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A Study on Mesh Size Dependency of Finite Element Blast Structural Analysis Induced by Non-uniform Pressure Distribution from High Explosive Blast Wave

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Abstract

Accurate computational blast analysis can be an effective replacement for costly blast field test. HFPB (high fidelity physics based) blast analysis method including high strain rate dependent material models and appropriate blast wave models is a representative precise blast analysis method. HFPB blast analysis method can be used for various analyses such as structural analysis, retrofitting design analysis, fragmentation analysis, and energy absorbing analysis. When analyzing the behaviour of structure under blast loading by HFPB finite element methods, load gradient differences occurs dependent on the mesh size. This causes gaps between the explosive energy and internal energy of structures and the results of analysis become mesh-size-dependent. In this study, the analytical considerations were presented for mesh sensitivity due to non-uniform pressure load distribution on the structure subjected to blast wave of high explosive with relatively close stand-off. Through the analysis results, the maximum element size which ensures the mesh-size independent analysis results is suggested.

Keywords: *HFPB (high fidelity physics based) blast analysis method, load gradient, mesh-size-dependent, mesh sensitivity, non-uniform pressure, the maximum element size*

1. Introduction

HFPB (high fidelity physics based) FEM (finite element method) can be used as a replacement for costly structure blast experiments where the experimental results verified HFPB FEM results can simulate actual blast experiments giving accurate information. Also, some conditions and data, which are impossible to generate and to obtain can easily be simulated and obtained in HFPB FEM as virtual experiment simulations and data measuring devices. Therefore, many nations and institutions without the capabilities to perform blast structure experiments for the defense against explosive terror and accidents can use HFPB FEM for the replacement purpose. For all of the reasons mentioned above, it is vital to develop effective HFPB FEM system for blast analysis and protective structure design. Recently, lots of studies on HFPB blast analysis method including material models reflecting strength increasing and strain rate effect due to extremely fast blast wave pressure have been carried on (Bangash, 1993; Byun *et al.*, 2006; Landry, 2003; Malvar *et al.*, 1997; Nam *et al.*, 2007; Shugar *et al.*, 1992).

When blast wave from explosion imposes pressure loads on the structures, the severe stress gradient on the surface of structure can be produced due to large amount of peak pressure from blast wave and non-uniformly distributed blast wave pressure. Even though the uniformly distributed pressure is loaded to the surface of structure, this severe stress gradient can be produced dependent on stiff boundary conditions.

When analyzing the behaviour of structure subjected to explosive blast loading by finite element methods, load gradient differences occurs dependent on the mesh size. This causes gaps between the explosive energy and internal energy of structures and the results of analysis become mesh-size-dependent. These problems can be solves by using smaller size element which satisfy the consistent results. Using small size element for blast analysis can also control the hourglass effect which usually occurs when using solid element. However, the smaller the element size is, the longer calculating time becomes. This can make the weak point of explicit integration method weaker in aspects of calculating effectiveness.

In this study, the analytical considerations were presented for mesh sensitivity due to non-uniform pressure load distribution on the structure subjected to blast wave of high explosive with

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relatively close stand-off. Through the analysis results, the maximum element size which ensures the mesh-size independent analysis results is suggested. The mesh size independent border point is considered as the element number of steep variation of structural behavior under blast load. This point coincides with the convergence point which makes the difference between internal energy and hourglass energy zero. Target structures for blast analysis are determined as plate type structure since it has wide surface and the element type is determined as solid element since it has tendency to cause hour glass effect.

2. Blast Wave from High Explosives

Blast wave is generated by an explosive event, when an unconfined charge detonates in air, it gives rise to a shock wave with a practically discontinuous pressure front that propagates with supersonic speed. The shock wave is initiate by the very rapid release of a large amount of energy in the surrounding medium with a sudden increase of pressure at the shock front followed by a gradual decrease of pressure as shown in Fig. 1. The maximum overpressure that occurs at the shock front is called the peak overpressure. As this wave moves away from the centre of the explosion, the overpressure in the shock front decreases steadily. The pressure behind the front also falls off exponentially.

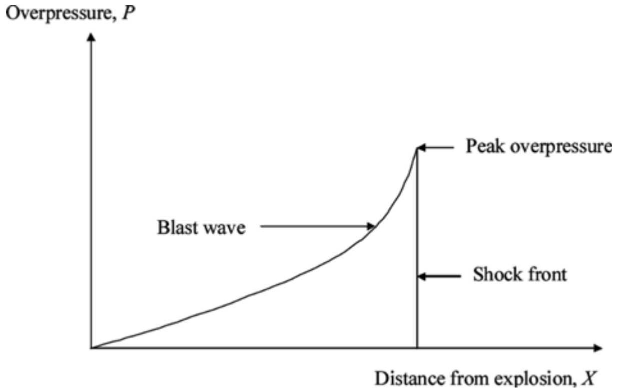


Fig. 1. Variation of Overpressure with DIstance in a Shock Wave (Zineddin, 2002)

After a short time, the overpressure behind the shock front drops rapidly and becomes smaller than that of the surrounding atmosphere. This pressure domain is known as the negative phase. The front of the blast wave weakens as it progresses outward and its velocity drops toward the velocity of sound in the undisturbed atmosphere. This sequence of events is shown in Fig. 2 (ASCE, 1985). The overpressure in the curves marked t_1 through t_3 has not fallen below that of the atmospheric pressure. In the curve marked t_4, the overpressure becomes negative at some distance behind the shock front.

The observed characteristics of air blast waves are found to be affected by the physical properties of the explosion source.

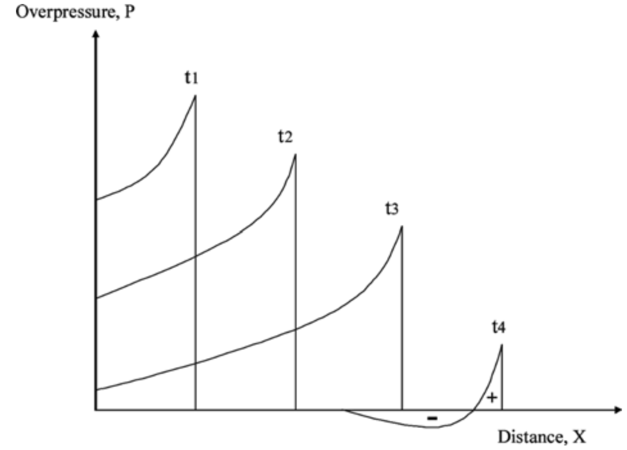


Fig. 2. Variation of Overpressure with Distance at Various Times (ASCE, 1985; Glasstone, 1977)

However, regardless of the explosive source, at a certain distance from the explosion center, all blast waves have almost the same configurations. Typical blast pressure profile is illustrated in Fig. 3 (TM5-1300, 1990; ASCE, 1999). At an arrival time of t_A after the explosion, pressure at that position suddenly increases to a peak value of overpressure, P_{so}, over the ambient pressure, P_o. The pressure then decays to ambient in time to, till it reaches a partial vacuum of negative peak pressure, P_{so}-, and eventually returns to ambient in time $t_o + t_o$-. The quantity P_{so} is usually referred to as the peak side-on overpressure, or merely as peak over pressure. The pressure versus time variation curve is divided into two segments: (a) positive phase and (b) negative phase. Generally, when considering structural design for blast load, negative pressure is neglected as its loading effect is too small. Meanwhile, the integration area under the pressure curve represents impulse, is, as shown in the Fig. 3 (TM5-1300, 1990).

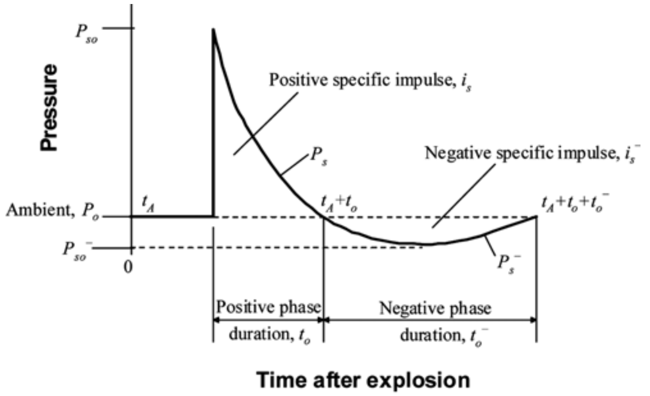


Fig. 3. Free-field Pressure vs. Time Variation (TM5-1300, 1990)

The characteristics of a blast wave resulting from an explosion depend mainly on the physical properties of the source and the medium through which blast waves propagate. To create reference blast experiments, some controlled explosions have been conducted under ideal conditions. To relate other explosions with non-ideal conditions to the reference explosions, blast scaling laws can be employed. The most widely used approach to blast wave scaling is that formulated by Hopkinson, which is commonly described as the cube-root scaling law (Hopkinson, 1915; Tolba, 2001). The scaled distance, Z, is defined using the Hopkinson-Cranz's cube root law as (ASCE, 1999):

$$Z = R/E^{1/3} \text{ or } Z = R/W^{1/3} \tag{1}$$

where Z is scaling distance, R is stand-off distance from the target structure, E is total explosive thermal amount of energy, W is charge weight of equivalent TNT amount. The scaling distance is used for evaluation of blast wave characteristics. Once the scaling distance from Eq. (1) is determined, various values of blast wave parameters can be obtained through the referenced diagram shown in Fig. 4. The reference diagram is based on a number of experimental works from the past. The blast wave parameters in Fig. 4 are used for plotting the pressure-time curve and calculating peak over pressure for blast analysis and design.

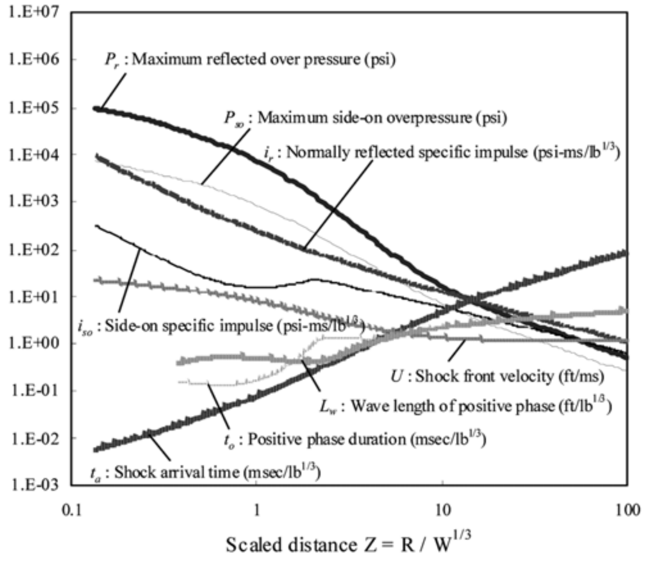


Fig. 4. Positive Phase Blast Wave Parameters for Hemispherical TNT Explosion on the Surface at Sea Level (TM5-1300, 1990; ASCE, 1999)

Meanwhile, there are several issues which should be considered in the process of blast analysis for the structures subjected to high explosive loading. The first issue to consider is a hourglass effect that causes singularity in global stiffness matrix from using a wrong element type. The second issue is a mesh size dependency that causes stress gradients to become inaccurate due to non-uniform distribution of blast pressure. The third issue is development of high strain rate dependent material model reflecting dynamic increasing effect and the fourth issue is modeling of second fragmentation of material due to the propagation of blast wave through the structural members. The final is reasonable determination of boundary conditions of structural members. From those issues, the problem about non-unifrom pressure distribution and mesh size dependency have not been studied enough even though it is the fundamental problem for the high fidelity physics based analysis.

3. Non-uniform Blast Pressure Loading on the Structure

Blast wave travels away from the centre of the explosive outward in a radial shape. This radial shape of blast wave induces different arrival times and peak values of over-pressures on the structure. And the blast pressure load distribution becomes non-uniform due to the different arrival times and peak over-pressures of blast wave. The non-uniformity of pressure distribution is dependent on the size and dimension of structure.

As the stand-off distance is relatively closer compared to structural size, this non-uniformity becomes distinct. For the same relative stand-off, the wider width induces more non-uniform pressure distribution. Among the structural members, the walls and slabs are more influenced by this non-uniform pressure due to their shape and dimension. Non-uniform pressure distribution on the slab structure is idealized as shown in Fig. 5.

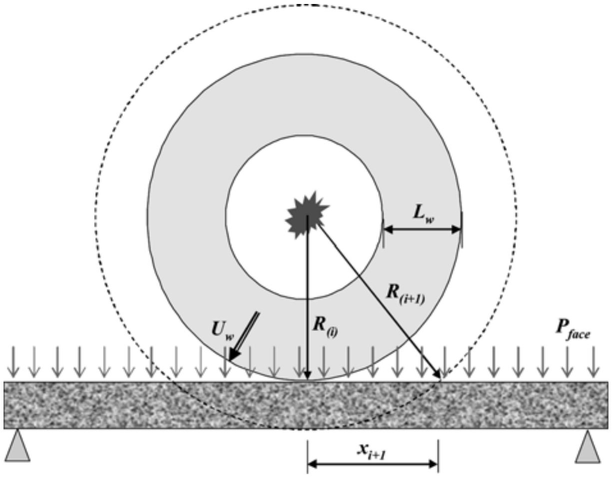


Fig. 5. Pressure Variation with Structural Geometry

Assume that the blast wave is traveling from the explosion at a distance of R from the structure, the blast wave with wave velocity of U_w and wave length of L_w imposes the peak pressure on the center of slab at an arrival time t_A (see Fig. 5). As the discrete duration time Δt goes by, blast wave imposes another different peak pressure at a distance of x_{i+1}. If the radius of blast wave is expanded from R_i to R_{i+1} during the time increment Δt, the traveling distance of blast wave can be expressed as

$$R_{(i+1)} = R_i + U_w \Delta t \tag{2}$$

And, the expanded radius R_{i+1} is calculated as

$$R_{(i+1)} = \sqrt{R_i^2 + x_{i+1}^2} \tag{3}$$

From Eqs. (2) and (3), the point x_{i+1} on which next peak pressure is imposed at a time $t_A+\Delta t$ can be obtained as shown in Eq. (4).

$$x_{i+1} = \sqrt{(U_w\Delta t) + 2RU_w\Delta t} \tag{4}$$

Meanwhile, the peak side-on overpressure on the point of x_{i+1} is determined by Eq. (5) based on the Henrych' equation (Henrych, 1979).

$$\begin{aligned} P_{so(i+1)} &= \left(\frac{1.4072}{R_{(i+1)}}W^{1/3} + \frac{0.5540}{R^2_{(i+1)}}W^{2/3} - \frac{0.0357}{R^3_{(i+1)}}W + \frac{0.000625}{R^4_{(i+1)}}W^{4/3}\right) \\ &\qquad 0.5W^{1/3} \le R \le 0.3W^{1/3} \\ P_{so(i+1)} &= \left(\frac{0.6194}{R_{(i+1)}}W^{1/3} + \frac{0.0326}{R^2_{(i+1)}}W^{2/3} - \frac{0.2132}{R^3_{(i+1)}}W\right) \\ &\qquad 0.3W^{1/3} \le R \le W^{1/3} \\ P_{so(i+1)} &= \left(\frac{0.0662}{R_{(i+1)}}W^{1/3} + \frac{0.405}{R^2_{(i+1)}}W^{2/3} - \frac{0.3288}{R^3_{(i+1)}}W\right) \\ &\qquad W^{1/3} \le R \le 10W^{1/3} \end{aligned} \tag{5}$$

where, P_{so} is the side-on overpressure at the wave front R is the stand-off distance and W is the charge weight. Finally, the peak overpressure on the point of x_{i+1} is determined with Eqs. (5) and (6).

$$P_{face(i+1)} = P_{so(i+1)} + C_D P_{d(i+1)} \tag{6}$$

where, P_{face} is front faced peak overpressure, C_D is front faced drag coefficient, and P_d is dynamic pressure which can be obtained from the experimental chart (TM5-1300, 1990). For implementation of non-uniform pressure distribution to the FEM analysis, the duration of positive phase is discretized into several reasonable time steps and the pressure loads on each point at time t_i are calculated using Eq. (6). The calculated pressure history is used as load function at each element.

4. Mesh Size Dependency of FE Blast Analysis

In this study, severe stress gradient on the structure due to non-uniformly distributed blast pressure is considered by applying the different time-load curves for each finite element. In addition, mesh size dependency is analytically investigated and reasonable mesh size is suggested for the solution of mesh size dependent problem. The reasonable mesh size which ensures the objectivity of analysis results is proved to control the hourglass effect. For the analysis, the explicit code LS-DYNA is used.

4.1 Comparison of Non-uniform and Uniform Peak Pressure on the Wall

For verification of the non-uniform pressure model, the blast pressure from the explosive is imposed on the plate uniformly and non-uniformly, respectively. To make the difference bigger, a simple metal square plate is chosen as an analysis model and the explosion is assumed as a 5 kg TNT free air burst and 1m above the 2 m × 2 m plate as shown in Fig. 6. The material properties are as follows: mass density of 7,830 kg/m^3; Young's modulus of 2,100 MPa; Poisson' ratio of 0.3; and yield stress of 6 MPa.

The acceleration of node is calculated at the different time and on the various points which are distributed from the center to corner. The acceleration history from the each analysis result has different trends as shown Fig. 7. Peak acceleration values of each analysis result are similar within an 8% difference, but the duration of reaction under uniform blast pressure is much longer than that of non-uniform blast pressure. So, the total impulse of uniform blast pressure is bigger than that of non-uniform blast pressure and deformation is almost uniform. This might lead to an overestimation of the deformation of the plate. However, even though the global deflection of the uniform blast pressure plate is relatively large, the concentration effect of a non-uniform blast pressure plate might result in worse of local behavior (See Fig. 8).

4.2 Verification of the Non-uniform Model

The blast load generation code, CONWEP, is widely used in the design and analysis of protective structures. CONWEP is based on the theory of the technical manuals such as TM5-855-1 (1986), and is known as somewhat conservative as far as

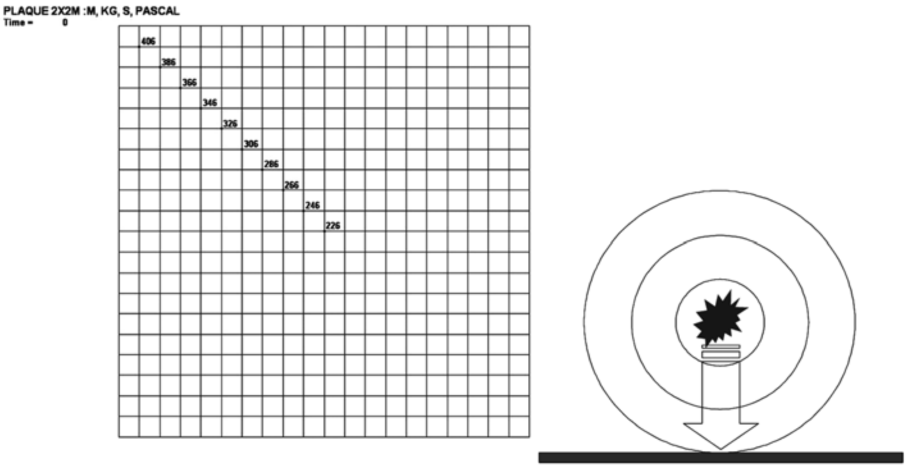
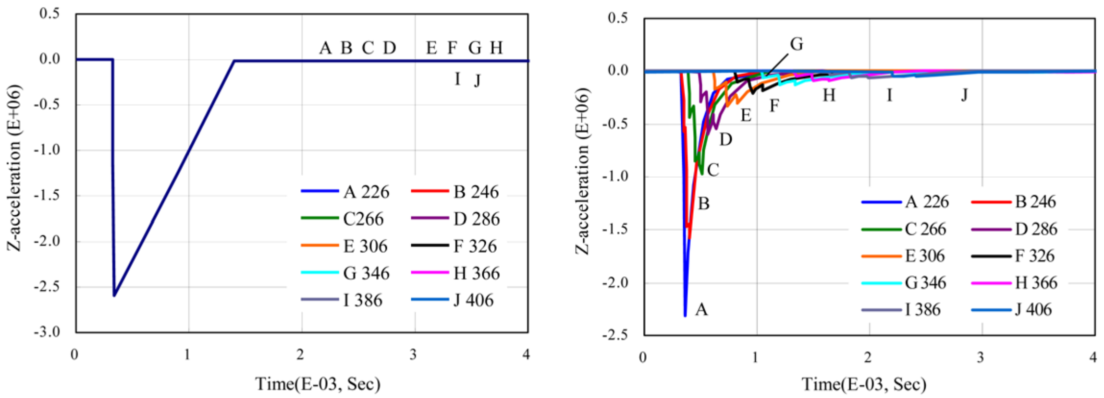
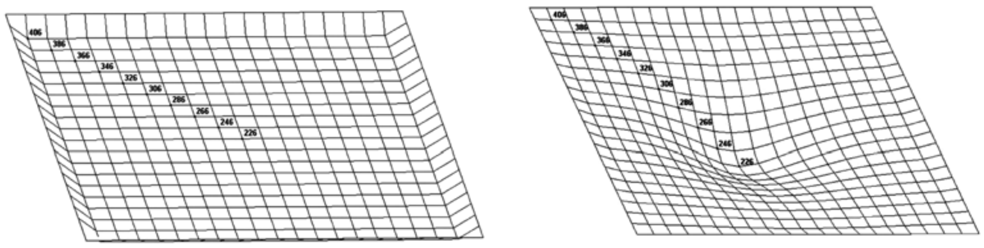


Fig. 6. Modeling of Plate Under Explosion



(a) Uniform blast pressure (b) Non-uniform blast pressure

Fig. 7. Acceleration History of Different Points in the Plate



(a) Uniform blast pressure (b) Non-uniform blast pressure

Fig. 8. Deformation Shape According to the Pressure Distribution Types

globally distributed pressure loads. This code calculates blast wave parameters using the embedded data and diagrams. The calculation result of the proposed model is compared with both CONWEP and some previous experimental research data and the pressure distribution coincides well on both accounts in Fig. 9.

4.3 Mesh Size Dependency

The analysis results to check the mesh size dependency from the severe stress gradients due to explosive blast wave are shown in Fig. 10. The point A of different node number in each graph of Fig. 10 is located on the exact center point of the differently meshed structure.

The front of blast wave applies the localized pressure to the loaded surface of structure and this can be resulted in mesh size dependency of FEM analysis. This mesh sensitivity has tendency to decrease with smaller mesh size and the explosive characteristics such as amount of explosive and stand-off distance can also affect to the mesh sensitivity of blast analysis. The results of mesh sensitivity analysis show that the severe stress gradients due to loaded blast pressure can be captured regularly under specific mesh size. In the end, it can be said that the mesh size used in analysis example of the paper is uniform but also enough to control the mesh sensitivity due to the severe stress gradients from blast loading.

In case of blast analysis using finite element method, the hourglass effect can come out easily in specific elements such as solid elements. LS-DYNA has 6 hourglass control options for solid elements (LSTC, 2006). In this study, Flanagan-Belytschko viscous form with exact volume integration for solid elements is used to control hourglass effect through the blast analysis.

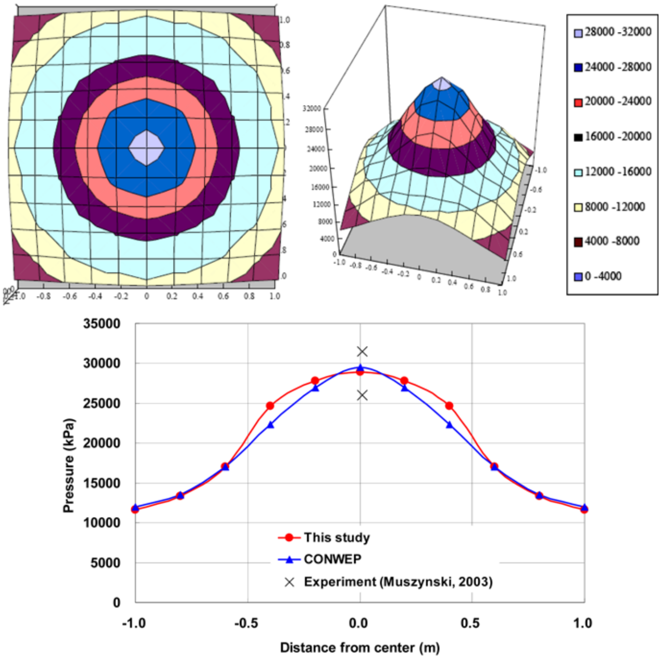


Fig. 9. Comparison of Results

Viscous hourglass control is known as proper for problems

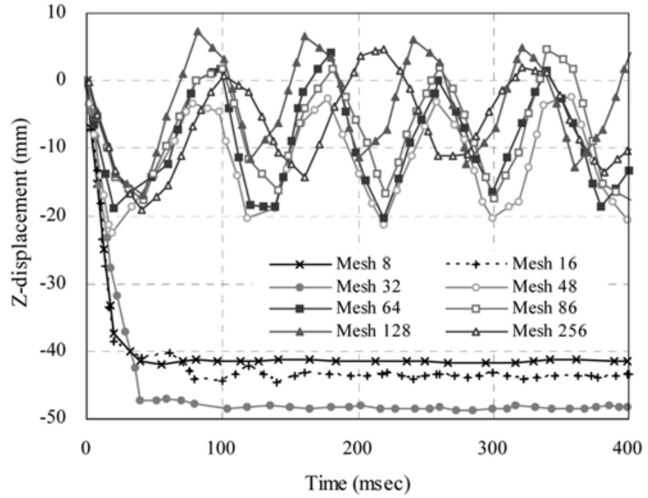


Fig. 10. Mesh Sensitivity Analysis

deforming with high velocities and stiffness control is known as proper for lower velocities, especially if the number of time steps are large. Since the blast analysis is in the regime of very high velocities problem, the viscous hourglass control is adopted in the paper. And also, for solid elements, the exact volume integration provides some advantage for highly distorted elements. Fig. 11 shows the comparison of internal energy and hourglass energy of target structural model. It can be said that the hourglass effect of elements is controlled effectively.

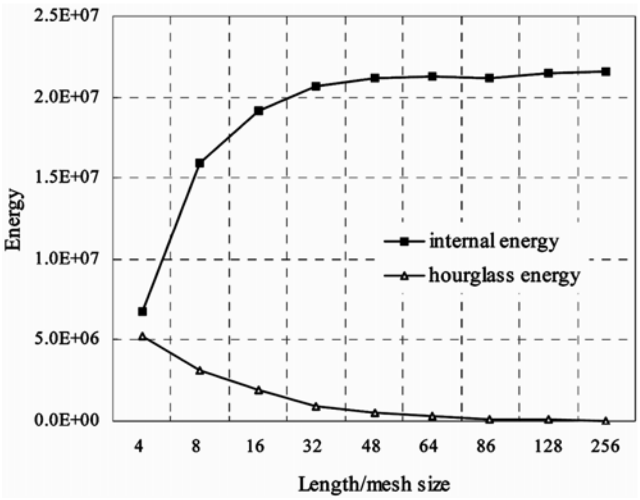


Fig. 11. Comparison of Internal Energy and Hourglass Energy According to Different Mesh Sizes

5. Conclusions

In this study, the high fidelity physics based blast analysis using non-uniform blast pressure distribution model is presented and mesh sensitivity analysis is carried out. Conclusions of this study are as follows:

1. The blast wave loading on structure is modeled considering the relative structure size and the relative stand-off. The suggested model can consider non-uniformly distributed pressure on the front face of structures.
2. The proposed non-uniform pressure model is verified with previous experimental data and test-data based program for calculating blast pressure. The results of proposed model well agree with the previous data. It is considered that the proposed model reflects more realistic phenomena than uniform pressure model which is used for simplified blast analysis.
3. Severe stress gradient on the structure due to non-uniformly distributed blast pressure is considered by applying the different time-load curves for each finite element. In addition, mesh size dependency is analytically investigated and reasonable mesh size is suggested for the solution of mesh size dependent problem. The reasonable mesh size which ensures the objectivity of analysis results is proved to control the hourglass effect.
4. For the further verification of HFPB blast analysis methods, lots of experimental studies are needed. The more experimental data for real size structures should be accumulated and the more precise high strain rate dependent material models should be developed.

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