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Reliability Analysis for the Competing Failure with Probabilistic Failure Threshold Value and Its Application to the k-out-of-n Systems

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Abstract: A method for reliability analysis of the competing failure with the probabilistic failure threshold value not the fixed threshold value is presented, which involves the random shocks and the degradation is independent and dependent respectively. Specifically, for the dependent condition, the effect due to the random shocks on the degradation is considered with a damage factor. In addition, the dependent competing failure model is applied to the reliability analysis of the k-out-of-n systems. Finally, two studied cases are presented to illustrate the proposed method, and the results show the proposed method is reasonable.
Key words: competing failure, random shocks, degradation, probabilistic failure threshold, k-out-of-n systems
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0 Introduction

Reliability analysis has becoming more and more important particularly in the design, safety assessments, and optimization of engineering materials and structures, and the reliability of a system is expressed by the probability that the item will perform its required function under given conditions for a stated time interval[1]. According to the failure mechanisms of many engineering structures and applications, there are generally two kinds of failure modes: one is the catastrophic failure which is caused by the external shocks; the other is the degradation failure which is resulted from the degradation and the cumulative random shocks, and if any one of the failure modes reaches the failure threshold value, the system or product will break down immediately, which is the competing failure[2]. Thus, it is necessary to evaluate the reliability based on the competing failure.

So far, there are lots of theoretical results and applications have been achieved on the competing failure. Klutke and Yang[3] developed an availability model for a system that deteriorates due to the Poisson shock and graceful degradation. Based on the copula method, Wang and Pham[4] put forward a novel dependent model for the multiple competing failure system which subjects to the degradation and random shocks. Li and Pham[5] investigated a reliability assessment model for a generalized multi-state degradation system under three independent failure processes, two degradation processes and a random shock process. Huang and Askin[6] gave an extension of reliability analysis for the electronic devices with multiple competing failure modes which involve the performance of aging degradation, and this approach can predict the dominant failure mode on the product. Peng et al.[7] studied a reliability modeling for the micro-electro-mechanical systems devices under two failure processes: one is the soft failure process that is caused by the continuous degradation and the additional abrupt degradation damages due to the random shocks process; the other is the catastrophic failure process which is caused by excessive shock magnitudes from the same shock process. However, in these models, the probabilistic failure threshold is not considered.

It should be noticed that the reliability analysis is usually based on the probabilistic failure threshold rather than pre-determined threshold. Therefore, in this paper, we will conduct reliability analysis under the dependent and independent competing failure process respectively based on the probabilistic failure threshold. Furthermore, the proposed dependent model is applied to the reliability analysis of the k-out-of-n

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systems, because the traditional way to obtain a high reliability system is the k-out-of-n systems. Thus, the reliability analysis of the k-out-of-n systems is also considered in this paper. And different from the conventional method is that the probabilistic failure threshold and the dependent relationship between the degradation and the random shocks are all considered.

1 The Random Shocks Analysis and the Degradation Analysis

1.1 The Random Shocks Analysis

It is obvious that the random shocks can cause the decreasing of component's life directly and may affect a degradation process indirectly as it often accelerates the speed of degradation process, thus it is one of the important subjects in the reliability modeling. In order to obtain the mathematical formulations for modeling the system reliability in random environments, shock models have been widely studied by Chen and Li[8], Mallor and Santos[9], Li and Zhao [10], Sgarbossa and Pham[11]. Traditionally, there are four principal categories of random shock model[12]: ① cumulative shock model, where a system is considered to fail when the cumulative damage from shocks exceeds the critical value; ② run shock model, which models a system operating normally until k consecutive shocks with critical magnitude occur prefixed critical level; ③ extreme shock mode, where a system will break down as soon as the magnitude of any shock exceeds a specified threshold; ④ δ-shock model, where a system failure occurs when the time lag between two successive shocks is shorter than a threshold[13]. Here, the cumulative shock model is considered to calculate the random shocks to the component, the arrival time of random shocks follow Poison distribution, and the magnitude of random shocks and time interval can be described in Fig. 1, where c_s is the magnitude of small damage, c_l is the magnitude of large damage, t_i $(i = 1, 2, \cdots, n)$ is the different time interval.

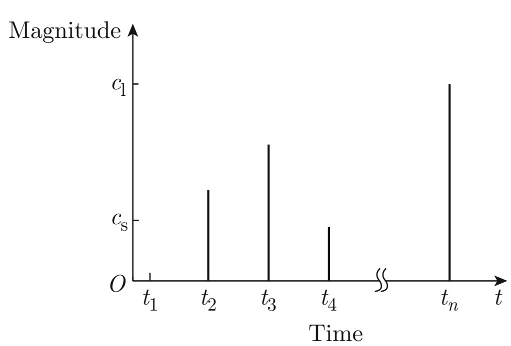

Fig. 1 The relationship between the magnitude of random shocks and time interval

If the random shocks are the cumulative shock model, then the probability of the total sum of the random shocks is less than the threshold can be represented as:

$$P[X(t) \leqslant S] = P\left[\sum_{i=1}^{N(t)} x_i(t) \leqslant S\right] = \sum_{n=0}^{\infty}\{P[N(t)=n]\times P[x_1 + x_2 + \cdots + x_{N(t)} \leqslant S \big| N(t)=n]\} = \sum_{n=0}^{\infty}\left\{\frac{(\lambda t)^n \mathrm{e}^{-\lambda t}}{n!} P[x_1 + x_2 + \cdots + x_{N(t)} \leqslant S]\right\}, \quad (1)$$

where P is the probability, $X(t)$ is the cumulative damage of random shocks at time, S is the critical value of random shock, x_i is the size of the shock load, and λ is the arrival rate of random shocks.

1.2 The Degradation Analysis

From the viewpoint of more reliable with a longer lifetime and higher quality of the product, it is really a significant challenge to obtain the sufficient and accurate time-to-failure data. To overcome the obstacles to obtain the information of the reliability specifications of the products under the normal operating conditions, the approach of the degradation analysis model is often used to evaluate the system reliability, which involves the measurement of the degradation of a product at various time points and then the degradation data can be used to estimate the eventual failure lifetime for the product. Furthermore, there are some advantages of performing reliability analysis in terms of the degradation data. On the one hand, the major advantage is that it relates the reliability analysis directly to the physics of failure mechanism. On the other hand, different physical deterioration processes such as cumulative wear, crack growth, fatigue, erosion, corrosion and so on can also be directly reflected. If the resistance of a deterioration structure and components decreases below the failure threshold, it will incur high cost losses. Therefore, many of papers have been public on the research of the reliability degradation model.

Based upon the methodology, the studies of the degradation analysis can be divided into three main categories. Firstly, one of the most widely used methods to model the degradation data is the General Path Model, in which the maximum likelihood estimation, Bayesian estimation are usually utilized to estimate the general path model and the distribution function of failure time respectively[14-15]. Then, the second method is employing the stochastic process including the Markov process, Brownian motion and Gamma process to perform the degradation analysis[16-17]. And the third approach is the statistical method, such as parametric and nonparametric estimation where the additive and multiplicative functions are often adopted to construct the degradation path function[18-19]. In this research, the traditional additive degradation path function is introduced to model the degradation D, and it can be expressed as

$$D(t, Y(t), \theta) = \eta(t, \theta) + Y(t), \tag{2}$$

where $\eta(t,\theta)$ is a mean degradation level, θ is the fixed effect parameter; and $Y(t)$ represents the random variation around a mean degradation level at time t.

Without loss of generality, if $Y(t)$ follows the Weibull distribution with the cumulative distribution function (CDF):

$$Y(t) = 1 - \exp\left[-\left(\frac{y}{\beta}\right)^{\alpha}\right], \tag{3}$$

then the CDF of the degradation process can be obtained, that is

$$P[\eta(t,\theta) + Y(t) \leqslant L] = P[Y(t) \leqslant L - \eta(t,\theta)] = \\ F(L - \eta(t,\theta)) = 1 - \exp\left[-\left(\frac{L-\eta(t,\theta)}{\beta}\right)^{\alpha}\right], \tag{4}$$

where L is the failure threshold of the degradation, α is the shape parameter of the Welbull distribution, and β is the scale parameter of the Welbull distribution.

Similarly, if the $Y(t)$ follows the normal distribution $Y \sim N(\mu_1, \sigma_1^2)$, the corresponding CDF of the degradation can also be obtained easily:

$$P[\eta(t,\theta) + Y(t) \leqslant L] = P[Y(t) \leqslant L - \eta(t,\theta)] = \\ F(L - \eta(t,\theta)) = \Phi\left(\frac{L - \eta(t,\theta) - \mu_1}{\sigma_1}\right), \tag{5}$$

where Φ is the CDF of a normal distribution.

2 The Random Shocks and the Degradation Pcesses are Independent and Dependent

2.1 The Random Shocks and the Degradation Pcesses are Independent

Assuming the component experience one degradation process and one random shocks process, and if the two processes are independent with each other, the reliability of component can be expressed by

$$R(t) = P[D(t) \leqslant L, X(t) \leqslant S] = \\ P[D(t) \leqslant L]P[X(t) \leqslant S]. \tag{6}$$

Considering the probabilistic failure threshold for each component, the probability of no degradation failure $R_{\rm D}$ is

$$R_{\rm D}(t) = P[D(t) \leqslant L(t)] = \int_0^{\infty} Y(t,u)\mathrm{d}L(u), \tag{7}$$

where u is a random variable.

And the probability of no shock cumulative damage failure $R_{\rm S}$ is

$$R_{\rm S}(t) = P[X(t) \leqslant S(t)] = \\ \sum_{n=0}^{\infty}\Big\{P[N(t)=n] \times \\ P\big[x_1 + x_2 + \cdots + x_{N(t)} \leqslant S(u)\big|N(t)=n\big]\Big\} = \\ \sum_{n=0}^{\infty}\Big[\frac{(\lambda t)^n \mathrm{e}^{-\lambda t}}{n!}\int_0^{\infty} F_X^n(u)\mathrm{d}S(u)\Big], \tag{8}$$

where $F_X^n(u)$ is the CDF of the sum of n independent and identically distributed X variables.

Substituting Eqs. (7) and (8) into Eq. (6), the reliability of component based on the probabilistic failure threshold can be obtained, that is

$$R(t) = R_{\rm S}(t)R_{\rm D}(t) = \int_0^{\infty} Y(t,u)\mathrm{d}L(u) \times \\ \sum_{n=0}^{\infty}\left[\frac{(\lambda t)^n \mathrm{e}^{-\lambda t}}{n!}\int_0^{\infty} F_X^n(u)\mathrm{d}S(u)\right]. \tag{9}$$

If the performance degradation follows Weibull distribution, and the cumulative damages caused by random shocks follows geometric process[10], the evaluation of system reliability can be achieved by

$$R(t) = \int_0^{\infty}\left\{1 - \exp\left[-\left(\frac{u}{\beta(t)}\right)^{\alpha(t)}\right]\right\}\mathrm{d}L(u) \times \\ \sum_{n=0}^{\infty}\left[\frac{(\lambda t)^n \mathrm{e}^{-\lambda t}}{n!}\int_0^{\infty}\prod_{m=1}^{n} F(a^{n-m}u)\mathrm{d}S(u)\right], \tag{10}$$

where a is a ratio of the geometric process.

In fact, it is not easy to solve the convolution in Eq. (9). Therefore, we utilize the following numerical methods to calculate the results.

$$R(t) = \int_0^{\infty} Y(t,u)\mathrm{d}L(u) \times \\ \sum_{n=0}^{\infty}\left\{\frac{(\lambda t)^n \mathrm{e}^{-\lambda t}}{n!} P\left[\sum_{i=1}^{N(t)} x_i \leqslant S(u)|N(t)=n\right]\right\} = \\ \int_0^{\infty}\left\{1 - \exp\left[-\left(\frac{u}{\beta(t)}\right)^{\alpha(t)}\right]\right\}\mathrm{d}L(u) \times \\ \sum_{n=0}^{\infty}\left\{\frac{(\lambda t)^n \mathrm{e}^{-\lambda t}}{n!}\Phi\left[\frac{S(u) - (\mu_1 + \mu_2 + \cdots + \mu_n)}{\sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2}}\right]\right\} =$$

$$\int_0^{\infty}\left\{1-\exp\left[-\left(\frac{u}{\beta(t)}\right)^{\alpha(t)}\right]\right\}\mathrm{d}L(u)\times$$
$$\sum_{n=0}^{\infty}\left\{\frac{(\lambda t)^n\mathrm{e}^{-\lambda t}}{n!}\varPhi\left[\frac{S(u)-\dfrac{\mu(1-a^{-n})}{1-a^{-1}}}{\sqrt{\dfrac{\sigma^2(1-a^{-2n})}{1-a^{-2}}}}\right]\right\}. \tag{11}$$

2.2 The Random Shocks and the Degradation Processes are Dependent

In the previous section, we developed an independent of the random shocks and the degradation model, in other words, the relationship between them is not considered. However, the effects on the degradation due to random shocks, especially those with serious damages, may not be neglected. Therefore, our objective in this section is to conduct dependent reliability analysis model with consideration of the random shocks effecting on the degradation under the probabilistic threshold.

Now assuming the component has two failure modes, which is similar to the former model, one is degradation process and the other is the cumulative damage of the random shocks. At the same time, we utilized the random variable C_i which is called "effect factor" to describe the effect factor of the ith shock, while C_i follows an normal distribution for simplify.

For the random shocks, if the damage x_i caused by every random shock follows the identical normal distribution with CDF

$$F(x_i)=\frac{1}{\sqrt{2\pi}\sigma_{\mathrm{s}}}\int_{-\infty}^{x_i}\mathrm{e}^{-\frac{(t-n\mu_{\mathrm{s}})^2}{2n\sigma_{\mathrm{s}}^2}}\mathrm{d}t, \tag{12}$$

where μ_{s} is the mean value of random shock, and σ_{s} is the variance of random shock.

According to the characteristic of normal distribution, $X(t)=\sum_{i=1}^{n}x_i$ still follows normal distribution with CDF

$$F(X)=\frac{1}{\sqrt{2\pi}n\sigma_{\mathrm{s}}}\int_{-\infty}^{x_i}\mathrm{e}^{-\frac{(t-n\mu_{\mathrm{s}})^2}{2n\sigma_{\mathrm{s}}^2}}\mathrm{d}t. \tag{13}$$

Taking into the probabilistic threshold account, the probability of no random shocks failure can be rewritten as:

$$R_{\mathrm{S}}=\sum_{n=0}^{\infty}\frac{(\lambda t)^n\mathrm{e}^{-\lambda t}}{n!}\int_0^{\infty}\varPhi\left(\frac{u-n\mu_{\mathrm{s}}}{\sqrt{n\sigma_{\mathrm{s}}^2}}\right)\mathrm{d}S(u). \tag{14}$$

Then for the issue of the degradation failure process under the probabilistic failure threshold, assuming the performance of the internal degradation follows the normal distribution $Y(t)\sim N(\mu_{\mathrm{d}},\sigma_{\mathrm{d}}^2)$, and the effect factor C_i also follows the normal distribution $C_i\sim N(\mu_{\mathrm{c}},\sigma_{\mathrm{c}}^2)$, then the compound effect of the degradation is $\sum_{i=1}^{N(t)}C_i+Y(t)\sim N(\mu_{\mathrm{d}}+n\mu_{\mathrm{c}},\sigma_{\mathrm{d}}^2+n\sigma_{\mathrm{c}}^2)$, therefore

$$\begin{aligned}R_{\mathrm{D}}(t)=P\left[Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\right]=\\ \sum_{i=1}^{N(t)}\Bigg\{P[N(t)=n]\times\\ P\left[Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\middle|N(t)=n\right]\Bigg\}=\\ \sum_{i=1}^{N(t)}\Bigg\{P[N(t)=n]\times\\ \int_0^{\infty}\varPhi\left(\frac{u-(\mu_{\mathrm{d}}+n\mu_{\mathrm{c}})}{\sqrt{\sigma_{\mathrm{d}}^2+n\sigma_{\mathrm{c}}^2}}\right)\mathrm{d}L(u)\Bigg\}.\end{aligned} \tag{15}$$

Then the reliability function of the component can be obtained, and it is given by:

$$\begin{aligned}R(t)=\\ P\left[\sum_{i=1}^{N(t)}x_i(t)\leqslant S(u),Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\right]=\\ \sum_{i=1}^{N(t)}\Bigg\{P[N(t)=n]P\Bigg[\sum_{i=1}^{N(t)}x_i(t)\leqslant S(u),\\ Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\Big|N(t)=n\Bigg]\Bigg\}=\\ \sum_{n=0}^{\infty}\Bigg[\frac{(\lambda t)^n\mathrm{e}^{-\lambda t}}{n!}\int_0^{\infty}\varPhi\left(\frac{u-n\mu_{\mathrm{s}}}{\sqrt{n\sigma_{\mathrm{s}}^2}}\right)\mathrm{d}S(u)\times\\ \int_0^{\infty}\varPhi\left(\frac{u-(\mu_{\mathrm{d}}+n\mu_{\mathrm{c}})}{\sqrt{\sigma_{\mathrm{d}}^2+n\sigma_{\mathrm{c}}^2}}\right)\mathrm{d}L(u)\Bigg].\end{aligned} \tag{16}$$

3 The Reliability Analysis of k-out-of-n Systems Under the Probabilistic Failure Threshold Value

Because the dependent competing failure is more realistic in practical, we apply the proposed dependent model to the k-out-of-n systems reliability analysis. As well known, the k-out-of-n systems is a system with n components such that the system is operational if and only if at least k of its n components are operational[19-20]. In other words, the system occurs failure as soon as $(n-k+1)$ components fail. In reality, applications of the k-out-of-n systems model, such as these systems which require more than one component to function in order for the entire system to operate. In this section, we will discuss the reliability analysis for the k-out-of-n systems under the probabilistic failure threshold.

It can be shown from literature[9], the estimation of reliability functions for a k-out-of-n identical and independent component systems can be represented as

$$R_{\mathrm{S}}(t)=\sum_{i=k}^{n}\binom{n}{i}R^{i}(t)[1-R(t)]^{n-i}. \tag{17}$$

Considering the special case of the dependent random shocks and the degradation, and substituting Eq. (16) into Eq. (17), we have

$$\begin{aligned}R_{\mathrm{S}}(t)=&\sum_{i=k}^{n}\binom{n}{i}\times\\&\left\{P\left[\sum_{i=1}^{N(t)}x_i(t)\leqslant S(u),Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\right]\right\}^{i}\left\{1-\right.\\&\left.P\left[\sum_{i=1}^{N(t)}x_i(t)\leqslant S(u),Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\right]\right\}^{n-i}=\\&\sum_{i=k}^{n}\binom{n}{i}\sum_{n=0}^{\infty}\left\{\frac{(\lambda t)^n\mathrm{e}^{-\lambda t}}{n!}\int_0^{\infty}\varPhi\left(\frac{u-n\mu_{\mathrm{s}}}{\sqrt{n\sigma_{\mathrm{s}}^2}}\right)\mathrm{d}S(u)\times\right.\\&\left.\int_0^{\infty}\varPhi\left(\frac{u-(\mu_{\mathrm{d}}+n\mu_{\mathrm{c}})}{\sqrt{\sigma_{\mathrm{d}}^2+n\sigma_{\mathrm{c}}^2}}\right)\mathrm{d}L(u)\right\}^{i}\times\\&\left\{1-\sum_{n=0}^{\infty}\left[\frac{(\lambda t)^n\mathrm{e}^{-\lambda t}}{n!}\int_0^{\infty}\varPhi\left(\frac{u-n\mu_{\mathrm{s}}}{\sqrt{n\sigma_{\mathrm{s}}^2}}\right)\mathrm{d}S(u)\times\right.\right.\\&\left.\left.\int_0^{\infty}\varPhi\left(\frac{u-(\mu_{\mathrm{d}}+n\mu_{\mathrm{c}})}{\sqrt{\sigma_{\mathrm{d}}^2+n\sigma_{\mathrm{c}}^2}}\right)\mathrm{d}L(u)\right]\right\}^{n-i}.\end{aligned} \tag{18}$$

It can be seen that the reliability function of the k-out-of-n systems can also be rewritten as follows.

$$R_{\mathrm{S}}(t)=\sum_{j=k}^{n}(-1)^{j-k}\binom{j-1}{k-1}\binom{n}{j}R^{j}(t). \tag{19}$$

Therefore, the reliability of the k-out-of-n systems, simply, is

$$\begin{aligned}R_{\mathrm{S}}(t)=&\sum_{j=k}^{n}(-1)^{j-k}\binom{j-1}{k-1}\binom{n}{j}\left\{P\left[\sum_{i=1}^{N(t)}x_i(t)\leqslant S(u),\right.\right.\\&\left.\left.Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\right]\right\}^{j}=\\&\sum_{j=k}^{n}(-1)^{j-k}\binom{j-1}{k-1}\binom{n}{j}\times\\&\left[\sum_{n=0}^{\infty}\frac{(\lambda t)^n\mathrm{e}^{-\lambda t}}{n!}\int_0^{\infty}\varPhi\left(\frac{u-n\mu_{\mathrm{s}}}{\sqrt{n\sigma_{\mathrm{s}}^2}}\right)\mathrm{d}S(u)\times\right.\\&\left.\int_0^{\infty}\varPhi\left(\frac{u-(\mu_{\mathrm{d}}+n\mu_{\mathrm{c}})}{\sqrt{\sigma_{\mathrm{d}}^2+n\sigma_{\mathrm{c}}^2}}\right)\mathrm{d}L(u)\right]^{j}.\end{aligned} \tag{20}$$

4 Number Examples

The studied case in this section aims to demonstrate the mathematical modeling and deduction of system reliability which subject to the competing failure under the independent and dependent between the random shocks and degradation in previous sections, respectively.

4.1 Case One

Although dependent competing model may be more suitable for practical product, of course, in some certain conditions, the independent model is more convenient to get the failure feature. Therefore, we utilize the data which can be found in literature[22], to illustrate the proposed independent competing failure model. That is, supposing a component suffers the internal degradation and the external random shocks process. On the one hand, for the internal degradation, follows the Weibull distribution with the shape parameter $\beta(t)$ and scale parameter $\alpha(t)$ is

$$\beta(t)=a_1t^{b_1}\exp(c_1t)$$

and

$$\alpha(t)=a_2\left(\frac{1}{t}+1\right)^{b_2}\exp\left(\frac{c_2}{t}\right)$$

respectively, the relative parameters are

$$a_1=0.665,\quad b_1=0.395,\quad c_1=0.003$$

and

$$a_2=6.546,\quad b_2=-528.965,\quad c_2=542.128.$$

On the other hand, for the random shocks, assume it follows a Poisson distribution with the arrival rate $\lambda=0.1$, which means the mean arrival time for a random shock is $1/\lambda=10$, the magnitude of the random shocks follow the $F(t)\sim N(1.0,0.05)$, and to overcome the difficulty of solving the convolution of the cumulative random shock, geometric process is adopted in Eq. (11) and $a=0.95$. In addition, the probabilistic failure threshold of the component is $L(u)\sim W(3.2,10)$ and $S(u)\sim W(2.5,35)$, and the fixed failure thresholds is $S=31.054$ and $L=8.956$, which are used to demonstrate the results of the proposed model through comparison.

Substituting these parameters into Eq. (11), we can obtain the reliability of the component subjected to the independent competing failure, that is:

$$R(t)=\int_0^{\infty}\left\{1-\exp\left[-\left(\frac{u}{0.665t^{0.395}\exp(0.003t)}\right)^{6.546\left(\frac{1}{t}+1\right)^{-528.965}\exp\left(\frac{542.128}{t}\right)}\right]\right\}\mathrm{d}L(u)\times$$

$$\sum_{n=0}^{\infty}\left[\frac{(0.1t)^n\mathrm{e}^{-0.1t}}{n!}\varPhi\left(\frac{S(u)-\dfrac{1.0\times(1-0.95^{-n})}{1-0.95^{-1}}}{\sqrt{\dfrac{0.05^2\times(1-0.95^{-2n})}{1-0.95^{-2}}}}\right)\right]. \tag{21}$$

Furthermore, the results of the component reliability for the probabilistic failure threshold and the fixed failure threshold are plotted in Fig. 2. At the same time, the reliability of corresponding to the internal degradation, external random shocks and are also shown in Fig. 3 respectively. From these results, we can see it clearly that the reliability of the fixed failure threshold model is higher than the probabilistic failure threshold model. And the probabilistic failure threshold model evaluating reliability may be more precisely than the fixed failure threshold since it is more realistic to practical phenomena. Thus, we can conclude that the reliability of fixed failure threshold may overestimate the reliability which is dangerous in the real world.

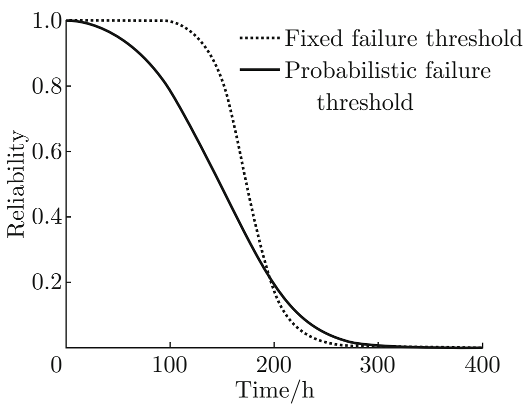

Fig. 2 The reliability of the probabilistic failure threshold and the fixed failure threshold

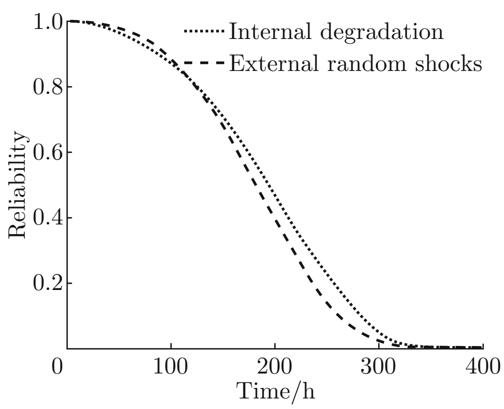

Fig. 3 The internal degradation and the external random shocks versus time

4.2 Case Two

In this subsection, we will firstly implement the reliability analysis under the independent and the dependent condition using the proposed method. And then, apply the dependent model to the k-out-of-n systems reliability analysis which is described in previous section. Now let internal degradation $Y(t)$ follows normal distribution with mean $\mu_{\mathrm{d}} = 0.016t^{0.37}$ and variance $\sigma_{\mathrm{d}} = 0.039t^{0.066}$; the random shocks follow the Poisson distribution with $\lambda = 0.25$ and the magnitude of the random shocks follow the normal distribution with $x_i \sim N(0.16,\ 0.01^2)$; the probabilistic failure threshold $S(u)$ and $L(u)$ follow the normal distribution with $N(0.6, 0.01)$, the fixed failure threshold of $S = 0.5$ and $L = 0.7$; finally, the effect factor is $C_i \sim N(0.03, 0.003^2)$.

According to Eq. (12), the reliability of independent competing failure can be rewritten as

$$\begin{aligned}R(t)=&\int_0^{\infty}Y(t,u)\mathrm{d}L(u)\times\\&\sum_{n=0}^{\infty}\left\{\frac{(\lambda t)^n\mathrm{e}^{-\lambda t}}{n!}P\left[\sum_{i=1}^{N(t)}x_i\leqslant S(u)\right]\right\}=\\&\int_0^{\infty}\varPhi\Big(\frac{u-0.016t^{0.37}}{0.039t^{0.066}}\Big)\mathrm{e}^{-\frac{(u-0.6)^2}{0.02}}\mathrm{d}u\times\\&\sum_{n=0}^{\infty}\left[\frac{(0.25t)^n\mathrm{e}^{-0.25t}}{n!}\int_0^{\infty}\varPhi\Big(\frac{u-0.016}{0.01}\Big)\times\right.\\&\left.\frac{1}{\sqrt{2\pi}\times0.1}\mathrm{e}^{-\frac{(u-0.6)^2}{0.02}}\mathrm{d}u\right].\end{aligned} \tag{22}$$

Meanwhile, substituting the corresponding parameters into Eq. (16), the reliability of the component under the dependent competing failure can be obtained by

$$R(t)=$$

$$P\left[\sum_{i=1}^{N(t)}x_i(t)\leqslant S(u),Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\right]=$$

$$\int_0^{\infty}\varPhi\Big(\frac{u-0.016t^{0.37}-0.03n}{\sqrt{0.003^2+(0.039t^{0.066})^2}}\Big)\times \frac{1}{\sqrt{2\pi}\times 0.1}\mathrm{e}^{-\frac{(u-0.6)^2}{0.02}}\mathrm{d}u\times \sum_{n=0}^{\infty}\left[\frac{(0.25t)^n\mathrm{e}^{-0.25t}}{n!}\int_0^{\infty}\varPhi\Big(\frac{u-0.016}{0.01}\Big)\times \frac{1}{\sqrt{2\pi}\times 0.1}\mathrm{e}^{-\frac{(u-0.6)^2}{0.02}}\mathrm{d}u\right]. \tag{23}$$

Finally, we apply the dependent model to the k-out-of-n systems (considering the special case 2-out-of-3), and substitute the relative parameters into the Eq. (20)

$$R_{\mathrm{S}}(t)=\sum_{j=k}^{n}(-1)^{j-k}\binom{j-1}{k-1}\binom{n}{j}\left\{P\left[\sum_{i=1}^{N(t)}x_i(t)\leqslant S(u), Y(t)+\sum_{i=1}^{N(t)}C_i\leqslant L(u)\right]\right\}^j = \sum_{j=2}^{3}(-1)^{j-k}\binom{j-1}{k-1}\binom{n}{j}\times$$

$$\left\{\int_0^{\infty}\varPhi\left(\frac{u-0.016t^{0.37}-0.03n}{\sqrt{0.003^2+(0.039t^{0.066})^2}}\times \frac{1}{\sqrt{2\pi}\times 0.1}\mathrm{e}^{-\frac{(u-0.6)^2}{0.02}}\right)\mathrm{d}u\times \sum_{n=0}^{\infty}\left[\frac{(0.25t)^n\mathrm{e}^{-0.25t}}{n!}\int_0^{\infty}\varPhi\left(\frac{u-0.016}{0.01}\right)\times \frac{1}{\sqrt{2\pi}\times 0.1}\mathrm{e}^{-\frac{(u-0.6)^2}{0.02}}\mathrm{d}u\right]\right\}^j. \tag{24}$$

The results of the dependent and the independent competing failure are shown in Fig. 4, from Fig. 4 we can see that the reliability result of independent competing failure model is lower at the early stage, and then it becomes higher than the dependent condition. We can see that the effect of the random shocks to the degradation is important, and if we will not consider the correlation between the degradation and shocks, the reliability estimation will not accurate and it may be too optimistic.

Furthermore, the reliability of the independent competing failure based on the probabilistic failure threshold and the fixed failure threshold are represented in Fig. 5 respectively. From the comparison, it is clear that the reliability of the probabilistic threshold model is lower than the fixed model.

Finally, for the reliability analysis of the dependent competing failure in the k-out-of-n systems which is based on the probabilistic failure threshold, we conduct the sensitivity analysis of the parameter k to the reliability, that is, given a fixed n, and changing the k, and the results are shown in Fig. 6. From the results we can see that the reliability shifts left with the increasing of the k, which means if k decreases, the working requirements of the system will be much lower. That is true, for example, when $k = 1$, the system will function if at least one component operates, and according to the reliability is higher than choosing other k.

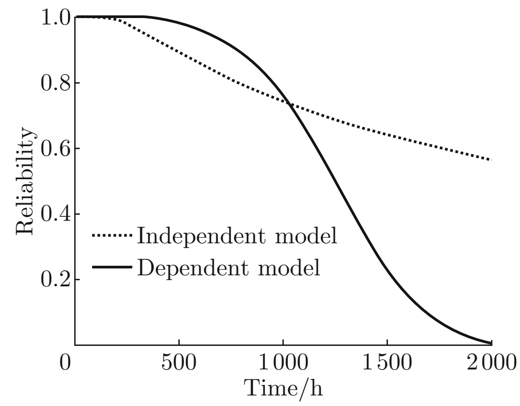

Fig. 4 Comparison of reliability between independent and dependent competing model

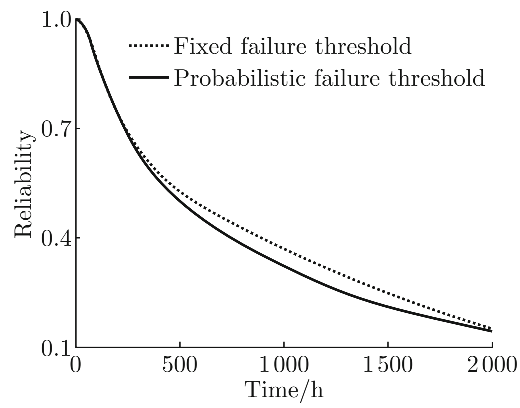

Fig. 5 The reliability of the probabilistic failure threshold and the fixed failure threshold

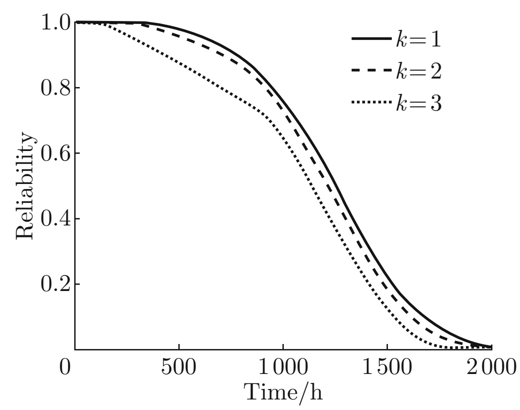

Fig. 6 The reliability of the different k

5 Conclusion

Reliability analysis plays an important role in evaluating the performance of a system and making maintenance decision in engineering. Generally, the system failure may be caused by the internal degradation and the external random shocks, both of them are important source to cause the system failure, and whenever one of them reaches the failure threshold, the system failure occurs. This paper firstly developed the reliability analysis for the competing failure process including the degradation and the random shocks based on the probabilistic threshold, and the independent and the dependent competing failure reliability analysis model is presented, respectively. And then the proposed dependent competing failure model is applied to the reliability analysis for the k-out-of-n systems. At last, two studied examples are given to illustrate the proposed model, and from the different comparison, we can clearly see that the reliability of the fixed failure threshold model is higher than the probabilistic failure threshold model. In addition, the results of the reliability under the independent and the dependent competing failure are different from each other, and the effect of random shocks on the degradation process is important. Furthermore, through the sensitivity analysis of the parameter k to the k-out-of-n systems reliability, it is shown that the reliability is shifts left with the increasing of the k. It is worth considering in the later research that to incorporate the different inspection and/or maintenance into the developed model, and how to minimize the cost of the inspection and/or maintenance and maximize the availability will also be considered.

References

[1] Alessandro B. Reliability engineering: Theory and practice [M]. New York, USA: Springer, 2010.

[2] Jiang L, Feng Q M, Coit D W. Reliability and maintenance modeling for dependent competing failure processes with shifting failure thresholds [J]. *IEEE Transactions on Reliability*, 2012, **61**(4): 932-948.

[3] Klutke G A, Yang Y J. The availability of inspected systems subject to shocks and graceful degradation [J]. *IEEE Transactions on Reliability*, 2002, **51**(3): 371-374.

[4] Wang Y P, Pham H. Modeling the dependent competing risks with multiple degradation processes and random shock using time-varying copulas [J]. *IEEE Transactions on Reliability*, 2012, **61**(1): 13-22.

[5] Li W J, Pham H. Reliability modeling of multi-state degraded systems with multi-competing failures and random shocks [J]. *IEEE Transactions on Reliability*, 2005, **54** (2): 297-303.

[6] Huang W, Askin R G. Reliability analysis of electronic devices with multiple competing failure modes involving performance aging degradation [J]. *Quality and Reliability Engineering International*, 2003, **19**: 241-254.

[7] Peng H, Feng Q, Coit D W. Reliability and maintenance modeling for systems subject to multiple dependent competing failure processes [J]. *IIE Transactions*, 2011, **43**: 12-22.

[8] Chen J Y, Li Z H. An extended extreme shock maintenance model for a deteriorating system [J]. *Reliability Engineering and System Safety*, 2008, **93**: 1123-1129.

[9] Mallor F, Santos J. Reliability of systems subject to shocks with a stochastic dependence for the damages [J]. *Test*, 2003, **12**(2): 427-444.

[10] Li Z H, Zhao P. Reliability analysis on the δ-shock model of complex systems [J]. *IEEE Transactions on Reliability*, 2007, **56**(2): 340-348.

[11] Sgarbossa F, Pham H. A cost analysis of systems subject to random field environments and reliability [J]. *IEEE Transactions on Systems, Man, and Cybernetics. Part C. Application and Reviews*, 2010, **40**(4): 429-437.

[12] Nakagawa T. Shock and damage models in reliability theory [M]. London, UK: Springer, 2007.

[13] Robinson M E, Crowder M J. Bayesian methods for a growth-curve degradation model with repeated measures [J]. *Lifetime Data Analysis*, 2000, **6**: 357-374.

[14] Yuan X X, Pandey M D. A nonlinear mixed-effects model for degradation data obtained from in-service inspections [J]. *Reliability Engineering and System Safety*, 2009, **94**: 509-519.

[15] Nicolai R P, Dekker R, van Noortwijk J M. A comparison of models for measurable deterioration: An application to coatings on steel structures [J]. *Reliability Engineering and System Safety*, 2007, **92**: 1635-1650.

[16] Xue J, Yang K. Dynamic reliability analysis of coherent multistate systems [J]. *IEEE Transactions on Reliability*, 1995, **44**(4): 683-688.

[17] Huang W, Dietrich D L. An alternative degradation reliability modeling approach using maximum likelihood estimation [J]. *IEEE Transactions on Reliability*, 2005, **54**(2): 310-317.

[18] Lu C J, Meeker W Q. Using degradation measures to estimate a time-to-failure distribution [J]. *Technometrics*, 1993, **35**(2): 161-174.

[19] Pham H. On the optimal design of k-out-of-n subsystems [J]. *IEEE Transactions on Reliability*, 1992, **41**(4): 572-574.

[20] Pham H, Wang H. Optimal (τ, T) opportunistic maintenance of a k-out-of-n: G system with imperfect PM and partial failure [J]. *Naval Research Logistics*, 2000, **47**: 223-239.

[21] Liu Y, Huang H Z, Pham H. Reliability evaluation of systems with degradation and random shocks [C]// *Proceedings of the Reliability and Maintainability Symposium*. Las Vegas, USA: IEEE, 2008: 328-333.