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Evaluation of Riverwater Quality by Entropy

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Abstract

As a measure of uncertainty of a system, entropy is employed as an indicator of the influx of pollutants from point as well as non-point sources of pollution to river systems in India. Most of the Indian rivers receive pollutants from municipal and industrial wastes, which are disposed without proper treatment. In addition, non-point source pollution also enters the receiving surface water diffusely at intermittent intervals. In this study, entropy is used to evaluate the water quality at different locations of six river systems in India: Baitarani, Brahmani, Malprabha, Pachin, Gomti and Yamuna. Two water quality variables, viz., dissolved oxygen (DO) and biochemical oxygen demand (BOD) are considered and their entropy values are determined at different locations for all the six river systems. However, due to the non-availability of data for Malprabha and Pachin river systems, entropy for electrical conductivity (Ec) and hardness (as $CaCO_3$) is determined only for Baitarani, Brahmani, Gomti and Yamuna river systems. The results indicate severe water quality conditions in River Yamuna and River Gomti (in the Ganga plains), moderate water quality condition in River Bairatani (eastern river), and marginal water quality conditions in River Brahmani (eastern river), River Malprabha (southern river) and River Pachin (north-eastern river). The entropy variation at different time scales (during 1990-91 and 1995-96) for River Yamuna and Gomti indicates a gradual increasing trend of pollution

Keywords: *entropy, dissolved oxygen, biochemical oxygen demand, electrical conductivity, hardness, pollution, water quality*

1. Introduction

River water pollution can be perceived as a result of discharge of pollutants into the river through human activities of production and consumption (point as well as non-point sources of pollution). When a pollutant is added to a river at any location, the pollutant will dissolve and diffuse throughout the river water. The dissolution and diffusion imply an increase in the entropy of the river water (by virtue of its definition and the second law of thermodynamics) (Singh, 1997, 2000). This suggests that an increase in entropy corresponds to an increase in water pollution.

The entropy concept was first introduced by Shannon (1948) for expression of information or uncertainty. Shannon's concept of a measure of uncertainty is based on the principle that greater the uncertainty about the outcomes, the more uniform should be the probabilities assigned to the outcomes. Expressed in terms of entropy, the greater the uncertainty about the outcome of a system, the greater the value of the entropy of that system. The Shannon entropy S of a river system can be expressed as:

$$S = -\sum_{i=1}^{m} p_i \ln(p_i) \tag{1}$$

in which, p_i is the probability of i^{th} event and m is the number of events. Further,

$$\sum_{i=1}^{m} p_i = 1 \tag{2}$$

Eq. (2) is needed to ensure the development of a complete probability distribution. The important characteristics of the formulation expressed by Eqs. (1) and (2) are:

(a) S takes on its maximum when all events have the same probability or uncertainty, i.e., $p_i = 1/m$.
(b) S takes on its minimum value (equal to 0), when there is certainty among the events.
(c) Any combination of probabilities will give a value of S between these two extremes.

The character of the entropy function described above also means that the probability distribution with the maximum entropy is the most unbiased distribution consistent with the information specified by the constraints. This observation in turn means that without any constraints other than Eq. (2), the distribution developed by the formulation is the most dispersed, i.e., it is a uniform distribution.

For a continuous variable, the Shannon entropy can be written as

$$S = -\int_0^{\infty} f(x) \ln f(x)\, dx \tag{3}$$

Jaynes (1957) developed the Principle of Maximum Entropy

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(POME) which states that the probability distribution with the greatest entropy value is the probability distribution that can be realized in the greatest number of ways. In other words, it is the most likely probability distribution. A complete treatment of POME is presented in a treatise by Levine and Tribus (1978), and Tribus (1969). The formulation of POME involving constraints is as follows:

$$\text{Max}S = -\sum_{i=1}^{m} p_i \ln(p_i) \tag{4}$$

subject to

$$\sum_{i=1}^{m} p_i = 1 \tag{5}$$

$$\sum_{i=1}^{m} p_i x_i = \mu \tag{6}$$

$$\sum_{i=1}^{m} p_i x_i^2 = \mu^2 + \sigma^2 \tag{7}$$

where p_i is the probability of event x_i; μ is the mean of the outcome of events x_i for all i; σ is the standard deviation of the outcome of events x_i for all i.

Different functional forms have been employed for water quality analysis, network design and management. Loftis and Ward (1980) attempted to identify 'regions' of frequencies of sampling in water quality monitoring. The criterion adopted in their study was based on the width of confidence intervals of water quality variables about the annual sample geometric means.

Tai and Goda (1980, 1985) and Goda *et al.* (1981) viewed water pollution as water initially containing a low value of entropy and then eventually discharged with a high value of entropy, which in turn increases the entropy of the environment. To extend the argument further, the diversity of species of organisms in water (or diversity index, DI, based on an ecological rule) was related to the degree of pollution; the DI was calculated using Eq. (1).

Palmer and Mackenzie (1985) discussed 'monitoring-effectiveness' and monitoring cost and the use of optimization methods (incorporated into an interactive computer program) for selecting the aquatic monitoring design that maximizes cost-effectiveness. They developed an approach for cost-effective design of aquatic monitoring networks in which the actual cost minimization issue was addressed by maximizing the statistical power for a specified financial budget or, conversely, minimizing the cost for a specified statistical power requirement.

Singh and Krstanovic (1987) applied the principle of maximum entropy for deriving a stochastic model for sediment yield from upland watersheds. By maximizing the conditional entropy subject to certain constraints, a probability distribution of sediment yield conditioned on the probability distribution of direct runoff volume was obtained.

Harmancioglu (1984) determined optimal sampling intervals for water quality monitoring using entropy. This work was subsequently extended to the assessment of network efficiency and cost effectiveness (Harmancioglu and Alpaslan, 1992). Entropy was used to quantify information contained within a set of water quality data obtained from a network. Using this information, the efficiency of a network was then analyzed by maximizing the amount of information collected from the network. Cost effectiveness, on the other hand, was evaluated by comparing the costs of monitoring versus the information gained via monitoring. It was shown by Harmancioglu and Alpaslan (1992) that entropy principle was applicable for network assessment, particularly in cases of rationalization of networks. One of the practical difficulties encountered in the application of entropy was the need to approximate the continuous probability distribution function by a discrete function. This discretization was found (Harmanciglou *el al.*, 1985) to be critical to the value of the entropy provided by the analysis and has the potential to change the decision arising from the entropy-based analysis. Singh (1997) evaluated the effect of discretization on entropy.

Harmancioglu *et al*. (1992a, b, c) presented a comprehensive review of design of water quality networks and discussed the potential of entropy in the design of water quality monitoring networks. Application of entropy to the design of monitoring network yields promising results, especially in the selection of technical design features, such as monitoring sites, time frequencies, variables to be sampled and sampling duration. Furthermore, it permits a quantitative assessment of the efficiency and benefit/cost parameters.

Kusmulyono and Goulter (1994, 1995) applied POME to develop a methodology for prediction of water quality values at discontinued sites. The methodology provides unbiased predictions of water quality levels at upstream tributaries and on the mainstream of a river, given observed changes in the distribution of the same water quality parameter at a downstream location. The method also has potential for identifying the location of causes of observed changes in water quality at a downstream location. The methodology employs the principle of minimum discrimination information (MDI) due to Kullback and Leibler (1951).

Harmancioglu and Singh (1998) reviewed the advantages as well as the limitations of the entropy method as applied to the design of water quality monitoring networks. Given an observed change in water quality levels at a downstream location, the entropy based formulation predicts the probabilities of each possible water quality level at each of the upstream stations.

Kaplan and Howitt (2002) developed a sequential entropy filter for disaggregating non-point sources from ambient data. A numerical simulation based on sediment loading was provided to illustrate the ability of the sequential entropy filter to recover the underlying parameters and optimally disaggregate ambient sediment load among non-point sources.

Jha *et al.* (2204) applied the principle of maximum entropy to simulate water quality of River Gomti, India. The analysis was done knowing the historical probability distribution at each unmonitored site, observed water quality values at all the continuing sampling stations, and an expression conveying information related to the historical relationship between continuing and unmonitored stations.

In the present work, the entropy method, based on Eqs. (4) to

(7), is applied to six river systems in India, which are located in different geographical regions of the country and have different human interventions in terms of point and non-point sources of pollution.

2. Study Areas and Data Collection

2.1 Study Areas

2.1.1 Baitarani River System

The Baitarani River, originating from Guptaganga hills in Keonjhar district of Orissa, India, at an elevation of 900 m, is located in the eastern part of India (Fig. 1). The drainage area of the river basin is 14218 sq. km. and is more or less oval shaped. The length of river is 360 km before it joins the Bay of Bengal. The geology of the area plays an important role in deciding the soil characteristics, the mineral resources, erosional potential and status, and the quality of surface and ground waters. The rock types are mainly granite, gneisses, Schistose, Khondalites and Quartzite. The middle Baitarani basin is partly hilly and partly plain and the lower Baitarani basin is the coastal area. The climate here is extreme in nature. The maximum recorded temperature in the basin in summer days is 45^0 centigrade and minimum temperature in winter days is 5^0 centigrade. The maximum and minimum annual rainfall varies from 3094 mm to 642 mm. The average annual rainfall of the basin is 1488 mm. Agriculture is the main source of livelihood of the people of the basin. Fishing is another important source of their livelihood.

2.1.2 Brahmani River System

The Brahmani River rises near Nagri village in Ranchi district of Jharkhand at an elevation of about 600 m, and is located in the eastern part of India (Fig. 1). The drainage area of the Brahmani is 39,268 km^2 and the basin lies in the states of Orissa, Jharkhand and Chattisgarh. About 37.6% the basin, 14768 sq. km., is a cropped area. The total length of the river is 799 km, and forms a

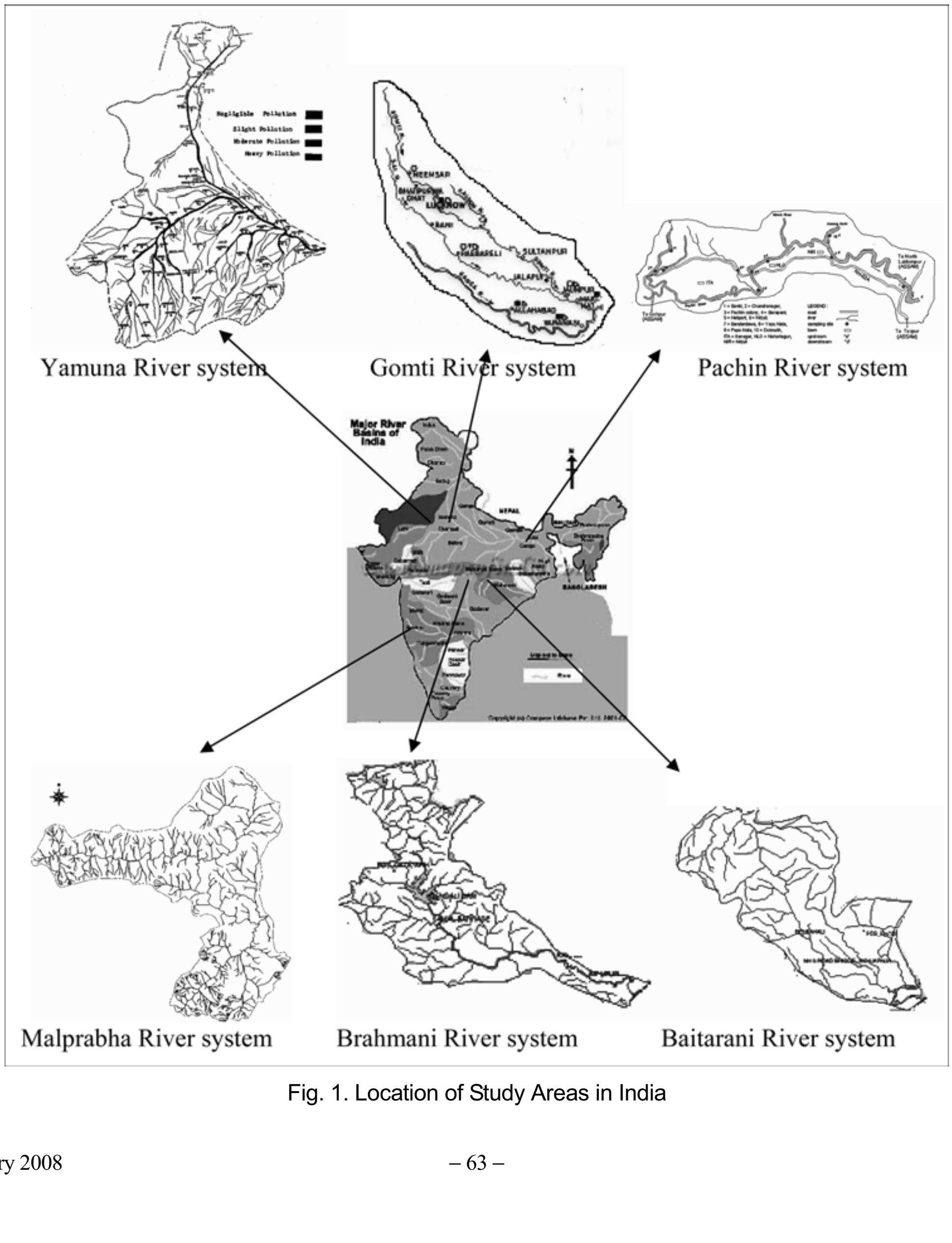

Fig. 1. Location of Study Areas in India

delta area before falling into the Bay of Bengal. The rock types are mainly granite, gneisses, Schistose, Khondalites and Quartzite. The mean annual rainfall of the basin is 1570 mm. The river plays an important role in the socio-economic, agricultural and industrial sector in Orissa.

2.1.3 Malprabha River System

The Malprabha River is an important right bank tributary of the Krishna River system, and is located in the southern part of India (Fig. 1). The river originates at an elevation of 1038 above mean sea level. The river traverses a course of about 306 km in the hardrock region before meeting the Krishna River. The total basin area of the Malprabha is 11549 sq. km. The variation of rainfall is significant, with the mean annual rainfall varying between 2000 to 3500 mm. The mean relative humidity is high during the south-west monsoon season and comparatively low during the non-monsoon period. The average annual temperature varies between 14 to 35°C. The mean annual discharge varies between 1000 to 2300 mm.

2.1.4 Pachin River System

The Pachin River is a perennial river originating from the foothills of lesser Himalayas, passes through the Itanagar Capital complex of Arunachal Pradesh, and finally drains into the Brahmaputra River in Assam. The river is located in the north-eastern part of India (Fig. 1). The drainage basin is characterized by sandstone, siltstone, shales, intrusions of gneiss and granites of tertiary to pleistocene formations (Wadia, 1981). Generally, the monsoon period sets in between May and August, and it accounts for about 80 percent of the total annual rain. The average annual rainfall and air temperature are 2578 mm, and 23.8°C, respectively. The mainstream of Pachin River is characterized by a variation of about 140 m in its altitude ranging between 245 m above the MSL in the west and 105 m in the east. The basin is subject to considerable deforestation for shifting cultivation, and setting up of residential places, townships, etc. In the upstream, the drainage basin is covered with forested mountains; and the river flows much below the bank. While in the downstream part, the effects of human activities, including agricultural, are prevalent.

2.1.5 Gomti River System

The Gomti River basin falls in the Gangetic plains of Uttar Pradesh, covers an area of 3028 sq. km, and is located in the northern part of India (Fig. 1). It is comprised of thick alluvium deposits of fluvial provenance, varying in thickness from 500 to 6000 m, the thickest being in the northern zone, having various grades of sand and clay in alternation. The basin physiography is uneven in north, smooth in central part and dissected in tail reaches, having several wetlands in depressions and palaeochannel areas. In the upper reaches the lithology is more sandy, whereas in central and tail reaches, the lithology is dominated by clay. The climate of the basin is suitable for 300 percent cropping intensity in three crop seasons: Kharif (July-September), rabi (October-March) Zaid (April-June) with evapotranspiration varying between 1500-1800 mm and rainfall between 900-1500 mm per annum. The water resources are obtained from rainfall, excessive ground water recharge and canal water supply for 96 percent irrigation intensity.

2.1.6 Yamuna River System

The River Yamuna is the largest tributary of the River Ganga and its catchment area comprises about 42% of the Ganga basin area in the Indian Territory. The river originates from the Yamunotri glacier near Bandarpunch at an elevation of about 6320 metres above mean sea level in the Tehri Garhwal district of Uttaranchal State, and is located in the northern part of India (Fig. 1). The basin area is 3,66,223 km^2. Before its confluence with the River Ganga at Allahabad, many tributaries, viz., the Hindon, the Karon, the Sangar and the Rind join the river on its left bank, while the

Table 1. Period of Observations on Water Quality Variables

River Basin	Sampling stations	Dissolved oxygen	Biochemical oxygen demand	Electrical conductivity	Hardness
Baitarani	Joda Anandpur Jajpur Chandabali Dhamra	25 years	25 years	25 years	25 years
Brahmani	Tilga Talcher Jenapur	25 years	25 years	25 years	25 years
Malprabha	Khanpur	5 years	5 years		
Pachin	Senki Chandranagar Pachin Colony Barapani Heliport Nirjuli Bandedewa	5 years	5 years		
Gomti	Lucknow (u/s) Lucknow (d/s) Jaunpur Maighat	10 years	10 years	10 years	10 years
Yamuna	Panipat Delhi (u/s) Delhi (c/l) Delhi (d/s) Mohana Mathura (u/s) Mathura (c/l) Mathura (d/s) Agra (u/s) Agra (d/s) Etawah (u/s) Etawah (d/s) Auraiya Hamirpur Pratappur (u/s) Pratappur (d/s)	10 years	10 years	10 years	10 years

Chambal, the Sind, the Betwa and the Ken on its right bank. The total length of Yamuna from its source at Yamunotri to its confluence with the Ganga at Allahabad is about 1376 km. The cultivable land in the basin in about 60% of the total area, while the forest cover is about 12.5%, leaving the non-arable land to about 27.5%, which in essentially steep and rocky. Modern scientific agricultural practices involving application of chemical fertilisers, pesticides and insecticides are extensively developed in the Yamuna basin. Important industries in the basin include fruit processing, breweries and distilleries in the hilly tracts, sugar mills, distilleries, automobile manufacturing, chemicals, drugs, electronics, thermal power station and food processing industries in plains.

2.2 Data Collection

Four important water quality variables, namely dissolved oxygen (DO), biochemical oxygen demand (BOD), electrical conductivity (EC) and hardness (TH), were considered. DO is a very important parameter of water quality and is an index of physical and biological processes occurring in water, whereas BOD is the most commonly used parameter to define the strength of a municipal or organic industrial wastewater. EC is the indicator of the presence of higher concentration of electrolytes (acids, bases and salts) in water. TH is another indicator of water quality, especially for domestic uses. It is the sum of concentration of alkaline earth metal cations present in water, which in high concentration is not suitable for bathing, washing and cooking. The availability of water quality data at various stations of a river system is shown in Table 1. Figs. 2, 3, 4 and 5 illustrate the minimum and

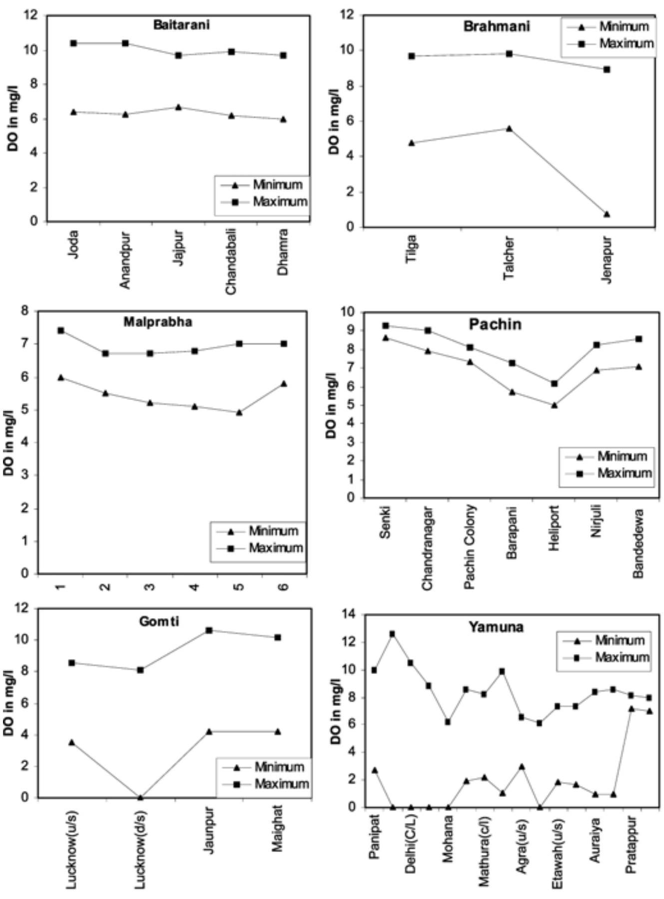

Fig. 2. Minimum and Maximum Dissolved Oxygen Values

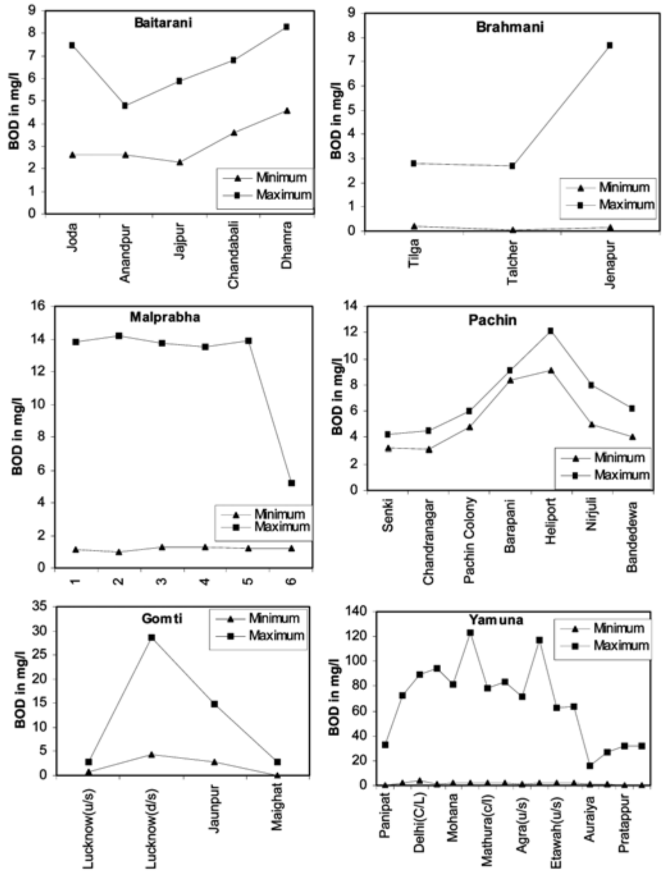

Fig. 3. Minimum and Maximum Biochemical Oxygen Demand Values

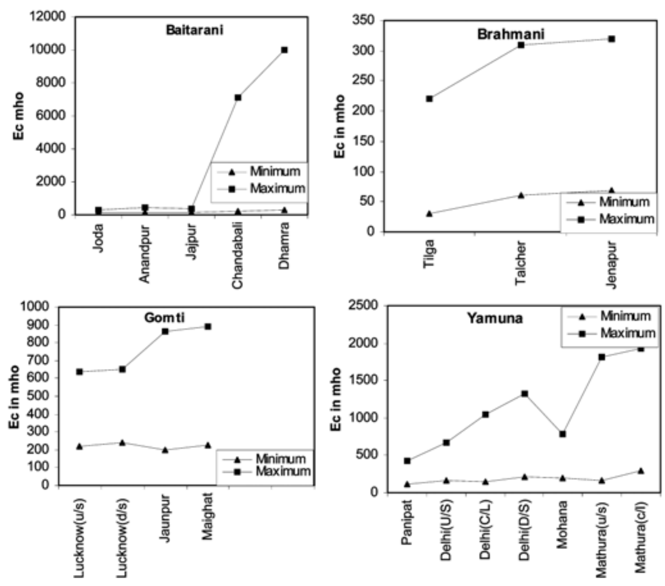

Fig. 4. Minimum and Maximum Electrical Conductivity Values

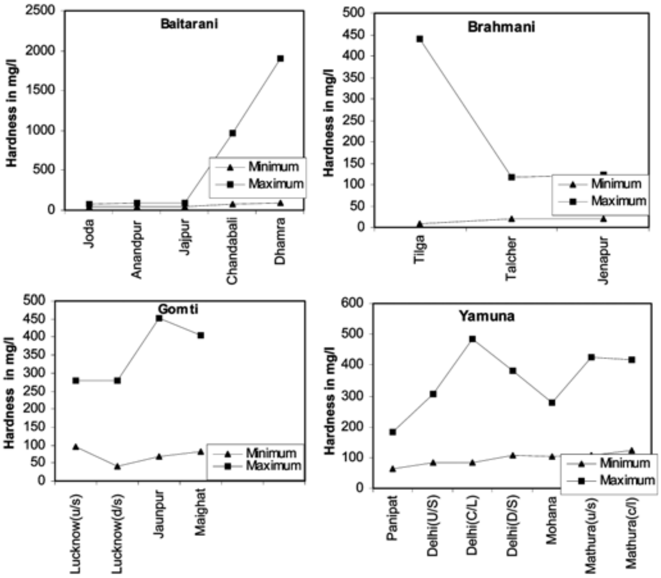

Fig. 5. Minimum and Maximum Hardness Values

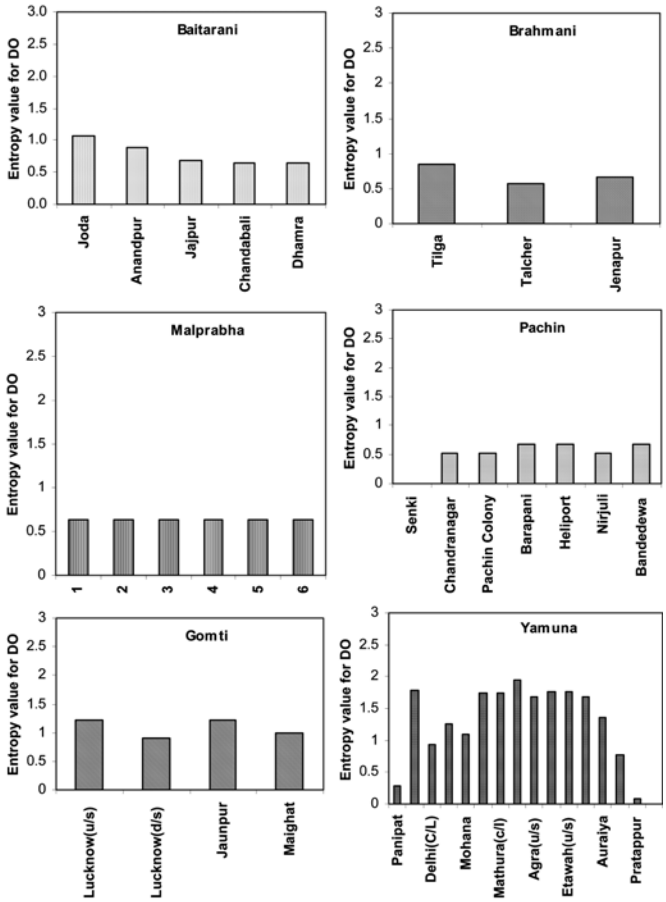

Fig. 6. Entropy Values for Dissolved Oxygen

maximum values of each water quality variable (DO, BOD, EC, TH). It is seen from these figures that there is a significant variation in the minimum and maximum values of each variable. In particular, the minimum and maximum BOD values of Yamuna River system are found to have significant differences, whereas very slight variation is found in the minimum and maximum values of BOD in Pachin River system.

3. Methodology

The water quality variables of DO, BOD, electrical conductivity and hardness collected from different sampling stations were used to assess the water quality of each river and its entropy values. Eqs. (4) to (7) were used to compute entropy at different sampling stations of Baitarani, Brahmani, Malprabha, Pachin, Gomti and Yamuna river systems. In order to discern the effect of level of discretization (i.e., the choice of the number of intervals) on the water quality values, Eqs. (4) to (7) were solved for different level of discretizations, namely, 8, 16, 24, 32, 40 and 48 (Kusmulyono and Goulter, 1995). It was found that the 24 number of intervals provided optimum results; this means that the entropy value does not changes with a higher number of intervals and the computation time reduces significantly. The method proposed by Kusmulyono and Goulter (1993) was intended primarily to predict the mean value of water quality over a selected period of time. However, in the present case it was found that the monthly data represents the seasonal variation and spatial variation accurately. Therefore there is no need to average the water quality data over a selected period of time.

4. Results and Discussion

4.1 Entropy of River Systems

The entropy values for DO, BOD, electrical conductivity and hardness for Baitarani, Brhamani, Malprabha, Pachin, Gomti and Yamuna river sytems were estimated using Eq. (4). Fig. 6 illustrates the Entropy for DO in all the six river basins. In case of DO, higher entropy implies physical and biological processes in water, including turbulence phenomena, low to high discharges and variation in the influx of pollutants in the river system. In Yamuna, Gomti and Baitarani river systems the entropy values were found to be higher (>1) for all the sampling stations.

For BOD, which defines the strength of a municipal or organic industrial wastewater, the Yamuna River system was found to have the highest entropy values, as shown in Fig. 7. Gomti River system also had high entropy values. These values indicate higher level of pollution at different sampling stations in the Yamuna and Gomti River systems, which are located in the Ganga plains in northern India. During field surveys it was found that human interventions in the form of point as well as non-point sources of pollution are maximum in both river basins and hence in the Ganga basin. The Malprabha River system, located in the southern region, and the Pachin River system, located in the north-east region, had low entropy values, indicating less pollution. This is expected because they are found to be less affected by human interventions. In the eastern region, the Baitarani River system was found to be more polluted than the Brahmani River system.

The electrical conductivity values of Yamuna and Gomti River systems further supplement the presence of acid, bases and salts in terms of higher entropy (Fig. 8). Fig. 9 illustrates the presence of calcium carbonate ($CaCO_3$) in terms of entropy in the Baitarani, Brahmani, Gomti and Yamuna river systems. In actual field condition the River Yamuna and River Gomti waters were not found suitable for bathing, washing, drinking and cooking purposes.

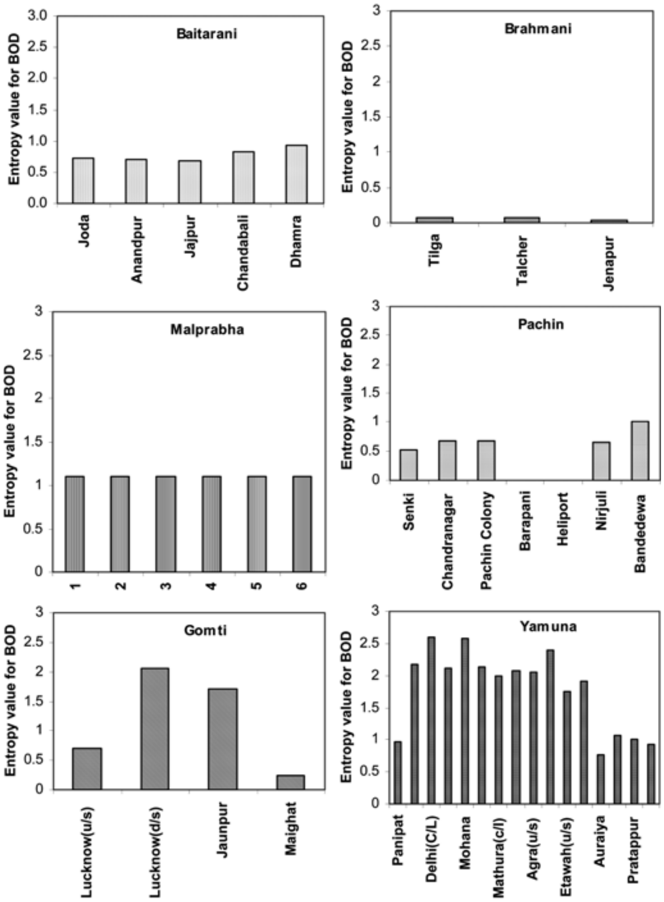

Fig. 7. Entropy Values for Biochemical Oxygen Demand

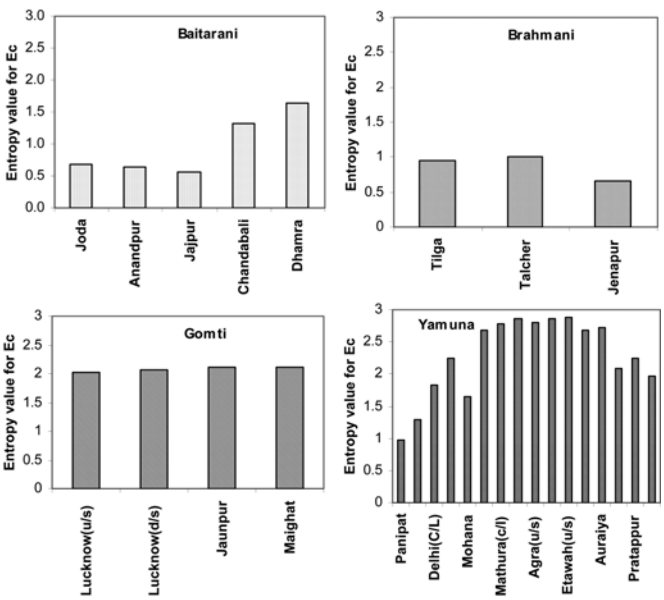

Fig. 8. Entropy Values for Electrical Conductivity

4.2 Temporal variation in Entropy of River Yamuna and River Gomti

The entropy values of all the four water quality variables at

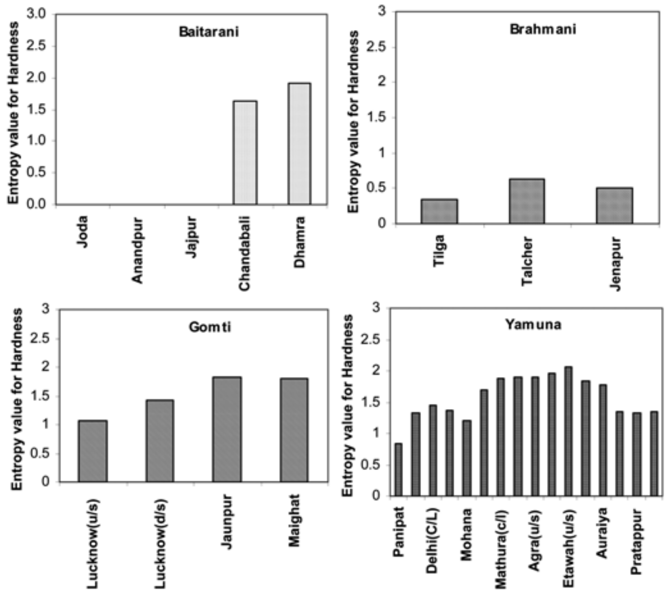

Fig. 9. Entropy Values for Hardness

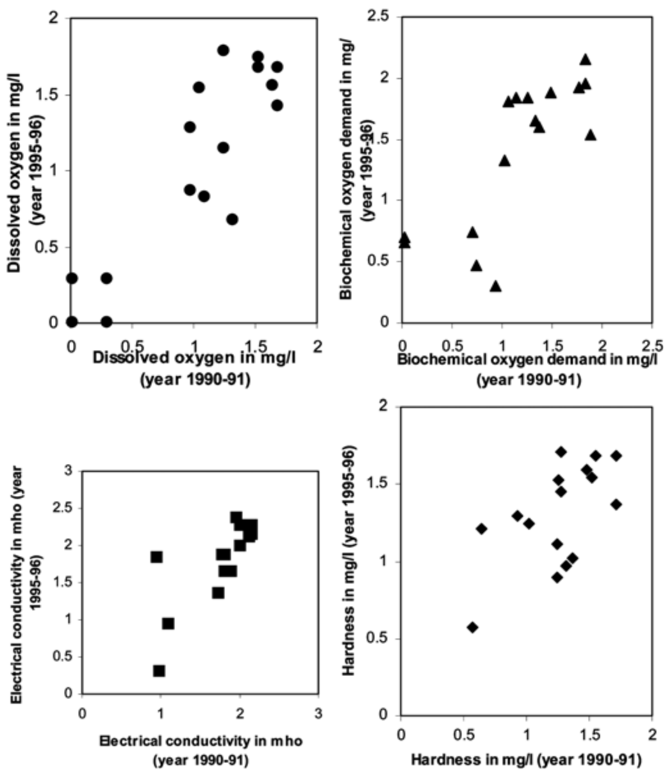

Fig. 10. Comparison of Entropy Values oF Different Water Quality Variables for the Years 1990-91 and 1995-96 in Yamuna River System

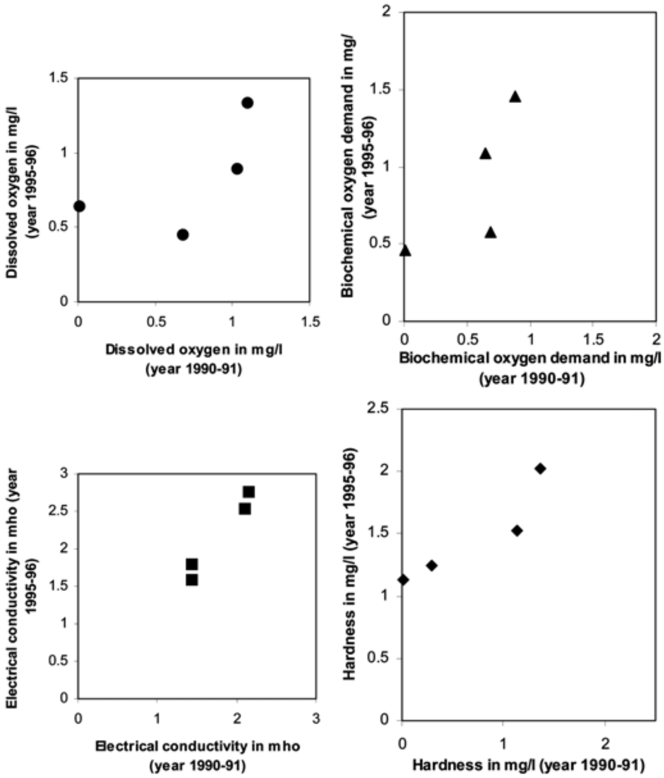

Fig. 11. Comparison of Entropy Values of Different Water Quality Variables for the Years 1990-91 and 1995-96 in Gomti River System

different sampling stations of River Yamuna at different time scales (during 1990-91 and 1995-96) indicate a gradual increase in entropy (water pollution) due to human interventions in the form of point as well as non-point sources of pollution, as shown in Fig. 10. This figure clearly indicates most of the values biased towards y-axis (entropy values for water quality variable for the year 1995-96). Similarly, the entropy values of all the four water quality variables at different sampling stations of River Gomti at different time scales (during 1990-91 and 1995-96) also indicate a gradual increase in entropy values, i.e., water pollution (Fig. 11).

5. Conclusions

Application of entropy to the assessment of water quality of Baitarani, Brahmani, Malprabha, Pachin, Gomti and Yamuna rivers reveals that entropy is an effective indicator of the pollution level at any section of any of the six river systems. It indicates that the Yamuna and Gomti River systems are highly polluted; Baitarani is moderately polluted; and Brahmani, Malprabha and Pachin river systems are marginally polluted. This information is extremely useful for water resources planners and managers. The temporal variation in the entropy values indicates a gradual increase entropy values and hence pollution.

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