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Numerical Modelling of the Motion of Rapid, Flow-like Landslides for Hazard Assessment

Oldrich Hungr*

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Abstract

Computer methods for analysis of stresses, strains and displacements of soil structures have made spectacular advances in recent years. Now, a new field of analysis with substantial practical significance is opening up. This is the quantitative analysis of motion of rapidly moving masses of earth and fragmented rock in landslides such as debris avalanches, debris flows, flow slides and rock avalanches. The new methods accept an unstable body of material on the slope as input (the "landslide source") and map its propagation down the slope in terms of flow depth and velocity. This allows outlining the extent of potential hazard areas and the distribution of hazard intensity parameters within them as needed for detailed hazard mapping, estimation of risks and design of remedial measures such as deflecting walls, dykes or barriers. Two advanced numerical solutions based on integrated "shallow-flow" Lagrangian theory of unsteady flow of non-Newtonian fluid are described in terms of their theoretical background, assumptions regarding the type of flow and rheological characteristics, as well as examples of a variety of results. The two models are the pseudo-three dimensional code DAN and its extension into three dimensions, DAN3D. Both codes have the following main capabilities: 1) Open rheological kernel, allowing the choice of a variety of rheological relationships to suit a particular character of the moving mass; 2) ability to change rheology during the flow; 3) non-hydrostatic, anisotropic internal stress distribution controlled by longitudinal strain; and 4) ability to entrain material during the flow. Both codes are highly efficient and both produce similar results when applied to the same example.

Keywords: *landslides, dynamic analysis, hazard assessment, remedial measures*

1. Introduction

Rapidly moving landslides such as debris flows, debris avalanches, flow slides and rock avalanches (for classification see Hungr *et al*., 2001) exact a steady toll of injury and damage on the populations of hilly and mountainous regions around the world. Gradually, in various countries, advances are being made to reduce landslide damage by predicting the extent and intensity of rapidly-moving landslides. In recent years, advances in computing technology have made it possible to model landslide motion over long distances using numerical models. The results from such "runout analyses" can be used to map expected velocities and depths of landslide motion, in order to estimate potential damage to structures and design remedial works such as dykes, barriers and walls.

Analytical techniques for prediction of landslide motion are rapidly developing at the present time. An excellent overview of the current state-of-the-art in this subject is contained in the Proceedings of the Benchmarking Exercise conducted at the International Forum on Landslide Disaster Management (Ho and Li, 2008), where a variety of competing numerical models were described and used.

The Geological Engineering research group at the University of British Columbia, Canada, has been conducting research on numerical landslide runout modelling for more than 10 years. We have developed two models: DAN, based on a one-dimensional solution of the equations of motion and DAN3D, using a two-dimensional integrated solution simulating motion on irregular three-dimensional surface described by a Digital Elevation Model (DEM). Both are utilized within the framework of the "equivalent fluid concept" which was described by Hungr (1995), but used tacitly by many other researchers. This concept does not attempt to analyse the micro-mechanics of landslide motion, with its complexities of material heterogeneity, varying degrees of saturation and complex pore-pressure response. Instead, a simple "equivalent fluid" is defined, a hypothetical material governed by simple internal and basal rheologies.

The models are calibration-based, meaning that the appropriate rheological parameters cannot be determined from laboratory tests, but must be constrained by trial-and-error back-analysis of known real landslides. The trials are judged in terms of their ability to simulate the bulk characteristics of the prototype event, including the total travel distance, the distribution of deposits and the velocities estimated along the path. A large number of case

*Professor, Dept. of Earth and Ocean Sciences, University of British Columbia, Vancouver V6T 1Z4, Canada (E-mail: ohungr@eos.ubc.ca)

studies have been analyzed and a valuable database of calibrated parameters has been created (cf. Hungr *et al*., 2005). For landslides of similar type and scale, the calibrated parameters are generally well-constrained, suggesting that accurate, first-order runout prediction is possible. Apart from calibration, it is also necessary to verify the solution algorithms, in order to demonstrate that the numerical theory is able to correctly account for the complex boundary conditions represented by landslides. Such "verification testing" usually relies on simulation of small-scale laboratory tests where the rheology of the material is simple (usually frictional) and can be represented by parameters obtained through independent testing.

2. Description of the Models

2.1 Model Features

The pseudo-three-dimensional model DAN, based on a one-dimensional integrated solution of the equations of motion, is described in Hungr (1995). Since that original publication, the model was upgraded by including the active/passive earth pressure equations of Savage and Hutter (1989), by adding the ability to entrain material from the path (for which the 1995 governing equations already made an allowance) and by providing a choice of normal and vertical reference columns. The 3D model DAN3D is completely described by McDougall (2006) and, in a simpler form by McDougall and Hungr (2004 and 2005).

The specific features of the two models are listed as follows:

DAN (Hungr, 1995) – This is an integrated one-dimensional Lagrangian solution, in which the condition of volumetric continuity takes account of narrowing or widening of the flow path, as pre-scribed by the user (Fig. 1).

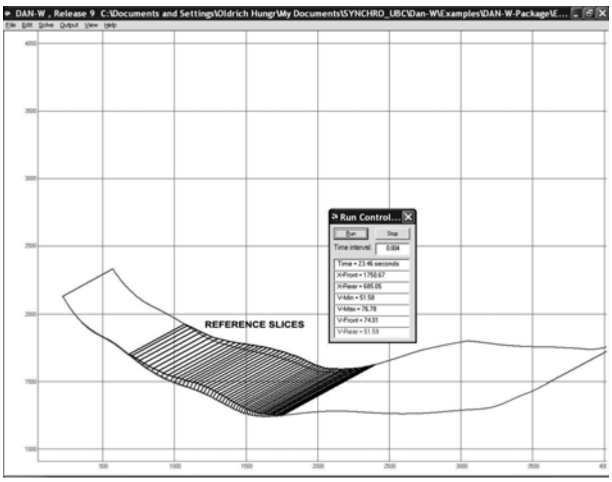


Fig. 1. Screen Capture of DAN Analysis, Animated in Calculation Time (The graphic represents the centreline profile of the landslide, while an isometry of the left half of the slide is also plotted. The reference slice boundaries are plotted in black where extension persists and in red for those columns subject to compression.)

Other features of the model include:

- Linear interpolation to maintain continuity (no numerical damping).
- Ability to vary the width of the flow by a user-prescribed path width function (pseudo-3D).
- Open rheological kernel, allowing the use of frictional (with constant pore-pressure ratio), plastic, Bingham, Voellmy (Voellmy, 1955) and other rheologies.
- Possibility to change material rheology along the path.
- Possibility to entrain material from the flow path, according to a user-prescribed erosion depth. The entrainment rate is proportional to flow depth and velocity and is set so that the full specified entrainment depth occurs once a given point on the path is over-run by the entire current volume of the landslide.
- Option to choose vertical or normal reference columns.
- Possibility to choose between Rankine, Savage – Hutter and Modified Savage – Hutter earth pressure equation (Hungr, 2008).

The model has been in use for more than 10 years and has been applied to more than 100 case histories, including cases solving professional design problems.

DAN3D (McDougall, 2006) – This is an integrated two-dimensional Lagrangian solution of unsteady flow over an irregular, three-dimensional surface. Its features are similar to those of DAN:

- Use of meshless Smooth Particle Hydraulics (SPH) to maintain continuity (no numerical damping).
- Accurate numerical approximation of arbitrary source volume geometry.
- Use of arbitrary, moderately-smoothed digital terrain model of the path.
- Open rheological kernel, allowing the use of frictional (with constant pore-pressure ratio) and Voellmy rheologies.
- Possibility to change material rheology along the path.
- Possibility to entrain material from the flow path, according to a user-prescribed erosion depth and erosion rate. The entrainment rate is proportional to flow depth and velocity and is set by the user by trial end error to ensure that full specified depth occurs where required (Mc Dougall and Hungr, 2005).
- Use of the Savage – Hutter earth pressure equation.
- "Steering" of the flowing mass is based on the assumption that the principal stress axes are parallel with the local direction of motion.

2.2 Rheological Relationships

Recognizing that various geological conditions produce different rheological response from the moving material, the models are capable of employing a variety of simple rheological relationships for the definition of the "equivalent fluid." Those most frequently used are:

1) Frictional resistance: Here, the resisting shear stress at the base of the flow, τ, equals:

$$\tau = \sigma(1-r_u)\tan\phi \tag{1}$$

where φ is the dynamic basal friction angle and σ is the total normal stress. The pore pressure ratio, r_u is assumed constant and equals u/σ, where u is the basal pore-pressure. As long as r_u can be assumed constant, the basal stress relationship remains frictional (i.e. the total normal stress and shear stress remain in proportion). Eq. (1) can then be simplified to include only one independent variable, a bulk basal friction angle φ_b, where:

$$\tan\phi_b = (1-r_u)\tan\phi \tag{2}$$

and:

$$\tau = \sigma\tan\phi_b \tag{3}$$

2) The Voellmy resistance model combines frictional and turbulent behaviour:

$$\tau = \sigma f + \frac{\rho g v^2}{\xi} \tag{4}$$

where f is the friction coefficient, v is the mean flow velocity, ρ is bulk density of the flowing material and ξ is the so-called turbulence parameter (equal to the square of the Chézy coefficient – Voellmy, 1955). The first term on the right side accounts for any frictional component of resistance and has the same form as Eq. (3) (f is analogous to $\tan\varphi_b$). The second term accounts for all possible sources of velocity-dependent resistance.

3) The Bingham flow resistance model combines a plastic yield strength with a viscous resistance dependent linearly on velocity. A Bingham fluid behaves as a rigid material below a threshold yield strength, but as a viscous material above. The following cubic equation must be solved to determine the basal shear resistance τ as a function of velocity, v:

$$v = \frac{h}{6\mu_{Bingham}}\left(2\,\tau - 3\,\tau_{yield} + \frac{\tau_{yield}^3}{\tau^2}\right) \tag{5}$$

where τ_{yield} is the Bingham yield stress and $\mu_{Bingham}$ is the Bingham viscosity.

2.3 Assumed Internal Stress Conditions

The "integrated" flow solutions, based on St.Venant Equations, assume shallow-flow conditions. This means that the flow-lines are taken as approximately parallel with the bed. Under these conditions, the differential equations of motion can be integrated in the direction perpendicular to bed, to reduce the problem by one dimension. What is then required, however, is to assume a distribution of bed-parallel stresses with depth. In hydraulics, it is commonly assumed that the pressure within the flowing sheet is hydrostatically distributed. In such a case, the bed-parallel normal stress can be calculated as the bed-normal stress, multiplied by a constant k, equal in this case to 1.0. The assumption of hydrostatic stress distribution is reasonable for fluids, which lack internal strength. Savage and Hutter (1989) recognized that a hydrostatic pressure distribution is not realistic when dealing with the flow of granular material, which has internal strength due to its frictional nature. The internal stress state is complex while the material remains coherent, like a solid body. However, once the granular material is strongly deformed and subject to generalized failure in shear, a simple relationship between the major and minor principal stresses (p_1 and p_3 respectively in plane strain) can be described according to Rankine's theory of frictional-plastic stress states.

When an element of soil is under compression or extension, the "passive or 'active' state" gives, respectively:

$$p_1 = \tan^2\left(\frac{\pi}{4} \pm \frac{\phi_i}{2}\right)p_3 = k_{p,a}p_3 \tag{6}$$

Where φ_i is the internal friction angle of the material and the major principal stress acts in the direction of maximum compressive strain. The "Passive Earth Pressure Coefficient", k_p, is always greater than 1.0 and ranges up to about 5.0 for typical granular soils. The "Active Coefficient", k_a, is less than 1.0 and ranges down to about 0.2.

The Rankine passive/active coefficients can be used directly for k in the pressure term of the momentum equation, as long as the friction angle between the base of the flow (φ_b) and the flowing material is negligibly small. This happens when the base of the flow is lubricated by a weak layer, such as a thin layer of liquefied saturated soil. Under these conditions, the principal stress axes are oriented approximately parallel with the base and p_1 is the bed-parallel normal stress in the passive state). In the active state, p_1 is the bed-normal stress. The distinction between the two states is set by the sign of the horizontal strain (Savage and Hutter, 1989). In the models proposed by Hungr (1995) and McDougall and Hungr (2004), there is a gradual transition between the two stress states, controlled by longitudinal strain within the moving mass.

In cases where the basal friction angle, φ_b is more significant relative to the internal friction angle, the principal stresses rotate. Savage and Hutter (1989) used the geometry of the Mohr's Circle to derive an approximate equation for the ratio between the normal stress parallel and perpendicular to the bed:

$$k_{(\min/\max)} = 2\left[\frac{1 \pm \sqrt{1-\cos^2\phi_i(1+\tan^2\phi_b)}}{\cos^2\phi_i}\right] - 1 \tag{7}$$

Here again, the plus sign corresponds to the passive state, where longitudinal compression is occurring and the minus sign to the active state. Eq. (7) forms the basis of what Pudasaini and Hutter (2007) refer to as the Savage-Hutter (SH) method. The theory relies on the shallow-flow assumption, i.e. that the flow lines are approximately parallel with the bed and that the depth gradient is small.

Recently, the Savage-Hutter Eq. (7) was modified by Hungr (2008), to compensate for contrary rotation of principal stresses, which occurs in problems involving large values of depth gradient, i.e. those departing too strongly from the shallow-flow assumption. The modification is achieved by reducing the tanφ_b term in Eq. (7) by an expression depending on the local value of the depth gradient (Hungr, 2008). This assumption is referred to henceforth as the "Modified Savage-Hutter" assumption and it has been implemented in a prototype version of the model DAN.

3. Example Solutions

3.1 The Dam Break Model in Granular Material (A Verification Problem)

Hungr (2008) describes laboratory experiments involving sudden removal of a dam, retaining a mass of dry sand. The dam-break experiments have been analysed using the program DAN, with four alternative assumptions regarding the distribution of the bed-parallel normal stress, as described above. The results, an example of which is shown in Fig. 2, indicate that both the hydrostatic (zero internal strength) and Savage-Hutter (SH) assumptions grossly overestimate the experimental runout for this particular problem configuration, which departs significantly from the shallow-flow assumptions. The Rankine assumption, on the other hand, underestimates runout. The Modified SH assumption produces reasonably accurate results. This divergence of results from different internal stress assumptions is observed only in problems similar to the one considered, where large values of depth gradients exist. True "shallow flow" problems with moderate depth gradients are much less affected by these assumptions.

3.2 Deflected Flow Experiment: A Model Verification Problem

As stated above, verification of the basic physics of the numerical algorithm requires comparison against controlled laboratory experiments, using simple rheologies. An ideal experiment of this type was completed recently by a research group at the EPFL in Lausanne, Switzerland for the International Forum on Landslides in Hong Kong, December, 2007 (Ho and Li, 2008). The experiment consists of releasing a volume of dry sand from a box onto a planar sloping surface. After moving some distance in a straight line, the sand flow is deflected sideways by another plane, similar to the action of a deflecting dyke placed obliquely across the path of an avalanche. The position and depth of the moving sand mass was surveyed at intervals, using an optical fringe projection technique (Manzella, 2008). Fig. 3(a) shows the contours of the final deposit from one of the tests, as reported by Manzella (2008). Fig. 3(b) shows the corresponding deposit contours obtained analytically using the model DAN3D.

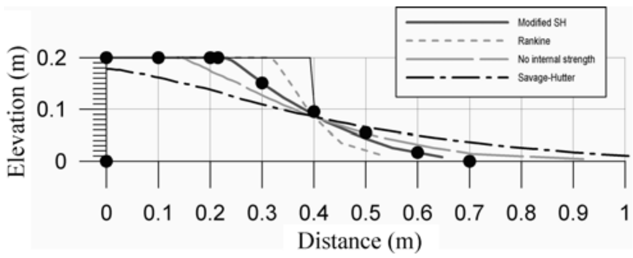


Fig. 2. An Analysis of a Sand Dam-break Experiment, Using DAN with Varying Internal Stress Assumptions (Hungr, 2008)

Similar to the dam-break experiment reported in Par. 3.1, the back analysis was conducted using frictional rheology, with both basal and internal friction angles (32º and 34º respectively) obtained independently from quasi-dynamic tilting tests. Thus, the analysis shown in Fig. 3 represents a very successful first-order prediction of the behaviour of the flowing sand in the experiment. Several other models demonstrated during the Hong Kong benchmarking exercise produced similarly good results (Ho and Li, 2008).

3.3 The Frank Slide (Rock Avalanche)

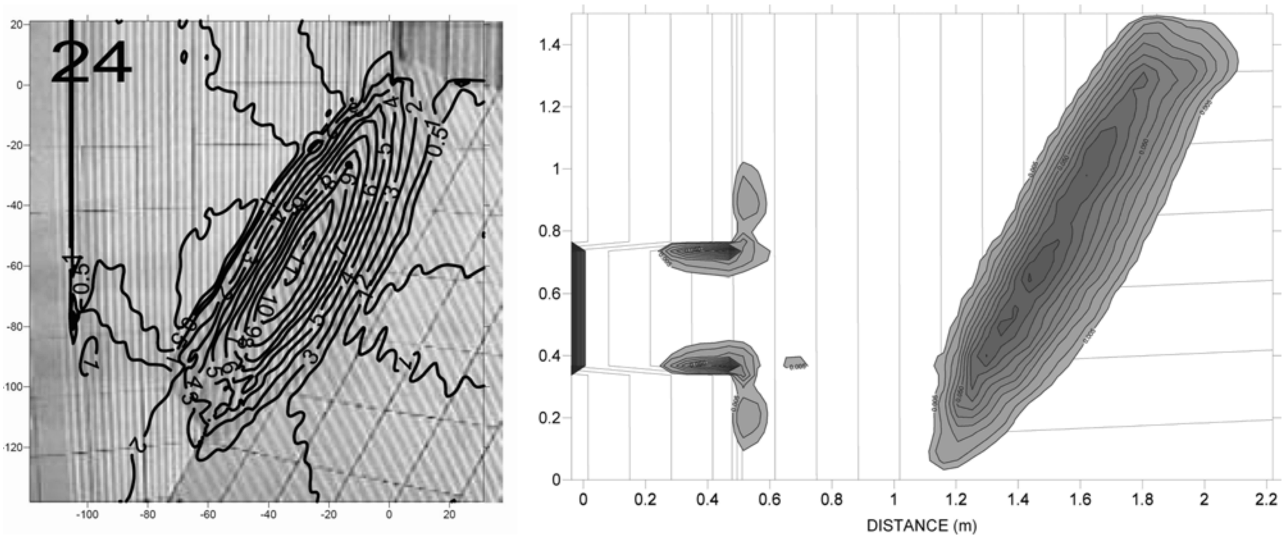
Both programs have now been applied to a number of real, long-runout landslides. The method of trial-and-error back-analysis is illustrated in Fig. 4, showing the analysis of the well-known, 36 million m^3 rock avalanche, the Frank Slide, that occurred in Southern Alberta in 1903 (Cruden and Krahn, 1978). Nine pairs of the Voellmy flow resistance coefficients have been used. Of these, the pair $f=0.1$ and $\xi=500\text{m/s}^2$ yield the best simulation of the debris distribution, while also showing good agreement in terms of the duration of motion. This type of methodology is used in calibrating the models against case histories. As is usually the case, the pseudo-three dimensional model DAN and the model DAN3D produce very similar results, when used with the same parameters and a compatible assumption regarding the changing width of the flow path. Because DAN runs much faster than DAN3D, the approximate correspondence between the two models enables the use of the former for efficient trials of a range of flow resistance parameters.

4. Use of the Models to Estimate Runup Against Protective Dykes

The models are sometimes used to simulate runup against terminal dykes that serve to stop the flow of a rapidly-flowing landslide. This involves an abrupt change of path slope and associated hydraulic shocks, which often induce instability in shallow-flow models. However, this problem has recently been overcome. The instability is prevented with the use of velocity averaging. At the end of each time step, the velocity of each of the reference columns in the 2D solution is adjusted using the following formula:

$$v_i' = \frac{v_{i-1}w_l + v_i + v_{i+1}w_r}{w_l + 1 + w_r} \tag{8}$$

Here, v_i' is the new velocity of Element i, adjusted by smoothing. The quantities v_i, v_{i-1} and v_{i+1} are the current velocities of reference element i and the elements that travel immediately



(a) (b)

Fig. 3. Back-analysis of the Sand Deflected Flow Experiment Using DAN3D (The flow thickness contour interval is 0.01 m (1 cm) and the axes are marked at 0.2 m (20 cm). (a) The Measured Experimental Result (Manzella, 2008), (b) The Final Deposit Obtained in the Analysis)

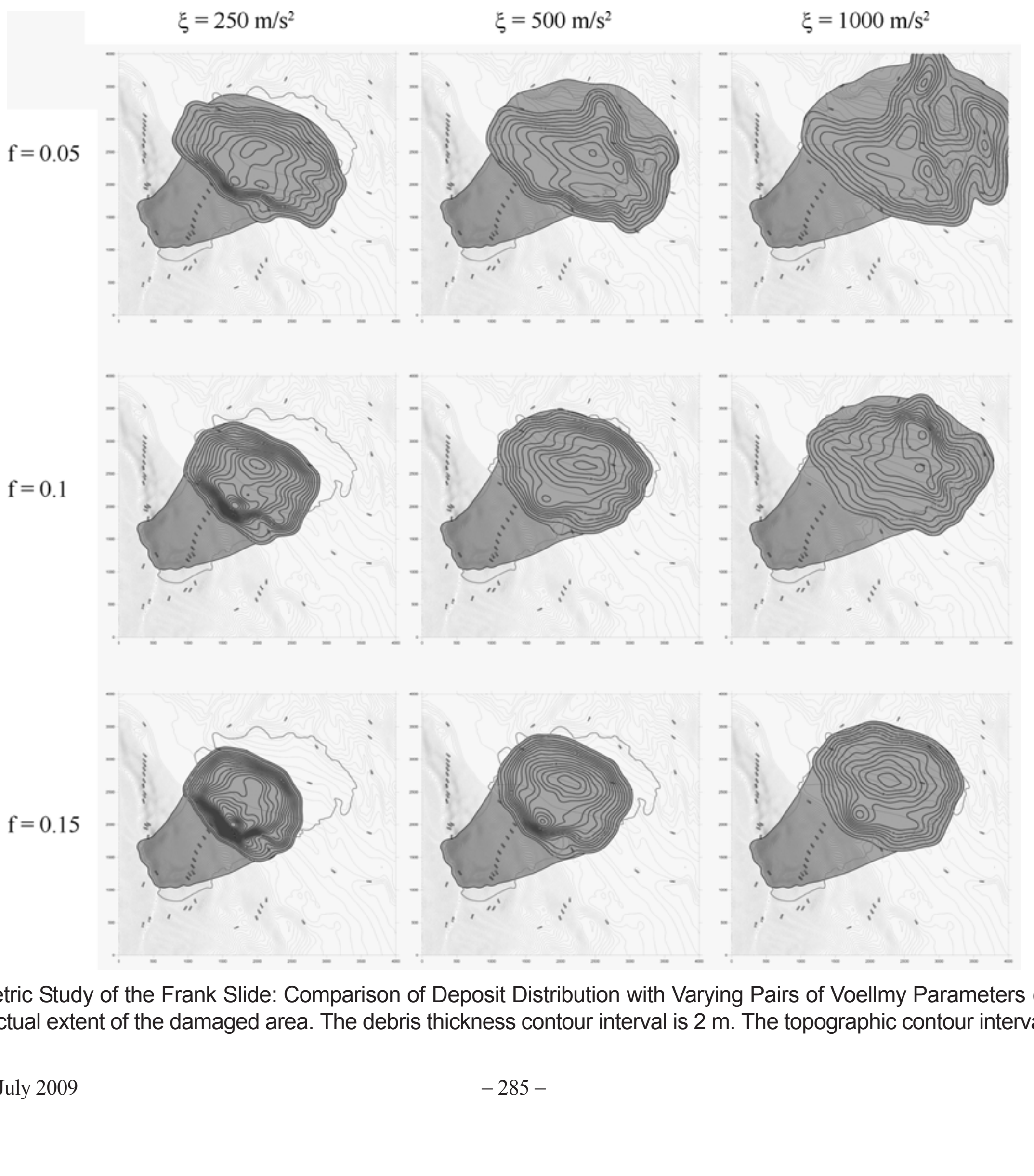


Fig. 4. Parametric Study of the Frank Slide: Comparison of Deposit Distribution with Varying Pairs of Voellmy Parameters (The gray line is the actual extent of the damaged area. The debris thickness contour interval is 2 m. The topographic contour interval is 10 m.)

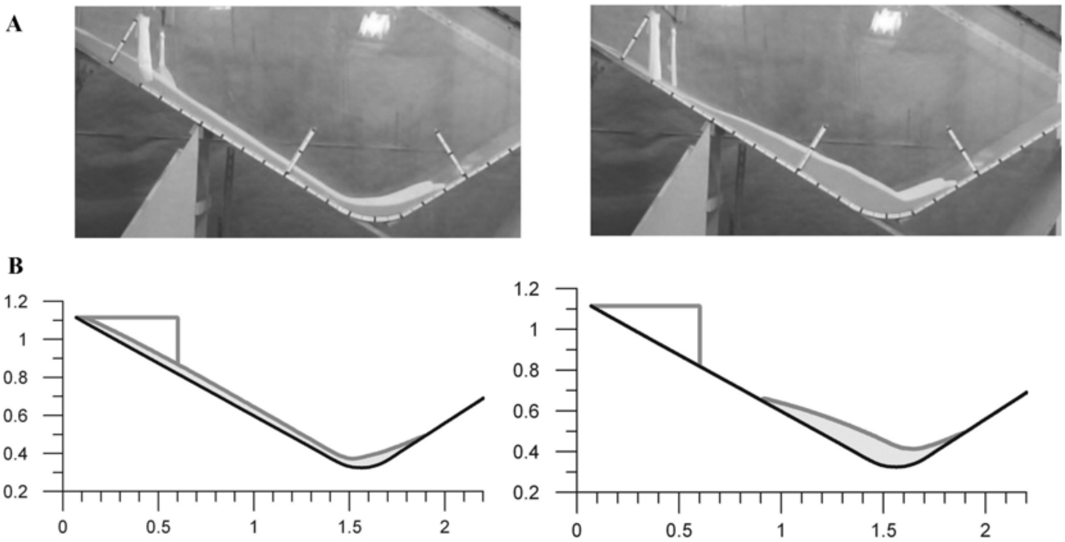


Fig. 5. An Analysis of Runup of a Sand Avalanche Using DAN (The marks are 10 cm apart. (a) Photos of the experiment at the point when the flow front reached the maximum run-up height and at the point when the entire avalanche came to a stop. (b) Corresponding images obtained from DAN analysis. The apparent distortion of the flow surface at the toe of the dyke is due to the use of normal reference elements. The experiment was conducted by Ing. D. Mancarella of the University of Taranto, Italy.)

before and behind it. The weighting factors w_l and w_r are inversely proportional to the distance between the adjacent elements and the element in consideration. Using empirical adjustment, these weights are dimensioned so that they assume significance only in cases where the adjacent elements are approaching too close, creating a potential for developing instability.

With this smoothing procedure, the model remains stable even in cases where an encounter of the landslide front with a steep adverse slope creates a shock wave. The new algorithm was verified using a series of sand flow experiments similar to that shown in Fig. 5a. In these experiments, sand was released from a reservoir, accelerated down a ramp and was arrested by an adverse slope of varying steepness. Fig. 5(b) shows one of the experiments at the point when the leading front of the landslide reached the highest runup point and at the moment of final deposition when the entire landslide came to a stop. An analysis of the same test configuration using DAN with independently-measured input friction angles is shown in Fig. 5(b). The comparison shows that both the maximum initial runup and the final deposition of debris are modeled well. The same was found for other test configurations, including those where the "dyke" slope was perpendicular to the approach path, like an impact wall.

These experiments show that the relatively simple shallow-flow model can be used to provide reasonable estimates of runup against dykes and barriers. In real applications, the ideal frictional model must be replaced by a calibrated model, using an appropriate rheology.

5. Conclusions

Numerical flow models based on the integrated shallow-flow equations are capable of realistic simulation of the movement of rapid, flow-like landslides. The models have been verified against a number of controlled laboratory landslide experiments. In order to make predictions in the field, the models must be calibrated against full-scale landslides in the field, as shown by the Frank Slide example. Such models will soon be available to make routine predictions of velocities and runout distances for practical uses in hazard assessment and mitigation.

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