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Event-Triggered Fixed-Time Consensus of Second-Order Nonlinear Multi-Agent Systems with Delay and Switching Topologies

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Abstract: To address fixed-time consensus problems of a class of leader-follower second-order nonlinear multi-agent systems with uncertain external disturbances, the event-triggered fixed-time consensus protocol is proposed. First, the virtual velocity is designed based on the backstepping control method to achieve the system consensus and the bound on convergence time only depending on the system parameters. Second, an event-triggered mechanism is presented to solve the problem of frequent communication between agents, and triggered condition based on state information is given for each follower. It is available to save communication resources, and the Zeno behaviors are excluded. Then, the delay and switching topologies of the system are also discussed. Next, the system stabilization is analyzed by Lyapunov stability theory. Finally, simulation results demonstrate the validity of the presented method.
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0 Introduction

Multi-agent systems (MASs) are the systems in which a number of agents complete large and complex tasks through limited local information exchange and coordination. In the last few years, the consensus problems of MASs are investigated in formation of multiple robots[1-2], track control of satellites[3-4] and distributed sensor networks[5-6]. One of the important issues is how to design an effective controller based on the information exchange among the agents. Some first-order fixed-time consensus results are obtained by researchers[7-8]. However, second-order and higher-order systems are common in practice. Unlike the first-order MASs, the consensus problems for second-order MASs are more challenging as the consensus of position and velocity are required simultaneously[9].

The convergence rate is a key performance index to evaluate the MASs consensus protocol. Compared with the control methods of traditional asymptotic convergence and finite-time convergence[10-11], the fixed-time consensus algorithms are more effective for solving the disadvantages of convergence time tending to infinity and dependence on the initial value. In Refs. [12-14], the consensus control algorithms are designed to achieve the fixed-time convergence for second-order MASs. The consensus problems of fixed-time and finite-time are considered under the unified framework[15]; the MASs converge in fixed-time and finite time by adjusting parameters, separately. In Refs. [16-17], fixed-time containment control for MASs is investigated, and the sufficient condition of convergence is obtained by employing the sliding-mode control method. In addition, the communication resources and computing power of agents are limited. This motivated the study of event-triggered mechanism to reduce the frequency of the controller updates. In Refs. [18-20], both first-order and second-order MASs under undirected topology are discussed. In Ref. [21], the fixed-time consensus problems of second-order MASs with uncertain bounded disturbances are investigated. Moreover in Ref. [22], based on the fixed-time consensus algorithm, the mechanisms of self-triggered and team-triggered are designed to avoid continuous communication. In Ref. [23], the problems of fixed-time consensus under continuous communication and intermittent communication are studied.

On the other hand, since some factors are out of control such as imperfect data transmission and changing

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communication environment, the delay and switching topologies are inevitable. Noticeably, it is meaningful to investigated fixed-time consensus of MASs with the delay and switching topologies. There are many results of consensus problems for first-order MASs with delay[24]. In Refs. [25-26], the system with delay is converted to a system without input delay by employing an extension of the Artstein's reducing transformation. In Ref. [27], the fixed-time consensus tracking control for second-order nonlinear MASs with delay is completed, and a sliding-mode controller is added in order to eliminate chattering. The convergence problem for high-order MASs with delay is transformed into Nash equilibrium seeking problem in Ref. [28]. The fixed-time convergence of nonlinear MASs with stochastically switching topologies and time-varying topologies is researched in Refs. [29-30]. In Refs. [31-32], the tracking control of second-order and high-order MASs with time-varying communication topologies can be achieved at fixed-time, respectively. In Ref. [33], the problem of fixed-time fault-tolerant control is studied, and the continuous control action can be provided in both switching and fixed topologies by employing the sliding-mode control.

In previous work, many results are obtained on the problems of continuous trigger and disturbance. However, there are few conclusions that discuss the fixed-time consensus of second-order MASs with delay and switching topologies, and these problems are frequent in practical applications. This paper is motivated by these factors to design a consensus algorithm to solve the problems. Inspired by the above discussion, event-triggered fixed-time consensus algorithms for second-order nonlinear MASs with delay and switching topologies are presented. Compared with fixed-time consensus algorithms in Ref. [12], an event-triggered mechanism is designed to save communication resources and avoid Zeno behaviors, and the control parameters are adjusted to reduce the threshold of the control input. In addition, the external disturbances and nonlinear term are also discussed. Motivated by Ref. [22], according to the fixed-time consensus algorithms for first-order MASs with delay and switching topologies, this paper extends the results to second-order MASs. In the case of switching topologies, all agents can connect arbitrarily if there is at least one path between followers and leader. Under the action of the controller, the velocity and position of all followers can reach agreement with the leader in fixed-time and the setting time can be obtained. In contrast to previous work, the control protocol designed in this study is effective to reduce the cost and improve the practicability of the MASs.

The main innovations can be summarized in two aspects. First, the event-triggered mechanism is used to reduce the frequency of the controller updates. Second, the fixed-time consensus problems of the second-order MASs with delay and switching topologies are solved.

The structure of this paper is given as follows. Some necessary basic knowledge of graph theory and system dynamic model is provided in Section 1. The event-triggered fixed-time consensus algorithms for second-order MASs with delay and switching topologies are given in Section 2. Finally, a simulation is presented and the result proves the availability of the controller in Section 3.

1 Problem Statement

1.1 Graph Theory

Suppose a system which contains $N+1$ agents. The communication topology is described by a digraph $G=(V,E,\boldsymbol{A})$, where $V=\{1,2,\cdots,N\}$ is a finite nonempty node set. The edge set is denoted as $E=V\times V$ and the set of neighbours of the ith node is denoted by $N_i=\{j:(i,j)\in E, j\neq i\}$. If node i can deliver a message to node j, there exists an edge between node i and node j, and the edge is described as (i,j). In an undirected graph that satisfies $(i,j)\in E$ and $(j,i)\in E$ for any $i,j\in V$, if the graph is directed, then the above equation does not hold. The adjacency matrix $\boldsymbol{A}=[a_{ij}]_{N\times N}$ is used to represent the connection relationship between the agent i and their neighbours. Assume that $a_{ii}=0$ for all $i\in V$. If there exists a path from i to j, then $a_{ij}>0$; otherwise, $a_{ij}=0$. The Laplacian matrix $\boldsymbol{L}=[l_{ij}]_{N\times N}$ of the graph is associated with the adjacency matrix $\boldsymbol{A}$, where $l_{ii}=\sum_{j=1}^{N}a_{ij}$ and the non-diagonal elements are $l_{ij}=-a_{ij}$. Herein, $\|\cdot\|$ represents the induced 2-norm for matrices and the Euclidean norm for vectors.

Consider there are N followers and 1 leader in the MASs. The agent is leader, and the followers are denoted as $1,2,\cdots,N$. The matrix $\boldsymbol{B}_{\mathrm{L}}\in\mathbf{R}^{N\times N}=\boldsymbol{B}+\boldsymbol{L}$, where $\boldsymbol{B}=\mathrm{diag}\{b_1,b_2,\cdots,b_N\}$ represents the input matrix as being a diagonal matrix. If the information from the leader can be received by the follower i, then $b_i>0$; otherwise, $b_i=0$.

Lemma 1[34] If and only if the adjacency matrix of graph G is irreducible, then G is strongly connected.

Lemma 2[35] Let $\boldsymbol{L}$ define as the Laplacian matrix of graph G and it is a positive semidefinite matrix. If graph G is undirected and connected, for any $\boldsymbol{x}=(x_1,x_2,\cdots,x_N)^{\mathrm{T}}$, the following equality $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{x}=\frac{1}{2}\sum_{j=1}^{N}a_{ij}(x_i-x_j)^2$, $i=1,2,\cdots,N$ holds. Here, 0 is the unique smallest eigenvalue of $\boldsymbol{L}$ and none of the other eigenvalues are negative. There are eigenvalues $0,\lambda_2,\cdots,\lambda_N$ satisfying $0\leqslant\lambda_2\leqslant\cdots\leqslant\lambda_N$ and 1 is the associated eigenvector of 0 eigenvalue. If $\mathbf{1}^{\mathrm{T}}\boldsymbol{x}=\mathbf{0}$, then $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{x}\geqslant\lambda_2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}$. Furthermore, it can be obtained that $\sum_{i=1}^{N}(a_{ij}(x_i-x_j))^2 = (\boldsymbol{L}^{\frac{1}{2}}\boldsymbol{x})\boldsymbol{L}(\boldsymbol{L}^{\frac{1}{2}}\boldsymbol{x}) \geqslant \lambda_2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{L}\boldsymbol{x}$ and $\sum_{i=1}^{N}(a_{ij}(x_i-x_j))^2 = (\boldsymbol{L}^{\frac{1}{2}}\boldsymbol{x})\boldsymbol{L}(\boldsymbol{L}^{\frac{1}{2}}\boldsymbol{x}) \leqslant 2\lambda_N V \leqslant 2\lambda_N V(0)$.

Lemma 3[36] If there exists a function $V:\mathbf{R}^n \rightarrow \mathbf{R}_+ \cup \{0\}$ and it is continuous and radially unbounded, some constants are denoted as $a>0$, $b>0$, $0<v<1<w$, such that:

(1) $V(x)=0 \leftrightarrow \boldsymbol{x}=\boldsymbol{0}$;

(2) For any $\boldsymbol{x}(t)$ that satisfies the inequality

$$\dot{V}(\boldsymbol{x}(t)) \leqslant -aV^v(\boldsymbol{x}(t)) - bV^w(\boldsymbol{x}(t)),$$

then the global fixed-time stability is reached. One has

$$T(\boldsymbol{x}_0) \leqslant T_{\max} = \frac{\pi\theta}{2\sqrt{ab}},$$

where $\theta>0$, $v=1-\dfrac{1}{\theta}$ and $w=1+\dfrac{1}{\theta}$. The MASs consensus is reached in settling time $T(\boldsymbol{x}_0)$, where $\boldsymbol{x}_0$ is the initial state.

Lemma 4[37] If there exists a set of constants $\kappa_1,\kappa_2,\cdots,\kappa_N \geqslant 0$, $0<p$. Then,

$$\sum_{i=1}^{N}\kappa_i^p \geqslant \Big(\sum_{i=1}^{N}\kappa_i\Big)^p,\quad 0<p\leqslant 1,$$
$$\sum_{i=1}^{N}\kappa_i^p \geqslant N^{1-P}\Big(\sum_{i=1}^{N}\kappa_i\Big)^p,\quad 0<p\leqslant \infty.$$

1.2 System Model

There exist N followers and 1 leader in the system. The node i is the representative of the agent in an undirected graph, and the nonlinear dynamic functions of the followers are given as follows:

$$\left.\begin{aligned}\dot{\boldsymbol{x}}_i(t) &= \boldsymbol{v}_i(t)\\ \dot{\boldsymbol{v}}_l(t) &= \boldsymbol{u}_i(t)+\boldsymbol{f}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)+\\ &\quad \boldsymbol{d}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)\end{aligned}\right\},\tag{1}$$

where, $i=1,2,\cdots,N$ represents the number of agents; $\boldsymbol{x}_i(t)\in\mathbf{R}^n$ is the position and $\boldsymbol{v}_i(t)\in\mathbf{R}^n$ is the velocity of the ith follower; $\boldsymbol{u}_i(t)\in\mathbf{R}^n$ is the input of the controller, and uncertain disturbance and nonlinear term of agent i are written as $\boldsymbol{d}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)\in\mathbf{R}^n$ and $\boldsymbol{f}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)\in\mathbf{R}^n$ respectively.

The dynamic of the leader is formulated as

$$\left.\begin{aligned}\dot{\boldsymbol{x}}_0(t) &= \boldsymbol{v}_0(t)\\ \dot{\boldsymbol{v}}_0(t) &= \boldsymbol{f}_0(\boldsymbol{x}_0(t),\boldsymbol{v}_0(t),t)\end{aligned}\right\},\tag{2}$$

where $\boldsymbol{x}_0(t)$ is the position of leader and $\boldsymbol{v}_0(t)\in\mathbf{R}^n$ is the corresponding velocity.

For convenience, some necessary definitions and assumptions are presented.

Assumption 1 The $\|\boldsymbol{f}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)\|$ and $\|\boldsymbol{d}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)\|$ in the system are bounded as F and D such as $\|\boldsymbol{f}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)\| \leqslant F$ and $\|\boldsymbol{d}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)\| \leqslant D$.

Assumption 2 The topology graph G is undirected and there is a spanning tree in graph G, i.e., $\boldsymbol{B}_{\mathrm{L}}$ is full rank.

Definition 1 For any initial value, the second-order MASs (1) and (2) can achieve a fixed-time stability if and only if the state of agent i satisfies

$$\left.\begin{aligned}\lim_{t\to T}\|\boldsymbol{x}_i(t)-\boldsymbol{x}_0(t)\|=0\\ \lim_{t\to T}\|\boldsymbol{v}_i(t)-\boldsymbol{v}_0(t)\|=0\end{aligned}\right\},\quad i\in\{1,2,\cdots,N\},\tag{3}$$

where T is bounded, and there exists $T_{\max}>0$ such that $T\leqslant T_{\max}$ is true.

Definition 2[36] If the system is Lyapunov stable and there is a set time $T(\boldsymbol{x}_0)>0$, such that the system converges to the equilibrium point within T. Then, the origin is said to be finite-time stable. Furthermore, for any initial state $\boldsymbol{x}_0$ if $\exists T>0$, such that $T\leqslant T_{\max}$, the origin of the system is fixed-time stable.

Definition 3 For each agent, the controller is updated if and only if $\inf\{t-t_k^i\}>0$.

For $i=1,2,\cdots,N$, define the error function as

$$\left.\begin{aligned}\bar{\boldsymbol{x}}_i(t) &= \boldsymbol{x}_i(t)-\boldsymbol{x}_0(t)\\ \bar{\boldsymbol{v}}_i(t) &= \boldsymbol{v}_i(t)-\boldsymbol{v}_0(t)\end{aligned}\right\}.\tag{4}$$

Remark 1 Assumption 1 is used for stability analysis in the following sections. Assumption 2 is used to ensure the communication between agents and it is a necessary condition for consensus of the MASs.

2 Main Results

The stability analysis of fixed-time consensus problems for MASs (1) and (2) is proposed in this part. Then, the availability of the event-triggered mechanism is also demonstrated. Finally, the consensus problems of the system in the case of delay and switching topologies are discussed as follows.

2.1 Event-Triggered Fixed-Time Consensus Algorithm

The fixed-time consensus controller is designed as follows:

First, the virtual velocity $\boldsymbol{v}_i^*$ is designed as follows:

$$\begin{aligned}\boldsymbol{v}_i^*(t) = &-k_1\mathrm{sig}\Big(\sum_{j=1}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))+\\ &b_i\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_0(t))\Big)^{\gamma_1}-\\ &k_2\mathrm{sig}\Big(\sum_{j=1}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))+\\ &b_i(\boldsymbol{x}_i(t)-\boldsymbol{x}_0(t))\Big)^{\gamma_2}+\boldsymbol{v}_0(t)=\end{aligned}$$

$$-k_1\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_1}-$$
$$k_2\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_2}+\boldsymbol{v}_0(t),\tag{5}$$

where k_1 and k_2 are positive constants, $0<\gamma_1<1$ and $\gamma_2>1$.

Remark 2 For the $\mathrm{sig}(\boldsymbol{x})^a$, there exist $\boldsymbol{x}\in\mathbf{R}$ and $a>0$ such that $\mathrm{sig}(\boldsymbol{x})^a=\mathrm{sign}(\boldsymbol{x})|\boldsymbol{x}|^a$. Additionally, the derivative satisfies $\frac{\mathrm{d}}{\mathrm{d}\boldsymbol{x}}\mathrm{sig}(\boldsymbol{x})^a=a|\boldsymbol{x}|^{a-1}$.

The tracking error is written as

$$\boldsymbol{e}_i(t)=\boldsymbol{v}_i(t)-\boldsymbol{v}_i^*(t).\tag{6}$$

From Eqs. (1), (5) and (6), one has

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t)=&\boldsymbol{u}_i(t)+\boldsymbol{f}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)+\\&\boldsymbol{d}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)+\\&k_1\gamma_1\Big|\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big|^{\gamma_1-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\boldsymbol{v}_i(t)-\boldsymbol{v}_j(t))\Big]+\\&k_2\gamma_2\Big|\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big|^{\gamma_2-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\boldsymbol{v}_i(t)-\boldsymbol{v}_j(t))\Big]-\\&\boldsymbol{f}_0(\boldsymbol{x}_0(t),\boldsymbol{v}_0(t),t).\end{aligned}\tag{7}$$

In order to simplify the process, let $\boldsymbol{f}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)$, $\boldsymbol{f}_0(\boldsymbol{x}_0(t),\boldsymbol{v}_0(t),t)$, and $\boldsymbol{d}_i(\boldsymbol{x}_i(t),\boldsymbol{v}_i(t),t)$ be replaced by $\boldsymbol{f}_i$ $\boldsymbol{f}_0$, and $\boldsymbol{d}_i$, respectively.

According to the above analysis, the event-triggered fixed-time consensus protocol is proposed:

$$\begin{aligned}\boldsymbol{u}_i(t)=&-k_3\mathrm{sig}(\boldsymbol{e}_i(t_k^i))^{\rho_1}-k_4\mathrm{sig}(\boldsymbol{e}_i(t_k^i))^{\rho_2}-\\&k_1\gamma_1\Big|\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t_k^i)-\boldsymbol{x}_j(t_k^i))\Big|^{\gamma_1-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\boldsymbol{v}_i(t_k^i)-\boldsymbol{v}_j(t_k^i))\Big]-\\&k_2\gamma_2\Big|\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t_k^i)-\boldsymbol{x}_j(t_k^i))\Big|^{\gamma_2-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\boldsymbol{v}_i(t_k^i)-\boldsymbol{v}_j(t_k^i))\Big]+\boldsymbol{f}_0-\boldsymbol{f}_i,\end{aligned}\tag{8}$$

where t_k^i is the latest triggered time of agent i, k_3 and k_4 are positive constants, $\rho_1\in(0,1)$ and $\rho_2>1$.

The measurement error is defined as

$$\begin{aligned}\boldsymbol{E}_i(t)=&k_3\mathrm{sig}(\boldsymbol{e}_i(t_k^i))^{\rho_1}+k_4\mathrm{sig}(\boldsymbol{e}_i(t_k^i))^{\rho_2}+\\&k_1\gamma_1\Big|\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t_k^i)-\boldsymbol{x}_j(t_k^i))\Big|^{\gamma_1-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\boldsymbol{v}_i(t_k^i)-\boldsymbol{v}_j(t_k^i))\Big]+\\&k_2\gamma_2\Big|\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t_k^i)-\boldsymbol{x}_j(t_k^i))\Big|^{\gamma_2-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\boldsymbol{v}_i(t_k^i)-\boldsymbol{v}_j(t_k^i))\Big]-\\&\boldsymbol{f}_0+\boldsymbol{f}_i-k_3\mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_1}-k_4\mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_2}-\\&k_1\gamma_1\Big|\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big|^{\gamma_1-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\boldsymbol{v}_i(t)-\boldsymbol{v}_j(t))\Big]-\\&k_2\gamma_2\Big|\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big|^{\gamma_2-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\boldsymbol{v}_i(t)-\boldsymbol{v}_j(t))\Big]+\boldsymbol{f}_0-\boldsymbol{f}_i.\end{aligned}\tag{9}$$

Based on Eqs. (7), (8) and (9), $\dot{\boldsymbol{e}}_i(t)$ can be expressed as

$$\begin{aligned}\dot{\boldsymbol{e}}_i(t)=&-\boldsymbol{E}_i(t)-k_3\mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_1}-\\&k_4\mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_2}+\boldsymbol{d}_i.\end{aligned}\tag{10}$$

According to the measurement error $\boldsymbol{E}_i(t)$ and the tracking error $\boldsymbol{e}_i(t)$, event-triggered mechanism can be designed as

$$\begin{aligned}&\Delta_i(\boldsymbol{E}_i(t),\boldsymbol{e}_i(t),t)=\\&\quad\|\boldsymbol{E}_i\|-\big(k_4 2^{1+\rho_2}N^{\frac{1-\rho_2}{2}}-D\big)\|\boldsymbol{e}_i\|^{\rho_2}\omega_i,\end{aligned}\tag{11}$$

where $\omega_i\in(0,1)$ is the triggered parameter. Then the next triggered moment of follower i can be obtained:

$$t_{k+1}^i=\inf\{t>t_k^i,\Delta_i(\boldsymbol{E}_i(t),\boldsymbol{e}_i(t),t)\geqslant 0\}.\tag{12}$$

From Definition 3, the Zeno behavior of the system can be avoided if and only if Eq. (12) is satisfied. The system cannot be triggered infinitely many times in finite time.

Remark 3 Any follower i updates its controller only at triggered moment $t_0^i,t_1^i,\cdots$. In addition, the leader has no event-triggered time.

Theorem 1 Suppose the undirected communication topology based on MASs (1) and (2) satisfies Assumptions 1 and 2. Under the action of controller (8) and event-triggered function (11), the consensus problem for MASs (1) and (2) can be addressed in a fixed-time. For any initial condition, the convergence time T can be obtained and the bound is as follows:

$$T = T_1 + T_2 \leqslant T_{1\max} + T_{2\max} = \frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}} k_4 N^{\frac{1-\rho_2}{2}}(1-\omega_i)}} + \frac{2\pi}{(\gamma_2-\gamma_1)\sqrt{k_1(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{1+\frac{\gamma_1+\gamma_2}{2}} k_2 N^{\frac{1-\gamma_2}{2}}}}. \tag{13}$$

Proof The Lyapunov function of the system is chosen as $V(t) = V_1(t) + V_2(t)$.

(1) The first step is to make the true velocity $\boldsymbol{v}_i$ converge to the virtual velocity $\boldsymbol{v}_i^*$ based on the backstepping control method. The corresponding Lyapunov function of this part is constructed by tracking error:

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{e}_i^2(t). \tag{14}$$

Take the derivative of the $V_1(t)$ and combine Eqs. (8)—(10):

$$\begin{aligned}\dot{V}_1(t) &= \sum_{i=1}^{N} \boldsymbol{e}_i(t)\dot{\boldsymbol{e}}_i(t) = \\ &\sum_{i=1}^{N} \boldsymbol{e}_i(t)(-\boldsymbol{E}_i(t) - k_3 \mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_1} - \\ &k_4 \mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_2} + \boldsymbol{d}_i) \leqslant \\ &\sum_{i=1}^{N} \boldsymbol{e}_i(t)(-\boldsymbol{E}_i(t) - k_3 \mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_1} - \\ &k_4 \mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_2} + D) = \\ &-k_3\sum_{i=1}^{N}|\boldsymbol{e}_i(t)|^{\rho_1+1} - k_4\sum_{i=1}^{N}|\boldsymbol{e}_i(t)|^{\rho_2+2} - \\ &-\sum_{i=1}^{N}|\boldsymbol{e}_i(t)|(\boldsymbol{E}_i(t)-D) \leqslant \\ &-k_3\Big(\sum_{i=1}^{N}\boldsymbol{e}_i^2(t)\Big)^{\frac{\rho_1+1}{2}} - k_4\Big(\sum_{i=1}^{N}\boldsymbol{e}_i^2(t)\Big)^{\frac{\rho_2+1}{2}} \\ &-\sum_{i=1}^{N}|\boldsymbol{e}_i(t)|(\boldsymbol{E}_i(t)-D).\end{aligned} \tag{15}$$

In the above equation,

$$\begin{aligned}&\sum_{i=1}^{N}|\boldsymbol{e}_i(t)|\,(\boldsymbol{E}_i(t)-D) \leqslant \|\boldsymbol{e}_i(t)\|(\|\boldsymbol{E}_i(t)\|-D) = \\ &\frac{\|\boldsymbol{e}_i(t)\|(\|\boldsymbol{E}_i(t)\|-D)}{V_1(t)^{\frac{\rho_2+1}{2}}}V_1(t)^{\frac{\rho_2+1}{2}} = \\ &\frac{\|\boldsymbol{e}_i(t)\|(\|\boldsymbol{E}_i(t)\|-D)}{2^{\frac{\rho_2+1}{2}}\|\boldsymbol{e}_i(t)\|^{\rho_2+1}} = V_1(t)^{\frac{\rho_2+1}{2}} = \\ &\frac{\|\boldsymbol{e}_i(t)\|^{-\rho_2}(\|\boldsymbol{E}_i(t)\|-D)}{2^{\frac{\rho_2+1}{2}}}V_1(t)^{\frac{\rho_2+1}{2}}.\end{aligned} \tag{16}$$

According to Lemma 4 and combine Eqs. (15) and (16):

$$\begin{aligned}\dot{V}_1(t) &\leqslant -k_3\Big(\sum_{i=1}^{N}\boldsymbol{e}_i^2(t)\Big)^{\frac{\rho_1+1}{2}} - k_4\Big(\sum_{i=1}^{N}\boldsymbol{e}_i^2(t)\Big)^{\frac{\rho_2+1}{2}} - \\ &\frac{\|\boldsymbol{e}_i(t)\|^{-\rho_2}(\|\boldsymbol{E}_i(t)\|-D)}{2^{\frac{\rho_2+1}{2}}}V_1(t)^{\frac{\rho_2+1}{2}} \leqslant \\ &-k_3 2^{\frac{\rho_1+1}{2}}V_1(t)^{\frac{\rho_1+1}{2}} - k_4 2^{\frac{\rho_2+1}{2}}N^{\frac{1-\rho_2}{2}}V_1(t)^{\frac{\rho_2+1}{2}} + \\ &\frac{\|\boldsymbol{e}_i(t)\|^{-\rho_2}(\|\boldsymbol{E}_i(t)\|+D)}{2^{\frac{\rho_2+1}{2}}}V_1(t)^{\frac{\rho_2+1}{2}}.\end{aligned} \tag{17}$$

From Lemma 3, the event-triggered condition is designed as

$$\|\boldsymbol{E}_i\| \leqslant (k_4 2^{1+\rho_2}N^{\frac{1-\rho_2}{2}} - D)\|\boldsymbol{e}_i\|^{\rho_2}\omega_i, \tag{18}$$

which has

$$\begin{aligned}\dot{V}_1(t) \leqslant &-k_3 2^{\frac{\rho_1+1}{2}}V_1(t)^{\frac{\rho_1+1}{2}} - \Big[k_4 2^{\frac{\rho_2+1}{2}}N^{\frac{1-\rho_2}{2}} - \\ &\frac{k_4 2^{1+\rho_2}N^{\frac{1-\rho_2}{2}}\omega_i + D(1-\omega_i)}{2^{\frac{\rho_2+1}{2}}}\Big]V_1(t)^{\frac{\rho_2+1}{2}} = \\ &-k_3 2^{\frac{\rho_1+1}{2}}V_1(t)^{\frac{\rho_1+1}{2}} - \Big[k_4 2^{\frac{\rho_2+1}{2}}N^{\frac{1-\rho_2}{2}} - \\ &\frac{k_4 2^{1+\rho_2}N^{\frac{1-\rho_2}{2}}\omega_i}{2^{\frac{\rho_2+1}{2}}} - \frac{D(1-\omega_i)}{2^{\frac{\rho_2+1}{2}}}\Big]V_1(t)^{\frac{\rho_2+1}{2}} = \\ &-k_3 2^{\frac{\rho_1+1}{2}}V_1(t)^{\frac{\rho_1+1}{2}} - \Big[k_4 2^{\frac{\rho_2+1}{2}}N^{\frac{1-\rho_2}{2}}(1-\omega_i) - \\ &\frac{D(1-\omega_i)}{2^{\frac{\rho_2+1}{2}}}\Big]V_1(t)^{\frac{\rho_2+1}{2}} \leqslant \\ &-k_3 2^{\frac{\rho_1+1}{2}}V_1(t)^{\frac{\rho_1+1}{2}} - \\ &\Big(k_4 2^{\frac{\rho_2+1}{2}}N^{\frac{1-\rho_2}{2}} - D\Big)(1-\omega_i)V_1(t)^{\frac{\rho_2+1}{2}}.\end{aligned} \tag{19}$$

Let $\rho_1 = 1 - \dfrac{1}{\theta}$, $\rho_2 = \dfrac{1}{\theta} + 1$, then $\theta = \dfrac{4}{\rho_2 - \rho_1}$, and the tracking time T_1 can be obtained:

$$T_1 \leqslant T_{1\max} = \frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}} k_4 N^{\frac{1-\rho_2}{2}}(1-\omega_i)}}. \tag{20}$$

Within the fixed-time T_1, the tracking error $\boldsymbol{e}_i(t)$ converges to 0 and the virtual velocity $\boldsymbol{v}_i^*(t)$ is tracked by true velocity $\boldsymbol{v}_i(t)$.

(2) When $\boldsymbol{e}_i(t)=\boldsymbol{0}$, we have

$$\boldsymbol{v}_i(t)=\boldsymbol{v}_i^*(t)= -k_1\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_1}- k_2\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_2}+\boldsymbol{v}_0(t). \quad (21)$$

Consider the Lyapunov function in the second part constructed by state error:

$$V_2(t)=\frac{1}{2}\bar{\boldsymbol{x}}_i^{\mathrm{T}}(t)\boldsymbol{B}_{\mathrm{L}}\bar{\boldsymbol{x}}_i(t). \quad (22)$$

According to Lemma 2 and take the derivative of $V_2(t)$:

$$\begin{aligned}\dot{V}_2(t)=&\bar{\boldsymbol{x}}_i^{\mathrm{T}}(t)\boldsymbol{B}_{\mathrm{L}}\dot{\bar{\boldsymbol{x}}}_i(t)=\\ &\sum_{i=1}^{N}\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\bar{\boldsymbol{v}}_i(t)=\\ &\sum_{i=1}^{N}\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))(\boldsymbol{v}_i(t)-\boldsymbol{v}_0(t))=\\ &\sum_{i=1}^{N}\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\times\\ &\Big[-k_1\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_1}-\\ &k_2\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_2}\Big]=\\ &-k_1\sum_{i=1}^{N}\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\times\\ &\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_1}-\\ &k_2\sum_{i=1}^{N}\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\times\\ &\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_2}\leqslant\\ &-k_1\sum_{i=1}^{N}\sum_{j=0}^{N}|a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))|^{\gamma_1+1}-\\ &k_2\sum_{i=1}^{N}\sum_{j=0}^{N}|a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))|^{\gamma_2+1}=\\ &-k_1\sum_{i=1}^{N}\sum_{j=0}^{N}\big|[a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))]^2\big|^{\frac{\gamma_1+1}{2}}-\\ &k_2\sum_{i=1}^{N}\sum_{j=0}^{N}\big|[a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))]^2\big|^{\frac{\gamma_2+1}{2}}\leqslant\\ &-k_1(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{\frac{\gamma_1+1}{2}}V_2(t)^{\frac{\gamma_1+1}{2}}-\\ &k_2N^{\frac{1-\gamma_2}{2}}(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{\frac{\gamma_2+1}{2}}V_2(t)^{\frac{\gamma_2+1}{2}}, \end{aligned}\quad (23)$$

where $\lambda_2(\boldsymbol{B}_{\mathrm{L}})$ is the smallest non-zero eigenvalue of the matrix $\boldsymbol{B}_{\mathrm{L}}$. Additionally, T_2 is expressed as

$$T_2\leqslant T_{2\max}=\frac{2\pi}{(\gamma_2-\gamma_1)\sqrt{k_1(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{1+\frac{\gamma_1+\gamma_2}{2}}k_2N^{\frac{1-\gamma_2}{2}}}}. \quad (24)$$

According to Lemma 3 and based on Eqs. (20) and (24), the consensus problem for MASs (1) and (2) is solved. In addition, convergence time T satisfies

$$T\leqslant\frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}}k_4N^{\frac{1-\rho_2}{2}}(1-\omega_i)}}+\frac{2\pi}{(\gamma_2-\gamma_1)\sqrt{k_1(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{1+\frac{\gamma_1+\gamma_2}{2}}k_2N^{\frac{1-\gamma_2}{2}}}}. \quad (25)$$

(3) The analysis of global stability is necessary to ensure that the agent state does not escape before the true velocity is tracked to the virtual velocity.

When $t\in[0,T_1]$, one can get that $\boldsymbol{v}_i^*(t)$ and $\boldsymbol{e}_i(t)$ are bounded from Eqs. (5), (6) and (21). In addition, $\boldsymbol{v}_i(t)=\boldsymbol{v}_i^*(t)+\boldsymbol{e}_i(t)$, so $\boldsymbol{v}_i(t)$ is bounded. According to the virtual velocity $\boldsymbol{v}_i^*(t)$ designed in Eq. (5) and the dynamic functions of Eq. (1), we can obtain

$$\dot{\boldsymbol{x}}_i(t)=\boldsymbol{v}_i(t)=\boldsymbol{v}_i^*(t)+\boldsymbol{e}_i(t)= \boldsymbol{e}_i(t)-k_1\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_1}- k_2\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\Big)^{\gamma_2}+\boldsymbol{v}_0(t). \quad (26)$$

Because $\boldsymbol{e}_i(t)$ is bounded at $t\in[0,T_1]$, $\boldsymbol{x}_i(t)$ is bounded. According to Lemma 3 and Eq. (23), the tracking of $\boldsymbol{x}_i(t)$ can be completed at $t\in[T_1,T_2]$. The proof is completed. □

Next, event-triggered mechanism is discussed with the following event-triggered condition:

$$\|\boldsymbol{E}_i\|\leqslant(k_4 2^{1+\rho_2}N^{\frac{1-\rho_2}{2}}-D)\|\boldsymbol{e}_i\|^{\rho_2}\omega_i. \quad (27)$$

The first trigger time is and $e(0)=0$. $\dfrac{\|\boldsymbol{E}_i\|}{\|\boldsymbol{e}_i\|^{\rho_2}}$ gets the maximum value $(k_4 2^{1+\rho_2}N^{\frac{1-\rho_2}{2}}-D)\omega_i$ when the MASs event-triggered condition (11) is violated. So the minimum interval between events is determined by the shortest time from $\dfrac{\|\boldsymbol{E}_i\|}{\|\boldsymbol{e}_i\|^{\rho_2}}=0$ to $\dfrac{\|\boldsymbol{E}_i\|}{\|\boldsymbol{e}_i\|^{\rho_2}}=(k_4 2^{1+\rho_2}N^{\frac{1-\rho_2}{2}}-D)\omega_i$.

Theorem 2 For the MASs (1) and (2), the Zeno behaviours are avoided under the action of controller (8) and event-triggered mechanism (11). The time-interval τ_i is a strictly positive number and expressed as

$$\tau_i = \frac{\left(k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} + \dfrac{D}{N^{1-\rho_2}}\right)\omega_i}{c_1(c_2+c_3)\left[c_2+c_3+\left(k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} + \dfrac{D}{N^{1-\rho_2}}\right)\omega_i\right]}. \tag{28}$$

Proof Let $\eta = \dfrac{\|\boldsymbol{E}\|}{\|\boldsymbol{e}^{\rho_2}\|}$. Combined with the measurement error in Eq. (9),

$$\begin{aligned} z = {}& k_3 \mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_1} + k_4 \mathrm{sig}(\boldsymbol{e}_i(t))^{\rho_2} + \\ & k_1\gamma_1 \left|\sum_{j=0}^{N} a_{ij}\mathrm{sig}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\right|^{\gamma_1-1} \times \\ & \left[\sum_{j=0}^{N} a_{ij}(\boldsymbol{v}_i(t)-\boldsymbol{v}_j(t))\right] + \\ & k_2\gamma_2 \left|\sum_{j=0}^{N} a_{ij}(\boldsymbol{x}_i(t)-\boldsymbol{x}_j(t))\right|^{\gamma_2-1} \times \\ & \left[\sum_{j=0}^{N} a_{ij}(\boldsymbol{v}_i(t)-\boldsymbol{v}_j(t))\right]. \end{aligned} \tag{29}$$

Take the derivative of the η and in every time period $[t_k^i, t_{k+1}^i)$ there exists

$$\begin{aligned} \dot{\eta} = {}& \frac{\mathrm{d}}{\mathrm{d}t}\frac{(\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E})^{\frac{1}{2}}}{((\boldsymbol{e}^{\rho_2})^{\mathrm{T}}(\boldsymbol{e}^{\rho_2}))^{\frac{1}{2}}} = \\ & \frac{1}{(\boldsymbol{e}^{\rho_2})^{\mathrm{T}}(\boldsymbol{e}^{\rho_2})}\left[(\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E})^{-\frac{1}{2}}(\boldsymbol{E})^{\mathrm{T}}(\boldsymbol{E})'((\boldsymbol{e}^{\rho_2})^{\mathrm{T}}(\boldsymbol{e}^{\rho_2}))^{\frac{1}{2}} - \right. \\ & \left. ((\boldsymbol{e}^{\rho_2})^{\mathrm{T}}(\boldsymbol{e}^{\rho_2}))^{-\frac{1}{2}}(\boldsymbol{e}^{\rho_2})^{\mathrm{T}}(\boldsymbol{e}^{\rho_2})'(\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E})^{\frac{1}{2}}\right] = \\ & -\frac{(\boldsymbol{E})^{\mathrm{T}}(z)'}{\|\boldsymbol{E}\|\|\boldsymbol{e}^{\rho_2}\|} - \frac{(\boldsymbol{e}^{\rho_2})^{\mathrm{T}}(\boldsymbol{e}^{\rho_2})'}{\|\boldsymbol{e}^{\rho_2}\|^2}\frac{\|\boldsymbol{E}\|}{\|\boldsymbol{e}^{\rho_2}\|^2} \leqslant \\ & -\frac{\boldsymbol{E}^{\mathrm{T}}(\boldsymbol{e}^{\rho_2})'}{\|\boldsymbol{E}\|\|\boldsymbol{e}^{\rho_2}\|} - \frac{(\boldsymbol{e}^{\rho_2})^{\mathrm{T}}(\boldsymbol{e}^{\rho_2})'}{\|\boldsymbol{e}^{\rho_2}\|^2}\frac{\|\boldsymbol{E}\|}{\|\boldsymbol{e}^{\rho_2}\|^2} = \\ & -\left(1+\frac{\|\boldsymbol{E}\|}{\|\boldsymbol{e}^{\rho_2}\|}\right)\left(\frac{\|(\boldsymbol{e}^{\rho_2})'\|}{\|\boldsymbol{e}^{\rho_2}\|}\right) \leqslant \\ & -(1+\eta)\frac{\rho_2\|\boldsymbol{e}\|^{\rho_2-1}\|\dot{\boldsymbol{e}}\|}{\|\boldsymbol{e}^{\rho_2}\|} \leqslant \\ & \rho_2\|\boldsymbol{e}\|^{\rho_2-1}(1+\eta)\left(\eta + \frac{k_3\mathrm{sig}(\boldsymbol{e})^{\rho_1} + k_4\mathrm{sig}(\boldsymbol{e})^{\rho_2} - \boldsymbol{d}}{\|\boldsymbol{e}^{\rho_2}\|}\right) \leqslant \\ & \rho_2\|\boldsymbol{e}\|^{\rho_2-1}(1+\eta)\left(\eta + \frac{k_3\|\boldsymbol{e}\|^{\rho_1} + k_4\|\boldsymbol{e}\|^{\rho_2} + D}{\|\boldsymbol{e}^{\rho_2}\|}\right) \leqslant \\ & \rho_2\|\boldsymbol{e}\|^{\rho_2-1}(1+\eta)\times \\ & \left(\eta + \frac{k_3}{N^{1-\rho_2}}\|\boldsymbol{e}\|^{\rho_1-\rho_2} + \frac{k_4}{N^{1-\rho_2}} + \frac{D}{\|\boldsymbol{e}^{\rho_2}\|}\right). \end{aligned} \tag{30}$$

According to Lemma 1, $\|\boldsymbol{e}\| = \sqrt{\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e}} \leqslant \sqrt{2V_1} \leqslant \sqrt{2V_1(0)}$; then, it can be obtained:

$$\begin{aligned} \dot{\eta} \leqslant {}& \rho_2\|\boldsymbol{e}\|^{\rho_2-1}(1+\eta)\times \\ & \left[\eta + \frac{k_3}{N^{1-\rho_2}}(2V_1(0))^{\frac{\rho_1-\rho_2}{2}} + \right. \\ & \left. D(2V_0(0))^{\frac{-\rho_2}{2}} + \frac{k_4}{N^{1-\rho_2}} + 1\right] \leqslant \\ & c_1(c_2+c_3+\eta)^2, \end{aligned} \tag{31}$$

where $c_1 = \rho_2\|\boldsymbol{e}\|^{\rho_2-1}$, $c_2 = \dfrac{k_3}{N^{1-\rho_2}}(2V_1(0))^{\frac{\rho_1-\rho_2}{2}} + \dfrac{k_4}{N^{1-\rho_2}}$ and $c_3 = D(2V_1(0))^{\frac{-\rho_2}{2}} + 1$. Let

$$\dot{\psi}_i = c_1(c_2+c_3+\psi_i)^2, \quad \psi_i(0,\psi_0^i) = \psi_0^i. \tag{32}$$

Therefore, $\eta_i(t)$ satisfies

$$\eta_i(t) \leqslant \psi_i(t,\psi_0^i), \tag{33}$$

where $\psi_i(t,\psi_0^i)$ is the solution of Eq. (32).

A solution can be obtained:

$$\psi_i(\tau_i, 0) = \frac{\tau_i c_1(c_2+c_3)^2}{1-\tau_i c_1(c_2+c_3)}. \tag{34}$$

According-to the event-triggered condition (11), we obtain

$$\frac{\|\boldsymbol{E}\|}{\|\boldsymbol{e}_i\|^{\rho_2}} \leqslant (k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} - D)\omega_i. \tag{35}$$

Based on Lemma 4, one has

$$\begin{aligned} \frac{\|\boldsymbol{E}\|}{\|\boldsymbol{e}\|^{\rho_2}} \leqslant {}& \frac{1}{N^{1-\rho_2}}\frac{\|\boldsymbol{E}\|}{\|\boldsymbol{e}_i\|^{\rho_2}} \leqslant \\ & k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}}\omega_i - D\omega_i\frac{1}{N^{1-\rho_2}} = \\ & \left(k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} - \frac{D}{N^{1-\rho_2}}\right)\omega_i = \\ & \frac{\tau_i c_1(c_2+c_3)^2}{1-\tau_i c_1(c_2+c_3)}. \end{aligned} \tag{36}$$

Then, it can be obtained that the minimum triggered interval is written as

$$\tau_i = \frac{\left(k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} - \dfrac{D}{N^{1-\rho_2}}\right)\omega_i}{c_1(c_2+c_3)\left[c_2+c_3+\left(k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} - \dfrac{D}{N^{1-\rho_2}}\right)\omega_i\right]}. \tag{37}$$

Then the minimum interval $\tau_i > 0$. The proof is completed. □

2.2 Fixed-Time Consensus Problem with Delay and Switching Topologies

Due to problems such as defective data transmission and processing after data reception, delay is frequent in MASs and the stability of systems will be affected. On the other hand, the communication between agents cannot remain unchanged, and the new communication connections are possible in practical applications. The consensus problem of the MASs under delay and switching topologies is discussed as follows.

2.2.1 *Event-Triggered Fixed-Time Consensus Algorithm with Delay*

For the MASs (1) and (2) with delay, the nonlinear dynamic functions of followers are redefined as

$$\left.\begin{aligned}&\dot{\hat{\boldsymbol{x}}}_i(t)=\hat{\boldsymbol{v}}_i(t)\\&\dot{\hat{\boldsymbol{v}}}_i(t)=\hat{\boldsymbol{u}}_i(t-h_i)+\boldsymbol{f}_i(\hat{\boldsymbol{x}}_i(t),\hat{\boldsymbol{v}}_i(t),t)+\\&\qquad\boldsymbol{d}_i(\hat{\boldsymbol{x}}_i(t),\hat{\boldsymbol{v}}_i(t),t)\end{aligned}\right\},\tag{38}$$

where h_i is delay and $h_i > 0$.

As mentioned above, the true velocity $\hat{\boldsymbol{v}}_i(t)$ is traced to the virtual velocity $\hat{\boldsymbol{v}}_i^*$ by backstepping design method, and the virtual velocity is adjusted as

$$\begin{aligned}\hat{\boldsymbol{v}}_i^*(t)=&-k_1\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\hat{\boldsymbol{x}}_i(t)-\hat{\boldsymbol{x}}_j(t))\Big)^{\gamma_1}-\\&k_2\mathrm{sig}\Big(\sum_{j=0}^{N}a_{ij}(\hat{\boldsymbol{x}}_i(t)-\hat{\boldsymbol{x}}_j(t))\Big)^{\gamma_2}+\\&\boldsymbol{v}_0(t)-\int_{t-h_i}^{t}\hat{\boldsymbol{u}}_i(T)\mathrm{d}T.\end{aligned}\tag{39}$$

The tracking error under the delay is expressed as

$$\hat{\boldsymbol{e}}_i(t)=\hat{\boldsymbol{v}}_i(t)-\hat{\boldsymbol{v}}_i^*(t).\tag{40}$$

The controller $\hat{\boldsymbol{u}}_i(t)$ is designed as

$$\begin{aligned}\hat{\boldsymbol{u}}_i(t)=&-k_3\mathrm{sig}(\hat{\boldsymbol{e}}_i(t_k^i))^{\rho_1}-k_4\mathrm{sig}(\hat{\boldsymbol{e}}_i(t_k^i))^{\rho_2}-\\&k_1\gamma_1\left|\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\hat{\boldsymbol{x}}_i(t_k^i)-\hat{\boldsymbol{x}}_j(t_k^i))\right|^{\gamma_1-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\hat{\boldsymbol{v}}_i(t_k^i)-\hat{\boldsymbol{v}}_j(t_k^i))\Big]-\\&k_2\gamma_2\left|\sum_{j=0}^{N}a_{ij}(\hat{\boldsymbol{x}}_i(t_k^i)-\hat{\boldsymbol{x}}_j(t_k^i))\right|^{\gamma_2-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\hat{\boldsymbol{v}}_i(t_k^i)-\hat{\boldsymbol{v}}_j(t_k^i))\Big]+\boldsymbol{f}_0-\boldsymbol{f}_i.\end{aligned}\tag{41}$$

According to the tracking error $\hat{\boldsymbol{e}}_i(t)$, the measurement error $\boldsymbol{E}_i(t)$ is represented as $\hat{\boldsymbol{E}}_i(t)$, and event-triggered mechanism is designed as

$$\begin{aligned}&\varDelta_i(\hat{\boldsymbol{E}}_i(t),\hat{\boldsymbol{e}}_i(t),t)=\\&\quad\|\hat{\boldsymbol{E}}_i\|-\big(k_4 2^{1+\rho_2}N^{\frac{1-\rho_2}{2}}-D\big)\|\hat{\boldsymbol{e}}_i\|^{\rho_2}\omega_i.\end{aligned}\tag{42}$$

Taking the derivative of $\hat{\boldsymbol{e}}_i(t)$ and combining Eq. (9), one can obtain

$$\begin{aligned}&\dot{\hat{\boldsymbol{e}}}_i(t)=\dot{\hat{\boldsymbol{v}}}_i(t)-\dot{\hat{\boldsymbol{v}}}_i^*(t)=\\&\hat{\boldsymbol{u}}_i(t-h_i)+\boldsymbol{f}_i(t)+\boldsymbol{d}_i(t)+\\&k_1\gamma_1\Big|\sum_{j=0}^{N}a_{ij}\mathrm{sig}(\hat{\boldsymbol{x}}_i(t)-\hat{\boldsymbol{x}}_j(t))\Big|^{\gamma_1-1}\times\\&\Big[\sum_{j=0}^{N}a_{ij}(\hat{\boldsymbol{v}}_i(t)-\hat{\boldsymbol{v}}_j(t))\Big]+\\&k_2\gamma_2\left|(\hat{\boldsymbol{x}}_i(t)-\hat{\boldsymbol{x}}_j(t))\right|^{\gamma_2-1}\Big[\sum_{j=0}^{N}a_{ij}(\hat{\boldsymbol{v}}_i(t)-\hat{\boldsymbol{v}}_j(t))\Big]-\\&\boldsymbol{f}_0(t)+\hat{\boldsymbol{u}}_i(t)-\hat{\boldsymbol{u}}_i(t-h_i)=\\&-\hat{\boldsymbol{E}}_i(t)-k_3\mathrm{sig}(\hat{\boldsymbol{e}}_i(t))^{\rho_1}-k_4\mathrm{sig}(\hat{\boldsymbol{e}}_i(t))^{\rho_2}+\boldsymbol{d}_i.\end{aligned}\tag{43}$$

Theorem 3 For the MAS (2) and Eq. (38), under the controller (41) and based on virtual velocity designed in Eq. (39), the fixed-time consensus with delay is achieved and the Zeno behaviour can be excluded by event-triggered mechanism (42). The bound of T is expressed as

$$\begin{aligned}T=&\hat{T}_1+\hat{T}_2\leqslant\\&\frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}}k_4N^{\frac{1-\rho_2}{2}}(1-\omega_i)}}+\\&\frac{2\pi}{(\gamma_2-\gamma_1)\sqrt{k_1(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{1+\frac{\gamma_1+\gamma_2}{2}}k_2N^{\frac{1-\gamma_2}{2}}}}+\\&\max(h_i).\end{aligned}\tag{44}$$

Proof Lyapunov function is constructed by tracking error:

$$\hat{V}_1(t)=\frac{1}{2}\sum_{i=1}^{N}\hat{\boldsymbol{e}}_i^2(t).\tag{45}$$

According to Eqs. (19) and (43), the derivative of $\hat{V}_1(t)$ is given:

$$\begin{aligned}\dot{\hat{V}}_1(t)=&\sum_{i=1}^{N}\hat{\boldsymbol{e}}_i(t)\dot{\hat{\boldsymbol{e}}}_i(t)=\\&\sum_{i=1}^{N}\hat{\boldsymbol{e}}_i(t)(-\hat{\boldsymbol{E}}_i(t)-k_3\mathrm{sig}(\hat{\boldsymbol{e}}_i(t))^{\rho_1}-\\&k_4\mathrm{sig}(\hat{\boldsymbol{e}}_i(t))^{\rho_2}+\boldsymbol{d}_i)\leqslant\\&-k_3 2^{\frac{\rho_1+1}{2}}\hat{V}_1(t)^{\frac{\rho_1+1}{2}}-\big(k_4 2^{\frac{\rho_2+1}{2}}N^{\frac{1-\rho_2}{2}}-D\big)\times\\&(1-\omega_i)\hat{V}_1(t)^{\frac{\rho_2+1}{2}}.\end{aligned}\tag{46}$$

The proof process of Lyapunov function can be given by Theorem 1. According to Lemma 3, the error $\hat{\boldsymbol{e}}_i(t)$ converges to 0 in time $\hat{T}_{1\max}$, and the convergence time is satisfied:

$$\hat{T}_{1\max} = \frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}} k_4 N^{\frac{1-\rho_2}{2}}(1-\omega_i)}}. \quad (47)$$

Remark 4 When $t = \hat{T}_{1\max}$, we can get that $\hat{V}_1(t) = 0$ and the error $\hat{\boldsymbol{e}}_i(t) = \boldsymbol{0}$, which implies $\left(k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} - D\right)\|\hat{\boldsymbol{e}}_i\|^{\rho_2}\omega_i = 0$. In this case, the consensus is not achieved if $\hat{\boldsymbol{u}}_i(t) \neq \boldsymbol{0}$. The event-triggered condition $\|\hat{\boldsymbol{E}}_i\| - \left(k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} - D\right)\|\hat{\boldsymbol{e}}_i\|^{\rho_2}\omega_i > 0$ can be satisfied, the system is updated, and $\hat{\boldsymbol{u}}_i(t)$ converges to 0. Therefore, $\hat{\boldsymbol{u}}_i(t) = \boldsymbol{0}$ is true before the consensus can be achieved. In addition, $\int_{t-h_i}^{t} \hat{\boldsymbol{u}}_i(T)\mathrm{d}T$ converges to 0 at time $\hat{T}_{1\max} + h_i$. Therefore, $\hat{T}_1$ can be obtained:

$$\hat{T}_1 \leqslant \hat{T}_{1\max} + \max(h_i) = \frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}} k_4 N^{\frac{1-\rho_2}{2}}(1-\omega_i)}} + \max(h_i). \quad (48)$$

According to the above analysis, Eq. (21) still holds and $\hat{\boldsymbol{v}}_i(t) = \hat{\boldsymbol{v}}_i^*(t) = -k_1\mathrm{sig}\Big(\sum_{j=0}^{N} a_{ij}\mathrm{sig}(\hat{\boldsymbol{x}}_i(t) - \hat{\boldsymbol{x}}_j(t))\Big)^{\gamma_1} - k_2\mathrm{sig}\Big(\sum_{j=0}^{N} a_{ij}(\hat{\boldsymbol{x}}_i(t) - \hat{\boldsymbol{x}}_j(t))\Big)^{\gamma_2} + \boldsymbol{v}_0(t)$. From Eqs. (4) and (21), it follows that

$$\dot{\bar{\hat{\boldsymbol{x}}}}_i(t) = \hat{\boldsymbol{v}}_i^*(t) - \boldsymbol{v}_0(t) = -k_1\mathrm{sig}\Big(\sum_{j=0}^{N} a_{ij}\mathrm{sig}(\hat{\boldsymbol{x}}_i(t) - \hat{\boldsymbol{x}}_j(t))\Big)^{\gamma_1} - k_2\mathrm{sig}\Big(\sum_{j=0}^{N} a_{ij}(\hat{\boldsymbol{x}}_i(t) - \hat{\boldsymbol{x}}_j(t))\Big)^{\gamma_2}. \quad (49)$$

Consider the Lyapunov function as

$$\hat{V}_2(t) = \frac{1}{2}\bar{\hat{\boldsymbol{x}}}_i^{\mathrm{T}}(t)\boldsymbol{B}_{\mathrm{L}}\bar{\hat{\boldsymbol{x}}}_i(t). \quad (50)$$

Take the derivative of $\hat{V}_2(t)$ as

$$\dot{\hat{V}}_2(t) = \bar{\hat{\boldsymbol{x}}}_i^{\mathrm{T}}(t)(\boldsymbol{B}_{\mathrm{L}})\dot{\bar{\hat{\boldsymbol{x}}}}_i(t) = \sum_{i=1}^{N}\sum_{j=0}^{N} a_{ij}(\hat{\boldsymbol{x}}_i(t) - \hat{\boldsymbol{x}}_j(t))\bar{\hat{\boldsymbol{v}}}_i(t) \leqslant -k_1(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{\frac{\gamma_1+1}{2}}\hat{V}_2(t)^{\frac{\gamma_1+1}{2}} - k_2 N^{\frac{1-\gamma_2}{2}}(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{\frac{\gamma_2+1}{2}}\hat{V}_2(t)^{\frac{\gamma_2+1}{2}}. \quad (51)$$

The position of followers converges to the leader in fixed-time and $\hat{\boldsymbol{T}}_2$ is written as

$$\hat{T}_2 \leqslant \hat{T}_{2\max} = \frac{2\pi}{(\gamma_2-\gamma_1)\sqrt{k_1(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{1+\frac{\gamma_1+\gamma_2}{2}} k_2 N^{\frac{1-\gamma_2}{2}}}}. \quad (52)$$

Under the action of controller (41) and event-triggered mechanism (42), the consensus problem of the MAS (2) and Eq. (38) is solved in fixed-time T, satisfying as follows:

$$T \leqslant \max(h_i) + \frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}} k_4 N^{\frac{1-\rho_2}{2}}(1-\omega_i)}} + \frac{2\pi}{(\gamma_2-\gamma_1)\sqrt{k_1(2\lambda_2(\boldsymbol{B}_{\mathrm{L}}))^{1+\frac{\gamma_1+\gamma_2}{2}} k_2 N^{\frac{1-\gamma_2}{2}}}}. \quad (53)$$

The proof is completed. □

2.2.2 *Event-Triggered Fixed-Time Consensus Algorithm with Switching Topologies*

Consider that the communication topology is time-varying and connected. For the MASs (1) and (2), a switching signal is introduced that $\delta(t) : [0, +\infty) \to Q$ and the topologies of the system can be determined by $\delta(t)$. The communication graph is fixed if and only if $\delta(t)$ is constant. Otherwise, $Q = \{1, 2, \cdots, N\}$ is a finite set and it is an index set for the set of undirected graphs $G_{\mathrm{s}} = (V, E, \boldsymbol{A}_{\delta(t)})$, where adjacency matrix $\boldsymbol{A}_{\delta(t)} = [a_{ij}^{\delta}]_{N\times N}$ and a_{ij}^{δ} is used to represent the adjacency relationship between the agent i and j. $G_{\delta(t)} \in G_{\mathrm{s}}$ is used to represent the topology at time t and $t_0, t_1, \cdots$ are the switching time series.

For the MASs under switching topologies, the nonlinear dynamic functions of followers are redefined as

$$\left.\begin{aligned} &\dot{\tilde{\boldsymbol{x}}}_i(t) = \tilde{\boldsymbol{v}}_i(t) \\ &\dot{\tilde{\boldsymbol{v}}}_i(t) = \tilde{\boldsymbol{u}}_i(t) + \boldsymbol{f}_i(\tilde{\boldsymbol{x}}_i(t), \tilde{\boldsymbol{v}}_i(t), t) + \\ &\qquad \boldsymbol{d}_i(\tilde{\boldsymbol{x}}_i(t), \tilde{\boldsymbol{v}}_i(t), t) \end{aligned}\right\}. \quad (54)$$

The virtual velocity (5) can be rewritten as

$$\tilde{\boldsymbol{v}}_i^*(t) = -k_1\mathrm{sig}\Big(\sum_{j=0}^{N} a_{ij}^{\delta}\mathrm{sig}(\tilde{\boldsymbol{x}}_i(t) - \tilde{\boldsymbol{x}}_j(t))\Big)^{\gamma_1} - k_2\mathrm{sig}\Big(\sum_{j=0}^{N} a_{ij}^{\delta}(\tilde{\boldsymbol{x}}_i(t) - \tilde{\boldsymbol{x}}_j(t))\Big)^{\gamma_2} + \boldsymbol{v}_0(t). \quad (55)$$

The error function is

$$\tilde{\boldsymbol{e}}_i(t) = \tilde{\boldsymbol{v}}_i(t) - \tilde{\boldsymbol{v}}_i^*(t). \quad (56)$$

We can obtain

$$\tilde{\boldsymbol{u}}_i(t) = -k_3\mathrm{sig}(\tilde{\boldsymbol{e}}_i(t_k^i))^{\rho_1} - k_4\mathrm{sig}(\tilde{\boldsymbol{e}}_i(t_k^i))^{\rho_2} - k_1\gamma_1\Big|\sum_{j=0}^{N} a_{ij}^{\delta}\mathrm{sig}(\tilde{\boldsymbol{x}}_i(t_k^i) - \tilde{\boldsymbol{x}}_j(t_k^i))\Big|^{\gamma_1-1} \times$$

$$\Big[\sum_{j=0}^{N} a_{ij}^{\delta}(\tilde{\boldsymbol{v}}_i(t_k^i) - \tilde{\boldsymbol{v}}_j(t_k^i))\Big] - k_2\gamma_2\Big|\sum_{j=0}^{N} a_{ij}^{\delta}(\tilde{\boldsymbol{x}}_i(t_k^i) - \tilde{\boldsymbol{x}}_j(t_k^i))\Big|^{\gamma_2-1}\times \Big[\sum_{j=0}^{N} a_{ij}^{\delta}(\tilde{\boldsymbol{v}}_i(t_k^i) - \tilde{\boldsymbol{v}}_j(t_k^i))\Big] + \boldsymbol{f}_0 - \boldsymbol{f}_i. \tag{57}$$

The measurement error $\boldsymbol{E}_i(t)$ is adjusted to $\tilde{\boldsymbol{E}}_i(t)$ and event-triggered mechanism is designed as

$$\varDelta_i(\tilde{\boldsymbol{E}}_i(t), \tilde{\boldsymbol{e}}_i(t), t) = \|\tilde{\boldsymbol{E}}_i\| - \big(k_4 2^{1+\rho_2} N^{\frac{1-\rho_2}{2}} - D\big)\|\tilde{\boldsymbol{e}}_i\|^{\rho_2}\omega_i. \tag{58}$$

Theorem 4 Under the Assumptions 1 and 2, the fixed-time consensus can be obtained by virtual velocity (55) and consensus protocol (57) for the MASs (54) and (2). And the event-triggered function (58) is used to avoid Zeno behavior. The bound of T is expressed as

$$T = \tilde{T}_1 + \tilde{T}_2 \leqslant \frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}} k_4 N^{\frac{1-\rho_2}{2}}(1-\omega_i)}} + 4\pi\big/\big\{(\gamma_2-\gamma_1)\big[k_1(4\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_1)}))^{\frac{\gamma_1+1}{2}}\times k_2 N^{\frac{1-\gamma_2}{2}}(4\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_2)}))^{\frac{\gamma_2+1}{2}}\big]^{\frac{1}{2}}\big\}, \tag{59}$$

where $\boldsymbol{L}_{2/(1+\gamma_1)}$, $\boldsymbol{L}_{2/(1+\gamma_2)}$ are the Laplacian matrices of communication topologies $G(\boldsymbol{A}^{\frac{2}{1+\gamma_1}})$ and $G(\boldsymbol{A}^{\frac{2}{1+\gamma_2}})$, and $\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_1)})$, $\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_2)})$ are the minimum eigenvalue of $\boldsymbol{L}_{2/(1+\gamma_1)}$ and $\boldsymbol{L}_{2/(1+\gamma_2)}$, respectively. In addition, there exist $\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_1)}) = \min\{\lambda_2(\boldsymbol{L}_{2/(1+\gamma_1)}(t_0)), \lambda_2(\boldsymbol{L}_{2/(1+\gamma_1)}(t_1)), \cdots\}$ and $\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_2)}) = \min\{\lambda_2(\boldsymbol{L}_{2/(1+\gamma_2)}(t_0)), \lambda_2(\boldsymbol{L}_{2/(1+\gamma_2)}(t_1)), \cdots\}$.

Proof Construct a Lyapunov function as

$$\tilde{V}_1(t) = \frac{1}{2}\sum_{i=1}^{N}\tilde{\boldsymbol{e}}_i^2(t). \tag{60}$$

Based on the measurement error $\tilde{\boldsymbol{E}}_i(t)$, take the derivative of $\tilde{V}_1(t)$ as

$$\dot{\tilde{V}}_1(t) = \sum_{i=1}^{N}\tilde{\boldsymbol{e}}_i(t)\dot{\tilde{\boldsymbol{e}}}_i(t) = \sum_{i=1}^{N}\tilde{\boldsymbol{e}}_i(t)(-\tilde{\boldsymbol{E}}_i(t) - k_3\mathrm{sig}(\tilde{\boldsymbol{e}}_i(t))^{\rho_1} - k_4\mathrm{sig}(\tilde{\boldsymbol{e}}_i(t))^{\rho_2} + \boldsymbol{d}_i) \leqslant \sum_{i=1}^{N}\tilde{\boldsymbol{e}}_i(t)(-\tilde{\boldsymbol{E}}_i(t) - k_3\mathrm{sig}(\tilde{\boldsymbol{e}}_i(t))^{\rho_1} - k_4\mathrm{sig}(\tilde{\boldsymbol{e}}_i(t))^{\rho_2} + D) \leqslant - k_3 2^{\frac{\rho_1+1}{2}}\tilde{V}_1(t)^{\frac{\rho_1+1}{2}} - (k_4 2^{\frac{\rho_2+1}{2}} N^{\frac{1-\rho_2}{2}} - D)\times (1-\omega_i)\tilde{V}_1(t)^{\frac{\rho_2+1}{2}}. \tag{61}$$

The tracking time $\tilde{T}_1$ can be obtained:

$$\tilde{T}_1 \leqslant \tilde{T}_{1\max} = \frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}} k_4 N^{\frac{1-\rho_2}{2}}(1-\omega_i)}}. \tag{62}$$

Lyapunov function $\tilde{V}_2(t)$ is constructed by position of agents. Take the derivative of $\tilde{V}_2(t)$:

$$\tilde{V}_2(t) = \frac{1}{2}\sum_{i=1}^{N}\bar{\tilde{\boldsymbol{x}}}_i^2(t), \tag{63}$$

$$\begin{aligned}\dot{\tilde{V}}_2(t) &= \sum_{i=1}^{N}\bar{\tilde{\boldsymbol{x}}}_i(t)\dot{\bar{\tilde{\boldsymbol{x}}}}_i(t) = \sum_{i=1}^{N}\bar{\tilde{\boldsymbol{x}}}_i(t)\bar{\tilde{\boldsymbol{v}}}_i(t) \leqslant \\ &\sum_{i=1}^{N}\bar{\tilde{\boldsymbol{x}}}_i(t)\Big(-k_1\sum_{j=1}^{N}a_{ij}^{\delta}\left|(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t))\right|^{\gamma_1} - k_2\sum_{j=1}^{N}a_{ij}^{\delta}\left|(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t))\right|^{\gamma_2}\Big) \leqslant \\ &-\frac{1}{2}k_1\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\delta}\left|(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t))\right|^{\gamma_1}(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t)) - \frac{1}{2}k_2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\delta}\left|(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t))\right|^{\gamma_2}(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t)) \leqslant \\ &-\frac{1}{2}k_1\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\delta}\left|(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t))\right|^{\gamma_1+1} - \frac{1}{2}k_2\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\delta}\left|(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t))\right|^{\gamma_2+1} \leqslant \\ &-\frac{1}{2}k_1\Big(\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\delta\frac{2}{\gamma_1+1}}\left|(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t))^2\right|\Big)^{\frac{\gamma_1+1}{2}} - \frac{1}{2}k_2\Big(\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}^{\delta\frac{2}{\gamma_2+1}}\left|(\bar{\tilde{\boldsymbol{x}}}_i(t)-\bar{\tilde{\boldsymbol{x}}}_j(t))^2\right|\Big)^{\frac{\gamma_2+1}{2}} \leqslant \\ &-\frac{1}{2}k_1(4\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_1)}))^{\frac{\gamma_1+1}{2}}\tilde{V}_2(t)^{\frac{\gamma_1+1}{2}} - \frac{1}{2}k_2 N^{\frac{1-\gamma_2}{2}}(4\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_2)}))^{\frac{\gamma_2+1}{2}}\tilde{V}_2(t)^{\frac{\gamma_2+1}{2}}.\end{aligned} \tag{64}$$

According to Lemma 3, the consensus time $\tilde{T}_2$ is satisfied as follows:

$$T_2 = \tilde{T}_{2\max} = 4\pi\big/\big\{(\gamma_2-\gamma_1)\big[k_1(4\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_1)}))^{\frac{\gamma_1+1}{2}}\times k_2 N^{\frac{1-\gamma_2}{2}}(4\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_2)}))^{\frac{\gamma_2+1}{2}}\big]^{\frac{1}{2}}\big\}. \tag{65}$$

Therefore, it can be concluded that the system consensus achieves within the fixed-time:

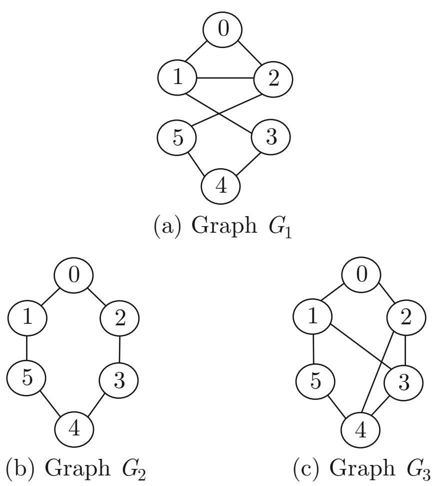
$$T \leqslant \frac{2\pi}{(\rho_2-\rho_1)\sqrt{k_3 2^{1+\frac{\rho_1+\rho_2}{2}} k_4 N^{\frac{1-\rho_2}{2}}(1-\omega_i)}} + 4\pi/\{(\gamma_2-\gamma_1)[k_1(4\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_1)}))^{\frac{\gamma_1+1}{2}} \times k_2 N^{\frac{1-\gamma_2}{2}}(4\lambda_2^{\min}(\boldsymbol{L}_{2/(1+\gamma_2)}))^{\frac{\gamma_2+1}{2}}]^{\frac{1}{2}}\}. \tag{66}$$

The proof is completed. □

Remark 5 The proof of event-triggered mechanism is similar to that of Theorem 2. The parameters in Eq. (31) are adjusted to $\tilde{c}_1 = \rho_2\|\tilde{\boldsymbol{e}}\|^{\rho_2-1}$, $\tilde{c}_2 = \frac{k_3}{N^{1-\rho_2}}(2\tilde{V}_1(0))^{\frac{\rho_1-\rho_2}{2}} + \frac{k_4}{N^{1-\rho_2}}$ and $\tilde{c}_3 = D(2\tilde{V}_1(0))^{\frac{-\rho_2}{2}} + 1$. It can be obtained that the result in Theorem 2 is still available. In addition, it is ensured that the topology graph is connected within the event-triggered interval. Therefore, event-triggered mechanism is still valid in the switching topologies.

3 Simulation Example

A simulation example is provided to validate the effectiveness of the event-triggered fixed-time consensus algorithm. The undirected connection topologies are given in Fig. 1, and the example in the fixed topology is represented by Fig. 1(a). In addition, Fig. 1 shows the switching topologies. Consider that the switching interval is 0.5 s and the switching sequence satisfies $(a) \rightarrow (b) \rightarrow (c) \rightarrow (a)$. Figure 2 shows the working principle of the controller. Assume that there is a leader and five followers in the system.



(a) Graph G_1 (b) Graph G_2 (c) Graph G_3

Fig. 1 Communication topology graphs

Some simulation parameters are given. Assume that the information of initial position and velocity for each follower is proposed as $\boldsymbol{x}(0) = [-1 \quad -2 \quad 2 \quad 1 \quad 3]$ and $\boldsymbol{v}(0) = [2 \quad 1 \quad 3 \quad 2 \quad 1]$. The corresponding information of the leader is $\boldsymbol{x}_0(0) = 1$ and $\boldsymbol{v}_0(t) = 0.4 + 0.3\sin t$. In addition, the other parameters are given as $\boldsymbol{f}_i(\boldsymbol{x}_i(t), \boldsymbol{v}_i(t), t) = 0.3\cos(0.5t)$, $\boldsymbol{d}_i(\boldsymbol{x}_i(t), \boldsymbol{v}_i(t), t) = 0.2\cos(\boldsymbol{x}_i(t))$, $k_1 = 2$, $k_2 = 1$, $k_3 = 3$, $k_4 = 2$, $\gamma_1 = 0.9$, $\gamma_2 = 1.1$, $\rho_1 = 0.3$, $\rho_2 = 1.05$, and $\omega = 0.5$. The choice of the parameters is derived by performing multiple tests within the given range.

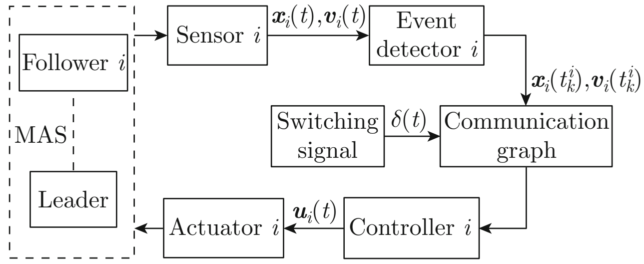


Fig. 2 Event-triggered fixed-time consensus controller

As shown in Figs. 3—8, it can be seen that the consensus of position and velocity can be achieved in these figures. Figures 3—5 are the path of position, the path of position with the event-triggered and the path of position with the delay and switching topologies, respectively. Figures 6—8 are the corresponding velocity information.

Figures 9—10 show the tracking error and measurement error of the system. It is clear that the errors

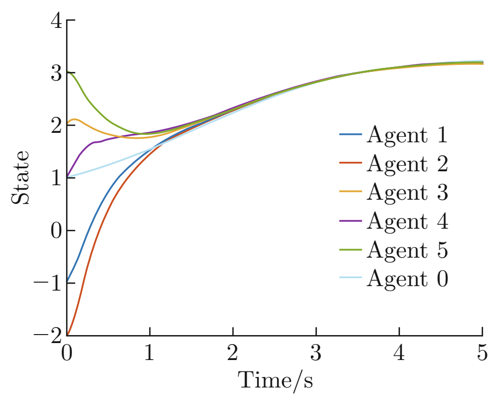


Fig. 3 Position of leader and followers

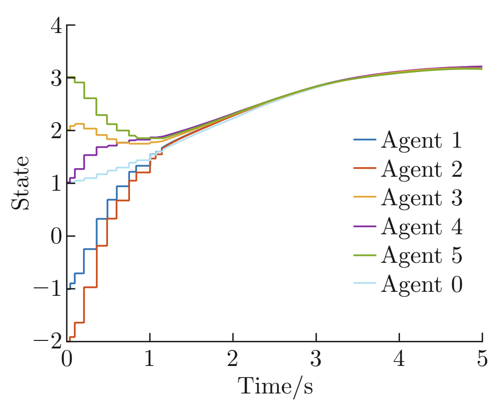


Fig. 4 Position under event-triggered mechanism

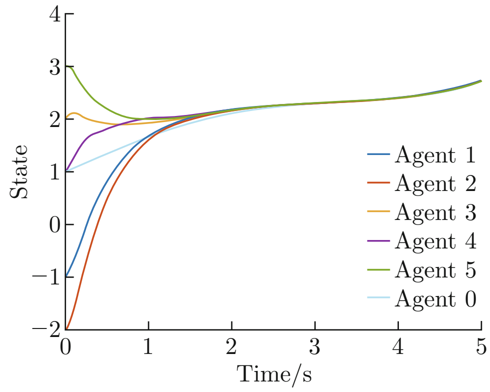


Fig. 5 Position of delay and switching topologies

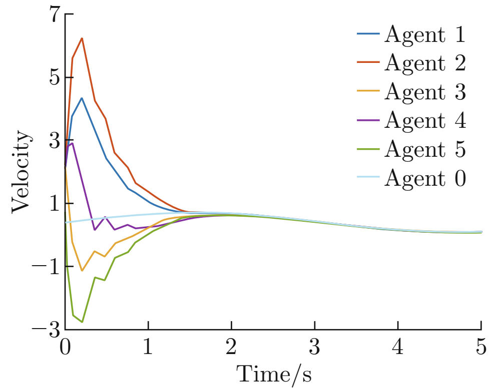


Fig. 6 Velocity of leader and followers

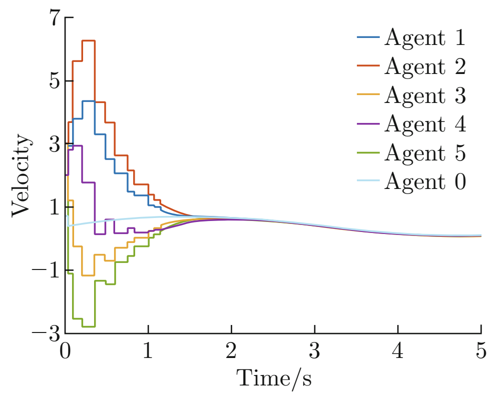


Fig. 7 Velocity under event-triggered mechanism

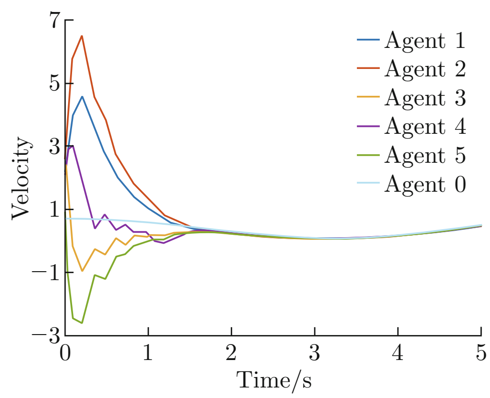
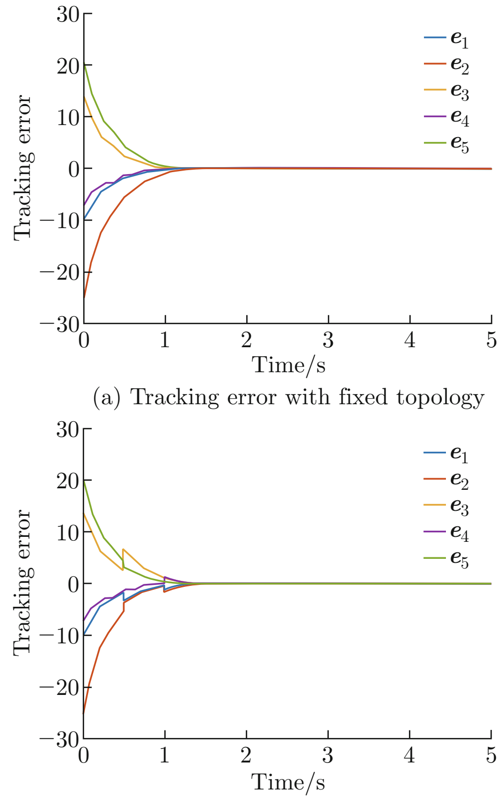


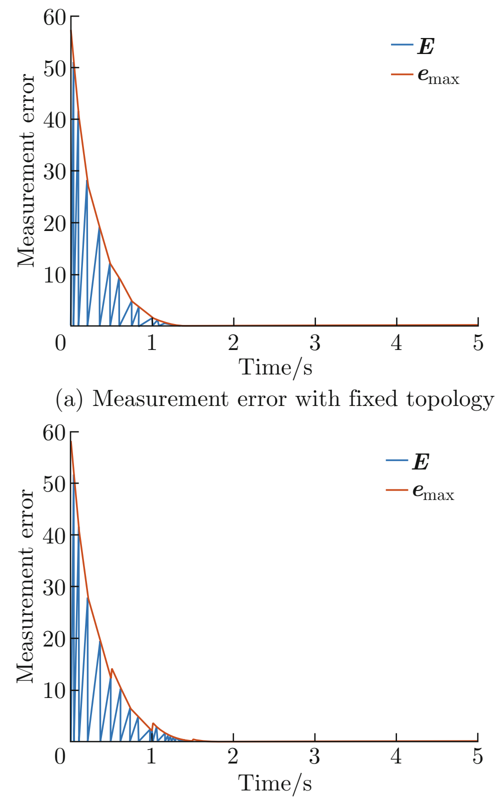
Fig. 8 Velocity of delay and switching topologies



(a) Tracking error with fixed topology

(b) Tracking error with delay and switching topologies

Fig. 9 Tracking error of true velocity and virtual velocity



(a) Measurement error with fixed topology

(b) Measurement error with delay and switching topologies

Fig. 10 Evolution of error norm and threshold converge to 0 in less than 2 s. In addition, the change of the switching topologies can be seen in the time nodes of Figs. 9(b) and 10(b).

For the MASs with delay and switching topologies, Figs. 11 and 12 show the inputs of agents and the sampling number of event-triggered mechanism. It can be seen that the inputs converge to 0 in around 2 s in Fig. 11, and the event-triggered interval is shown Fig. 12.

Figures 13 and 14 show the control inputs for Refs. [7] and [12] respectively. It can be seen from the results that the computational complexity of the controller designed in this paper is lower.

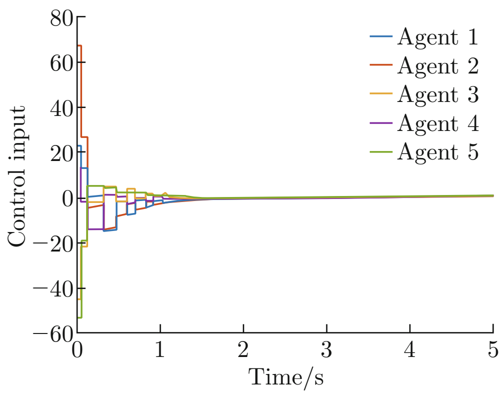


Fig. 11 Input of controller

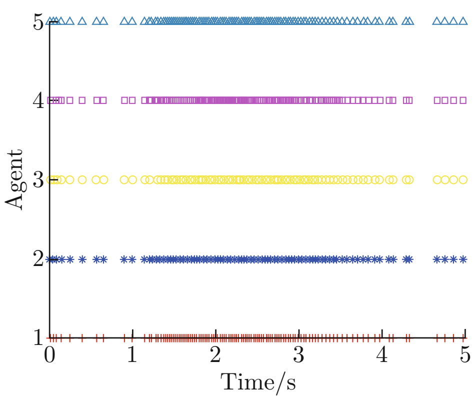


Fig. 12 Sampling of agents

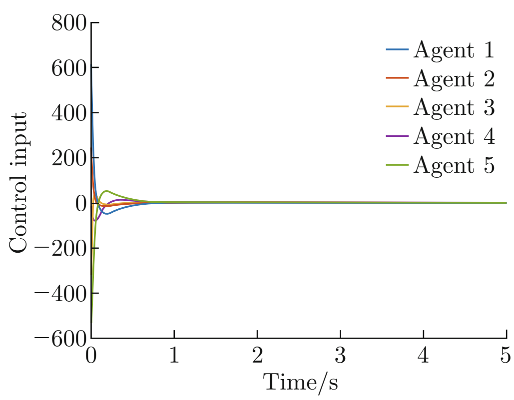


Fig. 13 Input of controller in Ref. [7]

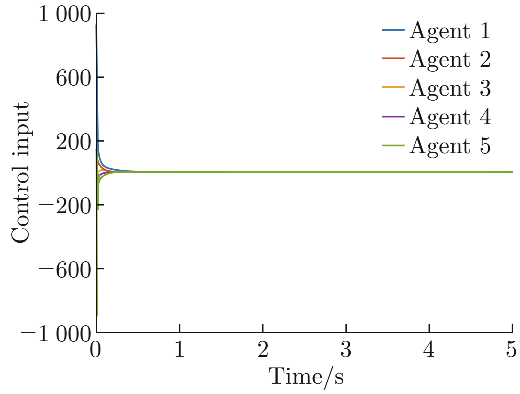


Fig. 14 Input of controller in Ref. [12]

4 Conclusion

A protocol of event-triggered fixed-time consensus for second-order nonlinear MASs with delay and switching topologies is proposed in this paper. The problem of convergence time dependence on initial value is addressed by employing the fixed-time consensus control. The control inputs are updated if and only if the measurement error of agents meets the trigger threshold condition. It is effective to avoid the Zeno behaviors. In addition, it is necessary to prove the availability of controller through stability analysis. The stabilization of the system is proved by Lyapunov stability theory. It can be seen from the results that the stability of the system can be ensured under the influence of external disturbances and nonlinear term. The controller is still valid in the case of delay and switching topologies. Therefore, the robustness and practicality of the system are improved. Finally, the simulation example indicates that the control algorithm is effective for solving the fixed-time consensus problem of the system. The object of this paper is second-order and homogeneous MASs. However, heterogeneous MASs are common in practical applications, and it is inevitable in the study of consensus problems. Furthermore, there are many other factors that affect the stability of the system, such as data packet loss and cyber-attack. The problems mentioned above are difficulties that need to be studied intensively in the future.

Conflict of Interest The authors declare no conflict of interest.

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