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Residual Life Prediction Based on Nonlinear Fatigue Damage Accumulation Model

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Abstract: When a nonlinear fatigue damage accumulation model based on damage curve approach is used to get better residual life prediction results, it is necessary to solve the problem caused by the uncertain exponent of the model. Considering the effects of load interaction, the assumption that there is a linear dependence between the exponent ratio and the loading ratio is established to predict fatigue residual life of materials. Three experimental data sets are used to validate the rightness of the proposition. The comparisons of experimental data and predictions show that the predictions based on the proposed proposition are in good accordance with the experimental results as long as the parameters that represent the linear correlativity are set an appropriate value. Meanwhile, the accuracy of the proposition is approximated to that of an existing model. Therefore, the proposition proposed in this paper is reasonable for residual life prediction.
Key words: fatigue, nonlinear, damage accumulation, residual life prediction
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0 Introduction

Fatigue is the main failure mode of many structures in engineering practices. Due to fatigue failure, serious consequences maybe come out. Especially in recent years, the increasingly hostile service environment of structures becomes a very striking problem, because it provides more convenient conditions for terrible accidents. Fatigue failure is a complex process and involves many subjects, such as mechanics, materials and thermodynamics. However, in general, its essence is just a damage accumulation process. For higher security and lower losses, it is necessary to find an accurate method to evaluate damage accumulation and further predict residual life of structures.

Till now, the models used to describe fatigue damage accumulation are mainly linear and nonlinear theories. Miner's rule is linear damage accumulation theory, which is widely used in engineering applications. But the basic assumptions of Miner's rule have some impractical respects which can bring about potential inaccuracy for fatigue life prediction, that is, the damage accumulation has nothing to do with the following respects: the loads less than fatigue limit, load sequences, and the interaction of loads. A lot of fatigue damage accumulation models were reported to remove or depress these limitations of Miner's rule. Zhu et al.[1-2] proposed a life prediction model by means of fuzzy sets theory based on Miner's rule. Some other theories were also proposed and developed and many of them used nonlinear fatigue damage accumulation models. The solutions of these nonlinear damage accumulation models include: damage curve approach[3-4], continuum damage mechanics[5-6], energy or thermodynamic entropy[7-8], physical property degradation or ductility exhaustion theories[9-10], load characteristics[4,11-12] and so on.

When a nonlinear damage accumulation model based on the damage curve approach is used to predict residue life, some procedures have been designed by some researchers. However, there are some problems in this process, for example, the predicted results sometimes have great dispersion, because the appropriate value or expression of the exponent in this model is needed. The aim of this paper is to solve this problem.

1 Nonlinear Fatigue Damage Accumulation Model Based on Damage Curve Approach

The definition of nonlinear cumulative fatigue damage was presented by Marco and Starkey in 1954[13].

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Subsequently, Manson and Halford [14] developed different damage curve approaches. They defined that the damage caused by one cycle under a certain loading stress is

$$D_i = \left(\frac{1}{N_i}\right)^{q_i}, \quad (1)$$

$$q_i = BN_i^{\mu}, \quad (2)$$

where D_i is the damage generated from one cycle, B and μ are material constants, and N_i is the fatigue life under the corresponding loading stress. The cumulative damage under multi-level loading stress is described as

$$D = \sum \left(\frac{n_i}{N_i}\right)^{q_i}, \quad (3)$$

where n_i is the cyclic number of the ith-level loading stress. Just like Miner's rule, fatigue failure is also considered to occur when $D = 1$ in this model.

The derivation of the residual life prediction using this nonlinear damage accumulation model can be described as follows. Firstly, the process of derivation under two-level loading stress is defined. Supposing the material is subjected to loading stress σ_1 for a certain cycles, subsequently, σ_1 stops working and σ_2 is applied, then material is always under the alternative action of σ_1 and σ_2 until fatigue failure occurs. The damage development curve is shown in Fig. 1.

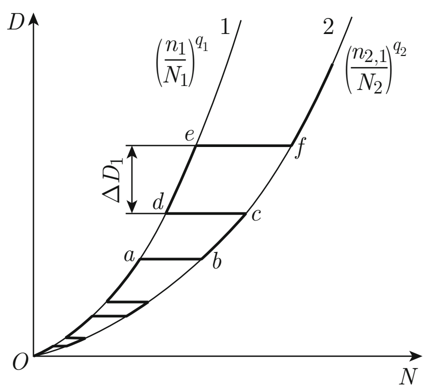

Fig. 1 Damage development curve under two-level loading stress

In Fig. 1, curves 1 and 2 represent the cumulative processes of damage variable under the action of loading stresses σ_1 and σ_2, respectively. Thus the damage development curve in the case of two-level loading stress should be along the bold line in Fig. 1. According to the nonlinear damage accumulation theory based on the damage curve approach, curve de can be expressed by

$$\Delta D_1 = \left(n_1/N_1\right)^{q_1}, \quad (4)$$

where, N_1 and q_1 are the fatigue life and exponent parameter at loading stress σ_1, respectively; n_1 is the total cyclic number when damage adds from points d to e along with curve 1.

If the same damage can be generated from the effects of loading stress σ_2 for $n_{2,1}$ times, that is the curve cf along with curve 2, the increment ΔD_2 can be also described as

$$\Delta D_2 = \left(\frac{n_{2,1}}{N_2}\right)^{q_2}. \quad (5)$$

Thus the following equation is true.

$$\left(n_1/N_1\right)^{q_1} = \left(\frac{n_{2,1}}{N_2}\right)^{q_2}. \quad (6)$$

The equivalent cyclic number $n_{2,1}$ is obtained through equation transformation.

$$n_{2,1} = N_2 \exp\left[\frac{q_1}{q_2} \ln\left(\frac{n_1}{N_1}\right)\right]. \quad (7)$$

If all cyclic numbers of loading stress σ_1 are transformed in this way, we can obtain the corresponding equivalent cyclic number of σ_2. Thus residual life under the action of σ_2 can be predicted through combining with failure criterion and stress-life curve (i.e. S-N curve) of the material, once the ratio of exponent parameter, i.e. q_1/q_2, is known. Certainly, the residual life of σ_1 can be obtained using the same method.

Similarly, suppose that the damage caused by the action of loading stress σ_1 and σ_2 for n_1 and n_2 times amounts to that after applying σ_3 for $n_{3,2}$ times. Then the following expressions are true.

$$\left(\frac{n_2 + n_{2,1}}{N_2}\right)^{q_2} = \left(\frac{n_{3,2}}{N_3}\right)^{q_3}, \quad (8)$$

$$n_{3,2} = N_3 \exp\left[\frac{q_2}{q_3} \ln\left(\frac{n_2 + n_{2,1}}{N_2}\right)\right]. \quad (9)$$

According to the same inference method, the equivalent cyclic number $n_{i,i-1}$ at σ_i which can generate an equal damage of applying σ_1, σ_2, $\cdots$, σ_{i-1} for n_1, $n_2, \cdots, n_{i-1}$ times is obtained as

$$n_{i,i-1} = N_i \exp\left[\frac{q_{i-1}}{q_i} \ln\left(\frac{n_{i-1} + n_{i-1,i-2}}{N_{i-1}}\right)\right]. \quad (10)$$

At this point, once the ratio of q_{i-1} to q_i is known, the residual life prediction of the material under loading stress σ_i can be calculated on the basis of failure criterion and S-N curve, that is to say, the key question of the residual life prediction is how to determine the appropriate value or expression of q_{i-1}/q_i. From Eq. (3), it is can be known that the ratio q_{i-1}/q_i satisfies

$$q_{i-1}/q_i = \left(\frac{N_{i-1}}{N_i}\right)^{\mu}. \quad (11)$$

Here μ is a material parameter and its value or expression is unknown, so q_{i-1}/q_i cannot be determined by using this nonlinear damage accumulation model based on damage curve approach. The following section will propose a method for determining the appropriate ratio for residual life prediction to solve this problem.

2 Method for Determining the Ratio of Exponent Parameters and Its Application of Residual Life Prediction

The ratio of exponent parameters in damage curve approach-based nonlinear fatigue damage accumulation model, i.e. q_{i-1}/q_i, is of great significance to the residual life prediction, so a method to determine the appropriate value or expression of q_{i-1}/q_i is designed.

The following considerations from literature and reasonable assumption should be noted to investigate the method for determining q_{i-1}/q_i.

(1) Load interaction has great impact on fatigue damage accumulation. Thus it plays an important role in residual life prediction.

(2) Load ratio has been used to describe load interaction effects by many researchers, and this method can make a good prediction[3,15].

(3) The ratio q_{i-1}/q_i has a connection with load interaction effects. Thus, the exponent ratio q_{i-1}/q_i and loading ratio σ_{i-1}/σ_i are somehow related.

We firstly try to adopt a simple form, i.e. linearity, to express the relationship between q_{i-1}/q_i and σ_{i-1}/σ_i. Thus the following expression is true.

$$\frac{q_{i-1}}{q_i} \propto \frac{\sigma_{i-1}}{\sigma_i}. \tag{12}$$

This linear relationship is characterized by an introduced parameter λ, that is to say, the value of q_{i-1}/q_i can be determined through the following equation.

$$\frac{q_{i-1}}{q_i} = \lambda\frac{\sigma_{i-1}}{\sigma_i}. \tag{13}$$

After Eq. (13) is substituted into Eq. (10), the effective cycles can be obtained as

$$n_{i-1} = N_i \exp\left[\lambda\frac{\sigma_{i-1}}{\sigma_i}\ln\left(\frac{n_{i-1}+n_{i-1,i-2}}{N_{i-1}}\right)\right]. \tag{14}$$

According to failure criterion, fatigue residual life at σ_i after applying $\sigma_1, \sigma_2, \cdots, \sigma_{i-1}$ for $n_1, n_2, \cdots, n_{i-1}$ is

$$N_{\text{res}} = N_i\left\{1-\exp\left[\lambda\frac{\sigma_{i-1}}{\sigma_i}\ln\left(\frac{n_{i-1}+n_{i-1,i-2}}{N_{i-1}}\right)\right]\right\}. \tag{15}$$

Equation (15) is the expression of fatigue residual life prediction model proposed in this paper. If the S-N curve of the material is known, the residual life under arbitrary-level load when the material has a degree of damage can be predicted by determining the ratio of exponent parameters q_{i-1} to q_i with Eq. (12) or Eq. (13). This model mainly considers the two ratios, q_{i-1}/q_i and σ_{i-1}/σ_i, as a kind of linear relationship, and load interaction effects can be accounted through this operation.

3 Experiments and Analyses

In this section, three experimental data sets obtained from three materials, including 45# steel, 16Mn steel and welded aluminum alloy joint, are used to validate the effectiveness of the proposed method. These three experiments are all carried out in the case of two-level loading conditions, experimental data are listed in Tables 1—3 and the detailed descriptions of experimental processes can be found in Refs. [4,16].

After the experimental data are substituted into Eq. (15), fatigue residual lives of these three materials can be calculated under the hypothesis that the parameter λ is equal to 0.1, 0.2, $\cdots$, 1.0. The comparisons of predicted results and experimental data of fatigue residual lives are shown in Figs. 2—4. From these figures, the predictions of the proposed model are in good agreement with the experiments, especially when λ is taken as 0.5 for 45# steel, 0.6 for 16 Mn steel, and 0.6 for welded aluminum alloy joint, respectively. Therefore, the assumed relationship between the exponent ratio q_{i-1}/q_i and the loading ratio σ_{i-1}/σ_i has certain rationality, and a nonlinear damage accumulation model based on this assumption expressed in Eq. (15) can provide satisfying results for predicting the material fatigue residual life.

The availability of the proposed model can be also seen from Fig. 5. By means of the model proposed in Ref. [10], the fatigue residual lives of 45# steel can be obtained for both high-to-low (H-L) and low-to-high (L-H) loading orders, as shown in Fig. 5 (up-triangles and lower-triangles, respectively). While asterisks and circles in Fig. 5 represent residual lives predicted by the proposed model when $\lambda = 0.5$ for H-L and L-H orders, respectively, that is the best satisfactory results in Fig. 2. Comparing the two-model predictions with

Table 1 Experiment data of 45# steel

Load mode	σ_1/MPa	σ_2/MPa	n_1	$\frac{n_1}{N_1}$	n_2	$\frac{n_2}{N_2}$
1	331.46	284.40	12 500	0.250 0	250 400	0.500 8
2	331.46	284.40	25 000	0.500 0	168 300	0.336 6
3	331.46	284.40	37 500	0.750 0	64 500	0.129 0
4	284.40	331.46	12 500	0.250 0	37 900	0.758 0
5	284.40	331.46	250 000	0.500 0	38 900	0.778 0
6	284.40	331.46	375 000	0.750 0	43 400	0.868 0

Table 2 Experiment data of 16Mn steel

Load mode	σ_1/MPa	σ_2/MPa	n_1	$\frac{n_1}{N_1}$	n_2	$\frac{n_2}{N_2}$
1	562.90	392.3	2	0.000 5	73 600	0.935 2
2	562.90	392.3	200	0.050 4	59 400	0.754 8
3	562.90	392.3	1 000	0.252 0	56 300	0.715 4
4	562.90	392.3	2 450	0.617 4	22 900	0.291 0
5	372.65	392.3	64 400	0.240 0	62 800	0.798 0
6	372.65	392.3	150 000	0.560 0	23 300	0.296 0

Table 3 Experiment data of welded aluminum alloy joint

Load mode	σ_1/MPa	σ_2/MPa	n_1	$\frac{n_1}{N_1}$	n_2	$\frac{n_2}{N_2}$
1	104	74	109 900	0.200 1	797 600	0.517 9
2	89	74	176 100	0.200 0	1 029 200	0.668 3
3	74	89	770 100	0.500 0	545 600	0.619 6
4	93	73	309 900	0.500 0	587 500	0.380 0
5	83	73	476 100	0.499 9	681 100	0.440 5
6	73	83	509 200	0.329 3	708 200	0.743 6

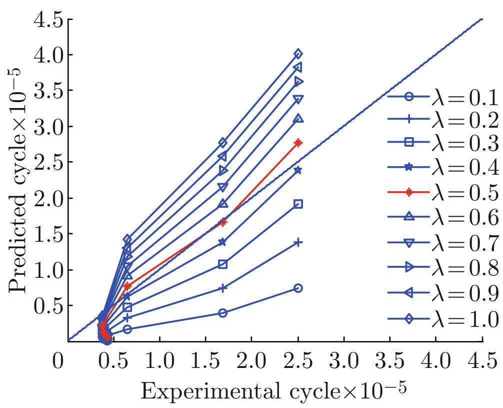

Fig. 2 Comparison of experimental data and predicted results for 45# steel

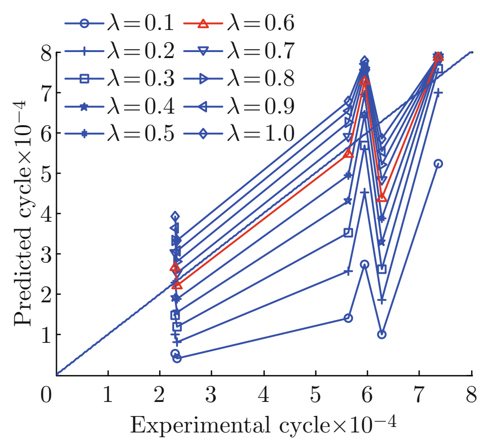

Fig. 3 Comparison of experimental data and predicted results for 16Mn steel

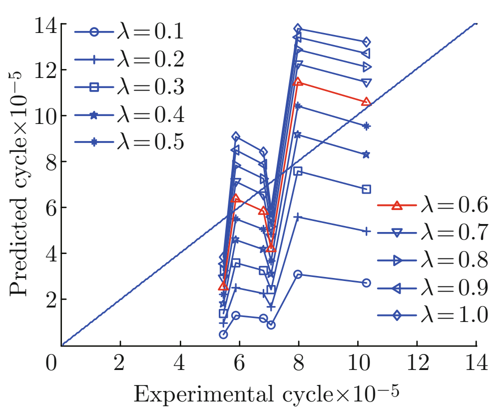

Fig. 4 Comparison of experimental data and predicted results for welded aluminum alloy joint

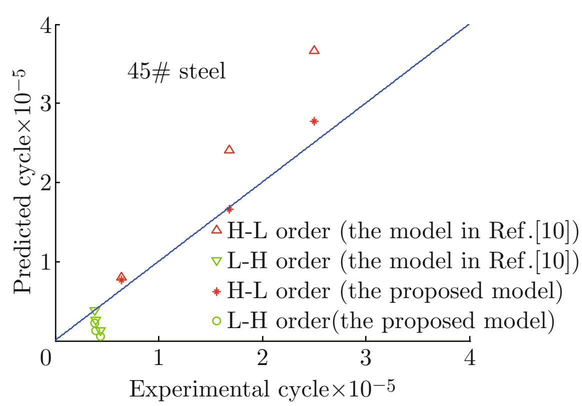

Fig. 5 Comparison of experimental data and two-model predictions under H-L and L-H loading conditions

the experimental data, we can see that the proposed model predictions for H-L order are better than those existing models, although less-accuracy for L-H order is also given by this proposed model. Generally, predicted lives calculated from Eq. (14) are in accordance with the practice, that is to say, there indeed is a parameter λ, and it can be used to describe the linear relationship between q_{i-1}/q_i and σ_{i-1}/σ_i. Meanwhile, a good result is obtained through this linear relationship.

4 Conclusion

The expression of the exponent in a nonlinear damage accumulation model based on the damage curve approach has not been determined, and this results in some difficulties in residual life prediction of materials. Through detailed deviation, once the exponent ratio is got, the residual life can be predicted subsequently. Considering that the damage relates to load sequence effect, there is connection between the exponent ratio and the loading ratio. A simple relationship between them, that is linearity, is the first assumption and attempt in this paper, and the residual life prediction model is established based on this assumption.

According to fatigue failure criterion and S-N curve of materials, three experimental data sets collected from three materials of 45# steel, 16Mn steel and welded aluminum alloy joint are used to validate the effectiveness of the assumption proposed in this paper. The comparison results show that for these three materials, when the parameter representing linear relativity is taken as 0.5 or 0.6, the difference between predicted and experimental lives are fall in a permitted range. Thus the assumption that the exponent ratio and loading ratio are linear correlation is reasonable for residual life prediction.

The comparison of experimental data from 45# steel and the prediction result further demonstrates the availability of the proposed model. It can be seen that a bit poor prediction is generated from the proposed model for L-H order, yet for H-L order, and the accuracy of the proposed model is higher than that from the existing model. However, the errors of the proposed model are close to those of the existing model and within an acceptable range in general. This further explains the reliability and rationality of the assumption proposed in this paper.

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