

Minimum Resistance Ship Hull Uncertainty Optimization Design Based on Simulation-Based Design Method

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Abstract: In the ship hull optimization design based on simulation-based design (SBD) technology, low precision of the approximate model leads to an uncertainty form of optimization model. In order to enable the approximate model with finite precision to maximize the effectiveness, uncertainty optimization method is introduced here. Wave resistance coefficient approximation model, built by back propagation (BP) neural network, is represented as a form of interval. Afterwards, a minimum resistance optimization model is established with the design space constituted by principal dimensions and ship form coefficients. Double-level nested optimization architecture is proposed: for outer layer, improved particle swarm optimization (IPSO) algorithm with learning factor improvement strategy is used to generate design variables, and for inner layer, modified very fast simulated annealing (MVFS) algorithm is used to solve the objective function interval with uncertainty region. Cases calculation proves the effectiveness and superiority of uncertainty optimization method for ship hull SBD optimization design, thus providing a good way for finding optimal designs.

Key words: ship hull optimization, approximation model, uncertainty optimization, interval programming

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0 Introduction

As a core part of ship overall design, ship hull form design based on simulation-based design (SBD) technology^[1], which is developed by combining the optimization technique and computational fluid dynamics (CFD) technique, plays a more and more important role in ship design field, and there are three crucial elements for SBD optimization design process, i.e., high precision simulation model (such as CFD solver), automatic modification of ship hull geometry, and optimization design platform.

Computation time constraints disenable the high precision CFD solver to execute simulation completely at every iteration step. Instead of CFD solver, the approximation model, which can greatly shorten computation time, is usually used in ship hull SBD process as an effective tool. The frequently used approximation models include response surface method (RSM), Kriging model and neural network, etc^[2-5].

Despite the high efficiency, approximation models have the disadvantage at the same time: the establish-

ment of approximation models needs amounts of precision results as input, and output results are sensitive to algorithm internal parameters, so inevitably, error of output will occur due to some uncontrollable causes, which is expressed as uncertainty. Although this error or uncertainty has a small value in most cases, large deviation of the whole system can also be generated by continuous iterative computation. Therefore, it has an important theoretical and practical significance for considering the uncertainty of approximate model.

According to the characteristics of uncertain parameters, kinds of uncertainty optimization methods existing basically fall into three categories: stochastic programming, fuzzy programming and interval programming^[6-8]. Among them, stochastic programming achieves optimum solution via random variable, whose probability density distribution needs to be given. Also, parameter included in fuzzy programming is fuzzy number, whose membership function needs to be got in advance. Actually, it is difficult to obtain the probability density distribution or membership function mentioned above. Therefore, there is a growing awareness that these two methods have restrictions on engineering application^[9]. By comparison, interval programming needs interval number, with its upper and lower bounds which are easily got in application, as an uncertain parameter. Thus uncertainty optimization

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method based on interval programming has found significant advantages and wide use in many engineering fields, such as profit maximization, wing design, automotive design and interior layout design^[10], but without any use in ship design field.

In this research, principal dimensions and ship form coefficients are combined to form the design space, and the restrictive condition (non-substantial changes of displacement) is integrated into the objective function as a form of penalty function. Thus, an optimization model with minimum resistance as the target is established. Wave resistance coefficient approximation model, built by back propagation (BP) neural network^[11], is represented as a form of interval for the ship hull uncertainty optimization.

1 Uncertainty Optimization Based on Interval Programming

1.1 Formulas

Interval number is a type of number expressed by an interval,

$$a^I = [a^L, a^U], \tag{1}$$

where $a^L, a^U \in \mathbf{R}$ and $a^L \leq a^U$; a^L and a^U are the lower and upper bounds of interval number a^I , respectively. When $a^L = a^U$, a^I turns out to be a real with deterministic value.

For interval numbers A_1^I and A_2^I , interval order relationship, expressed by \leq_{mw} , can be used to evaluate their degrees,

$$\left. \begin{aligned} A_1^I \leq_{mw} A_2^I, & \text{ only if } m(A_1^I) \geq m(A_2^I) \\ & \text{and } w(A_1^I) \geq w(A_2^I) \\ A_1^I <_{mw} A_2^I, & \text{ only if } A_1^I \leq_{mw} A_2^I \\ & \text{and } A_1^I \neq A_2^I \end{aligned} \right\}, \tag{2}$$

$$\left. \begin{aligned} m(A_1^I) &= \frac{A_1^L + A_1^U}{2}, & w(A_1^I) &= \frac{A_1^U - A_1^L}{2} \\ m(A_2^I) &= \frac{A_2^L + A_2^U}{2}, & w(A_2^I) &= \frac{A_2^U - A_2^L}{2} \end{aligned} \right\}, \tag{3}$$

where m is the midpoint of interval, and w is the radius.

After expressing the uncertainty number by interval number, uncertainty optimization problem can be described as

$$\left. \begin{aligned} \text{opt } \min \{ & f(\mathbf{x}, \mathbf{u}) \} \\ \text{s.t. } & g(\mathbf{x}, \mathbf{u}) \leq b^I = [b^L, b^U] \\ & \mathbf{x} \in \Omega^n, \mathbf{u} \in \mathbf{u}^I \end{aligned} \right\}, \tag{4}$$

where \mathbf{x} is a design variable with n dimensions, and its range is Ω^n ; \mathbf{u} is an uncertain vector with q dimensions, and its uncertainty is described by an interval number \mathbf{u}^I ; f and g are the objective function and restrictive

condition, respectively, which are related to \mathbf{x} and \mathbf{u} ; b^I is the allowable interval of uncertainty restrictive condition.

1.2 Transformation to Certainty

Objective function in Eq. (4) can be transformed to certainty based on interval order relationship,

$$\left. \begin{aligned} \min \{ & m(f(\mathbf{x}, \mathbf{u})), w(f(\mathbf{x}, \mathbf{u})) \} \\ m(f(\mathbf{x}, \mathbf{u})) &= \frac{1}{2}[f^L(\mathbf{x}) + f^U(\mathbf{x})] \\ w(f(\mathbf{x}, \mathbf{u})) &= \frac{1}{2}[f^U(\mathbf{x}) - f^L(\mathbf{x})] \\ f^L(\mathbf{x}) &= \min f(\mathbf{x}, \mathbf{u}) \\ f^U(\mathbf{x}) &= \max f(\mathbf{x}, \mathbf{u}) \end{aligned} \right\}. \tag{5}$$

Equation (5) shows that, the main idea of the transformation (from uncertainty optimization to certainty optimization) is to evaluate the design variables by midpoint and radius of interval objective function, and thus certainty objective function is obtained. Afterwards, weights of the two objectives (midpoint and radius) are given to achieve the single objective optimization model, whose objective function is

$$\text{opt } \min(1 - \beta)m(f(\mathbf{x}, \mathbf{u})) + \beta w(f(\mathbf{x}, \mathbf{u})), \tag{6}$$

where β is the weight, $0 \leq \beta \leq 1$, and is taken 0.5 normally.

Uncertainty restrictive condition in Eq. (4) can be converted to certainty restrictive condition via

$$\left. \begin{aligned} P(C^I \geq D^I) &\geq \lambda \\ C^I &= [g^L(\mathbf{x}), g^U(\mathbf{x})] \\ D^I &= [b^L, b^U] \end{aligned} \right\}, \tag{7}$$

where C^I is the possible range of uncertainty restrictive function at \mathbf{x} , D^I is the permissible restrictive interval number, $P(C^I \geq D^I)$ means the probability of C^I is greater than or equal to D^I , and λ is the probability threshold given in advance. The lower and upper bounds of restrictive interval, $g^L(\mathbf{x})$ and $g^U(\mathbf{x})$, are defined as

$$\left. \begin{aligned} g^L(\mathbf{x}) &= \min g(\mathbf{x}, \mathbf{u}), \\ g^U(\mathbf{x}) &= \max g(\mathbf{x}, \mathbf{u}) \end{aligned} \right\}. \tag{8}$$

Restrictive condition (in Eq. (7)) is integrated into the objective function (in Eq. (8)) as a form of penalty function, and thus a certainty-unconstrained optimization model is established as

$$\left. \begin{aligned} \text{opt } \min & f_p(\mathbf{x}, \mathbf{u}) \\ f_p(\mathbf{x}, \mathbf{u}) &= (1 - \beta)m(f(\mathbf{x}, \mathbf{u})) + \\ & \beta w(f(\mathbf{x}, \mathbf{u})) + \\ & \sigma \phi[P(C^I \geq D^I) - \lambda] \\ \phi[P(C^I \geq D^I) - \lambda] &= \\ & (\max\{0, -[P(C^I \geq D^I) - \lambda]\})^2 \end{aligned} \right\}, \tag{9}$$

where σ is the penalty factor with a large value, and φ is the penalty function.

1.3 Uncertainty Optimization Architecture and Its Algorithms

An illustration of ship hull SBD optimization design process is shown in Fig. 1. Double-level nested optimization architecture is used to solve the uncertainty optimization problem: the outer layer is used for generating the design variables, and the inner layer is used for calculating the interval objective function. Namely, amounts of design variables are generated via the outer layer, and inner layer optimization is called to obtain the uncertainty interval of objective function and restriction, and to convert the uncertainty intervals to the certainty ones. An illustration of the uncertainty optimization process is shown in Fig. 2.

Uncertainty optimization problem is usually non-

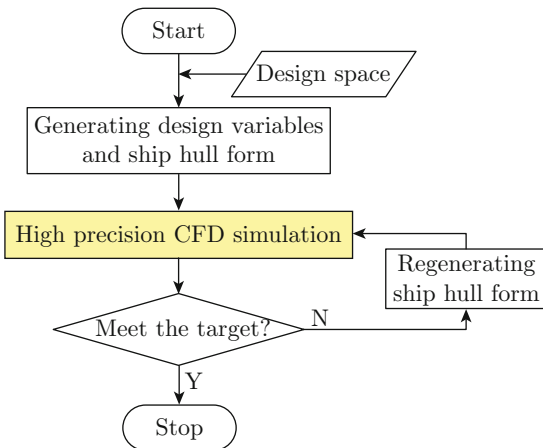


Fig. 1 Ship hull SBD optimization design process

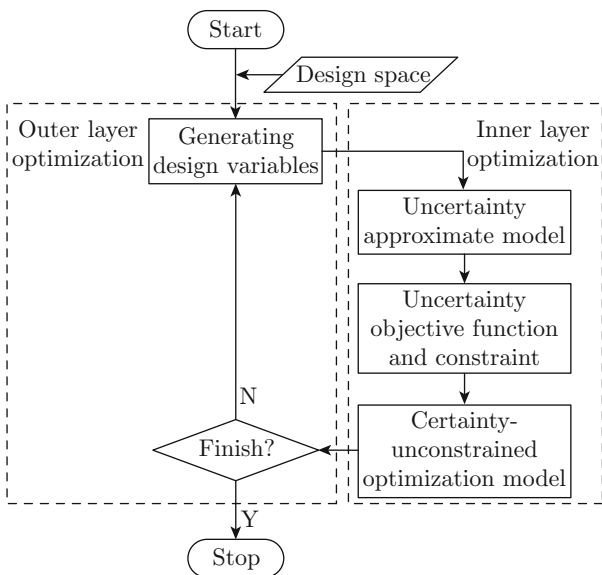


Fig. 2 Uncertainty optimization process with double-level nested architecture

continuous and non-differentiable, which can disable the application of traditional optimization algorithms based on gradient. Two optimization algorithms based on Monte-Carlo modern bionic theory are introduced here: particle swarm optimization (PSO) with learning factor improvement strategy, i.e., improved PSO (IPSO), and very fast simulated annealing (VFSA) with “annealing-tempering” mode, i.e., modified VFSA (MVFSA).

Purpose of outer layer optimization is to generate design variable individuals with wide coverage in the global scope, and this requires strong search ability for the algorithm. PSO algorithm has the advantage that optimization process has a wide-range, multi-direction and high-group collaboration. For traditional PSO, learning factors are usually set to be constant, while for IPSO, learning factors are set to be “S-type” with the iterative process. Improved learning factors are determined by

$$\left. \begin{aligned} c_1 &= \frac{4}{1 + \exp\left(a \frac{k}{k_{\max}} - 0.5\right)} \\ c_2 &= 4 - c_1 \end{aligned} \right\}, \quad (10)$$

where c_1 and c_2 are the learning factors that respectively represent the cognition part and social part of the algorithm; a is a parameter that controls the descent speed; k_{\max} and k are the maximum and current iteration times, respectively.

This learning factor improvement strategy can ensure that the particle swarm has a larger cognition part in the early iterations and a larger social part in the late. Meanwhile, changing trend of the two parts is smooth, which can enable the algorithm to converge to global optimum solution as possible. Thus, IPSO has stronger global search ability without increasing the computation time.

As the core of uncertainty optimization, inner layer has a high requirement for local search ability and efficiency of the algorithm. Through long-term research and application, simulated annealing (SA) proves to be strict and effective. However, SA needs enough model perturbation and iteration, and a suitable annealing plan. In order to overcome this shortcoming, MVFSA emerges^[12]. MVFSA introduces two new ideas based on SA: for high temperature, global random generator, which has a stronger ability than the traditional perturbation of SA and has nothing to do with initial temperature, is adopted; for low temperature, a certain restriction of perturbation is made to decrease perturbation space, thus the optimum solution can be found quickly, and its acceptance probability can also be increased.

The “annealing-tempering” mode of MVFSA is shown as follows.

Annealing Stage For the global searching, the temperature and perturbation are set as

$$\left. \begin{aligned} \Theta &= \Theta_0 \exp(-\alpha(j-1)^{1/2}) \\ M &= M_{\min} + C_{\text{per}}(M_{\max} - M_{\min}) \end{aligned} \right\}, \quad (11)$$

where Θ and Θ_0 are the current and initial temperatures, respectively; α is the attenuation coefficient; j is iteration number; M is the perturbation of current model; M_{\max} and M_{\min} are the maximum and minimum values of the perturbation, respectively; C_{per} is the perturbation coefficient.

Tempering Stage For the local searching, the temperature and perturbation are set as

$$\left. \begin{aligned} \Theta &= \Theta_0 \exp(-\alpha(j-k_0/\tau)^{1/2}) \\ M' &= M + (C_{\text{per}} - 0.5)(M_{\max} - M_{\min})/L_s(j) \end{aligned} \right\}, \quad (12)$$

where k_0 is iteration number of the annealing stage; τ is the temperature control factor; $L_s(j)$ is a factor of space search range, which makes the perturbation happens in a smaller and smaller range and thus enhances local searching ability.

“Annealing-tempering” mode makes MVFSA have higher temperature at the beginning, thus the searching space is bigger, and the reception probability of non-optimal solution is higher. Also, through enhancing the local searching ability, the new algorithm has more accuracy and efficiency with no changing of essence of SA algorithm (its unique form of receiving probability and Metropolis criteria).

2 Approximation Model

Approximation model, which can greatly shorten computation time, is used in ship hull SBD process as an effective tool, and the process is shown in Fig. 3.

2.1 BP Neural Network

BP neural network is a kind of multilayer feed forward network with error BP algorithm, and becomes one of the typical approximation technologies because of its excellent ability to approximate nonlinear function. A BP neural network model using tangent sigmoid as transfer function of neurons is represented by

$$\begin{aligned} \text{BPNN}_3 : O_i &= \\ & \sum_{j=1}^J W_{ij} \tanh \left(\sum_{k=1}^K W_{jk} \right. \\ & \left. \tanh \left(\sum_{n=1}^N W_{kn} \xi_n + b_{1k} \right) + b_{2j} \right) + b_{3i}, \end{aligned} \quad (13)$$

where ξ_n is input variable; O_i is output variable; W_{kn} , W_{jk} and W_{ij} are the weights of the layers between neurons; b_{1k} , b_{2j} and b_{3i} are the thresholds of neuron unit in each layer.

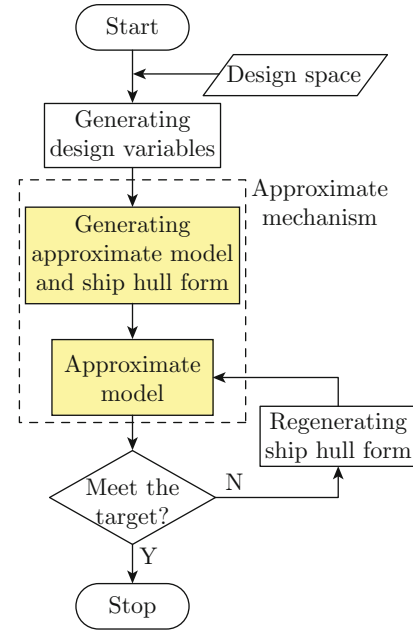


Fig. 3 Ship hull SBD optimization design process with approximate mechanism

Activation function of hidden layer neurons usually is S-type, and it can be achieved in any nonlinear mapping from input to output. The tangent and logarithm forms of activation function are respectively shown in

$$f(x) = \frac{2}{1 + \exp(-2x)} - 1, \quad (14)$$

$$f(x) = \frac{1}{1 + \exp(-x)}. \quad (15)$$

2.2 Uncertainty Handling of Approximate Model

BP neural network approximate model needs amounts of simulation results as inputs, it is very sensitive to the internal parameters, and thus error of output will occur due to some uncontrollable causes. Although this error or uncertainty has a small value in most cases, large deviation of the whole system can also be generated by continuous iterative computation. Therefore, it has an important significance for considering the uncertainty of approximate model.

A successfully trained neural network output is expressed as interval number,

$$\text{BPNN}^I = O_i^I = [O_i^L, O_i^U] = [O_i(1-\gamma), O_i(1+\gamma)], \quad (16)$$

where O_i^L and O_i^U are the lower and upper bounds of interval number O_i^I ; γ represents the uncertainty level of O_i^I , which is usually in terms of percentages. The larger the value of γ is, the greater the uncertainty degree of interval number will be.

3 Uncertainty Optimization of Ship Hull

3.1 Optimization Model

In this paper, Wigley hull is taken as example, and its hull function is^[13]

$$y = \frac{B}{2} \left[1 - \left(\frac{2x}{L} \right)^2 \right] \left[1 - \left(\frac{z}{T} \right)^2 \right], \quad (17)$$

$$-L/2 \leq x \leq L/2, \quad -T \leq z \leq 0,$$

where x, y and z are the coordinates of hull points. The numbers of waterline and station line are taken to 11. The design variables are identified by the whole ships' principal dimensions and the overall shape of a ship, in which the principal dimensions are represented by the waterline length L , waterline width B and draft T . The modification of the hull shape can be represented by the original data points multiplied hull modification function,

$$\left. \begin{aligned} y_f(x, z) &= y_{f0}(x, z)\omega(x, z) \\ y_a(x, z) &= y_{a0}(x, z)\omega(x, z) \end{aligned} \right\}, \quad (18)$$

$$\omega(x, z) = 1 - \sum_m \sum_n A_{m,n} \sin X \sin Z, \quad (19)$$

$$X = \pi \left(\frac{x - x_0}{x_{\max} - x_0} \right)^{m+2}, \quad Z = \pi \left(\frac{z_0 - z}{z_0 + T} \right)^{n+2},$$

where $y_f(x, z)$ and $y_a(x, z)$ respectively represent the former and after halves of the lateral data points of the hull after being changed, and both are in the mid ship-section of the interface; $y_{f0}(x, z)$ and $y_{a0}(x, z)$ are the initial ones; $\omega(x, z)$ is the modification function of the hull form; $A_{m,n}$ is to characterize the magnitude of the control variables, in this paper, $m, n = 1, 2, 3$.

Constraint condition is

$$\left. \begin{aligned} \frac{\nabla_0 - \nabla}{\nabla_0} &\leq \varepsilon \\ y(x, z) &\geq 0 \end{aligned} \right\}, \quad (20)$$

where ∇ and ∇_0 are the optimal and initial hull form's volumes, respectively, which can be calculated by Simpson method; ε is a small value for ensuring that the displacement volume of optimized ship is not below the lower limit.

The total resistance R_t is taken as the objective function. According to Hughes' viewpoint, the total resistance is divided into wave making resistance R_w , frictional resistance R_f , and viscous pressure resistance R_{pv} , i.e.,

$$R_t = R_w + R_f + R_{pv} = \frac{1}{2} \rho U^2 S (C_w + C_f + C_{pv}), \quad (21)$$

where U is the speed, and S is wet surface area.

Wave making resistance coefficient C_w is calculated by Michell's method^[14], which has a high precision on the low-speed and thin-hull resistance prediction. Through the interval number, BP neural network approximate model of C_w is established as

$$\text{BPNN}_{cw}^I = [\text{BPNN}_{cw}^L, \text{BPNN}_{cw}^U] = [\text{BPNN}_{cw}(1 - \gamma), \text{BPNN}_{cw}(1 + \gamma)]. \quad (22)$$

After integrating the constraint condition into the optimization objective, uncertainty optimization model is obtained as

$$\left. \begin{aligned} \text{opt min } f_p(\mathbf{x}, \mathbf{u}) \\ f_p(\mathbf{x}, \mathbf{u}) &= (1 - \beta)m(f(\mathbf{x}, \mathbf{u})) + \beta w(f(\mathbf{x}, \mathbf{u})) + \\ &\sigma \left(\max \left\{ 0, - \left[P \left(\frac{\nabla_0 - \nabla}{\nabla_0} - \varepsilon \geq 0 \right) - \lambda \right] \right\} \right)^2 \\ f(\mathbf{x}, \mathbf{u}) &= R_t^I = 0.5 \rho U^2 S (\text{BPNN}_{cw}^I + C_f + C_{pv}) \end{aligned} \right\}. \quad (23)$$

3.2 BP Neural Network Experiments

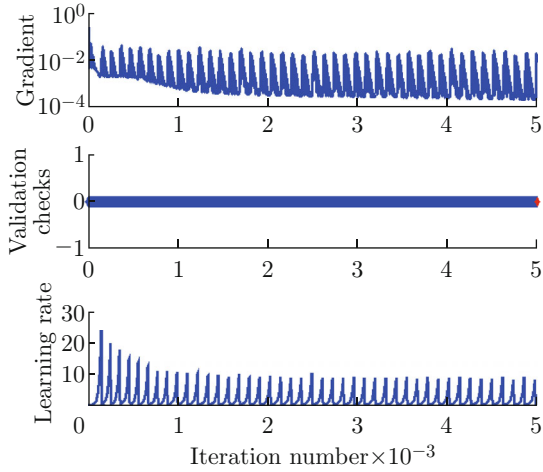
In order to get a suitable uncertainty level γ of Eq. (22), a series of experiments are done via changing numbers of hidden layer neurons and kind of activation functions, as shown in Table 1.

Table 1 Series of BP neural network experiments and results

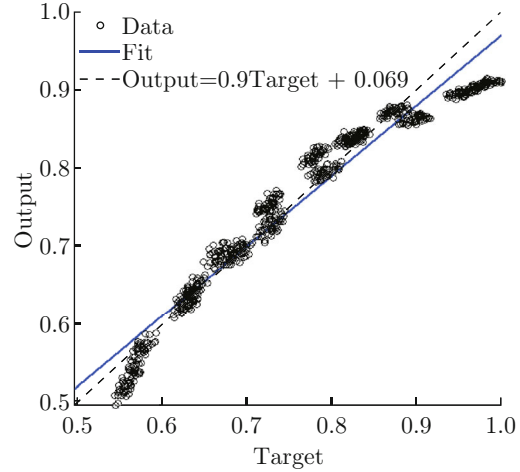
Case	Number of hidden layer neurons	Activation function	Iteration number $\times 10^{-3}$	Training residual $\times 10^2$	Training time/s
1	3	Tan-sigmode	5	97.146	56
2	6	Tan-sigmode	5	96.575	55
3	9	Tan-sigmode	5	97.059	60
4	12	Tan-sigmode	5	96.602	65
5	3	Log-sigmode	5	98.443	52
6	6	Log-sigmode	5	98.341	52
7	9	Log-sigmode	5	98.332	59
8	12	Log-sigmode	5	97.752	65

Training state and result regression of Case 1 are shown in Fig. 4.

Training residual from Table 1 represents the uncertainty of BP neural network approximate model, and



(a) Training state



(b) Result regression

Fig. 4 Training state and result regression of Case 1

3.3 Optimal Results and Analysis

For comparative analysis, certainty and uncertainty optimizations are done respectively to verify Eq. (23), and the results are shown in Table 2, where R_t/R_{t0} is

the resistance decrease ratio.

The uncertainty level γ of 5% is taken as an example, which can be seen as a typical situation. Optimal iterative curves are shown in Fig. 5, where f_{bj} is the objective function value.

Table 2 Result comparison of different methods

Optimization method	$\gamma/\%$	L/m	B/m	T/m	$A_{1,1}$	Time/s	$\frac{R_t}{R_{t0}}/\%$
Original value	—	2.0	0.20	0.125	0	—	—
Certainty	0	2.2	0.18	0.105	0.03	153	90.6
Uncertainty	3	1.9	0.18	0.107	0.12	352	92.8
Uncertainty	5	1.8	0.16	0.108	0.07	381	93.4
Uncertainty	7	1.8	0.17	0.116	0.08	406	93.5
Uncertainty	9	2.1	0.17	0.113	0.07	431	94.2

Note: take $A_{1,1}$ for example of $A_{m,n}$

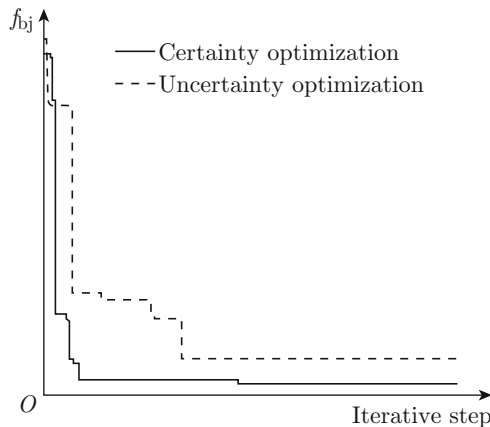


Fig. 5 Comparison of optimal iterative curves

The results show that, uncertainty optimization and certainty optimization can both effectively improve the ship speed performance. However, the result of uncertainty optimization is slightly worse than the result of certainty optimization, that is because the former considers the uncertainty influence of approximate model, which is closer to reality in engineering. Meanwhile, it is obvious that the calculation time of uncertainty optimization is more than that of certainty optimization, which can attribute to the double-level nested optimization architecture, but the extra calculation time is also reasonable and acceptable.

4 Conclusion

Aimed at ship hull form optimization design based

on SBD, a minimum resistance optimization model is established with the design space constituted by principal dimensions and ship form coefficients. Double-level nested architecture of the uncertainty optimization by IPSO and MVFSA algorithms is proposed. Some conclusions can be got as follows.

(1) Uncertainty ship hull optimization method with IPSO and MVFSA can obtain the optimum result with an acceptable calculation time. Thus, this method has the effectiveness.

(2) Under the same conditions, the result of uncertainty optimization is slightly worse than the result of certainty optimization, but the former considers the uncertainty influence of approximate model, so it can better reflect the reality and have the superiority.

(3) Further studies can be carried out from researching the properties of the uncertain parameters and applicability of optimization algorithms.

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