertical drainage, Eq. (4) was used for which the solutions proposed by Sivaram and Swamee (1977) and Zeng and Xie (1989) were applied for U_v and U_h, respectively. Fig. 4(a) shows the total degree of consolidation vs. consolidation time relationships for a given condition. It appears that vertical drainage affects U_{vh} within a very small percent (for example $\Delta U_{vh}(=U_{vh}-U_h)\cong 1\%$). The $\Delta U_{vh}(=U_{vh}-U_h)$ reaches the peak value at the early consolidation time and then rapidly decreases as the consolidation time increases. Fig. 4(b) shows ΔU_{vh} vs. consolidation time relationships with the drain spacing. It indicates that ΔU_{vh} and the consolidation time at the peak ΔU_{vh} increase rapidly as the spacing increases. However, even for the PVD spacing of 3 m, the ΔU_{vh} value is less than 4%.

In Fig. 5(a), ΔU_{vh} vs. the drain spacing (or n) relationships at the U_{vh} values of 60% and 90% and the peak ΔU_{vh} are shown. It appears that the peak ΔU_{vh} always takes place prior to reaching $U_{vh,60}$ and all degrees of consolidation increase linearly with the drain spacing. In the case where the spacing is less than 2 m and $H_d\leq 25$ m, the vertical drainage contributes to the total degree of consolidation with values as small as 2.27%. Fig. 5(b) shows the effects of c_h/c_v and H_d with the drain spacing. It appears that as the H_d and c_h/c_v values decrease, the ΔU_{vh} value increases. However, as the vertical drainage path is sufficiently large, the effect of c_h/c_v is insignificant (i.e., ΔU_{vh} at $U_{vh,80}$ is less than 2%).

Holtz *et al.* (1991) reported that the vertical drainage should be considered when the ratio T_v/T_h is greater than about 0.01~0.02. Assuming as $T_v/T_h=c_v/c_h$, the above cases are in the range $T_v/T_h=$ 0.1~0.5. However, the above results show that whether or not the vertical drainage should be considered depends upon not only T_v/T_h but the spacing and vertical drainage path as well.

In summary, the effect of vertical drainage is sufficiently insignificant for the general clay ground when incorporating PVD (probably greater than 10 m depth), particularly with a spacing of less than 3 m. This is probably the error caused by not consi-

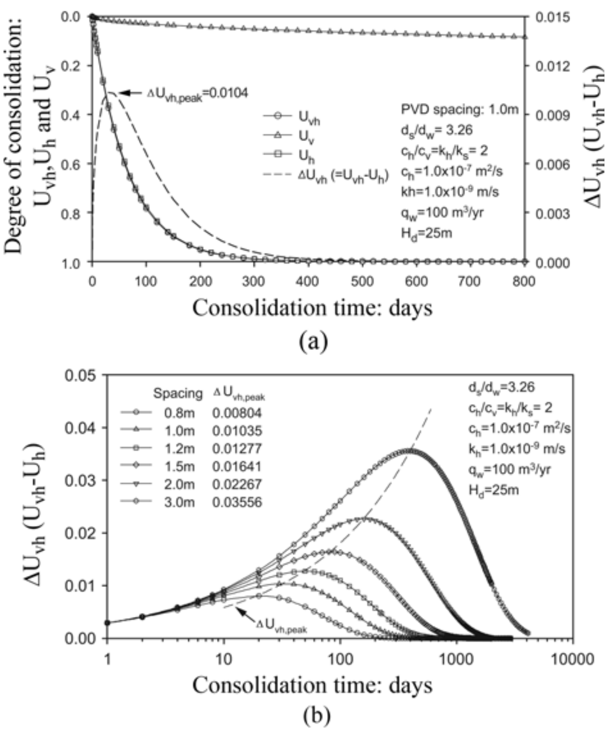

Fig. 4. Effect of Vertical Drainage on Total Degree of Consolidation: (a) Variation of ΔU_{vh}, (b) Spacing Effect on ΔU_{vh}

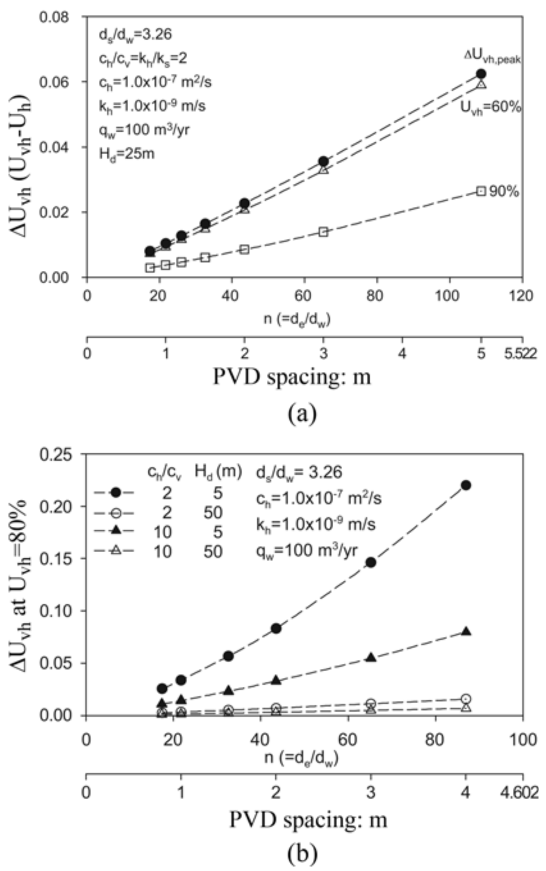

Fig. 5. Effects of Spacing, Vertical Drainage Path and c_h/c_v on ($U_{vh}-U_h$): (a) Effect of Spacing, (b) Effects of H_d and c_h/c_v

dering the fact that the vertical drainage is sufficiently small as compared to those used in estimating related parameters.

4.5 Effects of Each Factor on the Degree of Consolidation

According to the above results, the effect of vertical drainage can be negligible in the listed ranges of each parameter (Tables 2~4). To comprehensively investigate the effect of each factor on the total degree of consolidation, only the horizontal drainage was thus considered as shown below.

Fig. 6 shows the variation in F values presenting the effects of each factor. For this particular situation, the value was taken as $k_h/k_s=2$ and $k_h=10^{-9}$ m/sec, which is generally adopted for Busan clay (Chung *et al.*, 2006). It appears that $s(=d_s/d_w)$ affects the F values regardless of the PVD length when the spacing is 1.5m with a square pattern and $q_w=100$ m^3/yr; meanwhile, the installation depth significantly affects the value when $q_w=10$ m^3/yr. This means that when $q_w=100$ m^3/yr, either the spacing or $s(=d_s/d_w)$ is the major factor which is not significantly affected by well resistance. Similarly, only the spacing and smear effects can be considered as having a small or acceptable percent of error for design. However, when $q_w=10$ m^3/yr, $s=2$, and $k_h/k_s=2$, $F_{(n+s+r)}$ becomes approximately 1.5 times $F_{(n+s)}$ for l=30 m. As shown in Fig. 3(c), the magnitude in k_h can also be a governing factor as well as q_w.

Similarly, Fig. 7 shows the variation of $T_{h,80}$ (time factor at U_h= 80%) indicating the effects of the factors, i.e., the spacing, smear, and well resistance. Since T_h is directly related to F, T_h resulted to a similar trend as that of F as shown in Fig. 6.

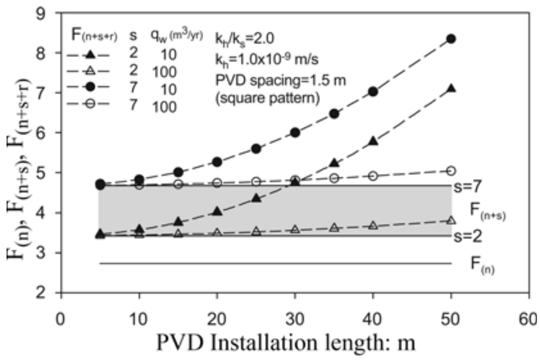

Fig. 6. Variation of Each Factor

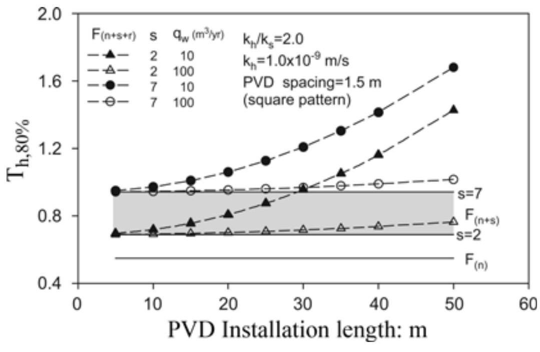

Fig. 7. Variation of $T_{h,80}$ According to Each Factor

5. Conclusions

Results obtained from the parametric study are summarized as follows:

1. In the theoretical solutions for horizontal drainage, the effects of spacing and smear are almost identical in all solutions, and the effect of well resistance is also analogous between the solutions except for Hansbo's approximate solution (1981). The effect of vertical drainage reaches the peak prior to $U_{vh,60}$ and then rapidly does down. Likewise, the vertical drainage has no significant effect on the total consolidation for example, less than 2% for the spacing of 1.5 m.

2. The extent of the equivalent diameter of PVD varies depending on the proposed methods, which affect the spacing ratio n and results to approximately 1.1 maximum difference in $F_{(n)}$ between methods.

3. The smear effect increases linearly at a fast rate as the d_s/d_w and k_h/k_s increase. However, when k_h/k_s is small (for example, 2.0), the effect of d_s/d_w is relatively insignificant (F_s=0.46~1.90 for d_s/d_w= 1.6~7.0).

4. The well resistance is affected predominantly by the k_h value and the discharge capacity of PVD rather than the installation depth. The effect is sufficiently insignificant when q_w>100 m^3/yr and k_h< 5×10^{-9} m/sec. However, this effect cannot be excluded in the case when a large settlement takes place for a long time.

Notations

d_e = Diameter of influence zone of each drain
d_m = Equivalent diameter of mandrel
d_s = Equivalent diameter of smear zone
d_w = Equivalent diameter of drain
$F = F_n+F_s+F_r$
F_n, F_s, F_r = Non-dimensional factors to account for drain spacing, smear effect and well resistance, respectively
H_d = Maximum vertical drainage path of compressible layer
k_h = Coefficient of horizontal permeability of undisturbed soil
k_s = Coefficient of horizontal permeability in smear zone
k_w = Coefficient of permeability of PVD
l = Maximum drainage length of PVD
n = Drain spacing ratio $=d_e/d_w$
q_w = Discharge capacity of PVD $=k_w iA$
$s = d_s/d_w$
T_v, T_r = Time factors for vertical and radial consolidations; $c_v t/H_d^2$ and $c_h t/d_e^2$
U_v, U_h = Average degree of consolidation induced by vertical and horizontal consolidations, respectively
U_{vh} = Average degree of consolidation due to combined vertical and horizontal consolidations
$\Delta U_{vh} = U_{vh} - U_h$

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