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Price-Based Power Control Algorithm in Cognitive Radio Networks Based on Monotone Optimization

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Abstract: This paper considers a price-based power control problem in the cognitive radio networks (CRNs). The primary user (PU) can admit secondary users (SUs) to access if their interference powers are all under the interference power constraint. In order to access the spectrum, the SUs need to pay for their interference power. The PU first decides the price for each SU to maximize its revenue. Then, each SU controls its transmit power to maximize its revenue based on a non-cooperative game. The interaction between the PU and the SUs is modeled as a Stackelberg game. Using the backward induction, a revenue function of the PU is expressed as a non-convex function of the transmit power of the SUs. To find the optimal price for the PU, we rewrite the revenue maximization problem of the PU as a monotone optimization by variable substitution. Based on the monotone optimization, a novel price-based power control algorithm is proposed. Simulation results show the convergence and the effectiveness of the proposed algorithm compared to the non-uniform pricing algorithm.
Key words: power control, cognitive radio, monotone optimization, price
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0 Introduction

With the increasing demand for wireless service, spectrum becomes scarce and increasingly crowded, and it needs to be used efficiently. The Federal Communications Commission (FCC) found the utilization of the spectrum is low at most of the time[1]. Thus, the technology of cognitive radio networks (CRNs)[2] is proposed to solve the problem of spectrum scarcity and improve the spectrum efficiency.

There are two basic methods that allow secondary users (SUs) to access the spectrum owned by the primary user (PU): overlay and underlay models[3]. In the overlay model, the SUs use spectrum sensing[4] to identify and exploit the spectrum availability. The SUs can access the spectrum when the PUs do not use it. In the underlay model, the SUs can coexist with the PU if the interference power constraint (IPC)[5] at the PU's receiver is not violent. We investigate the price-based power control problem in the CRNs under the IPC. The issue about pricing in the CRNs was investigated in Refs. [6-10]. The authors in Ref. [6] modeled the spectrum allocation problem in the CRNs as a non-cooperative game and proposed a price-based iterative water-filling algorithm to reach the Nash equilibrium. In Ref. [7], the authors investigated the pricing issue for the power control problem in the code division multiple access (CDMA) based CRNs. In Ref. [8], the authors proposed a joint-pricing and power allocation scheme for the CRNs. We proposed a novel price-based power control algorithm based on convex optimization to maximize the revenue of the CDMA based CRNs in Ref. [9]. However, the algorithm that we proposed in Ref. [9] could not handle the more general model as Ref. [10]. The authors only gave the optimal price for the case that there was no interference between the SUs. Since the utility of the PU was a non-convex function for more general case that there was interference between the SUs, the authors proposed a sub-optimal non-uniform pricing algorithm in Ref. [10]. We give the optimal price-based power control algorithm to improve the non-uniform pricing algorithm when there exists interference between the SUs. The main contributions of this paper are as follows. By characterizing the property of the transmit power of SUs under the optimal pricing scheme, we prove that the revenue maximization problem of PU can be expressed as an equivalent monotone optimization. Based on this result, a novel

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price-based power control algorithm based on monotone optimization is proposed. Simulation results show that the proposed pricing scheme can improve the revenue of the PU, and the sum revenue of the SUs compared with the non-uniform pricing algorithm proposed in Ref. [10].

1 System Model

We consider an uplink transmission for the CRNs. The system model is shown in Fig. 1. The PU is licensed to transmit, and the n SUs need to pay the PU for their transmissions. Link gain between SU_i (i.e. the ith SU) and the PU is denoted by $g_i\,(i = 1, 2, \cdots, n)$. Let h_{ij} denote the link gain from the jth SU's transmitter to the ith SU's receiver. The IPC of SUs to the PU is T. The PU will charge the ith SU price (i.e. λ_i) per unit interference power.

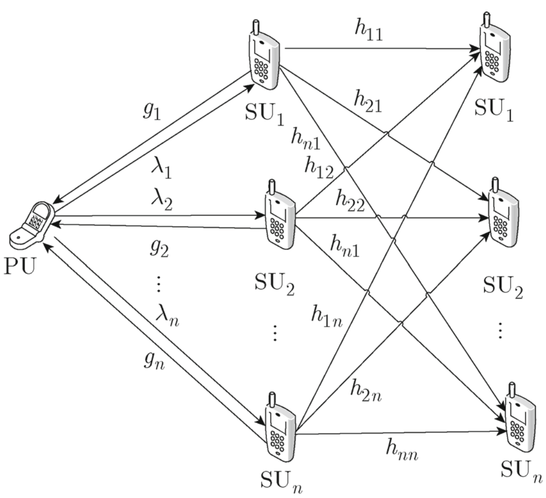

Fig. 1 System model

We model the strategy between the PU and the SUs as a Stackelberg game[11]. The PU is the leader in this game. It chooses a price for each SU to maximize its own revenue under IPC. The SUs are the followers of the game. After the PU chooses the price for each SU, the SUs will decide the transmit power to maximize its utility based on the non-cooperative power control game.

The problem of the PU is as follows:

$$\max \quad u_p(\lambda_1, \cdots, \lambda_n) = \sum_{i=1}^{n} \lambda_i g_i p_i, \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} g_j p_j \leqslant T, \tag{2}$$

$$p_j \geqslant 0, \quad j = 1, 2, \cdots, n, \tag{3}$$

where u_p denotes the revenue of the PU. Constraint (2) means that the total interference power made by SUs should be below a given threshold T to ensure the SUs' transmission not to cause unendurable interference to the PU. Constraint (3) means that the transmit power of each SU can only take on the non-negative value. The utility of the ith SU has two parts: one is the income from the transmit rate when it transmits at a given power p_i; the other is the payment to the PU. The signal-to-interference and noise ratio (SINR) of the ith SU is given by

$$\gamma_i(\boldsymbol{p}) = \frac{h_{ii} p_i}{\sum\limits_{j \neq i} h_{ij} p_j + \sigma_i^2}, \tag{4}$$

where p_i is the transmit power of the ith SU, $\boldsymbol{p} = [p_1 \ \cdots \ p_n]$ is the transmit power of all SUs, and σ_i^2 is the PU's interference plus the ambient noise at the ith SU's receiver. Thus, the utility of SU_i is given by

$$u_i(\boldsymbol{p}, \lambda_i) = w_i \log[1 + \gamma_i(\boldsymbol{p})] - \lambda_i g_i p_i, \tag{5}$$

where w_i is the equivalent utility per unit data rate valuation contributing to the ith SU's utility, which is a predefined coefficient that transforms the ith SU's transmission rate to a monetary utility. We refer to w_i as the preference factor of SU_i. Therefore, the optimization problem for the ith SU is as follows:

$$\begin{aligned} \max \quad & u_i(p_i, \boldsymbol{p}_{-i}, \lambda_i), \\ \text{s.t.} \quad & p_i \geqslant 0, \end{aligned} \tag{6}$$

where $\boldsymbol{p}_{-i}$ denotes the transmit power of the SUs except the ith SU.

2 Price-Based Power Control Algorithm

In this section, a novel pricing algorithm for the PU is given to maximize its revenue based on monotone optimization by the variable substitution. According to the property of the transmit power of SUs under the optimal price, we illustrate the relationship between the transmit power of SUs for the given price of λ_i.

Lemma 1 Let $(p_1, \cdots, p_n)$ be the transmit power of the SUs when the PU charges the ith SU for a given price λ_i such that λ_i is less than or equal to $\dfrac{w_i h_{ii}}{g_i \sigma_i^2}$ $(i = 1, 2, \cdots, n)$. Then the transmit power p_i of the SUs at the PU satisfies the following equation:

$$\begin{bmatrix} h_{11} & \cdots & h_{n1} \\ \vdots & & \vdots \\ h_{1n} & \cdots & h_{nn} \end{bmatrix} \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix} = \begin{bmatrix} w_1 h_{11}/(\lambda_1 g_1) - \sigma_1^2 \\ \vdots \\ w_n h_{nn}/(\lambda_n g_n) - \sigma_n^2 \end{bmatrix}. \tag{7}$$

Proof Using the optimal condition for the ith SU in Eq. (6), we have

$$\frac{\partial u_i(p_i,\boldsymbol{p}_{-i})}{\partial p_i}=\frac{w_i h_{ii}}{\sum_{j=1}^{n} h_{ij}p_j+\sigma_i^2}-\lambda_i g_i=0. \tag{8}$$

Then, we can get the following equation:

$$\sum_{j=1}^{n} h_{ij}p_j+\sigma_i^2=\frac{w_i h_{ii}}{\lambda_i g_i}. \tag{9}$$

For the $\mathrm{SU}_i\,(i=1,2,\cdots,n)$, Eq. (7) is derived by rewriting Eq. (9) in the matrix form.

According to Lemma 1, we have the following important identities:

$$\frac{w_i h_{ii} p_i}{\sum_{j=1}^{n} h_{ij}p_j+\sigma_i^2}=\lambda_i g_i p_i. \tag{10}$$

It means that the revenue of the PU gets from the ith SU can be expressed as $\dfrac{w_i h_{ii} p_i}{\sum_{j=1}^{n} h_{ij}p_j+\sigma_i^2}$ such that p_i satisfies Eq. (7) for a given price. Then substituting Eq. (10) into Eq. (1), we can rewrite the revenue of the PU:

$$\max\quad \sum_{i=1}^{n}\frac{w_i h_{ii} p_i}{\sum_{j=1}^{n} h_{ij}p_j+\sigma_i^2} \tag{11}$$

$$\text{s.t.}\quad \sum_{j=1}^{n} g_j p_j \leqslant T, \tag{12}$$

$$p_j \geqslant 0,\quad j=1,2,\cdots,n, \tag{13}$$

Since the objective function in Problem (11) is not a concave function, it is a non-convex optimization problem and thus cannot be solved globally and optimally by the convex optimization algorithm[12]. However, we introduce n auxiliary variables to change Problem (11) to a monotone optimization problem[13]. Hence, we propose a novel pricing scheme in order to find the global optimal solution to Problem (11) based on monotone optimization.

Let

$$z_i=w_i h_{ii} p_i\Big/\sum_{j=1}^{n} h_{ij}p_j+\sigma^2,$$
$$i=1,2,\cdots,n.$$

Problem (11) is equivalent to the following problem:

$$\max\quad u_p=\sum_{i=1}^{n} z_i \tag{14}$$

$$\text{s.t.}\quad \sum_{j=1}^{n} g_j p_j \leqslant T, \tag{15}$$

$$p_j \geqslant 0,\quad j=1,2,\cdots,n, \tag{16}$$

$$z_i=\frac{w_i h_{ii} p_i}{\sum_{j=1}^{n} h_{ij}p_j+\sigma_i^2},\quad i=1,2,\cdots,n. \tag{17}$$

Because the function of Problem (14) increases monotonically with z_i, it is equivalent as follows:

$$\max\quad u_p=f(z)=\sum_{i=1}^{n} z_i \tag{18}$$

$$\text{s.t.}\quad \sum_{j=1}^{n} g_j p_j \leqslant T, \tag{19}$$

$$p_j \geqslant 0,\quad j=1,2,\cdots,n, \tag{20}$$

$$0\leqslant z_i\leqslant \frac{w_i h_{ii} p_i}{\sum_{j=1}^{n} h_{ij}p_j+\sigma_i^2},\quad i=1,2,\cdots,n. \tag{21}$$

Let

$$S=\Big\{(p_1,\cdots,p_n)\Big|\sum_{i=1}^{n} g_i p_i\leqslant T,$$
$$p_i\geqslant 0,\quad i=1,2,\cdots,n\Big\}.$$

Feasible region defined by Formulas (19)—(21) can be written as

$$\varOmega=\bigcup_{\boldsymbol{p}\in S}\left\{(z_1,\cdots,z_n)\big|0\leqslant z_i\leqslant\frac{w_i h_{ii} p_i}{\sum_{j=1}^{n} h_{ij}p_j+\sigma_i^2}\right\}.$$

Therefore, the function of Problem (18) is equivalent to the following problem:

$$\max\ u_p=\sum_{i=1}^{n} z_i \tag{22}$$
$$\text{s.t.}\quad z\in\varOmega.$$

Next, we prove the function of Problem (22) is a monotone optimization problem over a normal set. First of all, we give some useful definitions as follows[13]. In this paper, for any two vectors $\boldsymbol{y}'\in\mathbf{R}^n$ and $\boldsymbol{y}\in\mathbf{R}^n$, $\boldsymbol{y}'\preceq\boldsymbol{y}$ means that $\boldsymbol{y}'$ is component-wise smaller than or equal to $\boldsymbol{y}$, i.e., $y_i'\leqslant y_i$, $\forall i=1,2,\cdots,n$.

Definition 1 A set $G \subset \mathbf{R}_+^n$ is called normal if for any point $x \in G$, all $x' \in \mathbf{R}_+^n$ with $x' \preceq x$ also satisfy $x' \in G$.

Definition 2 If $z \in \mathbf{R}_+^n$, the hyperrectangle

$$[0, z] = \{y \in \mathbf{R}_n^+ | 0 \preceq y \preceq z\}$$

is called a box.

Definition 3 Given any finite set $T \in \mathbf{R}_+^n$, the union of all the boxes $[0, z]$, $z \in T$, is called a polyblock with vertex set T.

According to the above definitions, we can verify that Problem (22) satisfies the following properties:

(1) The object function of Problem (22) is an increasing function with z.

(2) The feasible region Ω is a normal set. This is because Ω can be viewed as the union of a family of the normal set, and the union of a family of the normal set is normal set.

From the properties of Problem (1) and Constraint (2), Problem (22) maximizes an increasing function over a normal set. Therefore, the function of Problem (22) is a monotone optimization problem[13]. Moreover, the global optimal solution of the monotone optimization problem can be solved by a polyblock approximation approach[13]. Therefore, we use the polyblock approximation approach to give a novel price-based power control algorithm.

The proposed algorithm for price-based power control in the CRNs based on monotone optimization is described in pseudo-code.

Initialization Let tolerance $\varepsilon > 0$ and iterative number $k = 1$, construct the initial polyblock S_1 with vertex set $Z^{(1)} = \{z_1\}$, where the ith element of $\{z_1\}$ is given by

$$z_1^i = \max_{\boldsymbol{p} \in S} \frac{w_i h_{ii} p_i}{\sigma_i^2 + \sum_{j=1}^{n} h_{ji} p_j} = \frac{\dfrac{w_i h_{ii} T}{g_i}}{\dfrac{h_{ii} T}{g_i} + \sigma_i^2}.$$

Repeat

(1) Find optimal vertex $\boldsymbol{z}^{(k)}$ that maximizes the utility of PU based on $\boldsymbol{z}^{(k)} = \arg \max\limits_{z \in Z^{(k)}} f(z)$.

(2) Compute the intersection point $\boldsymbol{r}^{(k)} = \delta \boldsymbol{z}^{(k)}$ on the pareto boundary of Ω, where δ is determined by bisection method.

(3) Update the best intersection point $\boldsymbol{r}_{\text{best}}^{(k)}$ until the kth iteration as follows:

$$\boldsymbol{r}_{\text{best}}^{(k)} = \text{argmax}\{f(\boldsymbol{r}^{(k)}), f(\boldsymbol{r}_{\text{best}}^{(k-1)})\}.$$

(4) Generate n new vertices $\boldsymbol{z}^{(n),1}, \cdots, \boldsymbol{z}^{(n),n}$ adjacent to $\boldsymbol{z}^{(k)}$ by

$$\boldsymbol{z}^{(n),i} = \boldsymbol{z}^{(n)} - (z_i^{(n)} - r_i^{(n)}) \boldsymbol{e}_i,$$

where $\boldsymbol{e}_i$ denotes the vector whose every element equals 0 except the jth element being 1.

(5) Construct the vertex set $Z^{(k+1)}$ by

$$Z^{(k+1)} = Z^{(n)} \backslash \boldsymbol{z}^{(n)} \bigcup \{z^{(n),1}, \cdots, z^{(n),n}\}.$$

(6) $k = k + 1$.

Until $f(\boldsymbol{z}^{(k)}) - f(\boldsymbol{r}_{\text{best}}^{(k)}) \leqslant \varepsilon$.

Output Compute the optimal transmit power $(p_1, \cdots, p_n)$ of all the SUs by solving the equation

$$z_i = \frac{w_i h_{ii} p_i}{\sum_{j=1}^{n} h_{ji} p_j + \sigma_i^2}, \quad i = 1, 2, \cdots, n,$$

where z_i is the ith element of $\boldsymbol{r}_{\text{best}}^{(k)}$. The optimal price λ_i for the ith SU is given by

$$\lambda_i = \frac{w_i h_{ii}}{g_i \Big(\sum_{j=1}^{n} h_{ij} p_j + \sigma_i^2 \Big)}.$$

Let $\boldsymbol{z}^*$ be the optimal solution to Problem (22). The optimal revenue of the PU satisfies

$$f(\boldsymbol{r}_{\text{best}}^{(i)}) \leqslant f(\boldsymbol{z}^*) \leqslant f(\boldsymbol{z}^{(i)})$$

for each iteration i. It has been proven in Ref. [13] that the difference between $f(\boldsymbol{z}^{(i)})$ and $f(\boldsymbol{r}_{\text{best}})$ will be less than ε with a finite number of iterations for a given tolerance $\varepsilon > 0$. Therefore, utility $f(\boldsymbol{r}_{\text{best}}^{(i)})$ achieved by the PU from the proposed algorithm will converge to the optimal solution.

3 Simulation Results

We first show the proposed algorithm is convergent, and then we evaluate the performance of the proposed pricing algorithm by comparing it with the non-uniform pricing algorithm proposed in Ref. [10]. The non-uniform pricing algorithm proposed in Ref. [10] has two key steps. The first step is that each SU takes the worst interference power from other SUs as noise to remove the cross-interference power. The second step is that the PU computes the price and transmit power for each SU without cross-interference power by the non-uniform pricing algorithm. It is a suboptimal pricing scheme because it does not consider the cross interference channel gain by treating the worst interference power among the SUs.

The convergence performance of the proposed algorithms for three SUs is shown in Fig. 2. The interference power constraint is $T = 1\,\mathrm{mW}$, and the channel gain matrix among the SUs is

$$\boldsymbol{H} = \begin{bmatrix} 0.344\,5 & 0.001\,0 & 0.009\,2 \\ 0.007\,8 & 0.602\,2 & 0.083\,4 \\ 0.006\,8 & 0.003\,9 & 0.462\,4 \end{bmatrix}$$

The channel gain from the SUs to the PU is given by $\boldsymbol{g} = [0.016\,1 \;\; 0.039\,0 \;\; 0.023\,6]$, for all link $i\,(i = 1, 2, 3)$ there is $\sigma_i^2 = 1$, and the preference factor for all the SUs is 1. The tolerance ε for the proposed algorithm is set to be 0.05. Figure 2 shows the revenue of PU versus iteration. The proposed algorithm converges to 2.432 1 after about 70 iterations. The optimal revenue of PU obtained by exhaustive search is 2.433 8. Therefore, the proposed algorithm finds the global optimal solution within an allowable error of 0.001 7, which is less than the given tolerance $\varepsilon = 0.05$.

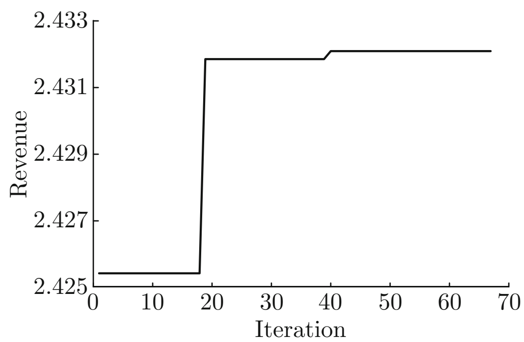

Fig. 2 The PU revenue versus iteration

We compare the system performance obtained by the proposed algorithm with the non-uniform pricing algorithm versus interference-to-noise ratio (INR), where $\mathrm{INR} = T/\sigma^2$. The SUs are randomly located in a square area with the PU located at the center. The radius of the square area is 1 km. The distance of the transmitter of the SU to its receiver is in the range of 100 m. The channel gains between the ith SU's transmitter to the jth SU's receiver and the PU are modeled as

$$h_{ij} = 10^{\alpha_{ij}/10} K d_{ij}^{-4},$$
$$g_i = 10^{\beta_i/10} K s_i^{-4},$$

where, d_{ij} and s_i are the distances of the ith SU's transmitter to the jth SU's receiver and the PU, respectively; α_{ij} and β_i are the random Gaussian variables with zero mean and 6 dB standard deviation; $K = 10^3$ indicates the system and transmission effect, such as antenna gain and carrier frequency. The number of SUs is 5, and the σ^2 equals 1 pW. The preference factor for all SUs is 1. The results are averaged over 10^4 independent realizations for the users' locations and fading channel coefficients. The INR changes from −20 to 20 dB, which means that the IPC changes from 0.01 to 100 pW.

Figure 3 shows the revenue of two algorithms versus the INR. The PU revenue obtained by the two algorithms increases as the INR increases. This is because that the pricing strategy for the PU increases as the INR increases. The PU revenue obtained by the proposed algorithm will gain more than 2% profit compared with the non-uniform pricing algorithm when the INR equals 20 dB. The revenue of the PU is saturated as the INR increases and is bounded by 5. The reason is that the revenue of the PU is bounded by $\sum\limits_{i=1}^{n} w_i$.

Figure 4 shows the sum revenue of the SUs versus INR. Both algorithms' sum revenues increase as the INR increases. The proposed algorithm is better than the non-uniform pricing algorithm for all the INR. When the INR equals 20 dB, the proposed algorithm improves 20% sum revenue of the SUs compared to the non-uniform pricing algorithm.

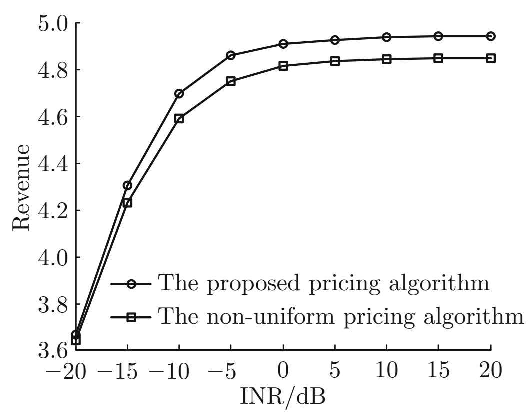

Fig. 3 The PU revenue versus INR

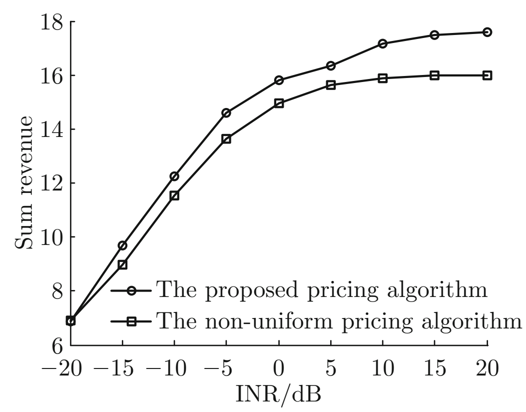

Fig. 4 Sum revenue of the SUs versus INR

4 Conclusion

We have investigated the price-based power control in the CRNs from the Stackelberg game. The revenue of the PU is expressed as a function of the transmit power of the SUs. Based on the variable substitution, the revenue of the PU is rewritten as a monotonic optimization. We propose a novel price-based power algorithm based on the monotonic optimization. Compared with the non-uniform pricing algorithm, the proposed pricing algorithm improves the revenue of both the PU and SUs.

We have assumed that the channel information is perfect in this paper. When the channel information is imperfect, the utility function and the interference power constraint condition for PU and SUs will be changed and the proposed work may not be applicable. The price-based power control in the CRNs under imperfect channel information will be part of our future work.

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