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Block Limit Analysis Method for Stability of Slopes During Earthquakes

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Abstract: Based on the limit analysis upper bound method, a new mechanism of soil slope failure has been proposed which was consisted of plastic shear zone and rigid block zone. The different zones interface were regarded as discontinuity lines. Two sliding blocks of the slope were also incorporated horizontal seismic force and vertical gravity force. The velocities and forces were analyzed in two blocks, and the expression of velocity discontinuities was derived according to the principle of incompressibility. The external force done work for the blocks and the internal energy dissipated of the plastic shear zone and the velocity discontinuous were solved. The stability ratios were derived for the height of two-level slope with different rates to involve seismic and no seismic. The present stability ratios were compared to the previous study, which showed the superiority of the mechanism and the rationality of the analysis. The critical height of the slope can provide a theoretical basis for slope support and design.
Key words: soil slope, stability ratio, earthquake, block limit analysis method
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Nomenclature

c— Soil cohesion, N
C_u— Undrained shear strength, kPa
H— Total height of the slope, m
H_c— Critical height of the soil slope, m
H_i— Height of different levels, $i = 1, 2$, m
K_c— Critical acceleration factor
K_cW_1—Seismic force in the horizontal direction of block $AMOB$, N
K_cW_2—Seismic force in the horizontal direction of block BOC, N
N— Stability ratio
N_v— Change rate of stability ratio
r_0— Width of block BCO base, m
r_1— Horizontal distance of slope top A to slope top C, m
$u(r)$— Horizontal velocity at horizontal distance r, m/s
$u_1(r)$— Uniform horizontal velocity across BO, m/s
v_0— Initial vertical velocity, m/s
v_1— Vertical velocity of the block zone, m/s
v_h— Horizontal velocity of the shearing zone, m/s
v_v— Vertical velocity of the shearing zone, m/s
V— Volume of the zone, m^3
W_1— Gravity force of block $AMOB$, N
W_2— Gravity force of the block BOC, N
α_i— Slope angle, $i = 1, 2$, °
β— Geometric parameter of the shearing zone
βr_0— Width of the slope base, m
γ— Bulk unit weight of the soil, kN/m^3
ε— Largest principal plastic strain rate

0 Introduction

There are mainly three general categories in seismic stability analysis of soil slopes: preudostatic analysis, stress-deformation analysis and permanent displacement analysis[1]. In 1950, Terzaghi[2] first proposed pseudostatic analysis which involved simply adding a permanent body force representing the earthquake shaking to a static limit-equilibrium analysis. The method is easy to use, and has a long history that provides a body of engineering judgment regarding its application, and provides a simple and scalar index of stability. However, its shortcoming tends to be over-conservative in some cases, it tells the researcher nothing about what happens after equilibrium is exceeded.

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Stress-deformation analysis involved much more complex modeling of slopes using a mesh in which the internal stresses and strains within elements are computed based on the applied external loads, including gravity force and seismic force. It provided the most realistic model of slope behavior, but it is very complex and requires a large quantity of soil-property datum as well as an accurate model of soil behavior. Permanent-displacement analysis bridges the gap between overly simplistic pseudostatic analysis and overly complex stress-deformation analysis. The method was firstly proposed by Newmark, which has given rise to a family of analyses commonly referred to as permanent-displacement analysis[3]. A key assumption of Newmark's method is that it treats a landslide as a rigid-plastic body, the mass does not deform internally, experiences no permanent displacement at accelerations below the critical level, and deforms plastically at constant stress along a discrete basal shear surface when the critical acceleration is exceeded. It has two advantages: firstly, it is much easier to use; secondly, it allows modeling of large displacements on discrete basal shear surfaces. Subsequently, many researchers[4-9] improved the method to allow for more complex and realistic field behaviors, which became so-called decoupled and fully coupled displacement analysis.

However, the most important feature of the seismic stability analysis of soil slope is the estimation of the seismic loads which will cause slippage of the soil mass and overall movements of the sliding soil mass throughout an earthquake. In fact, the factor of safety may drop below unity a number of times which will induce some movements of the failure section of a slope, but not cause the collapse of a slope. Limit analysis method doesn't need to compute a complete progressive failure analysis of stress and strain in stability problem of soil slope, and using stability ratio expresses the seismic stability of the slope. In 1984, Chang et al.[10] analyzed the stability of soil slope by upper bound presudo-static limit analysis, later several researchers[11-15] also have studied the stability of slope by limit analysis upper bound method. Combined rigid block method and limit bound analysis theory, the block limit analysis method (BLAM) is proposed to assess the stability of soil slope during earthquakes.

For the sake of simplifying problem, the study proposes a two sliding blocks of mechanism for soil slope failure, investigating the stability of soil slope is applied by BLAM described subsequently. The procedure is divided into three phases: firstly, the rate of internal energy dissipation and the external rate of work are computed; then, stability ratios of soil slope are obtained; finally, stability ratios with precious results are compared.

1 Mechanism of Soil Slope Failure

1.1 Definition of Problem

A two dimensional soil slope with two sliding blocks is considered as shown in Fig. 1. The mechanism is consisted of three block regions: the shearing region for block $AMOB$; the rigid block region for block BCO; the stable region for block $MQPGO$. Line MO, BO and CO are lines of discontinuity (the blocks interface).

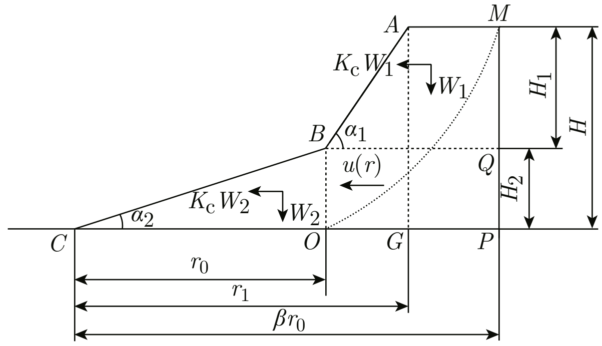


Fig. 1 Mechanism of soil slope failure

1.2 Model of Soil Slope Failure

1.2.1 *Velocity of Regions*

In the shearing zone, $u(r)$ is horizontal velocity at distance r and v_0 is initial vertical velocity. In the rigid block zone, $u_1(r)$ is horizontal velocity and v_1 (equal to 0) is vertical velocity. In the stable zone, velocity is zero. If the soil is assumed rigid-perfectly plastic behavior, with an uniform undrained shear strength C_u, and obeys Mohr-Coulomb failure criterion and an associated flow rule, the velocity fields of the collapse mechanism must satisfy the incompressibility condition

$$\frac{\mathrm{d}u}{\mathrm{d}r} + \frac{u}{r} + \frac{\mathrm{d}v}{\mathrm{d}z} = 0,$$

where u is horizontal velocity and v is vertical velocity.

The horizontal velocity at a horizontal distance r can be expressed as

$$u(r) = -\frac{v_0(\beta r_0^2 - r_0^2)}{Hr}. \tag{1}$$

1.2.2 *Forces of Regions*

The vertical component of the seismic acceleration is neglected since its effect on earthquake- induced slope collapse is generally not relevant. There is only gravity in the vertical direction.

2 Upper Bound Solution of Soil Slope Stability

The load, determined by equating the external rate of work to the internal rate of dissipation in an assumed deformation mode (or velocity field), satisfies velocity boundary conditions, strain rate and velocity compatibility conditions and is not less than the actual collapse load, the load is the upper bound solution[16].

2.1 Work Done of External Forces

The equations for each discontinuity may be derived by satisfying continuity of velocity in the direction normal to the discontinuity line.

$$Z_{MO} = \frac{H(r^2 - r_0^2)}{(\beta^2 - 1)r_0^2}. \tag{2}$$

Work done of external forces in the shearing zone and rigid zone is

$$\mathrm{d}W = v_\mathrm{v}\gamma\mathrm{d}V + v_\mathrm{h}K_\mathrm{c}\gamma\mathrm{d}V. \tag{3}$$

The work done by gravity in region $AMOB$ is given by

$$W_{\mathrm{G1}} = \gamma v_0\Big[\frac{1}{2}\tan\alpha_1(r_1 - r_0)^2 + r_0(\beta - 1)\Big(H_2 - H\frac{\beta^2 - 2\beta + 2}{3(\beta^2 - 1)}\Big)\Big]. \tag{4}$$

The work done by seismic force in region $AMOB$ is given by

$$W_{\mathrm{S1}} = K_\mathrm{c}W_{\mathrm{G1}}. \tag{5}$$

The work done by gravity force in region BOC is given by

$$W_{\mathrm{G2}} = 0. \tag{6}$$

The work done by seismic force in region BOC is given by

$$W_{\mathrm{S2}} = \frac{1}{2}r_0H_2\gamma K_\mathrm{c}u_1(r). \tag{7}$$

Therefore, the total work done is

$$W_\mathrm{T} = W_{\mathrm{G1}} + W_{\mathrm{S1}} + W_{\mathrm{G2}} + W_{\mathrm{S2}} = \gamma Hv_0\Big\{(K_\mathrm{c} + 1)\Big[\frac{1}{2H}\tan\alpha_1(r_1 - r_0)^2 + \frac{H_2}{H}(r_1 - r_0) + (\beta r_0 - r_1) - \frac{r_0}{3}\Big(\beta - \frac{2}{\beta + 1}\Big)\Big] + \frac{K_\mathrm{c}H_2(\beta r_0^2 - r_0^2)}{2H^2}\Big\}. \tag{8}$$

2.2 Energy Dissipated Along Discontinuity

The energy dissipated along a discontinuity of the mechanism of soil slope failure is

$$E_{MO} = C_\mathrm{u}\int_{r_0}^{\beta r_0}\frac{v_0 - v_1}{\sin\theta\cos\theta}\mathrm{d}r, \tag{9}$$

where $\tan\theta = \mathrm{d}Z_{MO}/\mathrm{d}r$.

The energies dissipated along discontinuity MO, CO and BO are

$$E_{MO} = C_\mathrm{u}v_0\Big(\frac{\beta^2r_0^2 - r_0^2}{2H}\ln\beta + H\Big), \tag{10}$$

$$E_{CO} = \frac{r_0^2v_0C_\mathrm{u}(\beta - 1)}{H_2}, \tag{11}$$

$$E_{BO} = H_2C_\mathrm{u}v_0. \tag{12}$$

Therefore, the total energy dissipated is

$$E_{\mathrm{Tdis}} = E_{CO} + E_{BO} + E_{MO} = C_\mathrm{u}v_0\Big[\frac{r_0^2(\beta - 1)}{H_2} + H_2 + \frac{\beta^2r_0^2 - r_0^2}{2H}\ln\beta + H\Big]. \tag{13}$$

2.3 Energy Dissipated with Shearing Zone

The energy dissipated with shearing zone is

$$E_{AMOB} = 2\int C_\mathrm{u}\varepsilon\mathrm{d}r, \tag{14}$$

where $\varepsilon = \mathrm{d}u(r)/\mathrm{d}r$.

Thus,

$$E_{AMOB} = C_\mathrm{u}v_0\frac{r_0^2(\beta - 1)}{H_2}\Big\{\tan\beta_1\Big[\lg\frac{r_1}{r_0} + \Big(\frac{1}{r_1} - \frac{1}{r_0}\Big)(r_0 - H_2)\Big] - \Big(\frac{H}{\beta r_0} - \frac{H}{r_1}\Big) - \frac{H}{(\beta^2 - 1)r_0}\Big[(\beta - 1) + \Big(\frac{1}{\beta} - 1\Big)\Big]\Big\}.$$

The total energy dissipated of the mechanism is

$$E_\mathrm{T} = E_{AMOB} + E_{\mathrm{Tdis}}. \tag{15}$$

2.4 Stability Ratio of Soil Slope

The stability ratio[16-17] $N = \gamma H/C_\mathrm{u}$ is obtained by equating the energy dissipated to the work done

$$N = \frac{r_0^2}{H_2H}(\beta - 1)\Big\{\tan\alpha_1\Big[\lg\frac{r_1}{r_0} + \Big(\frac{r_0}{r_1} - \frac{H_2}{r_1}\Big) - \Big(1 - \frac{H_2}{r_0}\Big)\Big] - \Big(\frac{H}{\beta r_0} - \frac{H}{r_1}\Big) - \frac{H}{r_0(\beta^2 - 1)}\times\Big[(\beta - 1) + \Big(\frac{1}{\beta} - 1\Big)\Big]\Big\} + \Big[\frac{r_0^2}{H_2H}(\beta - 1) + \frac{H_2}{H} + \Big(\frac{r_0}{H}\Big)^2\frac{(\beta^2 - 1)}{2}\ln\beta + 1\Big]\Big/\Big\{(K_\mathrm{c} + 1)\times\Big[\frac{1}{2}\cot\alpha_1\Big(\frac{H_1}{H}\Big)^2 + \frac{H_2H_1}{H^2}\cot\alpha_1 + \frac{H_2}{H}\cot\alpha_2(\beta - 1) - \frac{H_1}{H}\cot\alpha_1 - \frac{r_0}{3H}\Big(\beta - \frac{2}{\beta + 1}\Big)\Big] + \frac{K_\mathrm{c}H_2}{2H}\Big(\frac{r_0}{H}\Big)^2(\beta - 1)\Big\}. \tag{16}$$

3 Computing of Stability Ratio

3.1 Computing Method

According to several variables' relations in Eq. (16), the value of N will become a function with the following parameters.

$$N = F\Big(\frac{H_1}{H}, \frac{H_2}{H}, \alpha_1, \alpha_2, \beta\Big). \tag{17}$$

For a given geometry of the slope, the function F can be minimized with respect to variables (H_1/H, H_2/H, α_1, α_2, β).

The critical height of the soil slope (H_c) can be got when the $N_{\min}$ (the minimum value of N) is given.

$$H_c = \frac{c}{\gamma} F\left(\frac{H_1}{H}, \frac{H_2}{H}, \alpha_1, \alpha_2, \beta\right). \tag{18}$$

3.2 Results

According to Eq. (17), variation stability ratios with slope angle are given below.

3.2.1 $\alpha_1 = \alpha_2$

Figure 2(a) gives a model of soil slope when $\alpha_1 = \alpha_2$ and $H_1/H = 0$, which is the simplest model. Figure 2(b) gives a simple model of soil slope, when $\alpha_1 = \alpha_2$ and $H_1/H \neq 0$.

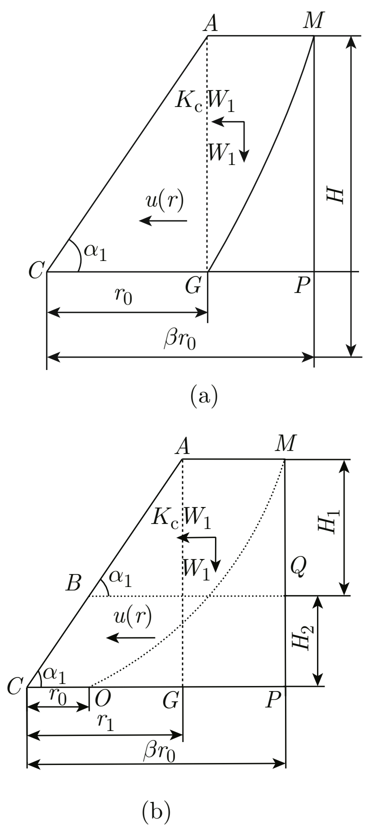


Fig. 2 Mechanism of soil slope failure ($\alpha_1 = \alpha_2$)

Figure 3 shows that variation of geometric parameters (β) ratio with slope angle.

Figures 4 and 5 indicate that variation of stability ratio (N) with slope angle (α) ($K_c = 0$ refers to no seismic force and $K_c = 0.1$ refers to seismic force).

3.2.2 $\alpha_1 > \alpha_2$

The stability ratios were given when $\alpha_1 = \alpha_2 + 15°$ in Figs. 6 and 7.

Figure 6 indicated the slope model when the value of H_1/H gradually increased.

Figure 7 indicated the stability ratios in different slope angles.

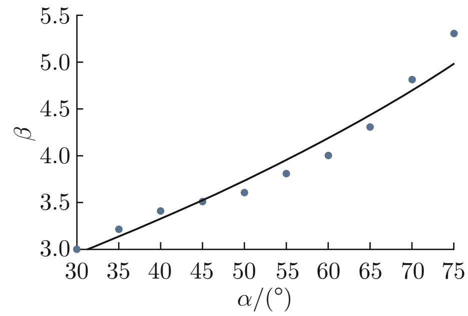


Fig. 3 Variation of geometric parameters ratio with slope angle

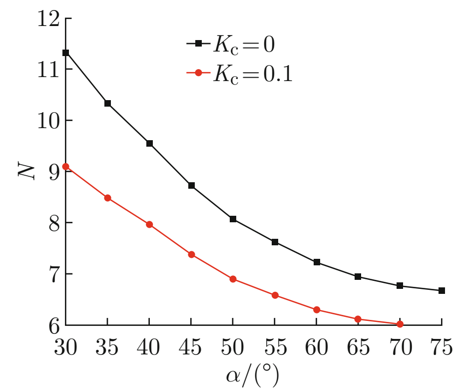
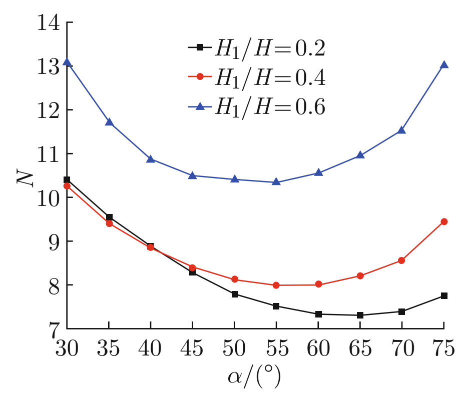
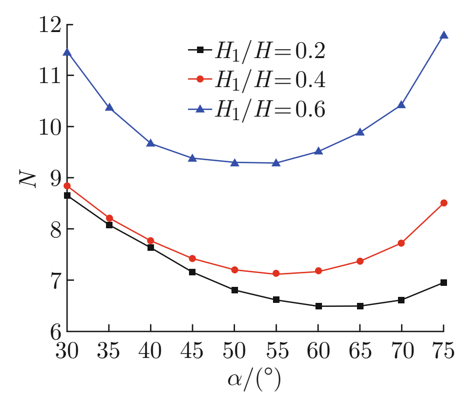


Fig. 4 Variation of stability ratio with slope angle



(a) K_c=0



(b) K_c=0.1

Fig. 5 Variation stability ratio with slope angle

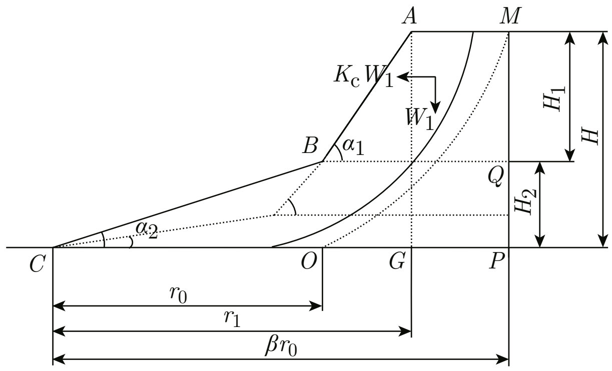
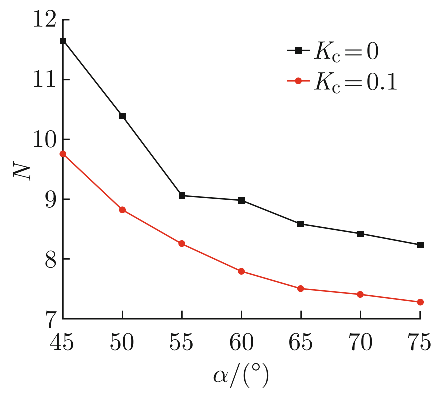
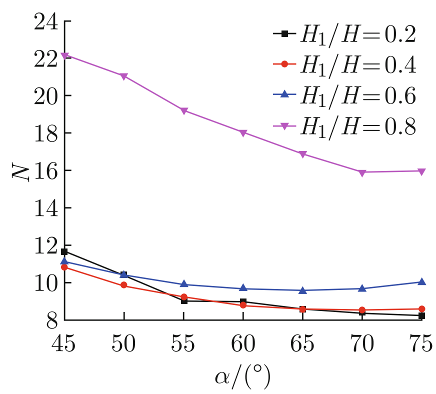


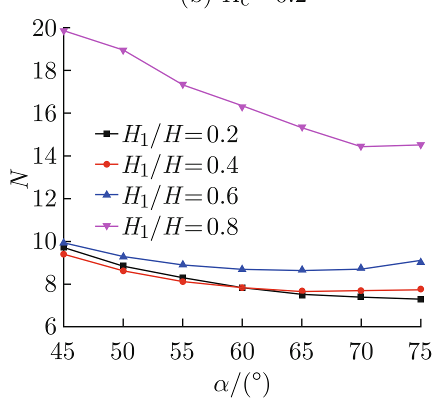
Fig. 6 Mechanism of soil slope failure



(a) $H_1/H=0.2$



(b) $K_c=0.2$



(c) $K_c=0.1$

Fig. 7 Variation of stability ratio with slope angle

4 Comparison of Stability Ratios

In order to validate the computational procedure, the obtained values of the stability ratios for soil slope when $H_1/H = 0$, from 45° to 60°($\alpha_1 = \alpha_2$) were compared with the results reported by Kumar[11], Michalowski[12] and Chen[18] on the basis of the upper bound limit analysis in Tables 1 and 2.

Table 1 Comparison of stability ratio ($K_c = 0$, $\alpha_1 = \alpha_2$)

$\alpha/(°)$	Stability ratio		
	Present study	Chen's study	Kumar's study
45	8.732	9.320	9.312
60	7.220	7.260	7.259
75	6.670	5.800	5.801

Table 2 Comparison of stability ratio ($K_c = 0.1, \alpha_1 = \alpha_2$)

$\alpha/(°)$	Stability ratio	
	Present study	Michalowski's study
45	7.372	7.300
60	6.289	6.300
75	5.946	5.200

5 Discussion

Figure 4 indicated N is gradually reduced and eventually obtained $N_{\min}$ when α ranges ($\alpha_1 = \alpha_2$) from 30° to 75° as well as $K_c = 0$ and $K_c = 0.1$, respectively. $N_{\text{vmax}} = 19.8\%$, $N_{\text{vmin}} = 10.8\%$, $N_{\text{v-aver}} = 14.6\%$ as $K_c = 0$ and $K_c = 0.1$, respectively.

Figure 5 shows the variation of N with the slope angle for the mechanism given in Fig. 2(b), the general trend of N decreases at first and then increases with the increase of slope angle. The mechanism obtains smaller value of N for $H_1/H = 0.2$, and $N_{\min}$ is achieved when $\alpha = 65°$, but $N_{\min}$ is achieved for $H_1/H = 0.4$ and 0.6 when $\alpha = 55°$. In the case of $K_c = 0$, the mechanism ($H_1/H = 0.4$) gives lower values of N when $\alpha < 40°$.

Figure 7 indicated that when $\alpha_1 \neq \alpha_2(\alpha_1 = \alpha_2 + 15°)$ for the mechanism given in Fig. 6: for the case of $H_1/H = 0.2$, N tends to decrease with the increase of slope angle, which is obtained $N_{\min}$ when $\alpha = 75°$. For the case of $H_1/H = 0.4$, 0.6, 0.8, N decreases at firstly and then increases. The mechanism ($H_1/H = 0.4$) gives lower values of N when $\alpha < 55°$, but the case of $H_1/H = 0.2$ obtains smaller values of N when $\alpha \geqslant 55°$. For the case of $H_1/H = 0.8$, N is far greater than the other cases.

Tables 1 and 2 show that N is much smaller than previous results when the slope angle equals to 45° and 60° respectively, although N is much larger than the previous results when the slope angle equals to 75°.

There are two main reasons for the abmoral change of N in Fig. 5: firstly, the collapse mechanism needs optimizing; secondly, $N_{\min}$ of soil slope is obtained in the certain condition.

Therefore, it needs more optimal slope collapse mechanism and accurate parameters of soil in order to obtain a more optimal upper bound solution. It is also a reason that N is too large to compute for the case of $H_1/H = 0.8$.

6 Conclusion

(1) Based on the limit analysis method and rigid block method, the BLAM has been proposed and may be used to analysis the stability ratios of soil slope during earthquakes.

(2) The present results are more optimized than the previous results in the certain range by comparing, which showed that the collapse mechanism is optimized and the analysis is reasonable.

(3) The critical height of soil slope can be got when the stability ratios obtain the minimum value for given parameter values.

(4) BLAM may be used to compute more complicated stability of soil slope subsequently.

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