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# Numerical Simulation on Solitary Wave Propagation and Run-up by SPH Method

**Nam Hyeong Kim* and Haeng Sik Ko****

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## Abstract

Smoothed Particle Hydrodynamics (SPH) is a meshless method which takes a Lagrangian approach. It is different from the traditional methods using mesh system. A number of numerical simulations of a solitary wave propagation using SPH method on a vertical wall and a sloping wall are carried out. The wave run-ups obtained this simulations are compared with the experimental date and resulted good agreement. The water surface profile on a sloping wall is also compared in detail with the experimental data and a good agreement is obtained. The results of particle configuration and velocity distribution at various times are visualized. It is shown that the method can simulate the sloshing phenomena of a solitary wave propagation, run-up, run-down and backwash with good accuracy.

Keywords: *SPH method, solitary wave, wave propagation, wave run-up, water surface profile*

## 1. Introduction

The damage of property and the loss of life have been occurred because of the great storm like tsunami. Indeed, most of the damage associated with tsunamis is related to their run-up on the shoreline. Therefore, it is necessary to analyze on a solitary wave used as a model a tsunami in nature. The study of a solitary wave run-up was preceded through Camfield and Street (1969), Synolakis (1987), Zelt (1991) and Li and Raichlen (2001).

Recently, numerical methods which do not use any mesh structures have been developed, such as meshless methods. Smoothed Particle Hydrodynamics (SPH) is one of meshless methods, and is a Lagrangian particle approach. SPH uses smoothed particles as interpolation points to represent materials at discrete locations, so it can easily trace material interfaces, free surfaces and moving boundaries due to large deformations since SPH uses particles or points rather than mesh as computational frame to interpolate. SPH was developed during the 1970's in astrophysics to study the collision of galaxies and the impact of bolides on planets (Gingold and Monaghan, 1977), and then the method has been successfully used for free surface hydrodynamics problems such as the study of gravity currents (Monaghan, 1994) and wave propagation (Monaghan and Kos, 1999). Recently, the SPH method has been used for wave impact studies of offshore structures (Gomez-Gesteira and Dalrymple, 2004). The paper related to the numerical simulation by SPH has been already published by the authors (Kim and Ko, 2007).

In this paper, the numerical simulations by this method on solitary wave run-up with different amplitudes on a vertical wall and a sloping wall are carried out. The results are compared with the experimental data. And then the solitary wave processes of propagating, run-up, run-down and backwash using the particle configuration and velocity distribution are also visualized. For the detail comparison, water surface profile tracking Lagrangian particles is compared with experimental data.

## 2. SPH

SPH method is based on integral interpolation. The concept of integral representation of a function used in the SPH method starts from the following function.

$$f(x) = \int f(x')\delta(x-x')dx' \tag{1}$$

where $\delta(x-x')$ is Dirac delta function. This equation, in discrete notation, lead to

$$f(x_i) = \sum_j \frac{m_j}{\rho_j} f(x_j) W(x_j - x'_j, h) \tag{2}$$

where $h$ is smoothing length, $j$ is particle, and $m_j$ and $\rho_j$ are mass and density, respectively.

### 2.1 Governing Equation

Continuity equation and Momentum equation by SPH are

*Member, Professor, Major of Civil and Environmental Engineering, Cheju National University, Jeju 690-756, Korea (Corresponding Author, E-mail: nhkim@cheju.ac.kr)

**Graduate Student, Dept. of Civil and Ocean Engineering, Cheju National University, Jeju 690-756, Korea (E-mail: kalbread@nate.com)

expressed as

$$\frac{d\rho_i}{dt} = -\rho_i \sum_j (\nu_j - \nu_i) \nabla W_{ij} \frac{m_j}{\rho_j} \tag{3}$$

$$\frac{d\nu_i}{dt} = -\sum_j m_j \left( \frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij} \right) \nabla_i W_{ij} + F_i \tag{4}$$

where $\nu_i$ and $x_i$ are velocity and position of particle $i$, respectively, $F_i$ is body force, and $\nabla_i$ is the gradient of the kernel for particle $i$. $P$ as the weak compressibility is calculated from Eq. (6).

### 2.2 Kernel Function

Kernel function is an important role in SPH method because analytical particles decide the affected area. In this study, the cubic spline function is used, which is generally used and proposed by Monaghan and Lattanzio (1985).

$$W(R,h) = \frac{15}{7\pi h^2} \times \begin{cases} \frac{2}{3} - R^2 + \frac{1}{2}R^3, & 0 \le R \le 1 \\ \frac{1}{6}(2-R)^3, & 1 \le R < 2 \\ 0, & R > 2 \end{cases} \tag{5}$$

where $R=r/h$, and $r$ is the distance between $x$ and $x'$.

### 2.3 State Equation

The pressure is calculated from the following state equation for flow of weak compressible free surface in SPH method (Batchelor, 1967; Monaghan, 1994).

$$P = B\left[ \left( \frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right] \tag{6}$$

where $B$ is coefficient should be taken with a speed of sound $c$, $\gamma$=7, and $\rho_0$ is initial density.

### 2.4 Artificial Viscosity

The artificial viscosity is used in SPH method to vanish a numerical diffusion in a boundary face. The artificial viscosity has the general form (Monaghan and Gingold, 1983).

$$\Pi_{ij} = \begin{cases} \dfrac{-\alpha \bar{c}_{ij}\mu_{ij} + \beta\mu_{ij}^2}{\bar{\rho}_{ij}}, & (\nu_{ij})(x_{ij}) < 0 \\ 0, & (\nu_{ij})(x_{ij}) \ge 0 \end{cases} \tag{7}$$

where $\alpha$=0.001 and $\beta$=0, $\eta=0.1h_{ij}$, $\bar{c}_{ij}=(c_i+c_j)/2$, $\mu_{ij}=h\nu_{ij}\cdot x_{ij}/(x_{ij}^2+\eta^2)$, and $\rho_{ij}=(\rho_i+\rho_j)/2$.

### 2.5 Boundary Condition

In this study, boundary array is used the stagger array proposed Gomez-Gesteira and Dalrymple (2004), the following equation of Lennard-Jones form is used to prevent penetration to boundary wall.

$$f(r) = \begin{cases} D\left[ \left( \frac{r_0}{r_{ij}} \right)^{n_1} - \left( \frac{r_0}{r_{ij}} \right)^{n_2} \right] \frac{x_{ij}}{r_{ij}^2}, & \left( \frac{r_0}{r_{ij}} \right) \le 1 \\ 0, & \left( \frac{r_0}{r_{ij}} \right) > 1 \end{cases} \tag{8}$$

where $n_1$, $n_2$ and $D$ are coefficients which changes with the condition, and $r_0$ is initial particle spacing.

### 2.6 Stability Condition

The time step is also dynamically adjusted in the computation for acceleration based on the following Courant condition in Eq. (9) and the constraint of viscous diffusion in Eq. (10) (Shao, 2005).

$$\Delta t \le 0.1 \frac{l_0}{V_{\max}} \tag{9}$$

$$\Delta t \le 0.1 \frac{l_0^2}{\nu} \tag{10}$$

where $V_{max}$ is maximum velocity, $l_0$ is initial particle spacing and $\nu$ is kinetic viscosity. A constant time step ($\Delta t$=$10^{-4}$) is used in this simulation.

## 3. Analyses and Results

### 3.1 Model Setup

Fig. 1 shows a solitary wave generated by a piston-type wave board and run-up $R$ on a vertical wall or a sloping wall according to $\beta$. The solitary wave is a wave consisting of a single water surface elevation $\eta(x,t)$ according to the proportion of wave amplitude $a$ to water depth $d$. $X1$ is the distance seaward from the toe of the slope that is equal to $L/2$, and $L$ is a characteristic length of the wave defined here for the solitary wave as (Li and Raichlen, 2001)

$$L = \frac{2}{\sqrt{\frac{3a}{4d}}} \left[ \cosh^{-1}\left( \sqrt{\frac{1}{0.05}} \right) \right]. \tag{11}$$

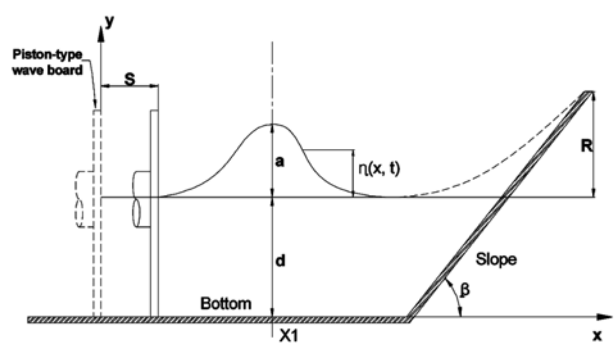


Fig. 1. Definition Sketch Showing a Solitary Wave

The time-dependent wave board trajectory $X_0$(t) for producing a solitary wave profile is used the following equation (Hughes, 1993).

$$X_0(t) = \frac{a}{dl}\tanh(ct - X_0) \tag{12}$$

where $X_0=S/2$ is instantaneous wave board position, $l = \sqrt{3a/4d^3}$, $c = \sqrt{g(d+a)}$, and $g$ is gravity acceleration.

### 3.2 Solitary Wave Run-up on a Vertical Wall

The maximum run-up of the solitary waves on vertical wall ($\beta$=90°) with different amplitudes is computed. The initial water depth $d$ is 0.1m, and 1490 fluid particles are involved in this simulation. The results are compared with the experimental data and the result of MAC method (Chan and Street, 1970) using the mesh as shown in Fig. 2. The agreement among the result of SPH method, the experimental data and the result of MAC method is satisfactory.

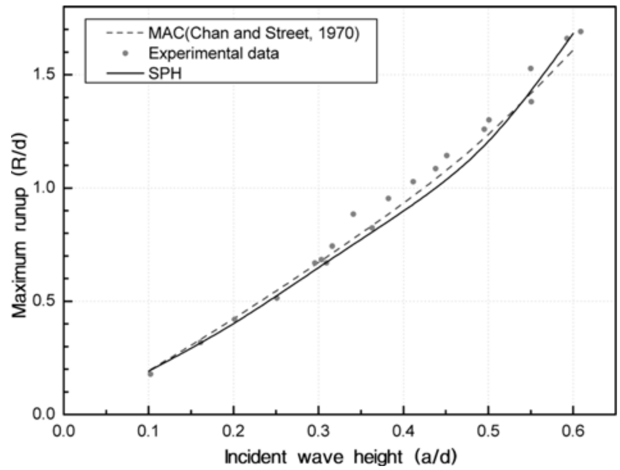


Fig. 2. Maximum Run-up Comparison of Solitary Wave with Different Amplitudes on a Vertical Wall

Figs. 3(a)-(f) are visualized by the particle configuration at various times of a solitary wave motion with an incident wave height $a/d$=0.5. Fig. 3(a) shows the still water surface before generating solitary wave. It is shown that the wave propagates to the right-hand vertical wall in Figs. 3(a)-(f), and Figs. 3(d)-(f) correspond to the maximum wave run-up, run-down and wave backwash processes, respectively. This calculation takes about 327sec on the 3.0GHz PC with 1GB memories.

The velocity distributions of solitary wave on vertical wall during the propagating processes are plotted in Figs. 4(a)-(f). It is shown that the velocity is fast where the solitary wave crest is located. When solitary wave run-up and run-down occur, velocity vectors propagate up and down along a vertical wall as seen in Figs. 4(c)-(e). Fig. 4(f) well shows that velocity vectors propagate backward as backwash phenomenon.

### 3.3 Solitary Wave Run-up on a Sloping Wall

The variation of the maximum run-up with different wave amplitudes $a/d$=0.163 for a slope of 1:2.08 ($\beta$≈26°) is presented

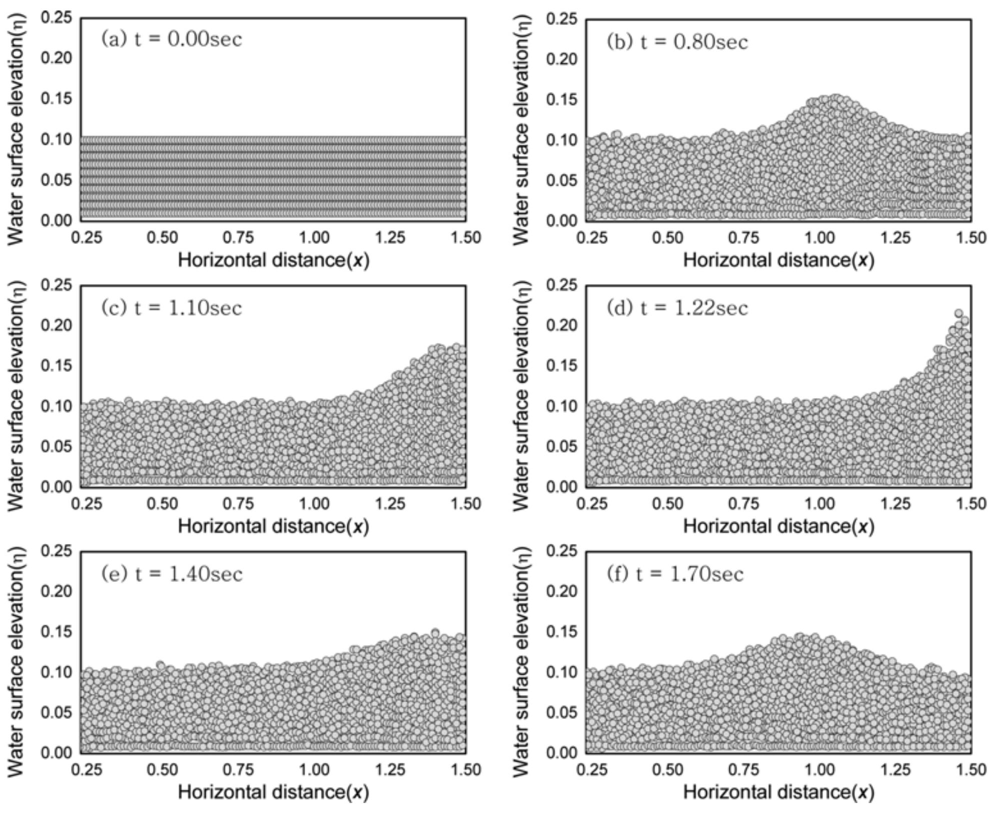


Fig. 3. Particle Configurations on Solitary Wave Propagation on a Vertical Wall

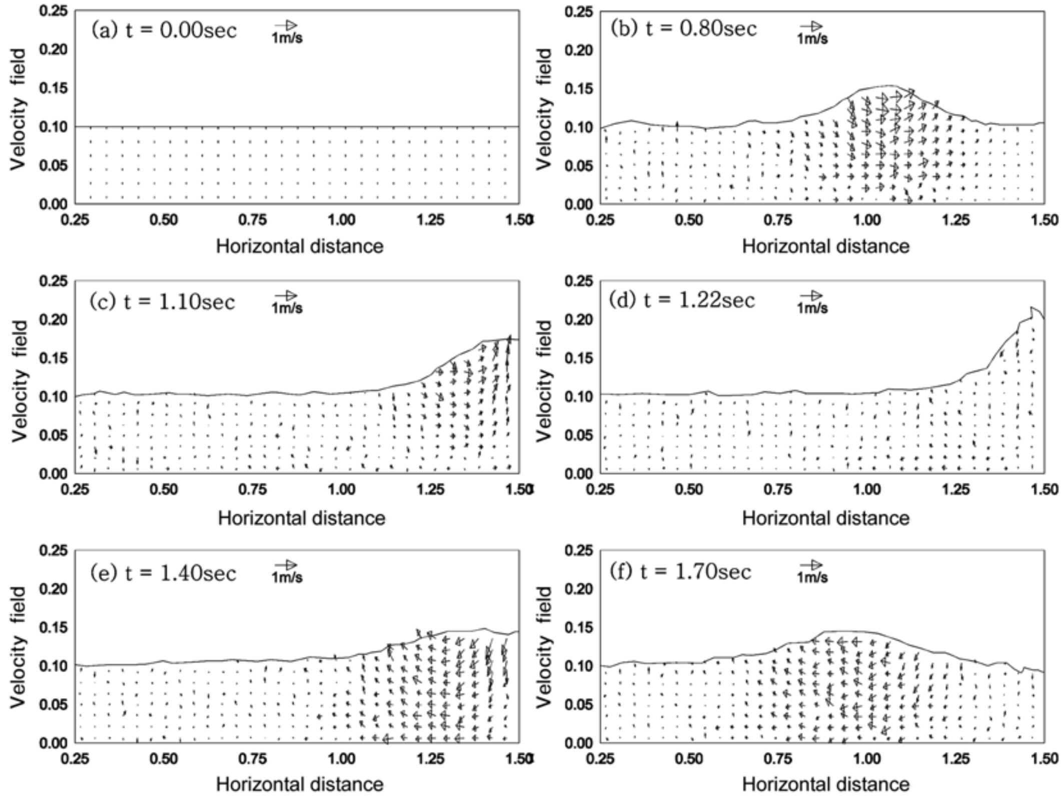


Fig. 4. Velocity Distributions on Solitary Wave Propagation on a Vertical Wall

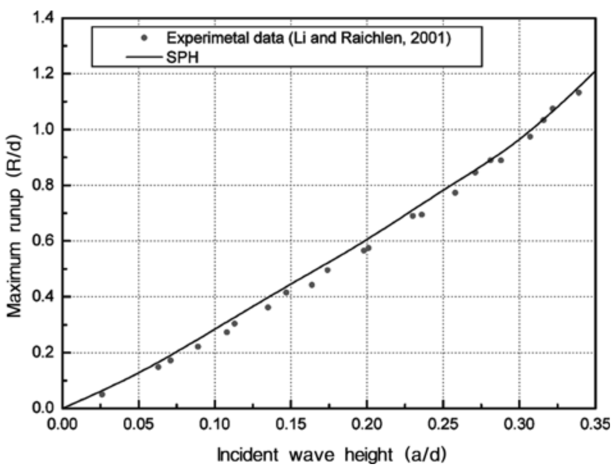


Fig. 5. Maximum Run-up Comparison of Solitary Wave with Different Amplitudes on a Slope of 1:2.08

in Fig. 5. Experimental results (Li and Raichlen, 2001) and SPH results are presented for comparison. It is also shown that the results are agreed, strongly.

The particle configurations of solitary wave propagation on a sloping wall of 1:2.08 at various times are shown in Figs. 6(a)-(e). As previously stated, this solitary wave is also generated by a piston-type wave board, the initial water depth $d$ is 0.21m and 4781 fluid particles are involved in this simulation. It is noted that $x$=0 refers to location of the original shoreline, negative values are offshore and positive values are onshore of position offshore. Therefore, $x$=-0.43 is located at the toe of the slope for this beach. Fig. 6(a) shows location of fluid particles as initial condition. Figs. 6(c)-(e) are shown well the processes of maximum run-up, run-down and backwash. This calculation takes about 20022 sec on the 3.0 GHz PC with 1 GB memories.

Figs. 7(a)-(f) present the velocity distributions about solitary wave propagation on a sloping wall. Fig. 7(b) shows that velocity vectors propagate on plane bottom. When solitary wave run-up and run-down occur, velocity vectors propagate up and down along a sloping wall as seen in Figs. 7(c)-(e). It can be seen that the run-down velocity in addition to gravity acceleration is faster than run-up velocity. Then the vectors propagate backward as backwash phenomenon as shown in Fig. 7(f).

For more detail comparison, water surface profiles on a slope of 1:2.08 are presented at the indicated non-dimensional times in Figs. 8(a)-(d). It is noted that solitary wave crest is located at $X1$, when $t$=0. The results of SPH method are compared to the experimental results (Li and Raichlen, 2001). In Fig. 8(c), the result of this method about run-down is slightly faster than the results of experimental data. It is seen that the numerical results about the entire process from run-up to run-down agree with experimental data.

## 4. Conclusions

In this paper, the numerical simulations on solitary wave

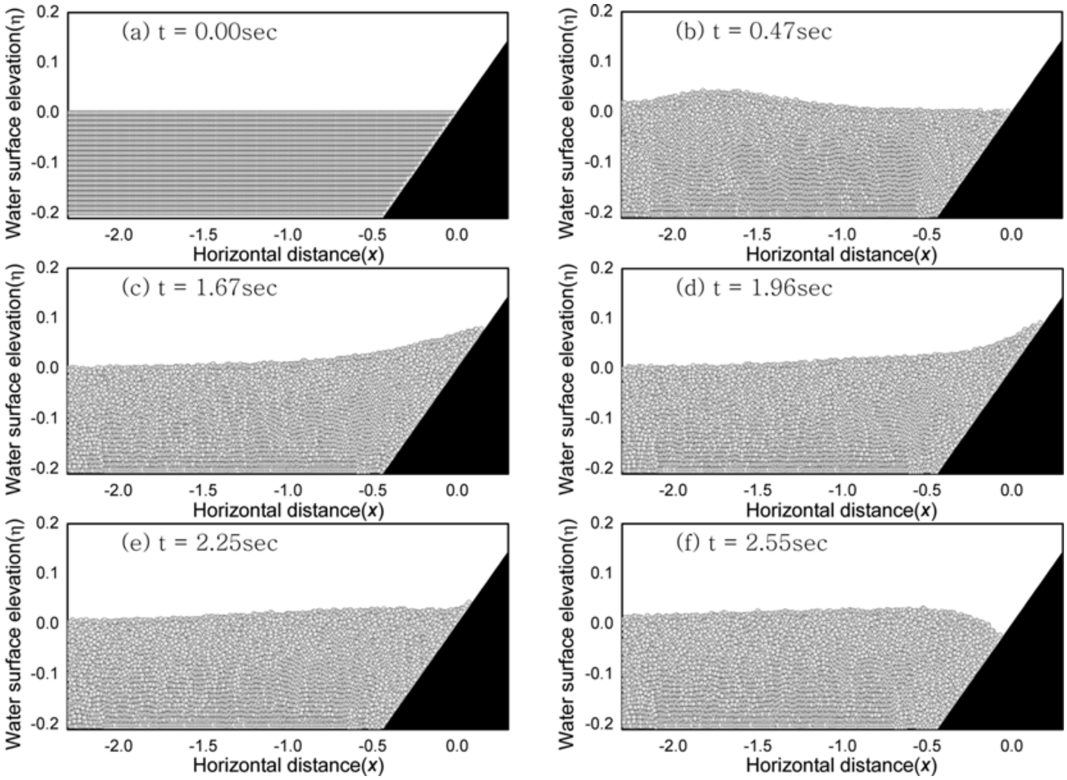


Fig. 6. Particle Configurations on Solitary Wave Propagation on a Sloping Wall of 1:2.08

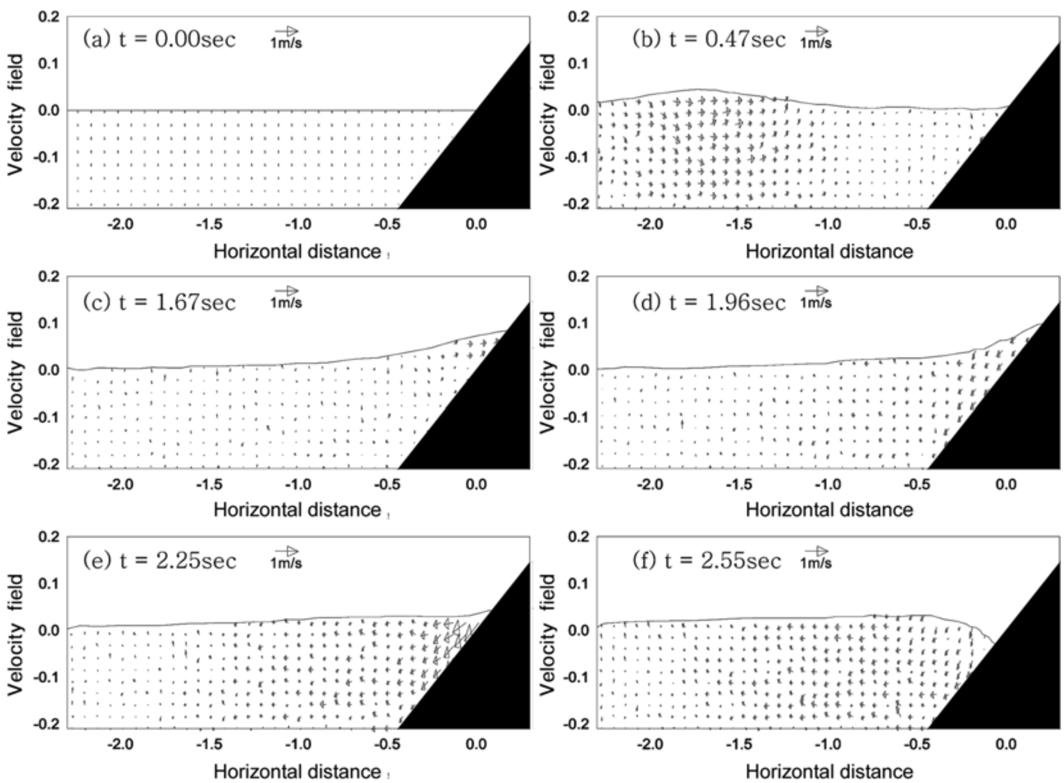


Fig. 7. Velocity Distributions on Solitary Wave Propagation on a Sloping Wall of 1:2.08

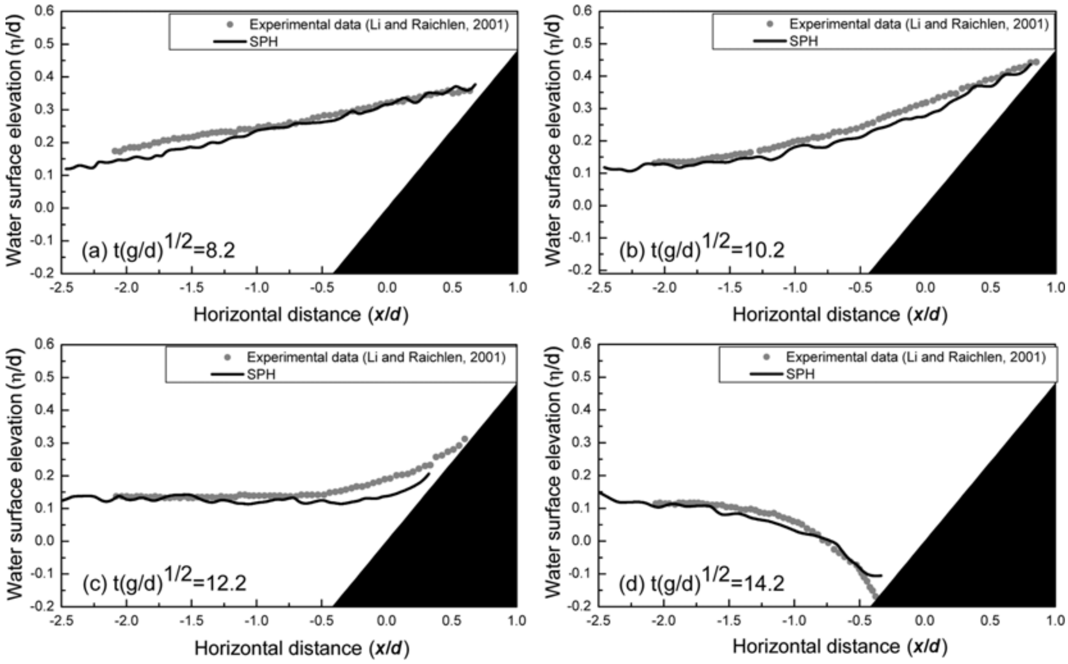


Fig. 8. Comparison of SPH and Experimental Data about Water Surface Profiles of Solitary Wave on a Sloping Wall of 1:2.08

propagation using SPH method as a meshless method are carried out. To verify this numerical simulation, maximum run-up of solitary wave with different amplitudes on a vertical wall and a sloping wall is compared with experimental data. And a good agreement result is obtained. For more detail comparison, water surface profile on a sloping wall is compared with the experimental data, and it is also obtained the satisfactory agreement. The particle configurations and the velocity distributions at various times are visualized. It is shown to be capable of simulating about sloshing phenomena of a solitary wave propagation, run-up, run-down and backwash on a vertical wall and a sloping wall. Therefore, this numerical simulation will be applied to the model of ocean waves like tsunami or storm surge.

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