J. Shanghai Jiao Tong Univ. (Sci.), 2018, 23(4): 515-526
https://doi.org/10.1007/s12204-018-1973-2

Tool for Predicting the Ultimate Bending Moment of Ship and Ship-Shaped Hull Girders

VU Van Tuyen[1,3], *YANG Ping*[1,2]* (杨　平)
(1. School of Transportation, Wuhan University of Technology, Wuhan 430063, China;
2. Key Laboratory of High Performance Ship Technology, Ministry of Education,
Wuhan University of Technology, Wuhan 430063, China;
3. Faculty of Shipbuilding, Vietnam Maritime University, Hai Phong 180000, Vietnam)

Abstract: The present paper presents a historical review associated with the research works on hull girder strength of ship and ship-shaped structures. Then, a new program is developed to determine the ultimate vertical bending moment of hull girder by applying direct method, stress distribution method, and progressive collapse analysis method. Six ships and ship-shaped structures used in the benchmark study of International Ship and Offshore Structures Congress (ISSC) in 2012 are adopted as examples. The calculation results by applying the developed program are analyzed and compared with the existing results. Finally, the roles of the developed program and its further development are discussed.
Key words: ship structure, hull girder, ultimate bending moment (UBM), ultimate strength
CLC number: U 661　　**Document code:** A

Nomenclature

A_{D}, A_{B}, A'_{B}—Total sectional areas of deck, outer bottom, and inner bottom, respectively, m^2
A_i—Total cross-sectional area of the ith element, m^2
A_{S}—Half-sectional area of all sides (including longitudinal bulkheads and inner sides), m^2
D—Hull depth, m
D_{B}—Height of double bottom, m
g—Neutral axis position from the base line in the sagging condition or from the deck in the hogging condition, m
H—Depth of hull section in linear elastic state, m
M_{p}—Fully plastic bending moment of hull section, MN · m
M_{u}—Ultimate bending moment (UBM) of hull section, MN · m
$M_{\mathrm{uh}}, M_{\mathrm{us}}$—UBMs in hogging and sagging conditions, respectively, MN · m
z_i—Vertical distance from base line to horizontal neutral axis of the ith element
$z_{\mathrm{NA\_cur}}$—Vertical distance from base line to horizontal neutral axis of cross-section
Z—Elastic section modulus at the compression flange, m^3
Z_{D}, Z_{B}—Elastic section moduli at deck and bottom, respectively, m^3
σ_{u}—Ultimate buckling strength of the compression flange, MPa
σ_{uD}, σ_{uS}, σ_{uB}, σ'_{uB}—Ultimate buckling strength of deck, side, outer bottom, and inner bottom, respectively, MPa
$\sigma_{\mathrm{u},i}$—Ultimate stress of the ith element
σ_{y}—Yield strength of the material, MPa
σ_{yD}, σ_{yS}, σ_{yB}, σ'_{yB}—Yield strength of deck, side, outer bottom, and inner bottom, respectively, MPa

0 Introduction

Facts indicate that there are a large number of vessel casualties. With regard to commercial ships, a casualty scenario of 13 ships with several damages (such as sank, hold flooding, broke in two, crushing, bow bent down, and local hull girder failure) during the period from 1968 to 1974 was drawn[1]. The statistics of the Royal Institution of Naval Architects (RINA) showed that there were 150 bulk carriers which were lost with a loss of more than 1 200 lives[2]. The loss of ships is classified into three categories: loss of buoyancy (or floating capacity), hull girder collapse loss, and loss of stability. Among them, hull girder loss consists of increase of hull girder loads and decrease of hull girder strength. Hold flooding, loading and unloading conditions or sea states lead to the change of hull girder loads, while the factors related to corrosion, cracks and dents lead to the degradation of hull girder strength, even hull girder collapse[3].

According to the common structural rule (CSR)

Received date: 2017-06-06
***E-mail:** pyangwhut@163.com

requirements of the International Association Classification of Society (IACS), all of the oil tankers and bulk carriers equal to or greater than 150 m in length must be checked for the seagoing condition to ensure that the residual strength checking criteria are satisfied[4]. Following the requirements, the hull girder residual strength is represented by hull girder ultimate bending capacity (HGUBC) or hull girder ultimate strength (HGUS).

To determine the HGUS, researchers proposed several methods in the field of naval architecture and ocean engineering. The existing methods are classified into three groups: ship accident investigation and model test, direct method, and progressive collapse analysis[5]. Direct method includes linear method, empirical formulas and analytical method. Specifically, empirical formulations which were identified in the studies of Vasta[6], Mansour and Faulkner[7], Faulkner and Sadden[8], Viner[9], Frieze and Lin[10], and Valsgaard and Steen[11]usually employ the ultimate strength of stiffened panels such as deck and outer bottom to determine the HGUS. Whereas, the others applied the assumption of stress distribution to establish the appropriate formulations to predict the ultimate hull girder bending moment in both hogging and sagging conditions, such as Caldwell[12], Paik and Mansour[13], Qi and Cui[5], and Paik et al.[14]. As regards the progressive collapse analysis method, it is known as several methods such as simplified method (or Smith's method)[15], idealized structural unit method (ISUM)[16], non-linear finite element method (NFEM)[17], and incremental-iterative method by IACS-CSR[4].

The above mentioned methods were employed in several research works to assess the UBM of ship or ship-shaped hull girder associated with different conditions. Hansen[18] conducted a study to investigate the influence of uncertain parameters such as plate deflection, stiffener deflection, and residual stresses on the uncertainty of hull beam strength. The study carried out an assessment on four different ships and models, namely Nishihara model, Dow's frigate model, container ship, and very large crude oil carrier (VLCC) using progressive collapse analysis method and finite element analysis with MARC code. Akhras et al.[19] performed an experiment to study the structure behaviors of a 1/5 full-size frigate model. The purpose of the experiment was to investigate the effect of initial imperfections on the ultimate strength of the selected box girder and make the comparison between experimental results and FABSTRAN code ones.

Paik et al.[20] suggested an advanced ultimate strength formulation for predicting the HGUS under vertical bending moment. The formulation covered not only influential parameters such as initial deflections, residual stresses, corrosion, collision and grounding but also all possible collapse modes of stiffened panels. And a capsize bulk carrier was employed to assess the accuracy of new proposed formulation. Sun and Bai[21] applied several hull girders which were used in benchmark study of International Ship and Offshore Structures Congress (ISSC) in 2000 to study the ultimate strength and reliability analysis. The simplified method and a time-variant reliability assessment method combined with influential factors such as corrosion and fatigue were employed in their study. Rigo et al.[22] investigated the effect of average strain-stress curve on the UBM and moment-curvature curve of three ships.

Yao[23-24] carried out a historical review associated with the hull girder strength. The author presented several considerations related to hull girder capacity of ship structures, state of the art, and future direction. Qi et al.[25] conducted a comparative study on the ultimate hull girder strength of a 300 000 DWT large double hull tanker. The study applied five different methods to determine the UBM capacity of hull girder, namely ISUM, simplified method, analytical method, elastic-plastic method (EPM), and incremental-iterative method by IACS-CSR. Ozguc et al.[26] used incremental-iterative approach to assess the ultimate hull girder strength of a bulk carrier under two load cases including pure vertical bending moment and the combination between vertical and horizontal bending moments. Paik et al.[27] did a comparative study by applying ANSYS finite element analysis (FEA) code, ALPS/HULL code, and IACS-CSR method to analyze the ultimate vertical bending moment of an AFRAMAX-class hypothetical double hull oil tanker.

Regarding the existing computer codes, ANSYS code, ABAQUS code, MARC code and MAESTRO code are currently used to analyze the UBM. Besides, there are several computer codes which were developed and employed by researchers or classification societies such as APLS/HULL code, Neptune code, ASSAS code, UMADS code, Poseidon code (Germanischer Lloyd), and NASTASS code[26,28-30].

The aim of this study is to develop a program for determining the vertical UBM of ship and ship-shaped hull girders. Six hull structures and three groups of ultimate hull girder strength methods are applied to calculate the vertical UBM. The calculation results obtained by the developed program are analyzed and compared with the existing results.

1 Applied Methods

1.1 Direct Method

Based on the assumptions that the hull girder will reach to the ultimate limit state when the upper deck panel in sagging condition or the bottom panel in hogging condition collapses, and the moment-curvature relationship is linear, several appropriate formulations are established as follows.

The simply-beam theory method is easy to apply; however, it does not take into account the effect of local failures of structural members[31]. With regard to this, the first failure hull girder is determined as

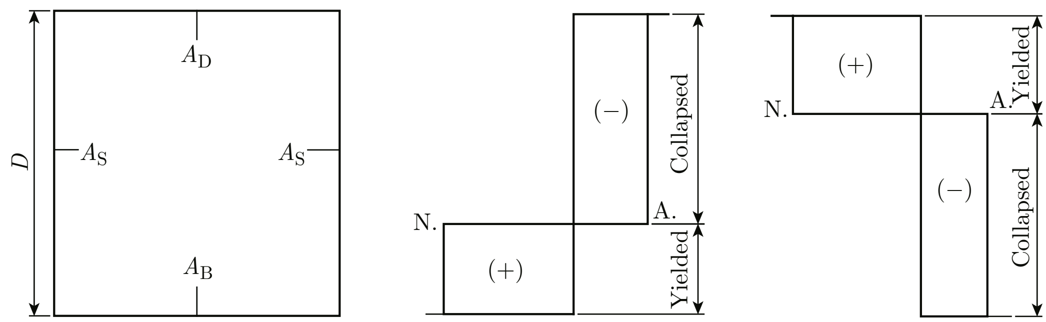
$$M_{\mathrm{us}} = Z_{\mathrm{D}}\sigma_{\mathrm{uD}}, \quad (1)$$

$$M_{\mathrm{uh}} = Z_{\mathrm{B}}\sigma_{\mathrm{uB}}. \quad (2)$$

The fully plastic bending moment of ship hull girder under vertical bending moment is usually covered in hull girder bending problem. According to the assumptions shown in Figs. 1 and 2, the cross-section of ship hull is idealized with equivalent sectional areas of deck, outer bottom, inner bottom, and side panel. It means that all longitudinal members are related to plating structure. The magnitude of fully plastic bending moment is calculated by

$$M_{\mathrm{p}} = A_{\mathrm{D}}(D-g)\sigma_{\mathrm{yD}} + A_{\mathrm{B}}g\sigma_{\mathrm{yB}} + A'_{\mathrm{B}}(g-D_{\mathrm{B}})\sigma'_{\mathrm{yB}} + \frac{A_{\mathrm{S}}}{D}[(D-g)^2\sigma_{\mathrm{ySU}} + g^2\sigma_{\mathrm{ySL}}], \quad (3)$$

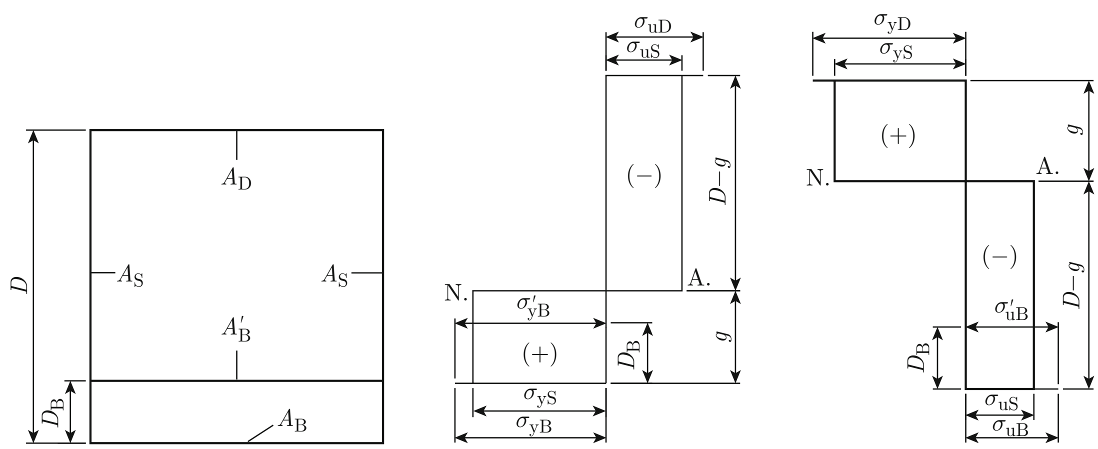
where σ_{ySU} is the yield strength of upper side panel, and σ_{ySL} is the yield strength of lower side panel.



(a) Equivalent cross section (b) Sagging (c) Hogging

(+) Tension, (−) Compression

Fig. 1 Original Caldwell's method (N.A. is neutral axis)



(a) Equivalent cross section (b) Sagging (c) Hogging

Fig. 2 Modified Caldwell's method

Vasta[6] suggested a formulation to predict the UBM of ship hull girder:

$$M_{\mathrm{u}} = Z\sigma_{\mathrm{u}}. \quad (4)$$

Mansour and Faulkner[7] modified the Vasta's expression, because they judged that the neutral axis would shift after buckling of the compression flange. The formulation is given as

$$M_{\mathrm{u}} = Z\sigma_{\mathrm{u}}(1+k), \quad (5)$$

where k is a function of the ratio of the areas of one side shell to the compression flange. For a frigate, the value of k is about 0.1.

Faulkner and Sadden[8] proposed an appropriate expression by taking into account the systematic errors associated with yield strength, ultimate compressive strength, and section effects:

$$M_{\mathrm{u}} = 1.15Z\sigma_{\mathrm{y}}\left[-0.1 + 1.4465\frac{\sigma_{\mathrm{u}}}{\sigma_{\mathrm{y}}} - 0.3465\left(\frac{\sigma_{\mathrm{u}}}{\sigma_{\mathrm{y}}}\right)^2\right]. \quad (6)$$

Based on the assumption that elastic behavior remains constant when the longitudinal stiffeners of the compression flange collapse. Viner[9] proposed a modified formulation:

$$M_{\mathrm{u}} = \alpha Z \sigma_{\mathrm{u}}, \tag{7}$$

where α is normally in a range of 0.92—1.05 (mean, 0.985).

Frieze and Lin[10] considered the relationship between the ultimate strength of hull girder and the ultimate strength of compression flange, and proposed a quadratic formula:

$$M_{\mathrm{u}} = M_{\mathrm{p}} \left[d_1 + d_2 \frac{\sigma_{\mathrm{u}}}{\sigma_{\mathrm{y}}} + d_3 \left(\frac{\sigma_{\mathrm{u}}}{\sigma_{\mathrm{y}}} \right)^2 \right], \tag{8}$$

where d_1, d_2 and d_3 are the constants. For sagging, $d_1 = -0.172$, $d_2 = 1.548$ and $d_3 = -0.368$ are set; for hogging, $d_1 = 0.003$, $d_2 = 1.459$ and $d_3 = -0.461$ are set.

Regarding the large elasto-plastic deflection of large-scale box girders and full-scale ship hulls, the hull girder strength has still enough capacity although the compression flange collapses. Valsgaard and Steen[11] suggested an empirical formula to predict the ultimate vertical bending moment of ship hull:

$$M_{\mathrm{u}} = B_{\mathrm{c}} Z \sigma_{\mathrm{u}}, \tag{9}$$

where B_{c} is a coefficient varying with the actual shape of the hull cross-section.

1.2 Stress Distribution Method

1.2.1 *Caldwell's Method*

The assumptions of stress distribution for hull cross-section have been suggested by several research groups. For instance, Caldwell[12] took into account the buckling of compressive members and yielding of tensile members. The original Caldwell's method does not take into account the double-hull cross-sections, but the modified Caldwell's method has considered the double-hull cross-sections and different material properties. The assumptions of equivalent cross-section and longitudinal stress distribution are described in Figs. 1 and 2, and listed as follows: cross-section is composed of different panels with equivalent thickness; entire material in tension flange will be fully yielding at the limit state; entire material in compression will reach ultimate buckling strength at the limit state; all structural members of a panel have the same yield strength; the ultimate strength of all sides and the compression flange is not the same; the change of neutral axis position is taken into account.

The UBMs in sagging and hogging conditions are determined, respectively, by

$$\begin{aligned} M_{\mathrm{us}} = & -A_{\mathrm{D}}(D-g)\sigma_{\mathrm{uD}} - A_{\mathrm{B}} g \sigma_{\mathrm{yB}} - \\ & A'_{\mathrm{B}}(g - D_{\mathrm{B}}\sigma'_{\mathrm{yB}} - \\ & \frac{A_{\mathrm{S}}}{D}[(D-g)^2\sigma_{\mathrm{uS}} + g^2\sigma_{\mathrm{yS}}], \end{aligned} \tag{10}$$

$$\begin{aligned} M_{\mathrm{uh}} = & A_{\mathrm{D}} g \sigma_{\mathrm{yD}} + A_{\mathrm{B}}(D-g)\sigma_{\mathrm{uB}} + \\ & A'_{\mathrm{B}}(D - g - D_{\mathrm{B}})\sigma'_{\mathrm{uB}} + \\ & \frac{A_{\mathrm{S}}}{D}[(D-g)^2\sigma_{\mathrm{uS}} + g^2\sigma_{\mathrm{yS}}]. \end{aligned} \tag{11}$$

However, Caldwell's method does not consider the post-buckling strength of structural members, and the actual UBM of hull girder may overestimate because the hull girder will collapse before the material in tension yields fully or that in compression collapses entirely. Moreover, the modern ships are often constructed from different modes of material, so this method does not give the accurate results.

1.2.2 *Paik and Mansour's Method*

Paik and Mansour[13] assumed that the bending stress distribution over the hull cross-section at the limit state is shown in Fig. 3. It is obvious that the outer bottom panel and the upper deck panel in sagging condition reach yield stress (σ_x^{Y}) and ultimate stress (σ_x^{U}), respectively. However, the trends of the stress follow reverse way with yield stress for upper deck panel and ultimate stress for outer bottom panel. The other panels reach the elastic stress of materials (σ_x^{E}).

In order to determine the neutral axis position in both sagging and hogging conditions at the ultimate limit state, the summation of axial forces over the entire cross-section of the hull must be zero.

When yield stress and ultimate stress are determined, the elastic stress can be calculated via the stress distribution, as shown in Fig. 3. Simultaneously, the distance from the ship's baseline to the horizontal neutral axis of the cross-section of the ship hull at the ultimate limit state is obtained by

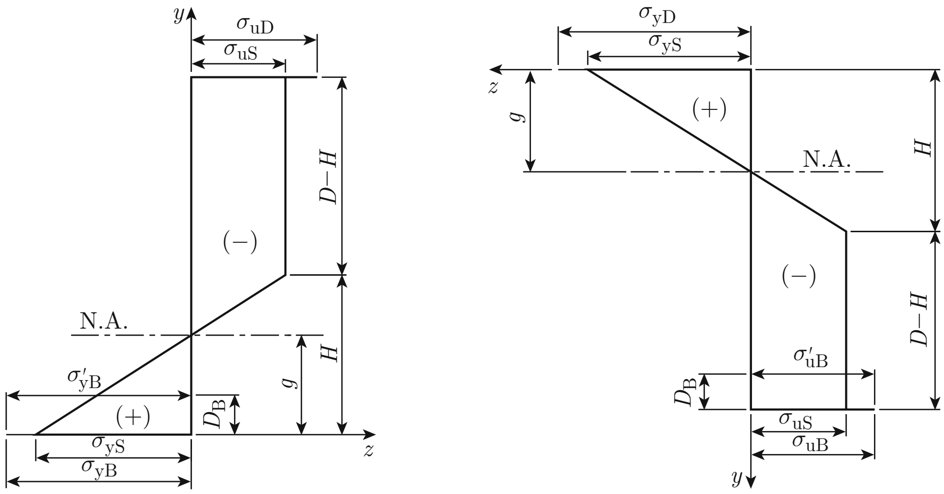
$$g_{\mathrm{u}} = \frac{\sum_{i=1}^{n} |\sigma_{x,i}| a_i z_i}{\sum_{i=1}^{n} |\sigma_{x,i}| a_i},$$

and the UBM is obtained by

$$M_{\mathrm{u}} = \sum_{i=1}^{n} \sigma_{x,i} a_i (z_i - g),$$

where $\sigma_{x,i}$ is the ultimate stress of the ith element, a_i is the total cross-sectional area of the ith element, and n is the total number of elements.

From the longitudinal stress distribution, the formulations to determine the UBMs for sagging and hogging



(a) Sagging (b) Hogging

Fig. 3 Paik and Mansour's method

conditions are given as

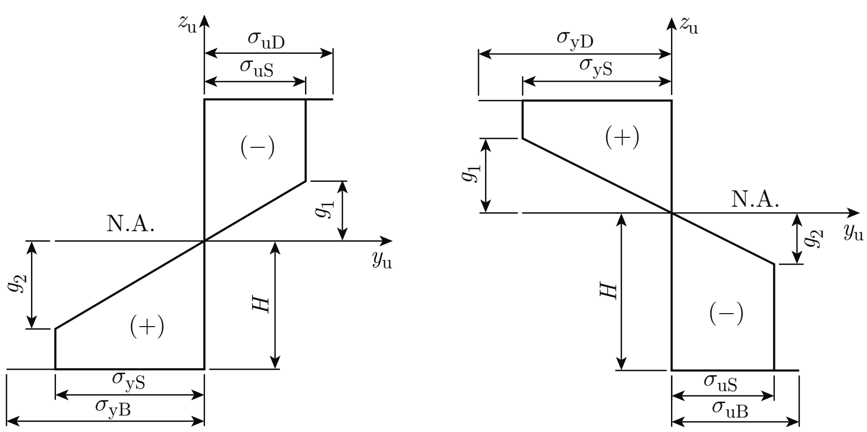
$$\begin{aligned}M_{\mathrm{us}} = & -A_{\mathrm{D}}(D-g)\sigma_{\mathrm{uD}} - \\ & \frac{A_{\mathrm{S}}}{D}(D-H)(D+H-2g)\sigma_{\mathrm{uS}} - A_{\mathrm{B}}g\sigma_{\mathrm{yB}} + \\ & \frac{A'_{\mathrm{B}}}{H}(g-D_{\mathrm{B}})[D_{\mathrm{B}}\sigma_{\mathrm{uS}} - (H-D_{\mathrm{B}})\sigma_{\mathrm{yS}}] - \\ & \frac{A_{\mathrm{S}}H}{3D}[(2H-3g)\sigma_{\mathrm{uS}} - (H-3g)\sigma_{\mathrm{yS}}], \end{aligned} \tag{12}$$

$$\begin{aligned}M_{\mathrm{uh}} = & A_{\mathrm{D}}g\sigma_{\mathrm{yD}} + A_{\mathrm{B}}(D-g)\sigma_{\mathrm{uB}} + \\ & A'_{\mathrm{B}}(D-g-D_{\mathrm{B}})\sigma'_{\mathrm{uB}} + \\ & \frac{A_{\mathrm{S}}}{D}(D-H)(D+H-2g)\sigma_{\mathrm{uS}} + \\ & \frac{A_{\mathrm{S}}H}{3D}[(2H-3g)\sigma_{\mathrm{uS}} - (H-3g)\sigma_{\mathrm{yS}}]. \end{aligned} \tag{13}$$

1.2.3 *Qi and Cui's Method*

Qi and Cui[5] proposed an advanced analytical method that is based on model test and FEA. This method is coupled with an elasto-plastic method (EPM, which is a combination of elastic large-deflection analysis and rigid plastic analysis) of buckling strength of stiffened panels. The assumptions and procedures of the proposed method are described as follows: the cross-section is divided into separate stiffened panels; the ultimate buckling strength of the stiffened panels is calculated by EPM; the tensile structural members will reach its yielding strength at the limit state; the compressive structural members will reach their ultimate buckling strength at the limit state; material in surrounding neutral axis position is assumed to remain in the elastic state; the elastic range is determined by the distance between tensile and compressive force central perpendicular to the neutral axis in Caldwell's ultimate strength model; the neutral axis position at the limit state is determined by equilibrium condition in the elastic-plastic ultimate strength model.

The assumed stress distribution is shown in Fig. 4,



(a) Sagging (b) Hogging

Fig. 4 Qi and Cui's method

and the UBMs in sagging and hogging conditions are determined, respectively, as

$$M_{\mathrm{us}} = -A_{\mathrm{D}}\sigma_{\mathrm{uD}}(D-H) - \frac{A_{\mathrm{S}}\sigma_{\mathrm{uS}}}{D}\left[(D-H)^2 - \frac{g_1^2}{3}\right] - \frac{A_{\mathrm{S}}\sigma_{\mathrm{yS}}}{D}\left(H^2 - \frac{g_2^2}{3}\right) - A_{\mathrm{B}}\sigma_{\mathrm{yB}}H, \tag{14}$$

$$M_{\mathrm{uh}} = A_{\mathrm{D}}\sigma_{\mathrm{yD}}(D-H) + \frac{A_{\mathrm{S}}\sigma_{\mathrm{yS}}}{D}\left[(D-H)^2 - \frac{g_1^2}{3}\right] + \frac{A_{\mathrm{S}}\sigma_{\mathrm{uS}}}{3}\left(H^2 - \frac{g_2^2}{3}\right) + A_{\mathrm{B}}\sigma_{\mathrm{uB}}H. \tag{15}$$

1.3 Progressive Collapse Analysis Method

The present paper applies the incremental-iterative method by IACS-SCR[4]. With regard to this method, the transverse section is divided into three types of elements: hard corner element, stiffener element, and stiffened plate element. All relevant failure modes for individual structural elements are considered to identify the weakest inter-frame failure modes. They are elasto-plastic collapse, beam column buckling, torsion buckling, Web local buckling of flanged profiles, Web local buckling of flat bars, and plate buckling. In addition, several assumptions are given as follows: the ultimate strength is calculated at hull transverse sections between two adjacent transverse webs; the hull girder transverse section remains plane during each curvature increment; the hull material has an elasto-plastic behavior; the hull girder transverse section is divided into a set of elements which are considered to act independently.

Figure 5 shows the relationship of the moment M versus the curvature χ. The main steps are generally made as follows.

Step 1 Divide the transverse section of hull into stiffened plate elements.

Step 2 Define stress-strain relationships for all elements.

Step 3 Initialize curvature χ_1 and neutral axis for the first incremental step.

Step 4 Calculate the corresponding strain ε_i and stress σ_i for each element.

Step 5 Determine the current neutral axis position of entire cross-section at each incremental step, i.e., $z_{\mathrm{NA\_cur}}$.

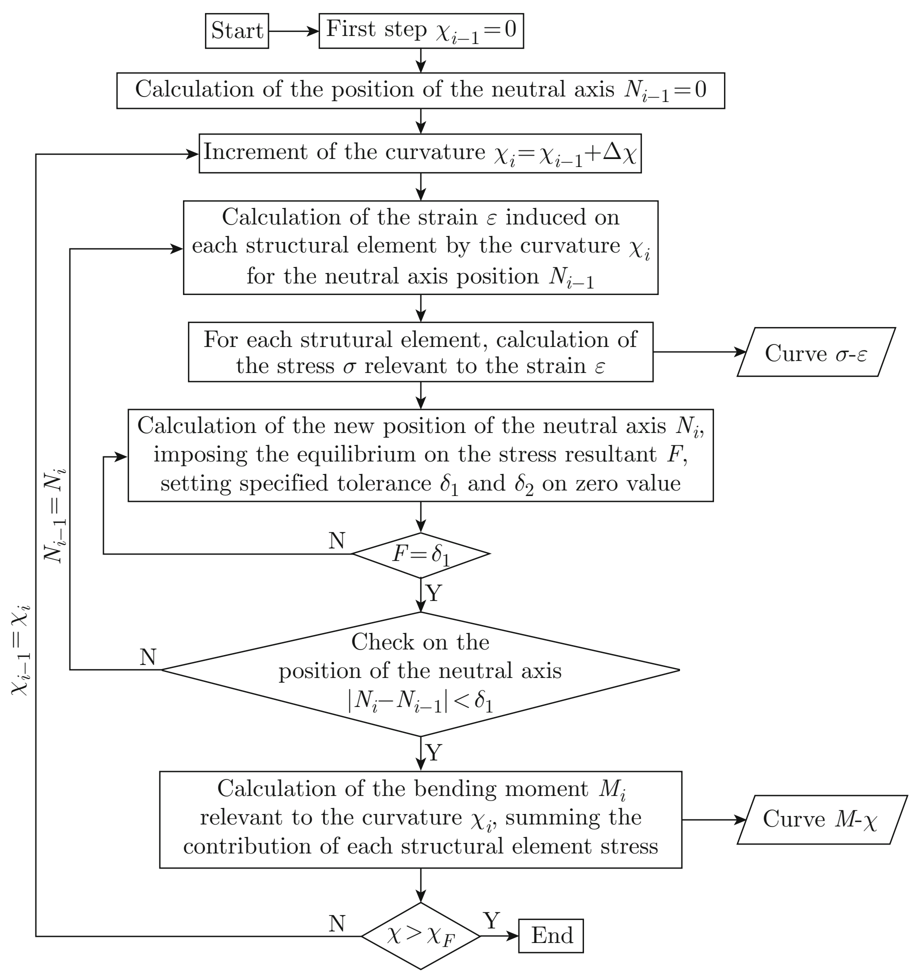


Fig. 5 Flow chart of the procedure for the evaluation of the curve M-χ

Step 6 Calculate the corresponding moment.

Step 7 Compare the moment in the current incremental step with the moment in the previous once. If the slope in M-χ relationship is less than a negative fixed value, terminate the process and define the peak value. Otherwise, increase the curvature by the amount of $\Delta\chi$ and go to Step 4.

Following the above procedure, the UBM capacity M_{u} is the peak point of the M-χ curve which is made by the bending moment components of all incremental steps:

$$M_{\mathrm{u}} = \sum \sigma_{\mathrm{u},i} A_{i,\mathrm{n}50}(z_i - z_{\mathrm{NA\_cur}}), \tag{16}$$

where $A_{i,\mathrm{n}50}$ is the net total cross-sectional area of the ith element.

1.4 Checking Criteria

The ships equal to or greater than 150 m in length must satisfy checking criteria of HGUBC[4]. With regard to this, the HGUBC at any hull transverse section is checked for hogging and sagging conditions, and satisfies the following equation:

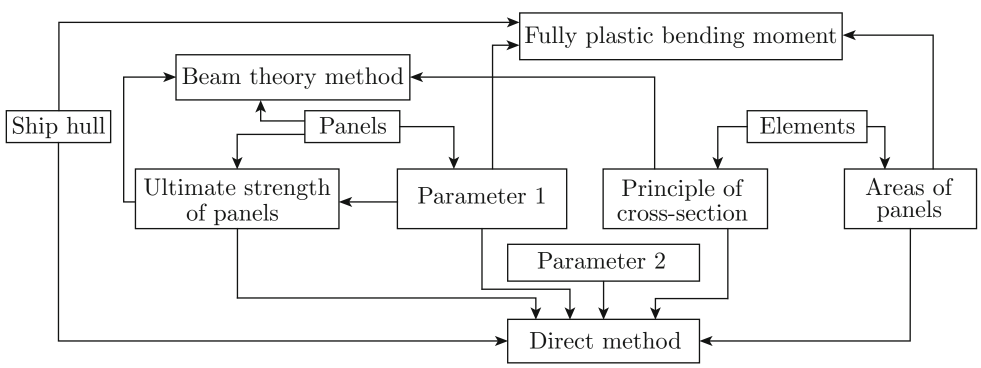
$$\gamma_{\mathrm{S}} M_{\mathrm{SW,U}} + \gamma_{\mathrm{W}} M_{\mathrm{WV}} \leqslant \frac{M_{\mathrm{u}}}{\gamma_{\mathrm{R}}}, \tag{17}$$

where $M_{\mathrm{SW,U}}$ is the permissible still water bending moment, M_{WV} is the vertical wave bending moment, γ_{S} is the partial safety factor for the still water bending moment, γ_{W} is the partial safety factor for the vertical wave bending moment, and γ_{R} is the partial safety factor for the vertical HGUBC.

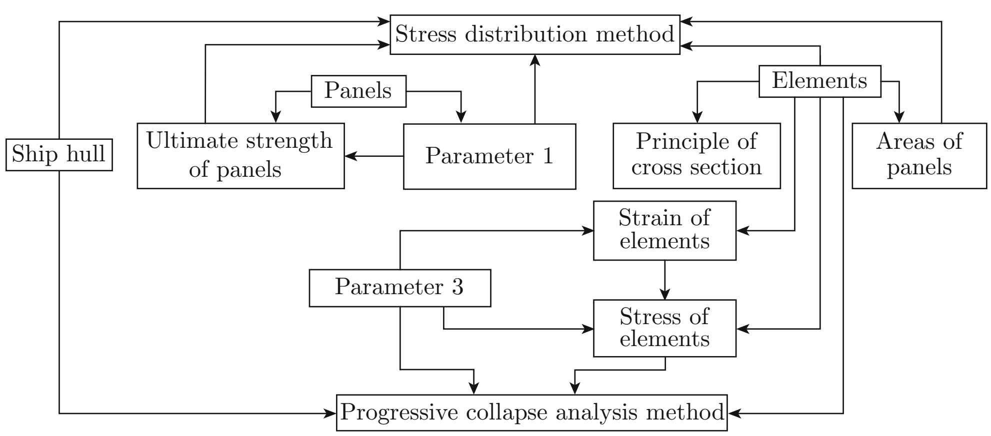
2 New Developed Program

The UBM is easily obtained by applying direct method, but it is more complex in stress distribution method. It is because of not only the requirement of axial forces' summation equivalent condition of entire cross-section but also the element stresses. When the progressive collapse analysis method is applied, the completion is more difficult with the loop in loop and the checking condition inside of each loop. In order to make a convenience in predicting the UBM, two algorithm schemes are drawn, as shown in Figs. 6(a) and 6(b). The former is applied for direct method, simply-beam theory method, and fully plastic bending moment, while the latter is used for stress distribution method and progressive collapse analysis method.

The structure of the developed program is divided into five modules: data (including ship hull, cross-section, panels and elements), ultimate strength of unstiffened plates, ultimate strength of stiffened plates, ultimate hull girder strength, and checking criteria. The interfaces of these modules are shown in Figs. 7—10.



(a) Direct method, simply-beam theory method, and fully plastic bending moment



(b) Stress distribution method and progressive collapse analysis

Fig. 6 Algorithm schemes for predicting UBM

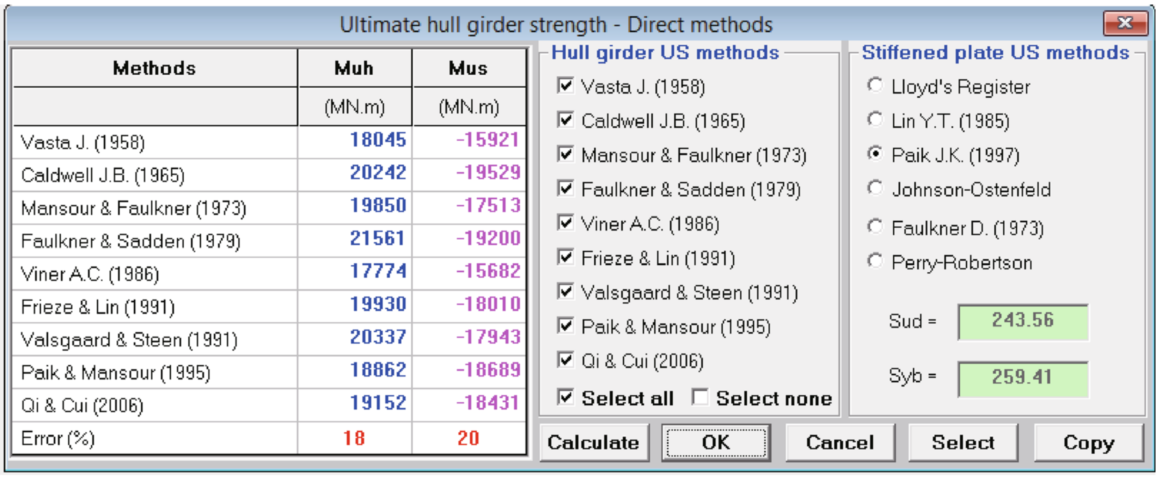


Fig. 7 Snapshot for the module of direct method

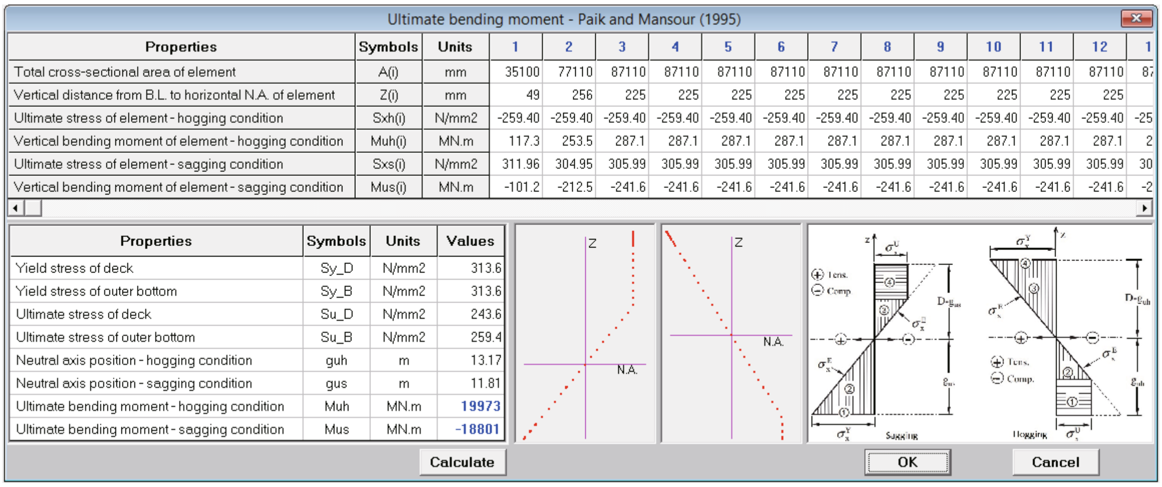


Fig. 8 Snapshot for the module of stress distribution method

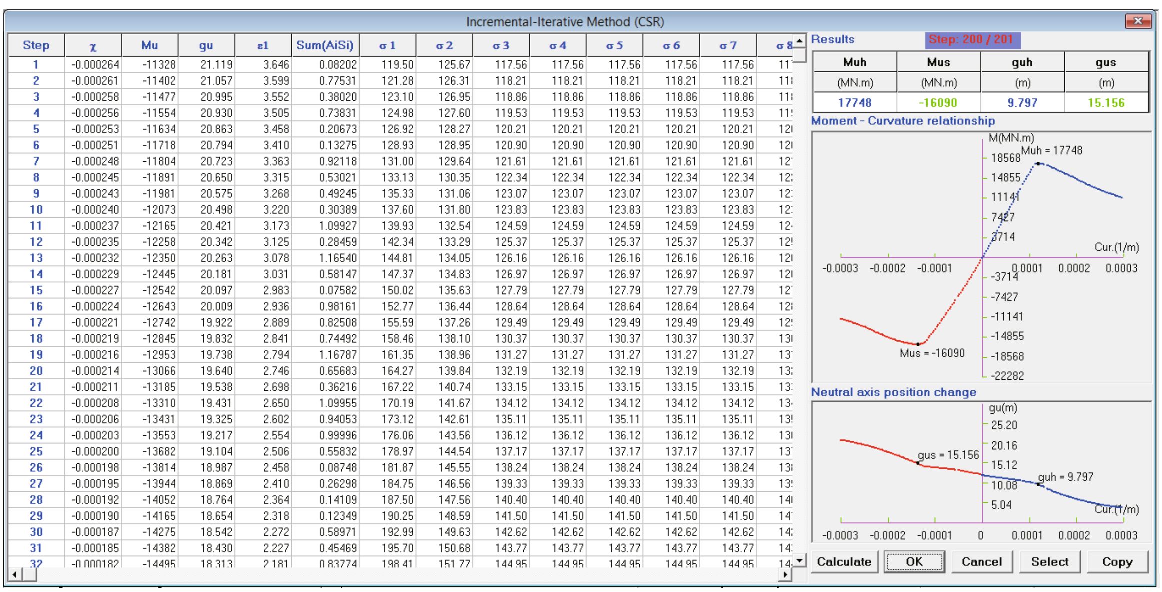


Fig. 9 Snapshot for the module of progressive collapse analysis method

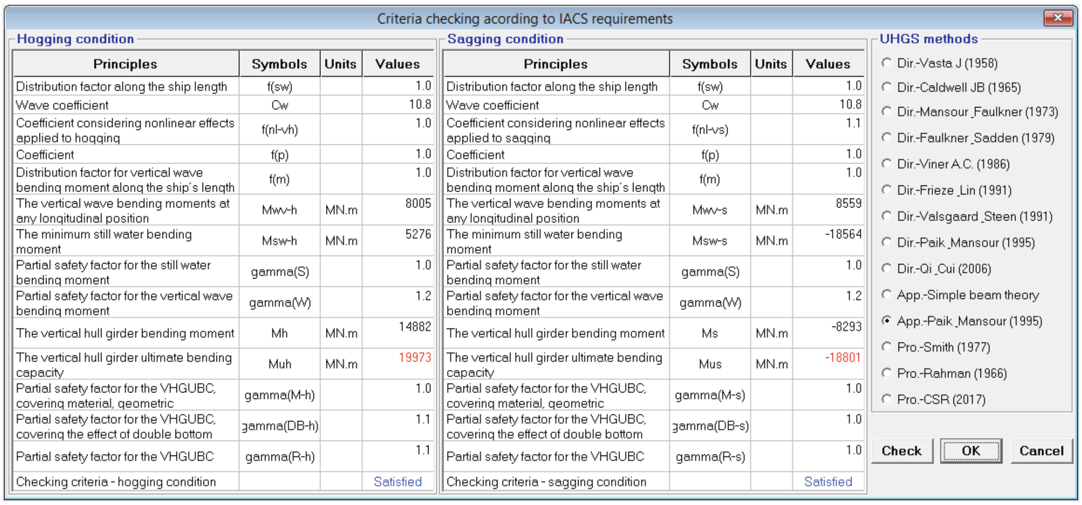


Fig. 10 Snapshot for the module of checking criteria

3 Selected Ships and Results

Six ships and model tests used in benchmark study of ISSC[32] are applied to calculate the vertical UBM with the developed program. The principal dimensions combined with the cross-section properties of the selected ships and models are described in Table 1.

By applying direct method, stress distribution method and progressive collapse analysis method, the UBMs of hull girders are obtained and collected, as shown in Table 2. The UBMs are determined in hogging and sagging conditions by a total of thirteen methods.

Table 1 Principal dimensions and cross-section properties

Case	L_{pp}/m	B/m	D/m	D_B/m	C_B	A/m^2	I/m^4	Z_D/m^3	Z_B/m^3	P_{NA}/m
I	230	32.2	21.5	1.88	0.60	3.84	243.52	18.17	27.30	8.59
II	313	48.2	25.2	0	0.83	7.84	857.06	62.53	69.94	12.25
III	18	4.2	2.8	0	0.45	0.056	0.061	0.044	0.043	1.416
IV	300	48.2	23.2	2.55	0.85	6.59	560.36	40.76	54.72	10.31
V	285	50	26.7	2.79	0.82	5.66	690.30	42.70	61.77	11.17
VI	315	58	30.3	3.00	0.86	9.62	1358.6	72.65	103.32	13.15

Note: Case I is container ship, Case II is single hull VLCC, Case III is Dow's frigate, Case IV is Suezmax-class double hull oil tanker, Case V is bulk carrier, Case VI is double hull VLCC, L_{pp} is the length between perpendiculars, B is the breadth, C_B is the block coefficient, A is the total cross-sectional area, I is the moment of inertia, and P_{NA} is the original neutral axis position from base line.

Table 2 UBMs by the developed program

Case	Condition	M_u/M_p												M_p/(MN·m)
		Ref. [6]	Ref. [12]	Ref. [7]	Ref. [8]	Ref. [9]	Ref. [10]	Ref. [11]	Ref. [13]	Ref. [5]	Stress distribution	Ref. [4]	Beam theory	
I	Hog.	0.775	0.870	0.852	0.942	0.763	0.817	0.873	0.691	0.664	0.783	0.815	0.776	8 005
	Sag.	0.683	0.795	0.751	0.808	0.672	0.898	0.769	0.778	0.666	0.912	0.903	0.684	
II	Hog.	0.810	0.908	0.891	0.968	0.798	0.894	0.913	0.847	0.860	0.896	0.797	0.811	22 282
	Sag.	0.715	0.876	0.786	0.862	0.704	0.808	0.805	0.839	0.827	0.844	0.722	0.715	
III	Hog.	0.631	0.901	0.694	0.750	0.622	0.913	0.711	0.814	0.855	0.739	0.740	0.632	14.43
	Sag.	0.460	0.758	0.506	0.567	0.453	0.634	0.518	0.740	0.710	0.619	0.575	0.460	
IV	Hog.	0.834	0.925	0.917	1.001	0.821	0.878	0.940	0.806	0.721	0.804	0.629	0.835	17 278
	Sag.	0.617	0.766	0.679	0.746	0.608	0.800	0.696	0.761	0.694	0.791	0.792	0.618	
V	Hog.	0.924	0.966	1.017	1.117	0.910	0.857	1.042	0.795	0.746	1.014	0.766	0.925	16 085
	Sag.	0.753	0.699	0.828	0.914	0.742	0.767	0.849	0.759	0.702	0.911	1.051	0.754	
VI	Hog.	0.855	0.931	0.940	1.025	0.842	0.882	0.963	0.769	0.729	0.784	0.612	0.856	31 089
	Sag.	0.561	0.775	0.617	0.683	0.552	0.747	0.632	0.757	0.680	0.753	0.819	0.561	

Note: Sag. is denoted for sagging; Hog. is denoted for hogging.

According to the report of ISSC committee[32], several candidates from different organizations and research groups had together given the UBM of six hull structures by multiform of methods and computer codes such as ANSYS code, ABAQUS code, and ALPS/HULL code. The summary of UBMs of benchmark study is shown in Table 3.

Table 3 UBMs by benchmark study of ISSC 2012

Case	Condition	M_u/M_p										M_p/ (MN · m)
		ANSYS (PNU)	ANSYS (ISR)	ABAQUS (CR)	ALPS/HULL (PNU)	CSR (BV)	CSR (CR)	CSR (PNU)	RINA Rules (UoG)	ISSC 2000	Modified Paik and Mansour[14]	
I	Hog.	0.871	0.936	0.957	0.864	0.809	0.984	0.969	0.857	0.949	0.800	8 005
	Sag.	0.868	0.896	0.953	0.829	0.758	0.948	0.856	0.737	0.814	0.884	
II	Hog.	0.779	0.951	0.981	0.778	0.785	0.929	0.902	0.890	0.898	0.839	22 282
	Sag.	0.726	0.907	0.926	0.775	0.719	0.834	0.816	0.829	0.876	0.800	
III	Hog.	0.779	—	0.856	0.741	—	0.824	0.773	0.806	0.919	0.716	14.43
	Sag.	0.736	—	0.742	0.689	—	0.708	0.681	0.655	0.657	0.647	
IV	Hog.	0.814	—	0.935	0.770	—	1.102	0.909	—	—	0.808	17 278
	Sag.	0.645	—	0.825	0.642	—	0.845	0.719	—	—	0.707	
V	Hog.	1.070	1.121	1.125	1.015	0.907	1.122	1.123	1.069	1.145	1.014	16 349
	Sag.	0.966	1.084	1.031	0.941	0.705	0.913	0.887	0.853	0.877	0.905	
VI	Hog.	0.879	0.968	0.997	0.823	0.754	0.960	0.914	0.907	0.911	0.826	31 089
	Sag.	0.724	0.906	0.804	0.707	0.577	0.805	0.712	0.698	0.630	0.720	

4 Comparative Analysis

The UBMs of six different hull girders are obtained by the developed program. The ratio of UBM to fully plastic bending moment and the magnitude of fully plastic bending moment are shown in Table 2. The discrepancy of UBMs by different methods is significant, up to 40%. The case III (sagging) and the case VI (hogging) have the biggest disparity, while the case II (hogging) has the smallest one. The difference between UBMs by direct method is in a range of 18%—32%, while this value by stress distribution method ranges from 6% to 27%. It can be noted that Viner's method, Qi and Cui's method, and IACS-CSR method underestimate the ultimate hull girder strength in several cases, while Mansour and Faulkner's method, Faulkner and Sadden's method, Valsgaard and Steen's method, and stress distribution method overestimate the UBMs of several hull girders.

The discrepancy of UBMs by the developed program is in a range of 18%—40%; meanwhile, the disproportion of UBMs by benchmark study ranges from 13% to 36%. Table 4 illustrates a comparison of the results obtained by the developed program and ISSC. The comparison is made between the stress distribution method (namely Caldwell's method, Paik and Mansour's method, Qi and Cui's method, and stress distribution method) and modified Paik and Mansour's method[14]. Moreover, the coefficient of variation (CoV) is also determined. Case II has the smallest CoV, while Case V has the biggest one.

Table 4 Comparison of the developed program and ISSC

Case	Condition	M_u/M_p					Standard deviation	Mean value	CoV/%
		Developed program				Modified Paik and Mansour's			
		Caldwell's	Paik and Mansour's	Qi and Cui's	Stress distribution				
I	Hog.	0.870	0.691	0.664	0.783	0.800	0.164	0.761	21.58
	Sag.	0.795	0.778	0.666	0.912	0.884	0.183	0.807	22.62
II	Hog.	0.908	0.847	0.860	0.896	0.839	0.055	0.870	6.37
	Sag.	0.876	0.839	0.827	0.844	0.800	0.045	0.837	5.38
III	Hog.	0.901	0.814	0.855	0.739	0.716	0.135	0.805	16.73
	Sag.	0.758	0.740	0.710	0.619	0.647	0.113	0.695	16.19
IV	Hog.	0.925	0.806	0.721	0.804	0.808	0.146	0.813	17.91
	Sag.	0.766	0.761	0.694	0.791	0.707	0.077	0.744	10.35
V	Hog.	0.966	0.795	0.746	1.014	1.014	0.237	0.907	26.14
	Sag.	0.699	0.759	0.702	0.911	0.905	0.189	0.795	23.72
VI	Hog.	0.931	0.769	0.729	0.784	0.826	0.154	0.808	19.03
	Sag.	0.775	0.757	0.680	0.753	0.720	0.073	0.737	9.97

5 Conclusion

A historical review associated with the ultimate hull girder strength is presented, and a program for predicting UBM of ship hull girders is also developed. This program is built by the Visual Basic code which can determine the UBM of every type of ship or ship-shaped structures such as single or double side, and with or without inner bottom. Moreover, the developed program also provides the checking criteria module which is applied to check the satisfaction of UBM in seagoing condition under IACS-CSR requirements. The comparison between applied methods is discussed. With regard to the discussion, the users may have the suitable consideration in selecting and applying the method for their special problems.

The calculation results obtained by the developed program are analyzed and compared. The developed program can save the time of calculation. Thirteen methods to determine the UBM of hull girders are available. The developed program will be a helpful tool to designers and/or researchers to assess the ultimate hull girder strength of ship structures including intact and/or aging ones.

However, the applied methods in this study do not take into account the influential factors which affect considerably the ultimate strength of ship hull girders. Hence, it is necessary to improve the methods to take into account the initial geometric imperfections (such as initial distortions and residual stresses) and the age-related damage (such as corrosion wastage, fatigue cracks, and local dents). This is also the future work of the present study.

References

[1] BUCKLEY W H. Hull girder structural design: The case for new loading conditions for extreme waves [J]. *Naval Engineers Journal*, 1978, **90**(1): 95-105.

[2] PAIK J K, KIM S K, LEE S K. Probabilistic corrosion rate estimation model for longitudinal strength members of bulk carriers [J]. *Ocean Engineering*, 1998, **25**(10): 837-860.

[3] YAMAMOTO N. Updating of probabilistic corrosion model based on Bayesian procedure [C]//*Proceedings of the ASME 2015 34th International Conference on Ocean, Offshore and Arctic Engineering*. Newfoundland, Canada: ASME, 2015: 1-7.

[4] IACS. Common structural rules for bulk carriers and oil tankers [R]. London, UK: IACS, 2017.

[5] QI E, CUI W. Analytical method for ultimate strength calculations of intact and damaged ship hulls [J]. *Ships and Offshore Structures*, 2006, **1**(2): 153-163.

[6] VASTA J. Lessons learned from full-scale structural tests [J]. *Transaction of Society of Naval Architects and Marine Engineers*, 1958, **66**: 165-243.

[7] MANSOUR A E, FAULKNER D. On applying the statistical approach to extreme sea loads and ship hull girder [J]. *Transactions of the Royal Institution of Naval Architects*, 1973, **115**: 277-313.

[8] FAULKNER D, SADDEN S. Toward a unified approach to ship structural safety [J]. *Transactions of the Royal Institution of Naval Architects*, 1979, **121**: 1-28.

[9] VINER A C. Development of ship strength formulations [C]//*Proceedings of the International Conference on Advances in Marine Structures*. New York, USA: Elsevier Science Ltd., 1986: 152-173.

[10] FRIEZE P A, LIN Y T. Ship longitudinal strength modelling for reliability analysis [C]//*Proceedings of the Marine Structural Inspection, Maintenance, and Monitoring Symposium*. Virginia, USA: SNAME, 1991: III C.1-20.

[11] VALSGAARD S, STEEN E. Ultimate hull girder strength margins in present class requirements [C]//*Proceedings of the Marine Structural Inspection, Maintenance, and Monitoring Symposium*. Virginia, USA: SNAME, 1991: III B.1-19.

[12] CALDWELL J B. Ultimate longitudinal strength [J]. *Transactions of the Royal Institution of Naval Architects*, 1965, **107**: 411-430.

[13] PAIK J K, MANSOUR A E. A simple formulation for predicting the ultimate strength of ships [J]. *Journal of Marine Science and Technology*, 1995, **1**(1): 52-62.

[14] PAIK J K, KIM D K, PARK D H, et al. Modified Paik-Mansour formula for ultimate strength calculations of ship hulls [J]. *Ships and Offshore Structures*, 2013, **8**(3/4): 245-260.

[15] SMITH C. Influence of local compressive failure on ultimate longitudinal strength of a ship's hull [C]//*Proceedings of the PRADS, International Symposium on Practical Design in Shipbuilding*. Tokyo, Japan: The Society of Naval Architects of Japan, 1977: 73-79.

[16] PAIK J K, SEO J K, KIM D M. Idealized structural unit method and its application to progressive hull girder collapse analysis of ships [J]. *Ships and Offshore Structures*, 2006, **1**(3): 235-247.

[17] GAO B G, DENG Y C, YANG S Q. A method based on non-linear FEM for ultimate limit state assessment of ship hull girders [C]//*International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering*. Chengdu, China: IEEE, 2013: 524-528.

[18] HANSEN A M. Strength of mid-ship sections [J]. *Marine Structures*, 1996, **9**(3): 471-494.

[19] AKHRAS G, GIBSON S, YANG S, et al. Ultimate strength of a box girder simulating the hull of a ship [J]. *Canadian Journal of Civil Engineering*, 1998, **25**(5): 829-843.

[20] PAIK J K, HUGHES O F, MANSOUR A E. Advanced closed-form ultimate strength formulation for ships [J]. *Journal of Ship Research*, 2001, **45**(2): 111-132.

[21] SUN H H, BAI Y. Ultimate strength and reliability assessment for the ship hull girders used in ISSC-2000 benchmark study [C]//*Proceedings of the 8th International Symposium on Practical Design of Ships and*

Other Floating Structures. Shanghai, China: Elsevier Science Ltd., 2001: 1005-1012.

[22] RIGO P, TODERAN C, YAO T. Sensitivity analysis on ultimate hull bending moment [C]//*Proceedings of the 8th International Symposium on Practical Design of Ships and Other Floating Structures*. Shanghai, China: Elsevier Science Ltd., 2001: 987-995.

[23] YAO T. Hull girder strength [J]. *Marine Structures*, 2003, **16**(1): 1-13.

[24] YAO T. Ultimate longitudinal strength of ship hull girder: Historical review and state of the art [J]. *International Journal of Offshore and Polar Engineering*, 1999, **9**(1): 1-9.

[25] QI E R, CUI W C, WAN Z Q. Comparative study of ultimate hull girder strength of large double hull tankers [J]. *Marine Structures*, 2005, **18**(3): 227-249.

[26] OZGUC O, DAS P K, BARLTROP N. A proposed method to evaluate hull girder ultimate strength [J]. *Ships and Offshore Structures*, 2006, **1**(4): 335-345.

[27] PAIK J K, KIM B J, SEO J K. Methods for ultimate limit state assessment of ships and ship-shaped offshore structures: Part III hull girders [J]. *Ocean Engineering*, 2008, **35**(2): 281-286.

[28] YONEYA T, KOBAYASHI H, RAHIM M A, et al. Total analysis system for ship structural strength [C]//*Proceedings of the 8th International Symposium on Practical Design of Ships and Other Floating Structures*. Shanghai, China: Elsevier Science Ltd., 2001: 971-977.

[29] TAYYAR G T, NAM J M, CHOUNG J M. Prediction of hull girder moment-carrying capacity using kinematic displacement theory [J]. *Marine Structures*, 2014, **39**: 157-173.

[30] NAM J M, CHOUNG J M, PARK S Y, et al. Assessment of residual ultimate strength of VLCC according to damage extents and average compressive strength of stiffened panel [C]//*Proceedings of the ASME 2014 33rd International Conference on Ocean, Offshore and Arctic Engineering*. California, USA: ASME, 2014: 1-8.

[31] HUGHES O F, PAIK J K. Ship structural analysis and design [M]. New Jersey, USA: SNAME, 2010.

[32] ISSC. ISSC committee III.1: Ultimate strength [C]//*Proceedings of the 18th International Ship and Offshore Structures Congress*. Rostock, Germany: ISSC, 2012: 1-107.