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Impact Angle/Time Constraint Guidance Design Based on Fast Terminal Error Dynamics

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Abstract: Considering the problem that the optimal error dynamics can only converge at the terminal time, an impact angle/time constraint missile guidance law with finite-time convergence is designed in this paper, which is based on the pure proportional navigation (PPN) guidance law and the fast terminal error dynamics (FTED) approach. The missile guidance model and FTED equation are given first, and the dynamic equation of impact angle/time error based on PPN is also derived. Then, the guidance law is designed based on FTED, and the guidance error can converge to 0 in a finite time. Furthermore, considering the field of view constraint, the guidance law is improved by using the saturation function mapping method. Finally, a numerical simulation example is given to verify the effectiveness of the guidance law, which shows that the guidance law proposed in this paper can make the missile quickly adjust to the desired states in advance, and effectively relieve the overload saturation pressure of the actuator.
Key words: design of guidance law, fast terminal error dynamics, field-of-view angle, impact time, impact angle
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Nomenclature

a_{IA}— Biased term to regulate the terminal impact angle
a_{IT}— Biased term to regulate the terminal impact time
a_{m}— Guidance acceleration command of the missile
a_{PPN}— Pure proportional navigation (PPN) term of the acceleration command
$(\boldsymbol{e}_r, \boldsymbol{e}_\theta)$— Line-of-sight (LOS) frame
M—Missile
n— Design parameter controlling the convergence curvature of the saturation function
N—Proportional navigation gain
p, q— Regulation parameters of the nonlinear term, which are positive odd numbers and $p > q$
$\boldsymbol{r}$— Vector of the relative distance between the target and the missile
$\boldsymbol{r}_{\mathrm{m}}$— Position vector of the missile
$\boldsymbol{r}_{\mathrm{t}}$— Position vector of the target
t— Current flying time
t_0— Initial time
t_{f}— Terminal time
t_{d}— Desired impact time
$(\boldsymbol{t}_{\mathrm{m}}, \boldsymbol{n}_{\mathrm{m}})$— Velocity frame
t_{go}— Remaining flight time
T— Target
$\boldsymbol{v}_{\mathrm{m}}$— Velocity vector of the missile
(x_0, y_0)— Position of the missile at the initial time
$(x_{\mathrm{f}}, y_{\mathrm{f}})$— Position of the missile at the terminal time
(X, Y)— Inertial frame
α— Coefficient of the linear term
β— Coefficient of the nonlinear term
ε— Tracking error
ε_t— Impact time error
ε_φ— Impact angle error
θ_{m}— Velocity leading angle
$\theta_{\max}$— Maximum field of view (FOV) angle of the missile
σ— LOS angle
φ_0— Flight path angle of the missile at the initial time
φ_{d}— Desired terminal impact angle
φ_{f}— Flight path angle of the missile at the terminal time
φ_{m}— Flight path angle

0 Introduction

The proportional navigation guidance (PNG) law is widely used in various missile guidance problems due to its simple structure, easy implementation, and good robustness[1]. Modern warfare puts forward higher

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performance requirements and more terminal constraints for tactical missile guidance, which results in the performance reduction of the PNG and the emergence of advanced guidance laws, such as impact angle constraint guidance (IACG) law and impact time constraint guidance (ITCG) law. The IACG, which can improve the lethality of the missile by attacking the weak area of the target, can be designed based on the improved biased proportional navigation guidance (BPNG) laws. In Ref. [2], the Lyapunov stability theory was used to modify and improve the PNG, so that it could hit fixed or slow-moving targets at specific impact angles, and was further extended to hit mobile targets. A nonsingular impact angle constrained guidance law was developed by using the advanced terminal sliding mode control schemes and extended state observer in Ref. [3]. The effects of system lag on the performance of IACG were investigated in Ref. [4], and the effects of the guidance coefficients on the terminal misses are also discussed. Reference [5] proposed a guidance strategy with impact angle constraints based on the analytical solution of pure proportional navigation (PPN) guidance law against stationary targets. Aimed at the problem of the minimum field-of-view (FOV) angle constraint of the strap-down seeker in hypersonic missiles, Ref. [6] proposed a guidance strategy that can be applied to the guidance law with independent orders on longitudinal and lateral channels. In Ref. [7], the problem of guidance command design was transformed into the design of the flight path angle through mathematical model transformation, and a polynomial guidance law with impact angle constraint based on the Bézier curve was proposed. A look angle shaping guidance law with impact angle and seeker's FOV constraints was proposed in Ref. [8], which considers the time-varying speed. Reference [9] proposed a finite-time convergent sliding-mode guidance law with impact angle constraint, which can make the line-of-sight (LOS) rate converge to zero in finite time. Aimed at the singular problems of the guidance command, two nonsingular sliding mode guidance laws were proposed in Refs. [10] and [11].

Impact time constraint is another important problem in missile guidance law design. ITCG was first proposed in 2006 to solve the salvo attack of anti-ship missiles[12]. In recent years, many scholars have carried out extensive research on ITCG under the requirements of new combat modes such as attack window limitation and cooperative strike. In Ref. [13], a new three-dimensional optimal guidance law for impact time constraint with seeker's FOV constraint was proposed, which is devised in conjunction with the concept of BPNG. An impact time and angle constraint guidance law was proposed in Ref. [14], which originates from look-angle shaping using time-polynomial. A nonsingular sliding mode guidance law was proposed based on a remaining flight time estimation method suitable for the maneuverable targets in Ref. [15], which is used to solve the attack problem of time cooperation of multiple missiles for high value or large targets. In Ref. [16], a three-dimensional impact time constraint guidance law with FOV constraint was proposed to solve the saturation attack problem of guided munitions.

As is known to all, the guidance law design is a kind of finite-time error tracking problem, and its guidance instruction command is derived from the deviation between the actual states and the desired states of the missile (the so-called "guidance error"). In previous studies, many scholars mainly focused on the effective elimination of the guidance error[17], that is, the guidance accuracy, but few studied the convergence characteristics of the guidance error. In engineering practice, the convergence rate of guidance error is often limited by practical factors such as the saturation of the missile actuator. If the error convergence time is too short, the acceleration command, generated by the designed guidance law, may exceed the saturation of the missile actuator, leading to the degradation of guidance performance in application. If the error convergence time is too long, there is a risk that the tracking error cannot converge to 0 in the desired time. Therefore, the key to missile guidance design is to make the tracking error converge to 0 in a limited time without generating a too large guidance command. Inspired by this, a multi-constraint guidance law design method based on the fast terminal error dynamics (FTED) method is proposed in this paper first. And then, the dynamics equations of impact angle error and impact time error are derived based on the PPN. Finally, the guidance law which can make the guidance error converge to zero quickly in finite time is obtained. Further, the guidance law is improved by using the saturation function to solve the problem of the seeker's maximum FOV angle constraint effectively. The effectiveness of the proposed guidance laws is verified by numerical simulation.

The main innovations of this paper are summarized as follows:

(1) A guidance law design method based on the FTED is proposed by combining the fast terminal sliding mode control (FTSMC) theory and the traditional error dynamics (TED) method. The advantage of this method is that, as long as the tracking error of the missile guidance problem is properly defined, it can be applied to guidance law design under any constraint.

(2) The specific design process of the impact angle constraint and impact time constraint guidance laws with FTED are given in this paper and the effectiveness is verified by numerical simulation.

(3) The seeker's FOV angle constraint is also considered in the design of the impact time constraint guidance law. The guidance law is improved by adopting the saturation function and the numerical simulation results show its effectiveness.

1 Missile-Target Relative Kinematics

The planar homing engagement geometry is shown in Fig. 1, where M and T denote the missile and target, respectively. The notation of (X, Y) represents the inertial frame. The variables $\boldsymbol{r}_\mathrm{m}$ and $\boldsymbol{r}_\mathrm{t}$ stand for the position vectors of missile and target, respectively; $\boldsymbol{v}_\mathrm{m}$ is the time-varying velocity vector of the missile. The symbol $\boldsymbol{r}$ denotes the vector of the relative distance between the target and the missile. The LOS frame ($\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$) and the velocity frame ($\boldsymbol{t}_\mathrm{m}$, $\boldsymbol{n}_\mathrm{m}$) are built based on the vector of the relative distance between the target and the missile and the velocity vector of the missile, respectively. The variable σ stands for the LOS angle, which is measured from the reference line x-axis to LOS and the positive direction is counterclockwise. Similarly, φ_m and θ_m are the missile flight path angle and velocity leading angle respectively, as shown in Fig. 1.

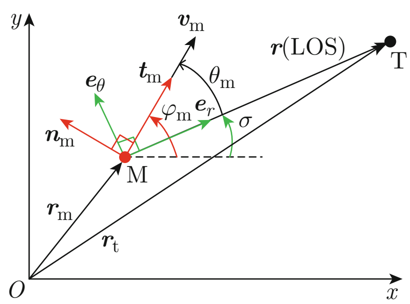


Fig. 1 Homing engagement geometry and parameter definitions

Based on the principles of kinematics, the kinematics equation of missile, depicted in Fig. 1, is formulated as

$$\left.\begin{aligned} \dot{x} &= v_\mathrm{m}\cos\varphi_\mathrm{m} \\ \dot{y} &= v_\mathrm{m}\sin\varphi_\mathrm{m} \end{aligned}\right\}, \tag{1}$$

and the equation of relative motion between missile and target can be given by

$$\left.\begin{aligned} \dot{r} &= -v_\mathrm{m}\cos\theta_\mathrm{m} \\ \dot{q} &= -\frac{v_\mathrm{m}\sin\theta_\mathrm{m}}{|\boldsymbol{r}|} \\ \dot{\varphi}_\mathrm{m} &= \frac{a_\mathrm{m}}{v_\mathrm{m}} \\ \varphi_\mathrm{m} &= \sigma + \theta_\mathrm{m} \end{aligned}\right\}, \tag{2}$$

where a_m is the guidance acceleration command of the missile that needs to be designed. And all of the variables without bold in the formula are scalars.

The initial and terminal constraints of flight time and impact angle control, respectively, are formulated as

$$\left.\begin{aligned} &x(t_0) = x_0, \quad y(t_0) = y_0, \quad \varphi_\mathrm{m}(t_0) = \varphi_0 \\ &x(t_\mathrm{f}) = x_\mathrm{f}, \quad y(t_\mathrm{f}) = y_\mathrm{f}, \quad \varphi_\mathrm{m}(t_\mathrm{f}) = \varphi_\mathrm{f} \\ &t_\mathrm{f} = t_\mathrm{d}, \quad \varphi_\mathrm{f} = \varphi_\mathrm{d} \end{aligned}\right\}, \tag{3}$$

where, t_0 is the initial time; t_f is the terminal time; (x_0, y_0) is the position of the missile at the initial time; $(x_\mathrm{f}, y_\mathrm{f})$ is the position of the missile at the terminal time; t_d is the expected flight time; φ_d is the expected terminal impact angle; φ_0 is the flight path angle of the missile at the initial time; φ_f is the flight path angle of the missile at the terminal time.

In the simplified missile guidance model, depicted in Fig. 1, the seeker's FOV is equivalent to the missile velocity lead angle. For the general guidance problem with FOV constraints, the FOV angle should satisfy

$$|\theta_\mathrm{m}| \leqslant \theta_\mathrm{max}, \tag{4}$$

where θ_max represents the maximum FOV angle of the missile, and it is generally less than 90°.

2 Fast Terminal Error Dynamics

Guidance law design is essentially a finite-time error tracking problem. In Ref. [18], the authors summarized the error dynamics (ED) method, gave the general form of the TED method, and put forward the optimal error dynamics (OED) method. Thus, a general method of guidance law design is given based on the OED equation. Referring to the ED method[18] and the FTSMC theory[19], the FTED method is proposed in this paper, which can make the error of the system converge to 0 in finite time and overcome the shortcoming that the TED method only guarantees asymptotic convergence and the OED method only converges when $t = t_\mathrm{f}$.

In Ref. [18], the TED equation is defined as

$$\dot{\varepsilon}(t) + \alpha\varepsilon(t) = 0, \tag{5}$$

where ε is the tracking error, and the constant $\alpha > 0$ is the coefficient of the linear term.

From Eq. (5), it is clear that the error $\varepsilon(t)$ converges asymptotically in the control process. Further, the OED equation, in Ref. [18], is given by

$$\dot{\varepsilon}(t) + \frac{\alpha}{t_\mathrm{go}}\varepsilon(t) = 0, \tag{6}$$

where $t_\mathrm{go} = t_\mathrm{f} - t$ denotes the remaining flight time, or the so-called time-to-go. However, the OED can only guarantee that the error $\varepsilon(t)$ converges at terminal time t_f.

Due to the limited time of missile hitting the target, it is hoped that the error $\varepsilon(t)$ can converge before the terminal time t_f. The FTSMC can make the system state converge to the equilibrium position in a finite time[20]. By referring to the FTSMC theory and the ED method, the terminal error dynamics (TeED) equation can be directly derived as

$$\dot{\varepsilon}(t) + \beta\varepsilon^{q/p}(t) = 0, \tag{7}$$

where, $\beta > 0$ is the coefficient of the nonlinear term; p and q are positive odd numbers, and $p > q$.

Through analysis, it can be concluded that the time required for the error amount $\varepsilon(t)$ to converge from the arbitrary initial state $\varepsilon(t_0)$ to the equilibrium state $\varepsilon(t) = 0$ is

$$T_{\mathrm{s}} = \frac{p}{\beta(p-q)} \left|\varepsilon(t_0)\right|^{(p-q)/p}. \tag{8}$$

Because of the introduction of the nonlinear part $\beta\varepsilon^{q/p}(t)$, the convergence rate of error convergence to 0 is improved, and the greater the systematic error, the faster the initial convergence rate.

However, TeED method still has defects in the convergence time, that is, when the systematic error converges near 0, the convergence speed of the nonlinear part $\beta\varepsilon^{q/p}(t)$ is less than that of the linear part $\alpha\varepsilon(t)$. Therefore, a fast terminal error dynamics method is proposed in this paper.

Combined with Eqs. (5) and (7), the FTED equation is determined as

$$\dot{\varepsilon}(t) + \alpha\varepsilon(t) + \beta\varepsilon^{q/p}(t) = 0, \tag{9}$$

where, α is defined in the same way as Eq. (5); β, p, and q are defined in the same way as Eq. (7).

The same thing can be said by analysis. It can be concluded that the time required for the error amount $\varepsilon(t)$ to converge from the arbitrary initial state $\varepsilon(t_0)$ to the equilibrium state $\varepsilon(t) = 0$ is

$$T_{\mathrm{s}} = \frac{p}{\alpha(p-q)} \ln \frac{\alpha\varepsilon^{(p-q)/p}(t_0) + \beta}{\beta}. \tag{10}$$

When the error $\varepsilon(t)$ is far away from the equilibrium position, the convergence time is mainly determined by the nonlinear part $\beta\varepsilon^{q/p}(t)$, while when the error quantity is close to the equilibrium position, the convergence time is mainly determined by the linear part $\alpha\varepsilon(t)$. Therefore, the FTED equation shown in Eq. (9) can not only make the error amount converge to the equilibrium position in a finite time but also do not lose the fast convergence near the equilibrium position, that is, the control command obtained based on the FTED method is smoother and the variation range is also small. The schematic diagram of FTED is shown in Fig. 2.

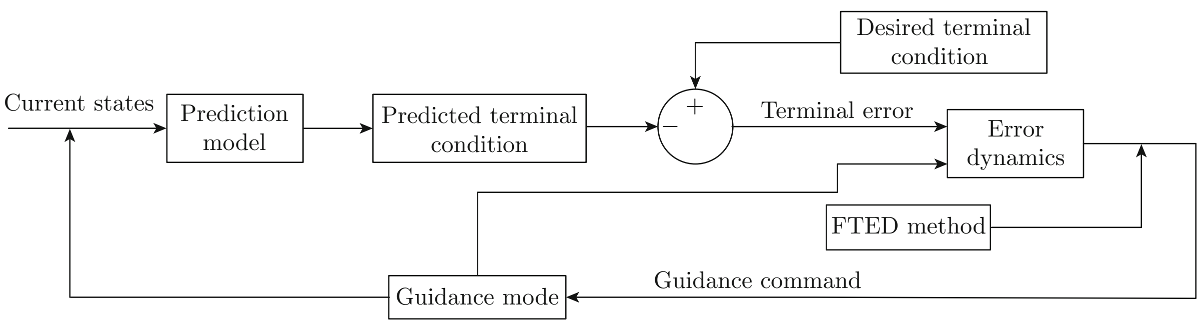


Fig. 2 Schematic diagram of FTED

3 Design of Guidance Law

In this section, the impact angle constraint and impact time constraint are considered respectively, and the specific process of corresponding guidance law design based on the FTED method is given.

3.1 FTED-IACG

The guidance command of the IACG can be written as

$$a_{\mathrm{m}} = a_{\mathrm{PPN}} + a_{\mathrm{IA}}, \tag{11}$$

where a_{PPN} is the PPN term, and a_{IA} is a biased term to regulate the terminal impact angle.

The terminal impact angle, when the missile is guided by PNG, can be expressed as

$$\varphi_{\mathrm{f}} = \frac{N}{N-1}\dot{\sigma} - \frac{1}{N-1}\varphi_{\mathrm{m}}. \tag{12}$$

Note that θ_{m} will converge to 0 eventually, namely $\varphi_{\mathrm{f}} = \sigma$.

Let the desired terminal impact angle be φ_{d}. Then the impact angle error can be defined as

$$\varepsilon_{\varphi} = \varphi_{\mathrm{d}} - \varphi_{\mathrm{f}}. \tag{13}$$

To achieve zero impact angle error, taking the derivative of ε_{φ}, substituting Eqs. (2), (11), and (12) in, sorting and simplifying, we have the impact angle error dynamics as follows:

$$\dot{\varepsilon}_{\varphi} = \frac{1}{(N-1)v_{\mathrm{m}}} a_{\mathrm{IA}}. \tag{14}$$

The FTED equation is selected as

$$\dot{\varepsilon}_{\varphi}(t) + \alpha\varepsilon_{\varphi}(t) + \beta\varepsilon_{\varphi}^{q/p}(t) = 0. \tag{15}$$

The convergence time of the impact angle error is shown in Eq. (10). Substituting Eq. (14) into Eq. (15), we can express the biased term to regulate the terminal impact angle as follows:

$$a_{\rm IA} = (N-1)v_{\rm m}[\alpha\varepsilon_\varphi(t) + \beta\varepsilon_\varphi^{q/p}(t)]. \tag{16}$$

Combining Eqs. (11) and (16), we have the FTED-IACG as follows:

$$a_{\rm m} = Nv_{\rm m}\dot{\sigma} + (N-1)v_{\rm m}[\alpha\varepsilon_\varphi(t) + \beta\varepsilon_\varphi^{q/p}(t)]. \tag{17}$$

Note that the FTED equation will reduce to the TeED equation when $\alpha = 0$. Hence, the FTED-IACG shown in Eq. (17) will reduce to the TeED-IACG as follows:

$$a_{\rm m} = Nv_{\rm m}\dot{\sigma} + \beta(N-1)v_{\rm m}\varepsilon_\varphi^{q/p}(t). \tag{18}$$

The convergence time of the TeED-IACG is shown in Eq. (8). Further, when $p = q$, the FTED-IACG reduces to the impact angle control guidance law based on optimal error dynamics in Ref. [18].

3.2 FTED-ITCG

For a stationary target, the guidance command of the ITCG can be written as

$$a_{\rm m} = a_{\rm PPN} + a_{\rm IT}, \tag{19}$$

where $a_{\rm PPN}$ is the PPN term, and $a_{\rm IT}$ is a biased term to regulate the terminal impact time.

The estimation of the total flight time, when the missile is guided by PNG, can be expressed as

$$t_{\rm f} \approx t + \frac{r}{v_{\rm m}}\left[1 + \frac{\theta_{\rm m}^2}{2(2N-1)}\right], \tag{20}$$

where t is the current flying time.

Denote the desired impact time as $t_{\rm d}$. Then the impact time error $\varepsilon_{\rm t}$ can be defined as

$$\varepsilon_{\rm t} = t_{\rm d} - t_{\rm f}. \tag{21}$$

By differentiating Eq. (21), and substituting Eqs. (2), (19), and (20) in, the error dynamics equation of the impact time is obtained as

$$\begin{aligned}\dot{\varepsilon}_{\rm t} = &-\dot{t}_{\rm f} = \\ &-\frac{\dot{r}}{v_{\rm m}}\left[1 + \frac{\theta_{\rm m}^2}{2(2N-1)}\right] - \frac{r\theta_{\rm m}\dot{\theta}_{\rm m}}{(2N-1)v_{\rm m}} - 1 = \\ &\cos\theta_{\rm m}\left[1 + \frac{\theta_{\rm m}^2}{2(2N-1)}\right] + \frac{(N-1)\theta_{\rm m}\sin\theta_{\rm m}}{2N-1} - \\ &\frac{r\theta_{\rm m}}{(2N-1)v_{\rm m}^2}a_{\rm IT} - 1.\end{aligned} \tag{22}$$

Taking the assumption of a small leading angle into consideration, namely, $\theta_{\rm m}$ is small, then we have the approximation as

$$\sin\theta_{\rm m} \approx \theta_{\rm m}, \quad \cos\theta_{\rm m} \approx 1 - \frac{\theta_{\rm m}^2}{2}. \tag{23}$$

By substituting Eq. (23) into Eq. (22), neglecting the higher-order terms, sorting and simplifying, then the impact time error dynamics can be obtained as

$$\dot{\varepsilon}_{\rm t} = -\frac{r\theta_{\rm m}}{(2N-1)v_{\rm m}^2}a_{\rm IT}. \tag{24}$$

Select the desired FTED equation for ε_t as

$$\dot{\varepsilon}_{\rm t}(t) + \alpha\varepsilon_{\rm t}(t) + \beta\varepsilon_{\rm t}^{q/p}(t) = 0. \tag{25}$$

Combining Eq. (24) and Eq. (25), we have the biased term to regulate the terminal impact time, which can be expressed as

$$a_{\rm IT} = \frac{(2N-1)v_{\rm m}^2}{r\theta_{\rm m}}[\alpha\varepsilon_{\rm t}(t) + \beta\varepsilon_{\rm t}^{q/p}(t)]. \tag{26}$$

The convergence time of the impact time error is shown in Eq. (10), which is related to the initial state of the missile.

Combining Eqs. (19) and (26), we have the FTED-ITCG as follows:

$$a_{\rm m} = Nv_{\rm m}\dot{\sigma} + \frac{(2N-1)v_{\rm m}^2}{r\theta_{\rm m}}[\alpha\varepsilon_{\rm t}(t) + \beta\varepsilon_{\rm t}^{q/p}(t)]. \tag{27}$$

Similarly, note that the FTED equation will reduce to the TeED equation when $\alpha = 0$. Hence, the FTED-ITCG shown in Eq. (27) will reduce to the TeED-ITCG as follows:

$$a_{\rm m} = Nv_{\rm m}\dot{\sigma} + \frac{\beta(2N-1)v_{\rm m}^2}{r\theta_{\rm m}}\varepsilon_{\rm t}^{q/p}(t). \tag{28}$$

The convergence time of the TeED-IACG is shown in Eq. (8). Further, when $p = q$, the FTED-IACG reduces to the impact angle control guidance law based on optimal error dynamics in Ref. [18].

To keep the missile locked to the target, the constraint of the missile's FOV angle is further considered based on the FTED-ITCG, which uses the idea of the saturation function to improve the guidance law.

In Eq. (27), the first term is the classical PPN term, whose main function is to control the missile hitting the target. The second is a biased term. Its main function is to control the flight time required by the missile to hit the target. Just because of the existence of the second term, the missile will produce a large leading angle when adjusting the flight time, so a saturation function is needed to limit it. The basic principle is as follows:

$$\left.\begin{array}{ll}\phi(\theta/\theta_{\max}) = 1, & \theta = 0 \\ \phi(\theta/\theta_{\max}) \in (0,1), & \text{else} \\ \phi(\theta/\theta_{\max}) = 0, & |\theta| = \theta_{\max}\end{array}\right\}. \tag{29}$$

Substituting Eq. (29) into Eq. (27), we obtain the FTED-ITCG with seeker angle constraint as

$$a_{\mathrm{m}} = N v_{\mathrm{m}} \dot{\sigma} + \frac{(2N-1)\phi(\theta/\theta_{\max}) v_{\mathrm{m}}^2}{r\theta_{\mathrm{m}}} [\alpha \varepsilon_{\mathrm{t}}(t) + \beta \varepsilon_{\mathrm{t}}^{q/p}(t)]. \quad (30)$$

From Ref. [17], the saturation function can be given as

$$\phi(x) = \frac{1}{1-\mathrm{e}^{-1}} (\mathrm{e}^{-|x|^n} - \mathrm{e}^{-1}), \quad (31)$$

where $n > 0$ is a design parameter controlling the convergence curvature of the saturation function.

It should be pointed out that, due to the limitation of the FOV angle, the expected total flight time of the missile t_{d} cannot be set to an arbitrarily large value, and its value range is

$$t_{\mathrm{d}} \in \left(\frac{r_0}{v_m}, \frac{r_0}{v_m \cos\theta_{\max}} \right), \quad (32)$$

where r_0 represents the relative distance between the missile and the target at the initial time.

Obviously, the nonlinear function $\phi(x)$ reduces the selectable range of the missile's expected flight time, but it also provides a method to shape the impact time error feedback command. The ability of the guidance law to adjust the impact time error decreases when the missile velocity leading angle approaches its maximum $\theta_{\max}$. At the same time, when $|\theta| = \theta_{\max}$, the proposed guidance law degenerates into the classical PNG. Meanwhile, because the leading angle of the missile starts to decrease gradually under the effect of PNG, the guidance law can keep satisfying the limited condition of the seeker's FOV. When a fixed nonlinear function $\phi(x)$ is selected, the time of error convergence is determined by α, β, p, q and the initial states of the missile.

4 Numerical Simulation

The guidance designed in the previous section is analyzed and demonstrated by numerical simulation in this section. Numerical simulations are conducted in an air-to-ground engagement scenario. In all the following simulations, the target is located at the origin of the reference frame. The relative states of the missile and target at the initial time are shown in Table 1.

4.1 Performance of FTED-IACG

In this subsection, a nonlinear numerical simulation is performed to validate the proposed impact angle constraint guidance law based on the FTED. For comparison, the TeED method is also applied in the simulations. The related parameters are summarized in Table 2, and the simulation results are shown in Fig. 3.

Figure 3(a) compares the missile flight trajectories obtained from these two different guidance laws. From this figure, it is clear that both guidance laws can successfully make the missile hit the target accurately. Compared with the TeED method, the missile guided by the proposed impact angle constraint guidance law based on the FTED method has a smoother trajectory. The acceleration commands of different guidance laws are presented in Fig. 3(b). As shown in this figure, the missile guided by the FTED-IACG has higher guidance acceleration in the early stage, which can effectively relieve the pressure of the actuator in the late stage.

Table 1 Initial states of the missile and target

Parameter	Value
Initial relative distance/km	20
Target initial position/m	(0, 0)
Missile initial position/km	(−14.142, 14.142)
Missile initial speed/(m · s^{-1})	500
Missile initial FOV angle/(°)	−45
Missile initial flight path angle/(°)	−30
Proportional navigation gain	3
Desired impact angle/(°)	−90
Desired impact time/s	45
Maximum seeker field angle/(°)	35
Maximum acceleration/(m·s^{-2})	200

Table 2 Simulation parameters of FTED-IACG

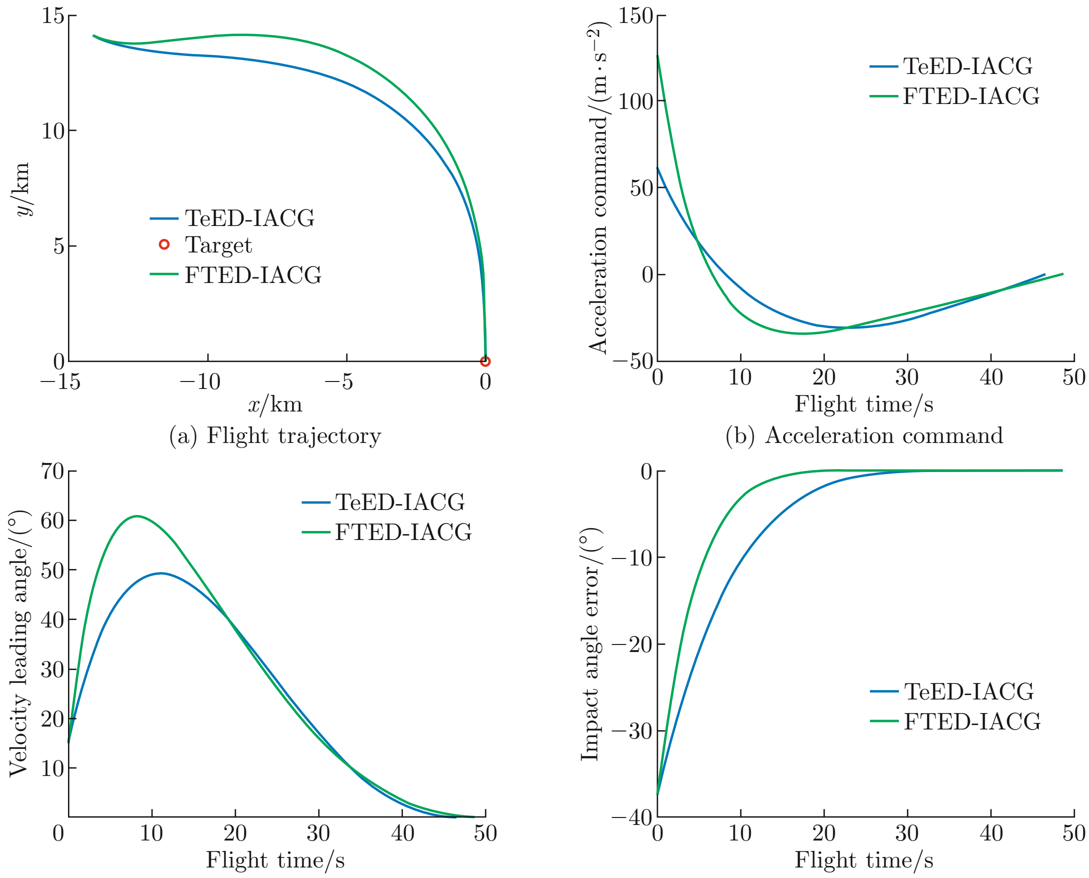
Parameter	Value
α	0.1
β	0.1
p	9
q	7

The velocity leading angle response comparison is shown in Fig. 3(c). As shown in this figure, the velocity leading angle of the missile under the two guidance laws is always within 90°, which conforms to the general assumption of the missile FOV angle. The comparison results of the impact angle error from both guidance laws are summarized in Fig. 3(d). From this figure, we can observe that the proposed guidance law is more agile and rapid in error convergence, which can make the error converge to 0 quickly in finite time. The quantitative comparison results are summarized in Table 3.

4.2 Performance of FTED-ITCG

Similar to the previous subsection, nonlinear numerical simulation is performed to validate the proposed impact angle constraint guidance law based on the FTED. For comparison, the TeED method is also applied in the simulations. The related parameters are summarized in Table 4, and the simulation results are shown in Fig. 4.

The flight trajectories and the velocity leading angle of the missile guided by these two different guidance laws are shown in Figs. 4(a) and 4(c), respectively. The



(a) Flight trajectory (b) Acceleration command

(c) Velocity leading angle (d) Impact angle error

Fig. 3 Comparison results of IACG between TeED and FTED

Table 3 Comparison results of IACG between TeED and FTED

Method	Error convergence time/s	Maximum velocity leading angle/(°)	Maximum guidance acceleration/$(\mathrm{m\cdot s^{-2}})$	Impact angle error/(°)
TeED	40.954 7	49.254 5	62.209 0	3.122×10^{-4}
FTED	29.122 1	60.674 7	127.658 9	1.995×10^{-4}

Table 4 Simulation parameters of FTED-ITCG

Parameter	Value
α	0.049
β	0.11
p	149
q	67

simulation results also show that both guidance laws can make the missile hit the target accurately with a smoother trajectory, and the velocity leading angle conforms to the general assumption of the maximum FOV angle of the missile. The quantitative comparison results are summarized in Table 5.

The guidance acceleration command and the impact time error of the missile guided by these two different guidance laws are shown in Figs. 4(b) and 4(d), respectively. As shown in these figures, the proposed guidance law still has a larger guidance acceleration in the early stage, and compared with TeED, it converges to the vicinity of the equilibrium position more quickly and enters the convergence mode dominated by the linear sliding mode earlier. It is worth noticing that the suspected discontinuous "cusps" at 25 s and 40 s in the guidance acceleration curve shown in Fig. 4(b) are continuous and smooth.

4.3 Performance of FTED-ITCG with FOV Constraint

This subsection verifies the effectiveness of the saturation function designed in the FTED-ITCG. The related parameters are summarized in Table 6, and the simulation results are shown in Fig. 5.

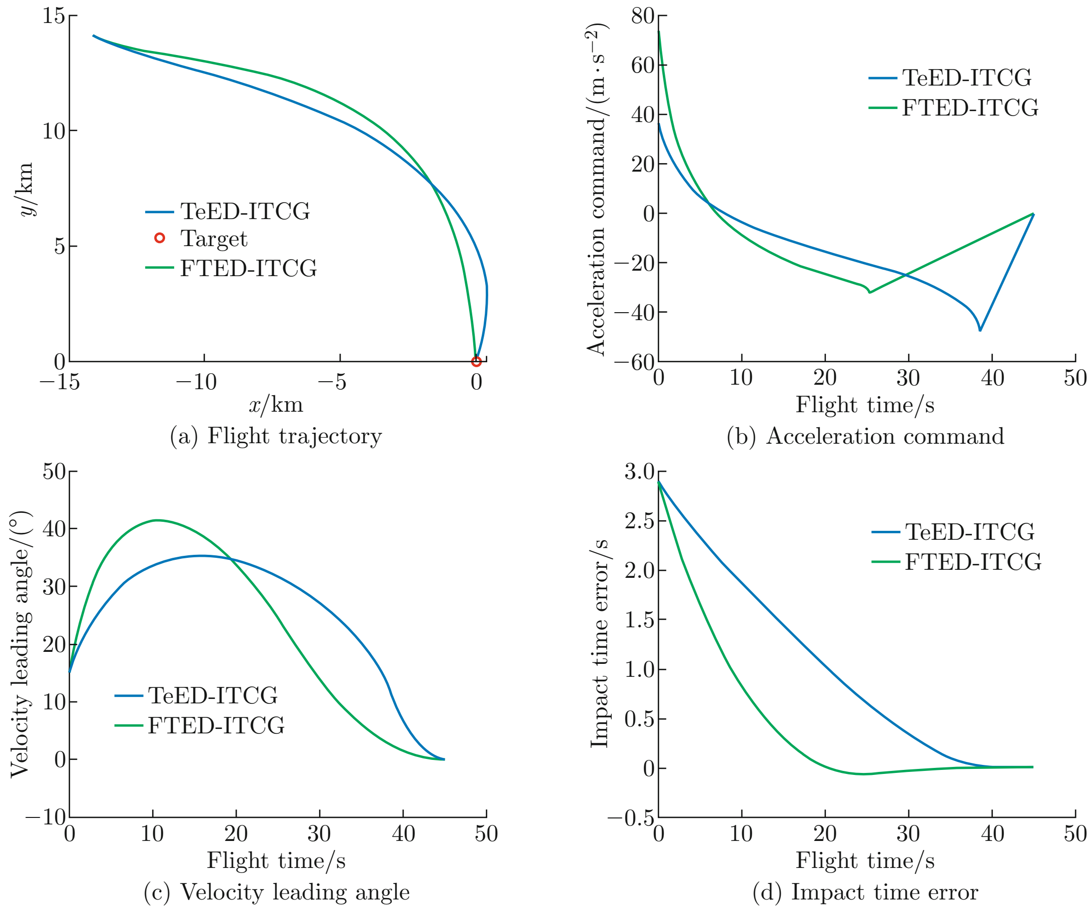


Fig. 4 Comparison results of ITCG between TeED and FTED

Table 5 Comparison results of ITCG between TeED and FTED

Method	Error convergence time/s	Maximum velocity leading angle/(°)	Maximum guidance acceleration/$(\mathrm{m \cdot s^{-2}})$	Impact time error/s
TeED	38.830 2	35.213 9	36.323 3	9.955×10^{-3}
FTED	26.567 5	41.321 7	73.828 7	9.995×10^{-3}

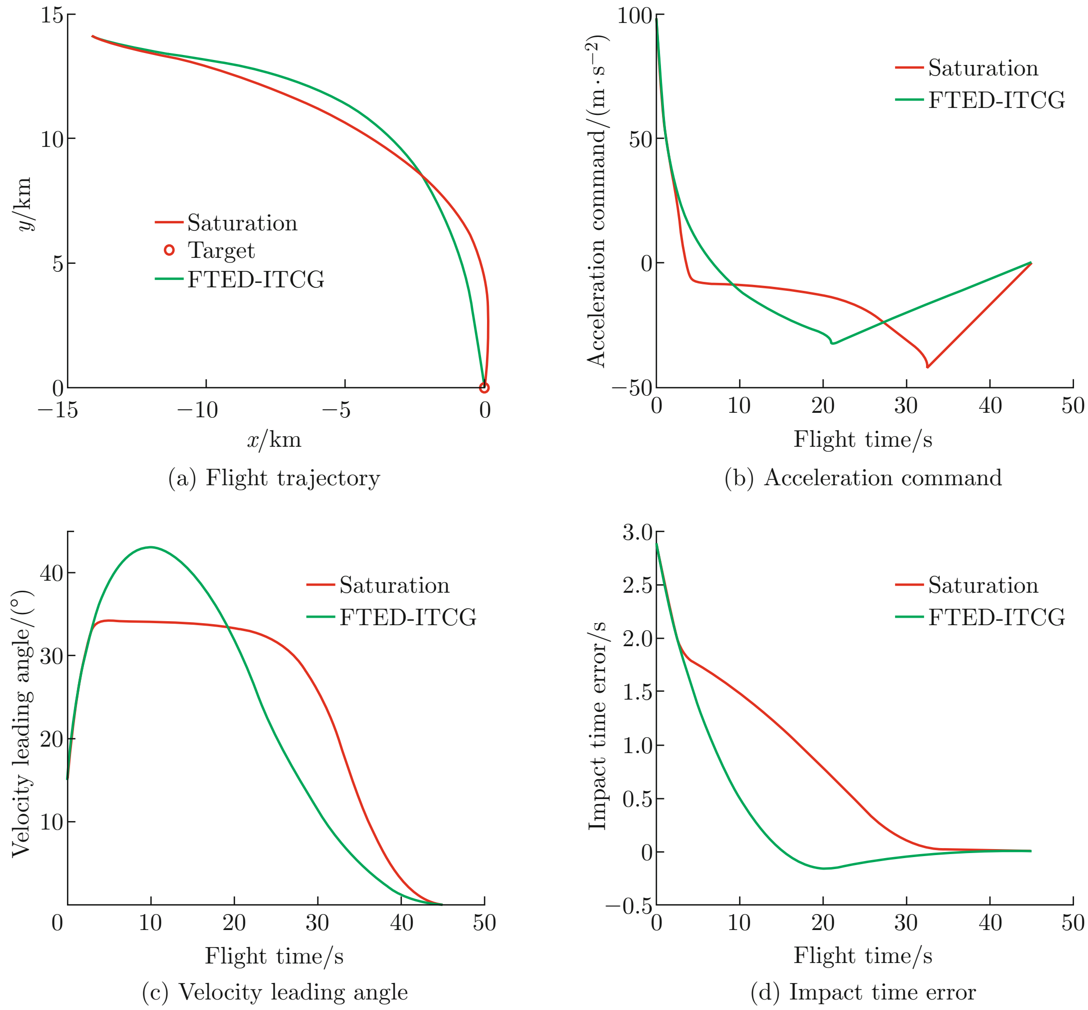
Table 6 Value of the simulation parameters

Parameter	Value
α	0.06
β	0.12
p	15
q	6.5
n	30

Figure 5 compares the flight trajectory, guidance acceleration command, velocity leading angle, and the impact time error of the missile obtained from these two conditions with/without FOV constraints. The simulation results show that the speed of error convergence is lower than before, but it is still better than that of TeED method. The quantitative comparison results are summarized in Table 7.

Table 7 Simulation results of FTED-ITCG with FOV constraint

Method	Error convergence time/s	Maximum velocity leading angle/(°)	Maximum guidance acceleration/$(\mathrm{m \cdot s^{-2}})$	Impact time error/s
FTED	26.567 5	42.950 4	98.200 6	9.988×10^{-3}
Saturation	—	34.227 1	98.200 6	9.982×10^{-3}



(a) Flight trajectory (b) Acceleration command

(c) Velocity leading angle (d) Impact time error

Fig. 5 Simulation results of ITCG with FOV constraint

The performance of the saturation function is shown in Fig. 5(c). From this figure, when the velocity leading angle of the missile is close to the seeker's maximum field angle, it can be stably kept below the maximum value without switching guidance laws, and the convergence rate can be adjusted by adjusting the parameters.

5 Conclusion

A guidance law design method based on FTED is proposed in this paper. On this basis, the guidance laws with impact angle/time constraints and FOV angle constraint are designed. Compared with the ordinary sliding mode control method, the TeED method overcomes the shortcoming of asymptotic convergence of errors by introducing a nonlinear function, so that the guidance error can converge to zero in finite time. Based on keeping the advantages of TeED, the FTED method overcomes its shortcoming of slow convergence speed when approaching equilibrium state, so that the system can converge to equilibrium state quickly in finite time. During the design of the guidance law, this advantage can make the missile quickly adjust to the desired states in advance, and effectively relieve the overload saturation pressure of the actuator. In addition, the acceleration command of the guidance law based on the FTED method is continuous without the switching term, which can effectively eliminate the chattering phenomenon. Numerical simulation shows that the designed guidance law has good performance and great engineering application potential.

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