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Reliability Modeling and Maintenance Policy Optimization for Deteriorating System Under Random Shock

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Abstract: Performance degradation and random shock are commonly regarded as two dependent competing risks for system failures. One method based on effective service age is proposed to jointly model the cumulative effect of random shock and system degradation, and the reliability model of degradation system under Nonhomogeneous Poisson processes (NHPP) shocks is derived. Under the assumption that preventive maintenance (PM) is imperfective and the corrective maintenance (CM) is minimal repair, one maintenance policy which combines PM and CM is presented. Moreover, the two decision variables, PM interval and the number of PMs before replacement, are determined by a multi-objective maintenance optimization method which simultaneously maximizes the system availability and minimizes the system long-run expect cost rate. Finally, the performance of the proposed maintenance optimization policy is demonstrated via a numerical example.
Key words: degradation, random shock, Nonhomogeneous Poisson processes (NHPP), imperfect preventive maintenance, multi-objective optimization
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0 Introduction

Performance degradation is one common characteristic of general product. As the service time increases, its performance degrade continuously and its failure rate increases gradually. At the same time, the system receives the random shock of different magnitudes, which can accelerate the system failure. Degradation and random shock are two major causes for system failure, and they also have a certain correlation. Random shock will accelerate degradation of system performance or result in failure directly. On the other hand, with the system degradation, the possibility of system failure due to the shock increases. In other words, the system with serious degradation is more vulnerable to fail.

System degradation has attracted great interests over the last decades in the reliability field. In Refs. [1-3], Gamma process is employed to capture the random character of the system performance degradation. Gamma process which has continuous nonnegative increment is naturally suitable to model degradation processes in which deterioration is considered to be a series of tiny increments. When the performance degrades to a pre-specified threshold, the system is considered to have failed. Reference [3] divides the operation cycle of the system into two stages through a predefined system degradation threshold. It assumes that there are different shock failure rates in different stages to solve the system reliability model under the combined action of shock and degradation. References [4–5] adapt the effective service age to model system reliability. The effective service age is linearly related to the working time. Since the service age threshold follows exponential distribution, the system failure rate in its whole life cycle is constant, which cannot capture system degradation feature. Also, due to the memoryless character of exponent distribution, random shock under the model will not affect the failure rate of the system, which fails to describe the acceleration of random shock on the system degradation. References [6–8] assume that the parameters in the degradation expression follow normal distribution and the random shocks follow the homogeneous Poisson process. When the sum value of inherent and shock-caused degradation exceeds the preset threshold, the system is considered to have failed. References [9–10] adopt the nonhomogeneous Markov process to describe the ageing characteristic of components and solve the system reliability using general generating function. However, when the state

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number of components is greater than three, the steady-state probability solution of the nonhomogeneous Markov process becomes very complicated.

Maintenance is recognized as being of both theoretical and practical importance in the reliability engineering field. Generally, there are two types of maintenance actions, preventive maintenance (PM) and corrective maintenance (CM)[9-11]. Maintenance strategy can also be divided into two candidates, the maintenance policy upon service age and upon working conditions[1-3]. Reference [5] only considers PM without CM. It means that if the system fails, it will stay on the failure state without any maintenance action until the next regular repair time. This type of maintenance strategy will cause highly system availability reduction due to the rising failure rate during the later stage. Reference [9] studies a service-age-based maintenance strategy, and only replacement action is taken into account. When service age of the system exceeds the predefined value, or the system performance reaches to the preset threshold, it will be replaced. It simplifies the service age as the actual service duration, and finally optimizes the system maintenance cost using the preset threshold of service time and degradation as the decision variables. The maintenance strategy of Refs. [6-8] belongs to the condition based maintenance (CBM) strategy which carries out the periodic examination on the system during the replacement cycle. When the working conditions of the system meet the requirements, no any maintenance measure is adopted. If any working condition fails to meet the requirements, the system is replaced. If the system fails at the interval of two examinations, it keeps the failure state until the next examination.

How to excellently schedule maintenance plan is an optimization issue. The commonly used optimization objectives include long run maintenance expected cost rate, system steady-state availability and system efficiency. They can also be divided into single objective optimization and multi-objective optimization according to the number of the optimization objectives. References [3], [6–8] and [10] adopt the optimization objective of long run maintenance cost rate, which belongs to the single objective optimization. Reference [5] takes the long run expected cost rate and availability as the optimization objectives and Ref. [6] takes expected cost rate and system efficiency as the optimization objectives, which belong to the multi-objective optimization.

To describe the impact of performance degradation and random shock on the system reliability, this paper first puts forward the concept of effective service age, and then models the system inherent degradation and external random shock respectively. The system reliability model is finally obtained by jointly considering these two competing risks accounting for system failure. Moreover, it proposes a maintenance strategy consisting of PM and CM, and presents a multi-objective optimization plan for maximizing system availability and minimizing long run expected cost rate. Finally, it analyzes the reliability of the system under different maintenance decisions and verifies the rationality of the models and the optimization effect of the maintenance decision based on one numerical example.

1 System Modeling

1.1 Reliability Model of Degradation System

The actual working time of the system is defined as t and its effective service age is denoted as $T_{\mathrm{v}}(t) = a + \int_0^t \delta(x)\mathrm{d}x$, in which a is the initial service time before delivery to customer and $\delta(x)$ is a nondecreasing function, representing the impact of the system working environment on its effective service age. In the ideal working environment, it can be assumed $\delta(x) = 1$, that means the effective service age is equal to the actual service time. When $\delta(x)$ is a monotone increasing function of variable x, it represents the accelerated aging process of the system under higher working load.

When the effective service age reaches a predefined threshold S, the system is considered to have failed. Assuming that the threshold S follows exponential distribution[4] with parameter of μ, S : $\exp(\mu)$, then the reliability function of the degradation system is

$$R_{\mathrm{o}}(t) = P\{T_{\mathrm{v}}(t) < S\} = P\left\{a + \int_0^t \delta(x)\mathrm{d}x < S\right\} = \exp\left(-\mu\left[a + \int_0^t \delta(x)\mathrm{d}x\right]\right). \tag{1}$$

1.2 Random Shock Modeling

It is assumed that the random shock follows the Nonhomogeneous Poisson process (NHPP), which is written as $\{N(t), t \geqslant 0\}$, the shock strength is assumed as $\lambda(t)$, mean value is $l(t) = E[N(t)] = \int_0^t \lambda(x)\mathrm{d}x$ and the arrival time of the ith shock is denoted as T_i. The impact of the shock on the system reliability is described by $(p(t),\ q(t))$ mixed shock model. Each shock makes the system directly fail with the probability of $p(t)$ and increases the change rate of effective service age with the probability of $q(t) = 1 - p(t)$, which means that shocks will accelerate aging performance of the system. The increment of the change rate of the effective system service age by the ith shock is expressed by the random variable w_i. $M(t)$ is the moment generating function of w_i and $M(t)$ is assumed to be derivable.

1.3 System Reliability Modeling Under Competing Risk of Degradation and Random Shock

Under the combined action of system degradation and external random shock, the system lifetime is expressed by the random variable Y, so the system reliability function is the conditional probability under the effect of shock, expressed as

$$P\{Y > t|N(t); w_1, w_2, \cdots, w_{N(t)}\} = \prod_{j=1}^{N(t)} q(T_j) \exp\Big\{ -\mu\Big(a + \int_0^t \Big[\delta(x) + \sum_{j=1}^{N(t)} w_j 1_{[T_j,\infty)}(x)\Big] dx\Big)\Big\} = \exp\left\{-\mu\left[a + \int_0^t \delta(x)\mathrm{d}x\right]\right\} \times \exp\left\{\prod_{i=1}^{N(t)} \ln q(T_i) - \mu \int_0^t \sum_{j=1}^{N(t)} w_j 1_{[T_j,\infty)}(x)\mathrm{d}x\right\}, \quad (2)$$

where $1_{[T_j,\infty)}(x) = \begin{cases} 1, & x \in [T_j, \infty) \\ 0, & x \notin [T_j, \infty) \end{cases}$ is the indicator function, and thus

$$\int_0^t \sum_{j=1}^{N(t)} w_j 1_{[T_j,\infty)}(x)\mathrm{d}x = \sum_{j=1}^{N(t)} w_j(t - T_j) = t\sum_{j=1}^{N(t)} w_j - \sum_{j=1}^{N(t)} w_j T_j,$$

and the system reliability is

$$P\{Y > t|N(t); w_1, w_2, \cdots, w_{N(t)}\} = R_{\mathrm{o}}(t) \exp\left\{\prod_{i=1}^{N(t)} \ln q(T_i) - t\mu \sum_{j=1}^{N(t)} w_j + \mu \sum_{j=1}^{N(t)} w_j T_j\right\}.$$

Hence, the system reliability function under the combined action of shock and degradation is expressed as

$$R(t) = P\{Y > t\} = R_{\mathrm{o}}(t) E\Big[\exp\Big\{\prod_{i=1}^{N(t)} \ln q(T_i) - t\mu \sum_{j=1}^{N(t)} w_j + \mu \sum_{j=1}^{N(t)} w_j T_j\Big\}\Big]. \quad (3)$$

It can be known from Eq. (3) that the system reliability function can be considered as being consisting of two parts which are connected in series. First part is the inherent reliability of the system denoted by $R_{\mathrm{o}}(t)$, the second part $E\Big[\exp\Big\{\prod_{i=1}^{N(t)} \ln q(T_i) - t\mu \sum_{j=1}^{N(t)} w_j + \mu \sum_{j=1}^{N(t)} w_j T_j\Big\}\Big]$ (written as $R_{\mathrm{s}}(t)$) represents the impact on system reliability caused by random shock. We assume that the random shock $N(t)$ follows NHPP. It can be readily converted into Homogeneous Poisson process (HPP) through time scale transformation. Define $N^*(t) = N(l^{-1}(t))$, $\{N^*(t) : t \geqslant 0\}$ is a HPP with intensity 1. Under this new time scale, the arrival time of the shock is $T_j^* = l(T_j), j \geqslant 1$. Given $s = l(t)$, following result can be obtained.

$$R_{\mathrm{s}}(t) = E\Big[\exp\Big\{\prod_{j=1}^{N^*(s)} \ln q(l^{-1}(T_j^*)) - l^{-1}(s)\mu \sum_{j=1}^{N^*(s)} w_j + \mu \sum_{j=1}^{N^*(s)} w_j l^{-1}(T_j^*)\Big\}\Big] = E\Big[E\Big[\exp\Big\{\prod_{j=1}^{N^*(s)} \ln q(l^{-1}(T_j^*)) - l^{-1}(s)\mu \sum_{j=1}^{N^*(s)} w_j + \mu \sum_{j=1}^{N^*(s)} w_j l^{-1}(T_j^*)\Big\}\Big| N^*(s) = n\Big]\Big] = \sum_{n=0}^{\infty} E\Big[\exp\Big\{\prod_{j=1}^{N^*(s)} \ln q(l^{-1}(T_j^*)) - l^{-1}(s)\mu \sum_{j=1}^{N^*(s)} w_j + \mu \sum_{j=1}^{N^*(s)} w_j l^{-1}(T_j^*)\Big\}\Big| N^*(s) = n\Big] P(N^*(s) = n). \quad (4)$$

Given $N^*(s)$ is equal to n, it is readily to know that $\{T_j^*\}$, $j = 1, 2, \cdots, n$ and $\{V_{(j)}^*\}$, $j = 1, 2, \cdots, n$ have the same joint distribution, and $\{V_{(j)}^*\}$, $j = 1, 2, \cdots, n$ are the order statistics of n independent random variables $\{V_j^*\}$, $j = 1, 2, \cdots, n$ which follow the uniform distribution in $[0, s]$[4]. Denote $U = V_1/s$, therefore U is a random variable with uniform distribution in [0, 1]. The conditional expectation in Eq. (4) can be solved as following.

$$E\left[\exp\Big\{\sum_{j=1}^{n} \ln q(l^{-1}(V_j)) - l^{-1}(s)\mu \sum_{j=1}^{n} w_j + \mu \sum_{j=1}^{n} w_j l^{-1}(V_j)\Big\}\right] = E\Big[\exp\Big\{\sum_{j=1}^{n} (\ln q(l^{-1}(V_j)) + w_j \mu(l^{-1}(V_j) - l^{-1}(s)))\Big\}\Big] = \{E[\exp\{\ln q(l^{-1}(V_1)) + w_j \mu(l^{-1}(V_1) - l^{-1}(s))\}]\}^n = \{E[\exp\{\ln q(l^{-1}(sU)) + w_j \mu(l^{-1}(sU) - l^{-1}(s))\}]\}^n = \{E[E[\exp\{\ln q(l^{-1}(sU)) + w_j \mu(l^{-1}(sU) - l^{-1}(s))\}|U = u]]\}^n = \Big\{\int_0^1 \exp\{\ln q(l^{-1}(su))\} E[\exp\{w_j \mu(l^{-1}(su) l^{-1}(s))\}]\mathrm{d}u\Big\}^n, \quad (5)$$

where $E[\exp\{w_j\mu(l^{-1}(su)-l^{-1}(s))\}] = M(\mu(l^{-1}(su)-l^{-1}(s)))$. $M(t)$ is the moment generating function of w_j. Through variable substitution $x = l^{-1}(l(t)u)$, $l(t)u = l(x)$, $u = \dfrac{l(x)}{l(t)}$, $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{l'(x)}{l(t)}$, then the following can be obtained.

$$\begin{aligned}&\int_0^1 \exp\{\ln q(l^{-1}(su))\}\times\\&E[\exp\{w_j\mu(l^{-1}(su)-l^{-1}(s))\}]\mathrm{d}u=\\&\int_0^1 \exp\{\ln q(l^{-1}(su))\}M(\mu(l^{-1}(su)-l^{-1}(s)))\mathrm{d}u=\\&\int_0^t \exp\{\ln q(x)\}M(\mu(x-t))\mathrm{d}x=\\&\frac{1}{l(t)}\int_0^t q(x)\lambda(x)M(\mu(x-t))\mathrm{d}x.\end{aligned}\tag{6}$$

Substituting Eq. (6) into Eq. (5) and Eq. (5) into Eq. (4), $R_{\mathrm{s}}(t)$ can be obtained, which represents the impact part of random shock on the system reliability.

$$\begin{aligned}R_{\mathrm{s}}(t)=&\sum_{n=0}^{\infty}\left[\frac{1}{l(t)}\int_0^t q(x)\lambda(x)M(\mu(x-t))\mathrm{d}x\right]^n\times\\&P\{N^*(s)=n\}=\\&\sum_{n=0}^{\infty}\left[\frac{1}{l(t)}\int_0^t q(x)\lambda(x)M(\mu(x-t))\mathrm{d}x\right]^n\frac{s^n}{n!}\mathrm{e}^{-s}=\\&\mathrm{e}^{-s}\sum_{n=0}^{\infty}\frac{1}{n!}\left[\int_0^t q(x)\lambda(x)M(\mu(x-t))\mathrm{d}x\right]^n=\\&\exp\Big\{-l(t)+\int_0^t q(x)\lambda(x)M(\mu(x-t))\mathrm{d}x\Big\}.\end{aligned}$$

Therefore the combinative reliability of the system is:

$$\begin{aligned}R(t)=&R_{\mathrm{o}}(t)R_{\mathrm{s}}(t)=\\&\exp\left\{-\mu\left[a+\int_0^t\delta(x)\mathrm{d}x\right]\right\}\times\\&\exp\left\{-l(t)+\int_0^t q(x)\lambda(x)M(\mu(x-t))\mathrm{d}x\right\}=\\&\exp\Big\{-\mu a-l(t)+\int_0^t(-\mu\delta(x)+\\&q(x)\lambda(x)M(\mu(x-t)))\mathrm{d}x\Big\}.\end{aligned}\tag{7}$$

And the failure rate function is

$$\begin{aligned}h(t)=&-\frac{\mathrm{d}}{\mathrm{d}t}\ln(R(t))=\\&\lambda(t)+\mu\delta(t)-\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t q(x)\lambda(x)M(\mu(x-t))\mathrm{d}x.\end{aligned}$$

As assumed in section 1.2, the derivative of $M(t)$ exists, so

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\int_0^t q(x)\lambda(x)M(\mu(x-t))\mathrm{d}x=\\&\int_0^t\frac{\mathrm{d}}{\mathrm{d}t}(q(x)\lambda(x)M(\mu(x-t)))\mathrm{d}x+q(t)\lambda(t)=\\&\int_0^t q(x)\lambda(x)M'(\mu(x-t))\mathrm{d}x+q(t)\lambda(t).\end{aligned}$$

Moreover, the system corresponding failure rate function is

$$\begin{aligned}h(t)=&\lambda(t)+\mu\delta(t)-q(t)\lambda(t)-\\&\int_0^t q(x)\lambda(x)M'(\mu(x-t))\mathrm{d}x.\end{aligned}\tag{8}$$

2 System Maintenance Model and Optimization

This paper proposes a mixed maintenance strategy model including PM and CM, in which PM refers to regular repair and maintenance of the system and CM refers to the corrective maintenance after the failure occurs. The maintenance strategy optimization is the multi-objective optimization and the optimization objectives are maximization of availability and minimization of expected maintenance cost rate.

2.1 Maintenance Assumptions

(1) Maintenance interval of PM is expressed by T, the total maintenance cost is denoted by C_{p}, and maintenance time is expressed by T_{p}.

(2) The system will be replaced at the Nth times of PM with the renewal cost of C_{r}. T_{r} represents the time needed for system renewal and NT refers to the system renewal cycle.

(3) CM is implemented when the accidental failure occurs between two PMs. The cost of CM is denoted by C_{c}. The maintenance time is expressed by T_{c} which is a random variable. The time of CM does not affect the PM interval T. For example, a failure occurs between the intervals of PMs and the maintenance time is T'_{c}. PM interval remains unchanged and the effective working time of the system between the intervals of PM reduces to be $T-T'_{\mathrm{c}}$.

(4) We assume that PM is imperfect, system reliability after PM will be improved, but it will not recover to the new state. To capture the effect of the imperfect maintenance, the recover factor α is introduced. After each time of regular maintenance, the service age threshold of the system will increase. After the ith time of regular maintenance, it is written as S_i, $S_i = S/\alpha^i$, $\alpha < 1$, where α depends on the maintenance effect of PM. With the impact of degradation and random shock, system failure rate will rise gradually. PM can be adopted periodically to improve system failure rate. After N times of PM, the system will be replaced

by a new one, and the failure rate reduces to the initial state. At this time, the system is considered as entering a new cycle.

(5) We adopt the assumption that CM is minimum maintenance, which is carried out after the accidental failure. In other words, after CM, system returns to working state, and its failure rate is restored to the previous value before recent failure.

(6) The costs of different kinds of maintenance are assumed to meet $C_{\mathrm{r}} > C_{\mathrm{c}} > C_{\mathrm{p}}$ and the maintenance times meet $E[T_{\mathrm{c}}] > E[T_{\mathrm{r}}] > E[C_{\mathrm{p}}]$.

As mentioned above, S is an exponential distribution random variable with parameter μ, so S_i follows the exponential distribution with parameter $\mu\alpha^{(i-1)}$, namely, $S_i : \exp(\mu\alpha^{i-1})$, it is readily to get the system reliability function $R_i(t)$ and the failure rate function $h_i(t)$ in the ith regular maintenance cycle.

$$R_i(t) = \exp\{\alpha^{i-1}(-\mu a) - l(t) + \int_0^t (-\mu\alpha^{i-1}\delta(x) + q(x)\lambda(x)M(\mu\alpha^{i-1}(x-t)))\mathrm{d}x\}, \tag{9}$$

$$h_i(t) = \lambda(t) + \mu\alpha^{i-1}\delta(t) - q(t)\lambda(t) - \int_0^t (q(x)\lambda(x)M'(\mu\alpha^{i-1}(x-t)))\mathrm{d}x, \quad t \in (I_{i-1}, I_i]. \tag{10}$$

2.2 Maintenance Optimization

In this paper, we will discuss the maintenance optimization issue which includes two optimization objectives, maximization of steady-state availability and minimization of expected maintenance cost rate. After N times of PM, the system will be replaced directly and return to the new state. According to the renewal reward theory, the steady-state availability of the system can be expressed as the ratio of the working time expectation in a renewal cycle $E[T_{\mathrm{available}}]$ to the renewal period expectation $E[T_{\mathrm{renew}}]$, as shown in Eq. (11). The expected maintenance cost rate can be expressed as the ratio of the expected cost $E[C_{\mathrm{renew}}]$ to the renewal period expectation $E[T_{\mathrm{renew}}]$, as shown in Eq. (12). The decision variables are the PM interval T and the times of PM in one renewal cycle N.

$$A(N,T) = \frac{E[T_{\mathrm{available}}]}{E[T_{\mathrm{renew}}]} = \frac{NT - E[T_{\mathrm{c}}]E[N_{\mathrm{c}}]}{NT + (N-1)E[T_{\mathrm{p}}] + E[T_{\mathrm{r}}]}, \tag{11}$$

$$C(N,T) = \frac{E[C_{\mathrm{renew}}]}{E[T_{\mathrm{renew}}]} = \frac{(N-1)C_{\mathrm{p}} + C_{\mathrm{c}}E[N_{\mathrm{c}}] + C_{\mathrm{r}}}{NT + (N-1)E[T_{\mathrm{p}}] + E[T_{\mathrm{r}}]}, \tag{12}$$

where $E[T_{\mathrm{p}}]$, $E[T_{\mathrm{r}}]$ and $E[T_{\mathrm{c}}]$ represent the expectations of the times needed in PM, replacement and CM respectively, and they can be obtained from experience or expert knowledge. N and T are the maintenance decision variables. $E[N_{\mathrm{c}}]$ is the expectation of the system failure times in a renewal cycle when CM is the minimum maintenance[12]. The expression is:

$$E[N_{\mathrm{c}}] = \sum_{i=0}^{N-1} \int_{iT}^{(i+1)T} h_i(t)\mathrm{d}t = \sum_{i=0}^{N-1} \int_{iT}^{(i+1)T} \{\lambda(t) + \mu\alpha^{i-1}\delta(t) - q(t)\lambda(t) - \int_0^t (q(x)\lambda(x)M'(\mu\alpha^{i-1}(x-t)))\mathrm{d}x\}\mathrm{d}x. \tag{13}$$

The maintenance optimization can be expressed as the multi-objective optimization problem.

$$\begin{cases} \text{Objective} : f_1 = \max A(N,T), f_2 = \min C(N,T), \\ \text{subject to} : N \in [1, N_{\max}], \quad T \in [T_{\min}, T_{\max}]. \end{cases}$$

The constraint conditions restrict the value range of the decision variables N and T. The value range of the decision variable determines the search scope of the efficient solution of the optimization problem. It is set according to previous maintenance experience. The decision variable N takes an integer and T is a real value, belonging to the mixed integer programming problem[13]. The solution of the optimization problem can be completed by the multi-objective evolutionary algorithm.

This paper employs NSGA-II algorithm[13-14] to complete the solution of above optimization problem to obtain the Pareto optimal solution set of the maintenance decision. NSGA-II algorithm is a non-dominated sorting genetic algorithm with the elitist strategy. Excellent individuals are selected from the effective solution set according to the actual situation as the decision vectors to finally realize the optimal balance between the maintenance cost rate and the system availability.

3 Example Analysis

It is assumed that the effect of random shock, which is denoted by w_i, follows exponential distribution with the parameter of β, so the derivative of the moment generating function in Eq. (13) is

$$M'(\mu\alpha^{i-1}(x-t)) = \left(\frac{\beta}{\beta - \alpha^{i-1}(x-t)}\right)' = -\frac{\beta\alpha^{i-1}}{(\beta - \alpha^{i-1}(x-t))^2}.$$

In this example, we set $\beta = 200$, $\mu = 0.02$, $\alpha = 0.8$, $\delta(t) = 0.01t^{1.5}$, $q(t) = \exp(-0.001t)$ and $\lambda(t) = 0.02t^{0.8}$. In the example, the original failure rate curve of the degraded system and the compound failure rate curve of the system after introducing random shock are shown in Fig. 1.

In the example, assuming the recover factor of PM is $\alpha = 0.8$, its failure rate under different maintenance strategy is shown in Fig. 2.

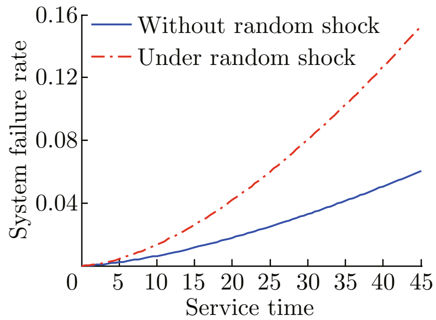

Fig. 1 Impact of random shock on the system failure rate

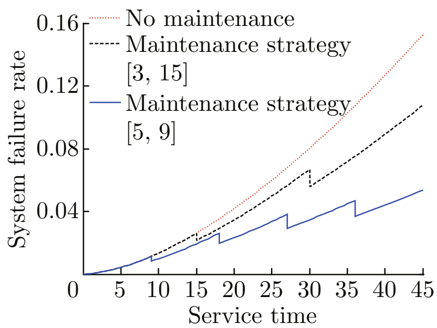

Fig. 2 Impact of different maintenance strategy on the system failure rate

The expected maintenance cost of replacement, CM and PM take 2 000, 200 and 20, and the expectations of their durations are set to 0.8, 0.2 and 0.05, respectively.

The optimization function "gamultiobj()" provided by MATLAB can only solve the optimization problem of real number encoding. However in this study, the decision variable N is an integer, so we redefine the variation function and cross function in the actual solving process. In the example, variable N adopts the binary coding method and the variation function adopts the uniform variation. After variation, if the result is beyond the value scope, re-variation is needed. The cross function uses the arithmetic crossover. After crossing, the variable N is taken the integer to get the variation individuals. Other relevant parameters are set as follows, population size: 60, Pareto boundary fraction: 0.5, value range of N: [1, 20], value range of T: [0.1, 50], long run cost rate fitness function: $C(N, T)$ and system unavailability fitness function: $1 - A(N, T)$. The Pareto optimal boundary of the maintenance decision problem after optimization is shown in Fig. 3, including a total of 30 Pareto optimal solutions. Table 1 lists the corresponding system availability and maintenance cost rate of these 30 Pareto solutions.

Pareto optimal solutions in Fig. 3 and Table 1 reflect

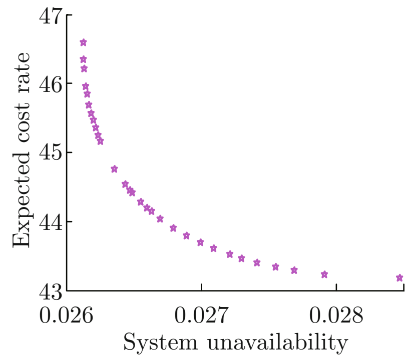

Fig. 3 Pareto optimal boundary of multi-objective optimization

Table 1 Pareto solutions of system availability $A(N, T)$ and maintenance cost rate $C(N, T)$

Index	Decision variable N	Decision variable T	$A(N,T)$/%	$C(N,T)$
1	5	12.187	97.387	46.587
2	5	13.543	97.387	46.342
3	5	13.667	97.387	46.214
4	5	13.928	97.385	45.955
5	4	15.052	97.384	45.838
6	4	15.214	97.383	45.689
7	4	15.351	97.381	45.568
8	4	15.470	97.380	45.466
9	4	15.604	97.378	45.353
10	3	19.737	97.376	45.246
11	3	19.855	97.374	45.153
12	3	20.402	97.364	44.757
13	3	20.747	97.356	44.534
14	3	20.878	97.353	44.455
15	3	20.959	97.351	44.407
16	3	21.192	97.344	44.276
17	3	21.358	97.340	44.188
18	3	21.458	97.337	44.137
19	3	21.672	97.330	44.034
20	3	21.961	97.320	43.904
21	3	22.237	97.311	43.791
22	3	22.521	97.300	43.686
23	3	22.770	97.291	43.603
24	3	23.068	97.278	43.514
25	3	23.278	97.269	43.458
26	3	23.541	97.258	43.396
27	3	23.832	97.244	43.337
28	3	24.128	97.230	43.286
29	3	24.582	97.208	43.228
30	3	25.611	97.152	43.175

that maintenance cost rate and system availability are two contradictory decision objectives. Increasing the system availability will increase the maintenance cost and reducing the maintenance cost will correspondingly reduce the system availability.

Multi-objective optimization results provide one decision set, and the final maintenance strategy can be selected from this set. For example, in the situation paying more attention to the system availability, it is feasible to select the decision variable [5, 12.187]. If the maintenance cost is limited, it is suggested to select the Pareto optimal solutions with low cost rate. The example in the paper selects the maintenance strategy with balanced availability and cost rate. N takes 3 and T takes 20.402. Under this maintenance strategy, the system failure rate function in a renewal cycle is shown in Fig. 4.

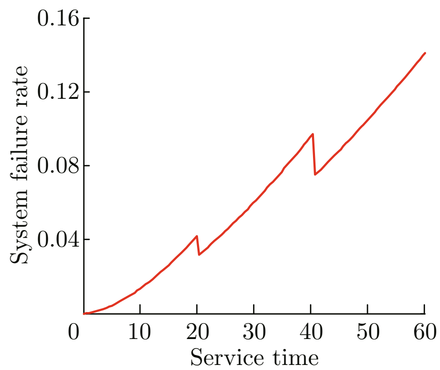

Fig. 4 The system failure rate curve under the maintenance strategy [3, 20.402]

4 Conclusion

This paper studies the reliability modeling and maintenance optimization issues of degradation system with considering random shock. First, we put forward the mathematical description method of the system performance degradation and random shock effects based on the effective service age and then establish the reliability model of the degradation system under the NHPP shock, and finally obtain the system reliability function and failure rate function. According to the operation characteristics, one maintenance strategy which combines imperfect periodic PM and minimal CM is given. And then we discuss the multi-objective maintenance optimization issues with PM maintenance cycle and PM times in a renewal cycle as the decision variables to maximize the long-time availability and minimize the steady-state maintenance cost rate. Finally, one simulation example is applied to illustrate the compound failure rate of the degradation system after introducing random shock and show the effect of different maintenance strategies. At last, realization of multi-objective optimization is given. The simulation results verify that our model and maintenance strategy proposed in this paper are available and effective.

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