J. Shanghai Jiaotong Univ. (Sci.), 2015, 20(6): 735-742
DOI: 10.1007/s12204-015-1684-x

Free Vibration Analysis of Cylindrical and Rectangular Sandwich Panels with a Functionally Graded Core

*LI Hua-dong** (李华东), *ZHU Xi* (朱 锡), *MEI Zhi-yuan* (梅志远), *ZHANG Ying-jun* (张颖军)
(Department of Ships Engineering, Naval University of Engineering, Wuhan 430033, China)

Abstract: Based on the Reissner assumptions, the free vibration analysis of simply supported cylindrical and rectangular sandwich panels with isotropic face sheets and a functionally graded core is concerned. Firstly, the expressions of the displacements, stresses and internal forces are presented according to the constitutive relations and stress states of the core and face sheets. Then, the dynamic stability and compatibility equations are given for cylindrical sandwich panels with functionally graded core, elastic modulus and density in which vary continuously in the thickness direction. Finally, the proposed solutions are validated by comparing the results of degenerate example with classical solutions, and a numerical analysis is performed on the example of simply supported cylindrical and rectangular sandwich panels. The elastic modulus and density of the functionally graded core are assumed to be graded by a power law distribution of volume fractions of the constituents, and the Poisson ratio is held constant. The effects of the distribution of functionally graded core's properties, the thickness-side ratios and ratio of radius (R) to length (l) $\kappa = R/l$ are also examined.
Key words: functionally graded materials, cylindrical sandwich panel, rectangular sandwich plate, simply supported, free vibration
CLC number: O 345, TB 332 **Document code:** A

0 Introduction

Functionally graded materials (FGMs) are composite materials that are microscopically inhomogeneous, and the mechanical properties vary continuously in one (or more) direction(s)[1]. Recently, as a new generation of sandwich structures, the FGMs concept was also applied to sandwich composites. The functionally graded (FG) sandwich construction commonly exists in two types: FGM face sheets-homogeneous core and homogeneous face sheets-FGM core[2]. In this paper, the later one is considered, where the core is functionally graded material, and the volume fraction of the FGM core constituents vary gradually, giving a non-uniform microstructure with continuously graded properties such as elastic modulus, damp, density, etc. This type of sandwich plates with FG cores may be utilized for many purposes, among them: dynamic or noise, vibration, and harshness (NVH) behaviour enhancement, thermal insulation, construction of light-weight structures with higher strength to weight ratios, and ease of manufacturing[3-4].

Heretofore, many researchers studied dynamic behaviour of pure functionally graded structures and functionally graded sandwich structures in the past few years. Pradyumna and Bandyopadhyay[5] carried out free vibration analysis of functionally graded curved panels employing the higher-order shear deformation theory (HSDT). Zhao et al.[6] presented a free vibration analysis of metal and ceramic functionally graded plates using the element-free k^p-Ritz method. Based on the 3D linear theory of elasticity, Li et al.[7] studied the 3D free vibration of multi-layer FGM system-symmetric and unsymmetrical FGM sandwich plates by Ritz method, and two common types of FGM sandwich plates were considered. Implementing a new model based on high-order sandwich panel theory, Rahmani et al.[8] has given a free vibration analysis of sandwich beams with syntactic foam as a functionally graded flexible core. Using element free Galerkin method and robust meshless method, the free vibration behaviour of sandwich beam with FG core was analyzed by Amirani et al.[9] and the penalty method was used for satisfaction of essential boundary condition and continuity of the beam. Neves et al.[10] derived a higher-order shear deformation theory for modelling functionally graded plates accounting for extensibility in the thickness direction, and

Received date: 2014-05-07
Foundation item: the National Natural Science Foundation of China (No. 50979110)
***E-mail:** lhd0727@163.com

studied the free vibration and buckling of functionally graded isotropic plates and functionally graded sandwich plates. Alibeigloo and Liew[11] examined the vibration behavior of FGM cylindrical sandwich panel using the Fourier series and state space technique.

To the best of the authors' knowledge, limited literature is available related to the study on the free vibration behaviours of functionally graded sandwich structures with a functionally graded core and isotropic face sheets, where the elastic modulus and density of the face sheets and core are not discontinuous at the interface. Therefore, in the present analysis, free vibration of FG cylindrical and rectangular sandwich panels is studied by employing assumptions in Reissner sandwich plate theory. The elastic modulus and density of the functionally graded core obey arbitrary function $E_{\mathrm{c}}(z)$ along the thickness direction, and the Poisson ratio ν keeps constant.

1 Basic Assumptions

As shown in Fig. 1, the sandwich cylindrical panel with length l and width b investigated has stiff face sheets and soft cores, the assumptions in Reissner sandwich plate theory are introduced as follows[12].

(1) Since the face sheet is thin, it is assumed to be in membrane-stress state, the normal stresses σ_x, σ_y through the thickness are uniform.

(2) Since the elastic modulus of the core is exceptionally low in comparison with the faces', the stresses in the core $\sigma_x = \sigma_y = \tau_{xy} = 0$.

(3) In the whole sandwich plate, the z-direction normal stress and strain are left out of account, i.e., $\sigma_z = 0$, $\varepsilon_z = 0$.

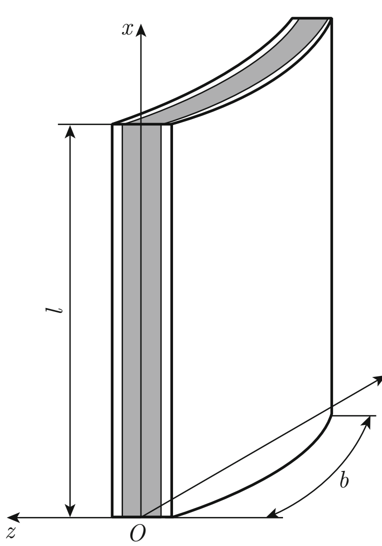

Fig. 1 FGM sandwich cylindrical panel

1.1 Stress and Displacement Fields in the Core

For the core, the stress equilibrium equations are (body forces are neglected):

$$\left.\begin{aligned} &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \\ &\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \\ &\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \sigma_z}{\partial z} = 0 \end{aligned}\right\}. \tag{1}$$

Considering the assumptions (2) and (3), the Eq. (1) may be simplified to be:

$$\left.\begin{aligned} &\frac{\partial \tau_{xz}}{\partial z} = 0 \\ &\frac{\partial \tau_{yz}}{\partial z} = 0 \end{aligned}\right\}. \tag{2}$$

It can be derived from the Eq. (2) that the shear stress is uniform through the thickness of the core. And according to the boundary conditions, the shear stresses at the top and bottom faces of the sandwich plate must vanish:

$$z = \pm\left(\frac{d_{\mathrm{c}}}{2} + d_{\mathrm{f}}\right), \quad \tau_{xz} = 0, \quad \tau_{yz} = 0, \tag{3}$$

where d_{c} and d_{f} are the thickness of the core and face sheet, respectively.

Since this is a thin-face sandwich, as shown in Fig. 2, the shear stress τ in the face can also be assumed to decrease uniformly across the thickness of each face to zero at the free surface.

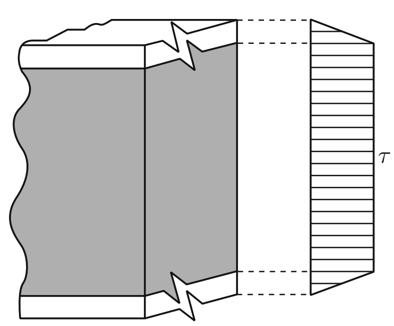

Fig. 2 The assumption of variation of transverse shear stress through the thickness

If the shear forces are F_x and F_y, the shear stresses τ_{xz} and τ_{yz} of the core can be given by[12]:

$$\left.\begin{aligned} \tau_{xz} &= \frac{F_x}{d_{\mathrm{c}} + d_{\mathrm{f}}} \\ \tau_{yz} &= \frac{F_y}{d_{\mathrm{c}} + d_{\mathrm{f}}} \end{aligned}\right\}, \tag{4}$$

The shear strains are obtained from the strain-strain relations of linear elasticity, i.e.,

$$\left.\begin{aligned}\gamma_{xz} &= \frac{F_x}{G_\mathrm{c}(z)(d_\mathrm{c}+d_\mathrm{f})}\\ \gamma_{yz} &= \frac{F_y}{G_\mathrm{c}(z)(d_\mathrm{c}+d_\mathrm{f})}\end{aligned}\right\}, \tag{5}$$

where $G_\mathrm{c}(z)$ is the shear modulus of the core, it is given by

$$G_\mathrm{c}(z) = \frac{E_\mathrm{c}(z)}{2(1+\mu)}, \tag{6}$$

and $E_\mathrm{c}(z)$ is the elastic modulus of the core.

According to stress-displacement relations, the shear strains can be also derived using the follow expressions:

$$\left.\begin{aligned}\gamma_{xz} &= \frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\\ \gamma_{yz} &= \frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\end{aligned}\right\}, \tag{7}$$

where u, v and w are the displacements along x, y and z axis, respectively. Substituting Eqs. (5) and (6) into Eq. (7) and integration with respect to z give

$$\left.\begin{aligned}u &= \frac{2(1+\mu)F_x}{(d_\mathrm{c}+d_\mathrm{f})}J_0(z) - z\frac{\partial w}{\partial x}\\ v &= \frac{2(1+\mu)F_y}{(d_\mathrm{c}+d_\mathrm{f})}J_0(z) - z\frac{\partial w}{\partial y}\end{aligned}\right\}, \tag{8}$$

where

$$J_0(z) = \int \frac{1}{E_\mathrm{c}(z)}\mathrm{d}z. \tag{9}$$

It can be seen from the Eqs. (8) and (9) that the displacements u and v are related to the variation of the functionally graded core's elastic modulus across the thickness.

1.2 Displacement and Stress Fields in the Face Sheets

With the assumption (1) and Eq. (8), the displacement field in face sheets of the sandwich plate is assumed to be given by

$$\left.\begin{aligned}u^\pm &= \frac{2F_x(1+\mu)}{(d_\mathrm{c}+d_\mathrm{f})}J_0\Big(\pm\frac{d_\mathrm{c}}{2}\Big) - \frac{d_\mathrm{c}+d_\mathrm{f}}{2}\frac{\partial w}{\partial x}\\ v^\pm &= \frac{2F_y(1+\mu)}{(d_\mathrm{c}+d_\mathrm{f})}J_0\Big(\pm\frac{d_\mathrm{c}}{2}\Big) - \frac{d_\mathrm{c}+d_\mathrm{f}}{2}\frac{\partial w}{\partial y}\end{aligned}\right\}, \tag{10}$$

where the superscripts (+) and (−) denote the bottom and top surfaces of the sandwich plate, respectively.

Since the strain field of the face sheets is:

$$\left.\begin{aligned}\varepsilon_x^\pm &= \frac{\partial u^\pm}{\partial x}\\ \varepsilon_y^\pm &= \frac{\partial v^\pm}{\partial y}\\ \gamma_{xy}^\pm &= \frac{\partial u^\pm}{\partial y}+\frac{\partial v^\pm}{\partial x}\end{aligned}\right\}. \tag{11}$$

The stress-strain relations of the face sheets can be expressed in the following expressions:

$$\left.\begin{aligned}\sigma_x^\pm &= \frac{E_\mathrm{f}}{1-\nu_\mathrm{f}^2}\Big(\frac{\partial u^\pm}{\partial x}+\nu_\mathrm{f}\frac{\partial v^\pm}{\partial y}\Big)\\ \sigma_y^\pm &= \frac{E_\mathrm{f}}{1-\nu_\mathrm{f}^2}\Big(\frac{\partial v^\pm}{\partial y}+\nu_\mathrm{f}\frac{\partial u^\pm}{\partial x}\Big)\\ \tau_{xy}^\pm &= \frac{E_\mathrm{f}}{2(1+\nu_\mathrm{f})}\Big(\frac{\partial u^\pm}{\partial y}+\frac{\partial v^\pm}{\partial x}\Big)\end{aligned}\right\}, \tag{12}$$

where E_f and ν_f are the elastic modulus and Poisson ratio of the face sheets.

1.3 The Internal Forces of Sandwich Plate

For the sandwich structures, the bending moments M_x, M_y and M_{xy} are defined as

$$\left.\begin{aligned}&M_x = (d_\mathrm{c}+d_\mathrm{f})d_\mathrm{f}(\sigma_x^+ - \sigma_x^-)/2\\ &M_y = (d_\mathrm{c}+d_\mathrm{f})d_\mathrm{f}(\sigma_y^+ - \sigma_y^-)/2\\ &M_{xy} = M_{yx} = (d_\mathrm{c}+d_\mathrm{f})d_\mathrm{f}(\tau_{xy}^+ - \tau_{xy}^-)/2\end{aligned}\right\}. \tag{13}$$

The substitution of the Eqs. (10) and (12) into Eq. (13) yields the expressions of the moment components:

$$\left.\begin{aligned}&M_x =\\ &\frac{E_\mathrm{f}}{1-\nu_\mathrm{f}^2}\Big[\chi\frac{\partial F_x}{\partial x} - \eta\frac{\partial^2 w}{\partial x^2} + \nu_\mathrm{f}\Big(\chi\frac{\partial F_y}{\partial y} - \eta\frac{\partial^2 w}{\partial y^2}\Big)\Big]\\ &M_y =\\ &\frac{E_\mathrm{f}}{1-\nu_\mathrm{f}^2}\Big[\chi\frac{\partial F_y}{\partial y} - \eta\frac{\partial^2 w}{\partial y^2} + \nu_\mathrm{f}\Big(\chi\frac{\partial F_x}{\partial x} - \eta\frac{\partial^2 w}{\partial x^2}\Big)\Big]\\ &M_{xy} =\\ &\frac{E_\mathrm{f}}{2(1+\nu_\mathrm{f})}\Big(\chi\frac{\partial F_x}{\partial y} - 2\eta\frac{\partial^2 w}{\partial x\partial y} + \chi\frac{\partial F_y}{\partial x}\Big)\end{aligned}\right\}, \tag{14}$$

where, the parameters χ and η are

$$\left.\begin{aligned}\chi &= (1+\mu)d_\mathrm{f}\Big[J_0\Big(\frac{d_\mathrm{c}}{2}\Big) - J_0\Big(-\frac{d_\mathrm{c}}{2}\Big)\Big]\\ \eta &= \frac{d_\mathrm{f}(d_\mathrm{c}+d_\mathrm{f})^2}{2}\end{aligned}\right\}. \tag{15}$$

2 Governing Equations

2.1 Equations of Motion

For the functionally graded sandwich cylindrical panel, the equations of motion are:

$$\left.\begin{aligned}&\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y} = 0\\ &\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y} = 0\\ &\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y} - F_x = 0\\ &\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y} - F_y = 0\\ &\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}+\frac{N_y}{R}+\rho_\mathrm{s}\bar{\omega}^2 w = 0\end{aligned}\right\}, \tag{16}$$

where $\bar{\omega}$ is the natural frequency of FG sandwich cylindrical panels, R is ratio of radius, and ρ_s is the areal density of functionally graded sandwich structure, which is calculated as follows,

$$\rho_s = 2d_f\rho_f + \int_{-\frac{d_c}{2}}^{\frac{d_c}{2}} \rho_c(z)\mathrm{d}z, \tag{17}$$

and ρ_f is the bulk density of face sheets, $\rho_c(z)$ is the distribution function of FGM core's density.

To solve the Eq. (16), we introduce the stress function Φ, and assume N_x, N_y, N_{xy} in terms of Φ as[12]

$$\left.\begin{aligned} N_x &= \frac{\partial^2\Phi}{\partial y^2} \\ N_y &= \frac{\partial^2\Phi}{\partial x^2} \\ N_{xy} &= -\frac{\partial^2\Phi}{\partial x\partial y} \end{aligned}\right\}. \tag{18}$$

Substituting Eq. (18) into Eq. (16), it can be found that the first two equations are satisfied. By substituting Eqs. (14) and (18) into the other three equations in Eq. (16) and simplifying, we obtain

$$\left.\begin{aligned} &H\chi\frac{\partial^2 F_x}{\partial x^2} + K\chi\frac{\partial^2 F_x}{\partial y^2} + (H\nu_f\chi + K\chi)\frac{\partial^2 F_y}{\partial x\partial y} - \\ &\quad H\eta\frac{\partial^3 w}{\partial x^3} - (H\nu_f\eta + 2K\eta)\frac{\partial^3 w}{\partial x\partial y^2} - F_x = 0 \\ &(K\chi + H\nu_f\chi)\frac{\partial^2 F_x}{\partial x\partial y} + H\chi\frac{\partial^2 F_y}{\partial y^2} + K\chi\frac{\partial^2 F_y}{\partial x^2} - \\ &\quad H\eta\frac{\partial^3 w}{\partial y^3} - (H\nu_f\eta + 2K\eta)\frac{\partial^3 w}{\partial x^2\partial y} - F_y = 0 \\ &\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{1}{R}\frac{\partial^2\Phi}{\partial x^2} + \rho_s\bar{\omega}^2 w = 0 \end{aligned}\right\}, \tag{19}$$

where

$$\left.\begin{aligned} H &= \frac{E_f}{1-\nu_f^2} \\ K &= \frac{E_f}{2(1+\nu_f)} \end{aligned}\right\}. \tag{20}$$

2.2 Compatibility Equation

For the functionally graded sandwich cylindrical panel, its deformation is still guided by the following compatibility equation:

$$\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} = \frac{\partial^2\gamma_{xy}}{\partial x\partial y}. \tag{21}$$

Strain-displacement relations are:

$$\left.\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} \\ \varepsilon_y &= \frac{\partial v}{\partial y} \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \end{aligned}\right\}, \tag{22}$$

and equations of non-moment elasticity theory for cylindrical panels are,

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= \frac{1}{B}(N_x - \nu_f N_y) \\ \frac{\partial v}{\partial y} &= \frac{1}{B}(N_y - \nu_f N_x) + \frac{w}{R} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} &= \frac{2(1+\nu_f)}{B}N_{xy} \end{aligned}\right\}, \tag{23}$$

where $B = 2E_f d_f$.

After substituting Eqs. (22), (23) and (18) into the compatibility Eq. (21), the deformation compatibility equation for cylindrical plate can be ultimately translated into:

$$\frac{1}{B}\Big(\frac{\partial^4\Phi}{\partial y^4} + \frac{\partial^4\Phi}{\partial x^4} + 2\frac{\partial^4\Phi}{\partial x^2\partial y^2}\Big) + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0. \tag{24}$$

3 Boundary Conditions and Solutions

3.1 Boundary Conditions

For panels which are simply supported all round the edges, the boundary conditions may be written as:

$$\left.\begin{aligned} x = 0,\ \ l: w = 0,\ \ M_x = 0,\ \ \sigma_x = 0,\ \ v = 0 \\ y = 0,\ \ b: w = 0,\ \ M_y = 0,\ \ \sigma_y = 0,\ \ u = 0 \end{aligned}\right\}. \tag{25}$$

According to the boundary conditions, we may assume

$$\left.\begin{aligned} w(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\sin\frac{m\pi x}{l}\sin\frac{n\pi y}{b} \\ F_x(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{x_{mn}}\cos\frac{m\pi x}{l}\sin\frac{n\pi y}{b} \\ F_y(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{y_{mn}}\sin\frac{m\pi x}{l}\cos\frac{n\pi y}{b} \end{aligned}\right\}. \tag{26}$$

where W_{mn}, $Q_{x_{mn}}$, $Q_{y_{mn}}$ are unknown functions.

3.2 Solution Procedure

Substituting Eq. (26) into the third formula in Eq. (19) gives:

$$\begin{aligned} \Phi = R\Big(\frac{l}{m\pi}\Big)^2 \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Big[-\frac{m\pi}{l}Q_{x_{mn}} - \\ \frac{n\pi}{b}Q_{y_{mn}} + \rho_s\bar{\omega}^2 W_{mn}\Big]\sin\frac{m\pi x}{l}\sin\frac{n\pi y}{b}. \end{aligned} \tag{27}$$

Then, the substitution of Eq. (27) into compatibility Eq. (24) yields:

$$\begin{aligned} \frac{m\pi}{l}\phi Q_{x_{mn}} + \frac{n\pi}{b}\phi Q_{y_{mn}} - \\ \Big[\phi\rho_s\bar{\omega}^2 - \frac{B}{R^2}\Big(\frac{m\pi}{l}\Big)^2\Big]W_{mn} = 0, \end{aligned} \tag{28}$$

where

$$\phi = \Big[\frac{l}{m\pi}\Big(\frac{n\pi}{b}\Big)^2 + \frac{m\pi}{l}\Big]^2. \tag{29}$$

At the same time, from the first two formulas of the Eq. (19), we can obtain:

$$\left.\begin{aligned}&\Big[H\chi\frac{m^2\pi^2}{l^2}+K\chi\frac{n^2\pi^2}{b^2}+1\Big]Q_{x_{mn}}+\\&\quad(H\nu_{\mathrm{f}}\chi+K\chi)\frac{mn\pi^2}{lb}Q_{y_{mn}}-\\&\quad\Big[H\eta\frac{m^3\pi^3}{l^3}+(H\nu_{\mathrm{f}}\eta+2K\eta)\frac{mn^2\pi^3}{lb^2}\Big]W_{mn}=0\\&(K\chi+H\nu_{\mathrm{f}}\chi)\frac{nm\pi^2}{lb}Q_{x_{mn}}+\\&\quad\Big(H\chi\frac{n^2\pi^2}{b^2}+K\chi\frac{m^2\pi^2}{l^2}+1\Big)Q_{y_{mn}}-\\&\quad\Big[H\eta\frac{n^3\pi^3}{b^3}+(H\nu_{\mathrm{f}}\eta+2K\eta)\frac{m^2n\pi^3}{l^2b}\Big]W_{mn}=0\end{aligned}\right\}. \tag{30}$$

The Eq. (30) can be also written in the following form:

$$\begin{bmatrix}A_{11} & A_{12}\\ A_{12} & A_{22}\end{bmatrix}\begin{bmatrix}Q_{x_{mn}}\\ Q_{y_{mn}}\end{bmatrix}=\begin{bmatrix}B_1\\ B_2\end{bmatrix}W_{mn}, \tag{31}$$

where

$$\left.\begin{aligned}A_{11}&=H\chi\frac{m^2\pi^2}{l^2}+K\chi\frac{n^2\pi^2}{b^2}+1\\A_{12}&=(H\nu_{\mathrm{f}}\chi+K\chi)\frac{mn\pi^2}{lb}\\A_{22}&=H\chi\frac{n^2\pi^2}{b^2}+K\chi\frac{m^2\pi^2}{l^2}+1\\B_1&=H\eta\frac{m^3\pi^3}{l^3}+(H\nu_{\mathrm{f}}\eta+2K\eta)\frac{mn^2\pi^3}{lb^2}\\B_2&=H\eta\frac{n^3\pi^3}{b^3}+(H\nu_{\mathrm{f}}\eta+2K\eta)\frac{m^2n\pi^3}{l^2b}\end{aligned}\right\}. \tag{32}$$

Then, by solving the Eq. (31), the coefficients $Q_{x_{mn}}$ and $Q_{y_{mn}}$ can be obtained in terms of W_{mn} as

$$\begin{bmatrix}Q_{x_{mn}}\\ Q_{y_{mn}}\end{bmatrix}=\begin{bmatrix}\dfrac{D_1}{D}\\ \dfrac{D_2}{D}\end{bmatrix}W_{mn}, \tag{33}$$

where

$$\left.\begin{aligned}D_1&=A_{22}B_1-A_{12}B_2\\D_2&=A_{11}B_2-A_{12}B_1\\D&=A_{11}A_{22}-A_{12}^2\end{aligned}\right\}. \tag{34}$$

Substitute Eq. (33) into Eq. (28), and consider of $W_{mn}\neq 0$, we can obtain:

$$\begin{aligned}&\Big(\frac{m\pi}{l}\Big)\phi\frac{D_1}{D}+\Big(\frac{n\pi}{b}\Big)\phi\frac{D_2}{D}-\\&\qquad\phi\rho_{\mathrm{s}}\bar{\omega}^2+\frac{B}{R^2}\Big(\frac{m\pi}{l}\Big)^2=0.\end{aligned} \tag{35}$$

3.3 Solutions

In the following, we will give the exact solutions for free vibrating of the functionally graded sandwich cylindrical panels and rectangle plates by solving Eq. (35). It will be straightforward because it is not needed to solve the eigenproblem. The solutions are as follows.

(1) Natural frequency of FG sandwich cylindrical panels

$$\bar{\omega}=\sqrt{\frac{m\pi}{l}\frac{D_1}{\rho_{\mathrm{s}}D}+\frac{n\pi}{b}\frac{D_2}{\rho_{\mathrm{s}}D}+\Big(\frac{m\pi}{l}\Big)^2\frac{B}{\rho_{\mathrm{s}}R^2\phi}}. \tag{36}$$

(2) Natural frequency of rectangle FG sandwich plates. Let $R=\infty$, then the calculation formula for the natural frequencies of rectangular functionally graded sandwich plates are acquired as:

$$\bar{\omega}=\sqrt{\frac{m\pi}{l}\frac{D_1}{\rho_{\mathrm{s}}D}+\frac{n\pi}{b}\frac{D_2}{\rho_{\mathrm{s}}D}}. \tag{37}$$

In the following discussions, the natural frequency f is defined as

$$f=\frac{\bar{\omega}}{2\pi}. \tag{38}$$

4 Numerical Results and Discussions

4.1 Free Vibration Analysis of Cylindrical and Rectangular Sandwich Panels with Homogeneous Cores

To verify the correctness of proposed method, as shown in Eqs. (1) and (2), the free vibration analysis of cylindrical sandwich panels and rectangular sandwich plates with a homogeneous core is given and the result is compared with the classical solutions[12].

(1) Rectangular sandwich plate: $l=2\,000\,\mathrm{mm}$, $b=1\,000\,\mathrm{mm}$.

(2) Cylindrical sandwich panels: $l=2\,000\,\mathrm{mm}$, $R=b=1\,000\,\mathrm{mm}$.

The thickness of the sandwich panels equals to 50 mm, and the thickness of face sheet and core is 5 and 40 mm respectively. The face sheet and core are both isotropic materials, the engineering elastic constants are as follows.

Face sheets: $E_{\mathrm{f}}=20\,\mathrm{GPa}$, $\nu_{\mathrm{f}}=0.2$, $\rho_{\mathrm{f}}=1.2\times10^{-6}\,\mathrm{kg/mm^3}$.

Core: $E_{\mathrm{c}}=0.1\,\mathrm{GPa}$, $\nu=0.45$, $\rho_{\mathrm{c}}=0.8\times10^{-6}\,\mathrm{kg/mm^3}$.

The results of natural frequencies f obtained by the present paper for sandwich cylindrical panels and rectangular plates are shown and compared with classical solutions in Tables 1 and 2. It is obvious that the results of the present model closely agree with classical solutions.

Table 1 Comparison of natural frequencies of rectangular sandwich plates

Mode	m	n	f/Hz		Error/%
			Presented	Classical[12]	
1	1	1	72.8	71.2	2.2
2	2	1	103.9	101.0	2.8
3	3	1	145.8	141.1	3.4

Table 2 Comparison of natural frequencies of cylindrical sandwich panels

Mode	m	n	f/Hz		Error/%
			Presented	Classical[12]	
1	1	1	99.5	98.3	1.2
2	1	2	175.0	169.1	3.5
3	2	1	198.9	197.4	0.8

4.2 Free Vibration Analysis of Cylindrical and Rectangular Sandwich Panels with Functionally Graded Cores

In this section, the free vibrations of simply supported functionally graded cylindrical and rectangular sandwich panels with functionally graded cores are analyzed. For the functionally graded core, the variation of elastic modulus along the thickness direction is assumed to be of the following form[13-14]:

$$E_c(z) = E_t\left[(1-\lambda)\left(\frac{1}{2}-\frac{z}{d_c}\right)^{n_0}+\lambda\right], \tag{39}$$

where E_t is elastic modulus at the upper surface of the core; λ is the ratio of elastic modulus at core's upper and lower surface; and n_0 is the volume fraction exponent (n_0 is also called grading index[12]). In this example, $E_t = 0.1\,\text{GPa}$, the Poisson ratio of the core is held constant $\nu = 0.45$, while the other properties (including thickness, density ect.) of the face sheets and core are the same as the case in Subsection 4.1. The length (l) and width (b) of the panels satisfy $l = b$.

In the following, the effects of parameters λ, n_0, the thickness-side ratios δ and κ on the natural frequencies of functionally graded cylindrical and rectangular sandwich panels are discussed. The ratios δ and κ are defined as:

$$\left.\begin{aligned} \delta &= \frac{l}{d_c+2d_f} \\ \kappa &= \frac{R}{l} \end{aligned}\right\}. \tag{40}$$

4.2.1 *Effect of FGM Core Material Parameters λ and n_0*

Based on the derived formulation, the natural frequency of functionally graded sandwich cylindrical panels ($\kappa = 1$) and rectangular plates are calculated and various gradient parameters $\lambda = 0.1$, 0.2, 1, 5, 10 and $n_0 = 0.5$, 1, 2 are considered in the analysis. The length and width of the panels satisfy $l = b = 2\,000\,\text{mm}$.

Tables 3 and 4 display the effects of λ and n_0 on the natural frequency of functionally graded cylindrical and rectangular sandwich panels when the density of FGM core keeps constant along the thickness direction. It can be seen that: ① the fundamental frequencies of functionally graded cylindrical and rectangular sandwich panels (f_f) will rise with the increase of λ; ② when $\lambda < 1$, the fundamental frequency of functionally graded sandwich panels become lower with the rise of n_0; when $\lambda > 1$, on the contrary.

Table 3 The fundamental frequency of FG cylindrical sandwich panels for different n_0 and λ when ρ_c is constant

λ	f_f/Hz		
	$n_0 = 0.5$	$n_0 = 1$	$n_0 = 2$
0.1	72.74	65.81	58.34
0.2	74.32	69.63	64.56
1	80.29	80.29	80.29
2	83.76	85.12	86.38
5	88.67	90.89	92.44
10	92.18	92.57	92.71

Table 4 The fundamental frequency of FG rectangular sandwich plates for different n_0 and λ when ρ_c is constant

λ	f_f/Hz		
	$n_0 = 0.5$	$n_0 = 1$	$n_0 = 2$
0.1	31.47	28.97	25.69
0.2	31.98	30.40	28.46
1	33.75	33.75	33.75
2	34.67	35.01	35.32
5	35.84	36.34	36.73
10	36.61	37.07	37.40

In practice, the density ρ_c of the FGM core in functionally graded sandwich structures varies along the thickness. The distribution of density ρ_c can be assumed to obey the same function as the elastic modulus $E_c(z)$:

$$\rho_c(z) = \rho_t\left[(1-\lambda)\left(\frac{1}{2}-\frac{z}{d_c}\right)^{n_0}+\lambda\right], \tag{41}$$

where ρ_t is the density at the upper surface of the core.

Tables 5 and 6 show the effects of λ and n_0 on the natural frequency when the density ρ_c varies the along the thickness direction accordance with the Eq. (41). It is obvious that the effects of λ and n_0 on the natural

Table 5 The fundamental frequency of FG cylindrical sandwich panels for different n_0 and λ when $\rho_c = \rho_c(z)$

λ	f_f/Hz		
	$n_0 = 0.5$	$n_0 = 1$	$n_0 = 2$
0.1	82.26	80.23	77.71
0.2	82.78	82.69	82.52
1	80.29	80.29	80.29
2	75.14	72.89	70.89
5	63.18	58.01	53.92
10	51.68	44.79	40.03

Table 6 The fundamental frequency of FG rectangular sandwich plates for different n_0 and λ when $\rho_c = \rho_c(z)$

λ	f_f/Hz		
	$n_0 = 0.5$	$n_0 = 1$	$n_0 = 2$
0.1	35.59	35.31	34.21
0.2	35.62	36.10	36.38
1	33.75	33.75	33.75
2	31.10	29.98	28.98
5	25.54	23.19	21.42
10	20.52	17.93	16.15

frequency are different from that of the case ρ_c keeps constant. It can be seen that: ① be contrary to the rise of fundamental frequencies in Tables 3 and 4, the frequencies become lower in Tables 5 and 6 with the rise of λ (except for $\lambda = 0.2$ in rectangular sandwich panels); ② the fundamental frequencies get lower with the rise of n_0, regardless of $\lambda < 1$ or not (except for $\lambda = 0.2$ in rectangular sandwich panels).

It can be seen from the above that when the density of the FGM core changes in the thickness direction, it is unable to conclude a uniform law of the effect of λ and n_0 on the natural frequency of the structures. In practical applications, it is needed to analyze the effects according to the specific distribution function of $E_c(z)$ and $\rho_c(z)$.

4.2.2 *Effect of Plate Side-to-Thickness Ratio δ*

Simply supported functionally graded cylindrical and rectangular panels are considered to study the effect of the plate side-to-thickness ratio on the natural frequency, where the density of FGM core keeps constant and different λ and n_0 are studied. The variation of fundamental frequency of functionally graded cylindrical and rectangular sandwich panels with ratio $\delta = \dfrac{l}{d_c + 2d_f}$ are shown in Figs. 3 and 4 Respectively. It can be seen that the natural frequencies always decrease with increased side-to-thickness ratio δ in all cases.

4.2.3 *Effect of Ratio $\kappa = R/l$*

Simply supported functionally graded cylindrical sandwich panels with $l = b = 2\,000$ mm are analyzed

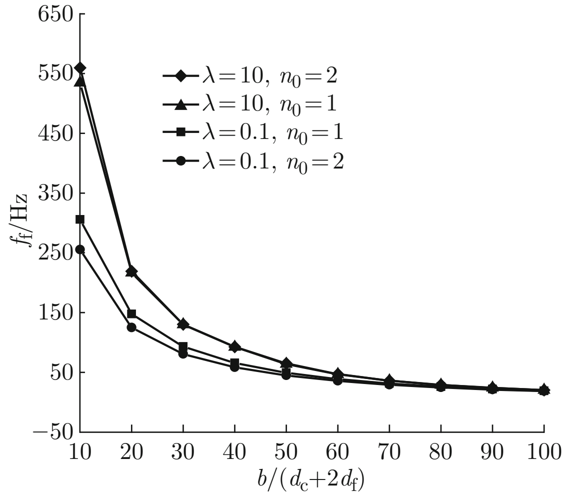

Fig. 3 Fundamental frequencies versus side-to-thickness ratio δ ($\kappa = 1$)

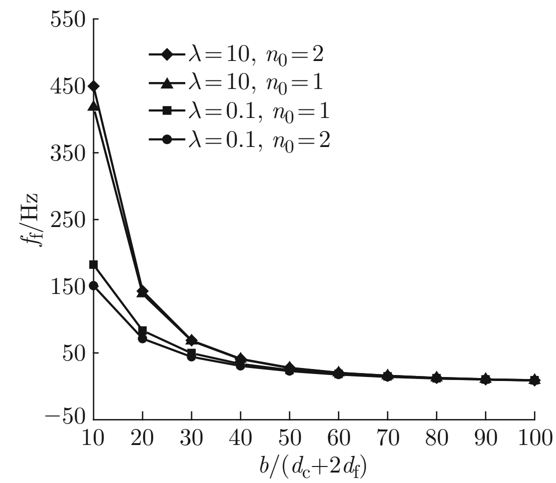

Fig. 4 Fundamental frequencies versus side-to-thickness ratio δ

to study the effect of ratios $\kappa = R/l$ on the natural frequency. The density of FGM core keeps constant and different λ and n_0 are considered. The variations of fundamental frequency with respect to the various parameters are given in Table 7. From Table 7 is seen that the fundamental frequencies decrease with increase in

Table 7 The fundamental frequency of FG cylindrical sandwich panels for different κ, n_0 and λ when $\rho_c = \rho_c(z)$

Condition	f_f/Hz			
	$\kappa = 1$	$\kappa = 10$	$\kappa = 100$	$\kappa = \infty$ (plate)
$\lambda = 0.1,\ n_0 = 1$	65.81	30.18	28.97	28.96
$\lambda = 0.1,\ n_0 = 2$	58.34	27.05	25.70	25.69
$\lambda = 10,\ n_0 = 1$	92.57	38.02	37.08	37.07
$\lambda = 10,\ n_0 = 2$	92.71	38.35	37.41	37.40

ratio κ.

5 Conclusion

The free vibration analysis of simply supported cylindrical and rectangle sandwich panels with isotropic face sheets and a functionally graded core is investigated in this paper based on the Reissner assumptions. The degenerate numerical results for sandwich structures with a homogeneous core are compared with the existing classical solutions and good agreement is displayed. For the functionally graded cylindrical and rectangle sandwich panels with constant density of FGM core along the thickness, the fundamental frequency of the structures always rises with the increase of λ and decrease with the increase in side-to-thickness ratios δ and κ; and when $\lambda < 1$, the fundamental frequency becomes lower with the rise of n_0, when $\lambda > 1$, on the contrary. However, if the density of the FGM core changed in the thickness direction, it is needed to analyze the effects according to the specific distribution function of $E_c(z)$ and $\rho_c(z)$.

References

[1] Nie G J, Zhong Z. Semi-analytical solution for three-dimensional vibration of functionally graded circular plates [J]. *Computer Methods in Applied Mechanics and Engineering*, 2007, **196**: 4901-4910.

[2] Abdelaziz H H, Atmane H A, Mechab I, et al. Static analysis of functionally graded sandwich plates using an efficient and simple refined theory [J]. *Chinese Journal of Aeronautics*, 2011, **24**: 434-448.

[3] Shariyat M. A generalized high-order global-local plate theory for nonlinear bending and buckling analyses of imperfect sandwich plates subjected to thermo-mechanical loads [J]. *Composite Structures*, 2010, **92**: 130-143.

[4] Wu Z, Chen W J. A C0-type higher-order theory for bending analysis of laminated composite and sandwich plates [J]. *Composite Structures*, 2010, **92**: 653-661.

[5] Pradyumna S, Bandyopadhyay J N. Free vibration analysis of functionally graded curved panels using a higher-order finite element formulation [J]. *Journal of Sound and Vibration*, 2008, **318**: 176-192.

[6] Zhao X, Lee Y Y, Liew K M. Free vibration analysis of functionally graded plates using the element-free *kp*-Ritz method [J]. *Journal of Sound and Vibration*, 2009, **319**: 918-939.

[7] Li Q, Lu V P, Kou K P. Three-dimensional vibration analysis of functionally graded material sandwich plates [J]. *Journal of Sound and Vibration*, 2008, **311**: 498-515.

[8] Rahmani O, Khalili S M R, Malekzadeh K, et al. Free vibration analysis of sandwich structures with a flexible functionally graded syntactic core [J]. *Composite Structures*, 2009, **91**: 229-235.

[9] Amirani M C, Khalili S M R, Nemati N. Free vibration analysis of sandwich beam with FG core using the element free Galerkin method [J]. *Composite Structures*, 2009, **90**: 373-379.

[10] Neves A M A, Ferreira A J M, Carrera E, et al. Static, free vibration and buckling analysis of isotropic and sandwich functionally graded plates using a quasi-3D higher-order shear deformation theory and a meshless technique [J]. *Composites: Part B*, 2013, **44**: 657-674.

[11] Alibeigloo A, Liew K M. Free vibration analysis of sandwich cylindrical panel with functionally graded core using three-dimensional theory of elasticity [J]. *Composite Structures*, 2014, **113**: 23-30.

[12] Institute of Mechanics, Chinese Academy of Science. The bending stability and vibration of sandwich plates and shells [M]. Beijing: Science Press, 1977.

[13] Chakraborty A, Gopalakrishnan S, Reddy J N. A new beam finite element for the analysis of functionally graded materials [J]. *International Journal of Mechanical Sciences*, 2003, **45**: 519-539.

[14] Bayat M, Sahari B B, Saleem M, et al. Bending analysis of a functionally graded rotating disk based on the first order shear deformation theory [J]. *Applied Mathematical Modelling*, 2009, **33**: 4215-4230.